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To cite this article: T G Nesterenko *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **516** 012009

IOP Conf. Series: Materials Science and Engineering **516** (2019) 012009 doi:10.1088/1757-899X/516/1/012009

Amplitude control system of drive-mode oscillations of MEMS gyroscopes

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Abstract. Two operating modes can be singled out for a gyroscope, namely drive mode and sense mode. During the drive mode, electrostatic forces of gyroscope enable drive-mode harmonic oscillations of a moving mass. With the angular velocity and translational acceleration of an object in the sense mode, a moving mass moves due to the Coriolis and inertial forces, thereby causing changes in capacitances of comb structures. Resultant capacitance changes are proportional to parameters to be measured, i.e. the rotational rate and linear acceleration. The aim of this work is to investigate and create a system of amplitude control of drive-mode oscillations. To improve the mems efficiency, drive-mode oscillations are excited at a resonant frequency.

1. Introduction

MEMS gyroscopes operates on the Coriolis principle. Two operating modes can be singled out for a gyroscope, namely drive mode and sense mode. During the drive mode, electrostatic forces of gyroscope enable harmonic oscillations of a moving mass. With the angular drift velocity and translational acceleration of an object in the sense mode, a moving mass moves due to the Coriolis and inertial forces, thereby causing changes in capacitances of comb structures. Resultant capacitance changes are proportional to parameters to be measured, *i.e.* the rotational rate and linear acceleration.

The aim of this work is to investigate and create a system of amplitude control of drive-mode oscillations. To improve the gyroscope efficiency, drive-mode oscillations are excited at a resonant frequency. The amplitude of drive-mode oscillations should be constant which is provided by the closed circuit system or a feedback circuit with the automatic level control (ALC) feedback [1–3]. The ALC feedback configuration is very important for the system stability based on a proportional-integral (PI) controller which is decisive for the amplitude control of drive-mode oscillations.

2. Mechanics of drive mode

The drive mode is provided by a mechanical resonator, whose simplified equivalent circuit model is given in Figure 1 [4].

The damping of drive-mode oscillations in the sensor is characterized by μ_{y1}, μ_{y2} and μ_y damping factors. Equaling in value and opposite in direction electrostatic forces $F_{el\,1}$ and F_{el2} are applied to moving masses. The motion equations for moving masses are as follows:

$$m_{y1} \cdot \ddot{y}_1 + (\mu_{y1} + \mu_y) \cdot \dot{y}_1 + \left(k_{y1} + k_y\right) \cdot y_1 - \mu_y \cdot \dot{y}_2 - k_y \cdot y_2 = F_{el1} \qquad (1)$$

$$m_{y2} \cdot \ddot{y}_2 + (\mu_{y2} + \mu_y) \cdot \dot{y}_2 + \left(k_{y2} + k_y\right) \cdot y_2 - \mu_y \cdot \dot{y}_1 - k_y \cdot y_1 = F_{el2}\,, \qquad (2)$$

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where m_{y1}, m_{y2} are moving masses which perform drive-mode oscillations in opposite directions; k_y is the rigidity of spring elements between moving masses directed along *Y*-axis of drive-mode oscillations; k_{y1}, k_{y2} are joint rigidities of spring elements suspending the outer and decoupling frames.

For the ideal system we assume that $m_{y1}=m_{y2}=m_y$, $\mu_{y1}=\mu_{y2}=\mu_{y0}$, $k_{y1}=k_{y2}=k_{y0}$. Then Equal (1) can be rewritten as

$$m_{y1}\cdot\ddot{y}_1+(\mu_{y0}+2\cdot\mu_y)\cdot\dot{y}_1+\left(k_{y0}+2\cdot k_y\right)\cdot y_1=F_{el1} \tag{3}$$

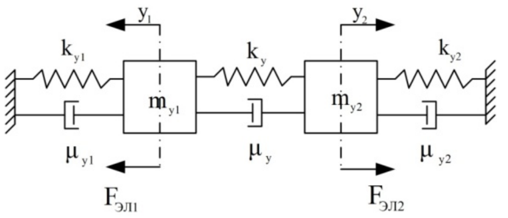

Figure 1. A simplified equivalent circuit model of mechanical resonator.

Harmonic oscillations usually occur in the drive mode, when an AC input signal $V_{in}=V\sin(\omega t)$ and DC voltage V_{dc} should be applied to the comb drive actuators *via* anchors and moving mass, as illustrated in Figure 2.

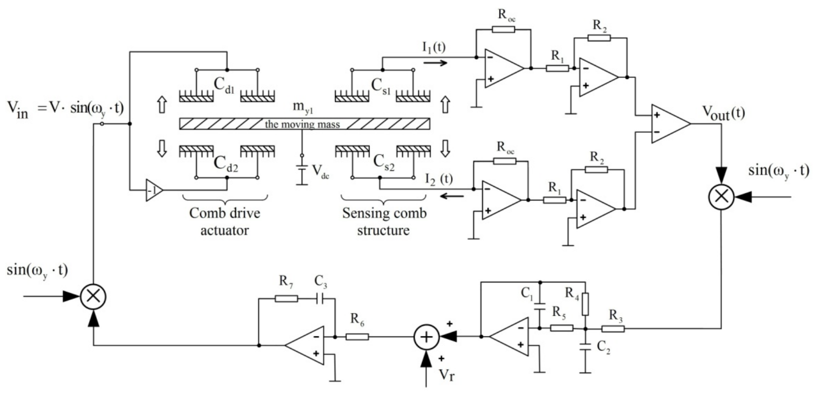

Figure 2. Interface electronics of the drive mode.

Electrostatic forces that act between two electrically conductive combs $C_{d1}$and C_{d2} are defined by the following relations [5–7]:

$$F_{el1}=2\cdot b_1\cdot V_{dc}\cdot V\cdot\sin(\omega_y\cdot t) \text{ and } F_{el2}=-2\cdot b_1\cdot V_{dc}\cdot V\cdot\sin(\omega_y\cdot t), \tag{4}$$

Where b_1 is the factor to approximate electrically conductive combs C_{d1} and C_{d2} [4].

The correlation between the mechanical motion $y_1(s)$ and applied force $F_{el1}(s)$ can be determined by

$$\frac{y_1(s)}{F_{el1}(s)}=\frac{1}{m_{y1}}\cdot\frac{1}{s^2+\dfrac{\mu_1}{m_{y1}}\cdot s+\dfrac{k_1}{m_{y1}}}, \tag{5}$$

where $\mu_1=\mu_{y0}+2\mu_y$ is the damping factor; $k_1=k_{y0}+2k_y$ is the rigidity of spring element.

The correlation between the motion and the input signal is obtained from

$$\frac{y_1(s)}{V_{in}(s)}=\frac{1}{m_{y1}}\cdot 2\cdot b_1\cdot V_{dc}\cdot\left[s^2+\left(\frac{\omega_y}{Q_y}\right)\cdot s+\left(\omega_y\right)^2\right]^{-1}, \tag{6}$$

where $\left(\omega_y\right)^2=k_1\cdot\left(m_{y1}\right)^{-1}$ and $Q_y=\left(\mu_1\right)^{-1}\cdot\sqrt{k_1\cdot m_{y1}}$.

Vibrations of the moving mass m_{y1} enable changes in the equivalent electrically conductive combs $C_{s1}=a_2+b_2\cdot y$ and $C_{s2}=a_2-b_2\cdot y$. These changes result in the generation of currents I_1 and I_2 passing between sensing electrodes, such that

$$I_1(t) = V_{dc} \cdot b_2 \cdot \dot{y} \text{ and } I_2(t) = -V_{dc} \cdot b_2 \cdot \dot{y} . \tag{7}$$

The voltage output V_{out} at the current to voltage converter can be written in the form

$$V_{out}(t) = 2 \cdot V_{dc} \cdot b_2 \cdot R_{oc} \cdot R_2 \cdot (R_1)^{-1} \cdot \dot{y} . \tag{8}$$

The correlation between the input and output signals is found from

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{m_{y1}} \cdot \frac{R_2}{R_1} \cdot R_{oc} \cdot 4 \cdot b_1 \cdot b_2 \cdot V_{dc}^2 \cdot s \cdot \left[s^2 + \left(\frac{\omega_y}{Q_y} \right) \cdot s + (\omega_y)^2 \right]^{-1} . \tag{9}$$

3. Amplitude controls of drive-mode oscillations

A feedback circuit used for the amplitude control in the drive mode is a nonlinear system. It is difficult to analyze the behavior of nonlinear system, especially when it achieves the third order [8]. However, the only important driver is a change in the amplitude response of the system rather than its complete performance. The dynamic equilibrium of the feedback circuit is provided by the force driven by the negative feedback.

According to Equal 6 we obtain the dynamic equation for the drive mode:

$$\ddot{y} + \frac{\omega_y}{Q_y} \cdot \dot{y} + \omega_y^2 \cdot y = \frac{1}{m_{y1}} \cdot 2 \cdot b_1 \cdot V_{dc} \cdot V_{in} . \tag{10}$$

This motion and the input signal can be expressed by

$$y \approx y_m(t) \cdot \sin(\omega_y \cdot t) , \qquad V_{in} \approx V_{m,in}(t) \cdot \cos(\omega_y \cdot t) , \tag{11}$$

Where $y_m(t)$, $V_{m,in}(t)$ are amplitudes of motion and input signal, both being changed in time.

Now we can derive the amplitude ratio between the input and output signals

$$\frac{V_{m,out}(s)}{V_{m,in}(s)} = \frac{1}{2} \cdot \frac{A \cdot K_{A/V}}{s + \frac{1}{2} \cdot \frac{\omega_y}{Q_y}} , \tag{12}$$

where $A = \frac{4 \cdot V_{dc}^2 \cdot b_1 \cdot b_2}{m_{y1}}$, $K_{A/V} = R_{oc} \cdot \frac{R_2}{R_1}$.

Therefore, instead of a total model we can use a simplified model of the feedback circuit presented in Figure 3 [9].

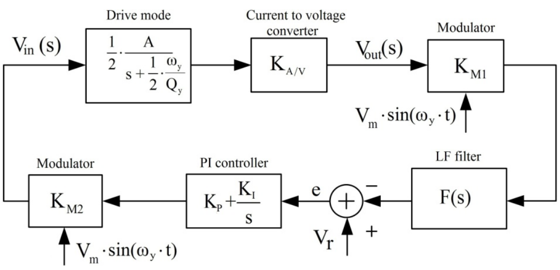

Figure 3. Schematic of the amplitude control feedback circuit for the drive mode.

As can be seen from Figure 3, the feedback circuit incorporates two signal modulators, the low-frequency (LF) filter and PI controller. The voltage output V_{out} at the current to voltage converter is compared in the summer to the reference voltage V_r. Mismatched signal *e* transfers to PI controller to apply a correction for the drive signal.

When the amplitude of the output signal is lower than that of the reference signal, the error signal of the control system becomes positive, thereby increasing the electrostatic force. When the amplitude of the output signal is higher than that of the reference signal, a reverse process occurs. Owing to this mechanism, the amplitude of the output signal is controlled by the reference value.

Let us accept two criteria for the PI controller design. The first is a short buildup time, and the second is a sufficient phase margin.

In order for the first criterion to be met, the pole cancellation [8] should be performed in the simplified model of the feedback circuit, *i.e.* the following condition must be satisfied:

$$s+\frac{1}{2}\cdot\frac{\omega_y}{Q_y}=s+\frac{K_I}{K_P}\rightarrow\frac{K_I}{K_P}=\frac{1}{2}\cdot\frac{\omega_y}{Q_y} \quad (13)$$

Where K_P, K_I are gain factors for PI controller.

After the pole cancellation, the transmission gain in the feedback circuit is set to provide the system maximum bandwidth with the sufficient phase margin (approx.. 60 degrees). The phase margin has a direct effect on setting time and transient overshoot. Hence, the optimum phase margin is a necessary criterion for constructing the system for the amplitude control of drive-mode oscillations. After the pole cancellation in the simplified model of the feedback circuit, the system order depends on the order of the LF filter. The second criterion is met in conformity with the following equation [9]:

$$K_P=4\cdot\pi\cdot f\cdot\left(A\cdot K_{A/V}\cdot K_{M1}\cdot K_{M2}\cdot K_F\right)^{-1}, \quad (14)$$

where F(s) is a transfer function of the LF filter; f is the frequency for 30 degree phase shift; $K_F=|F(s)|_f$ is the transmission gain of LF filter; K_{M1}, K_{M2} are transmission gains of modulators;

Figure 4 presents a total model of the amplitude control system of drive-mode oscillations. The modeling is performed using design factors of the simplified model of the feedback circuit. Using Eqns (10) and (11), we find the transmission gains of K_P= 2 and K_I= 360 for PI controller.

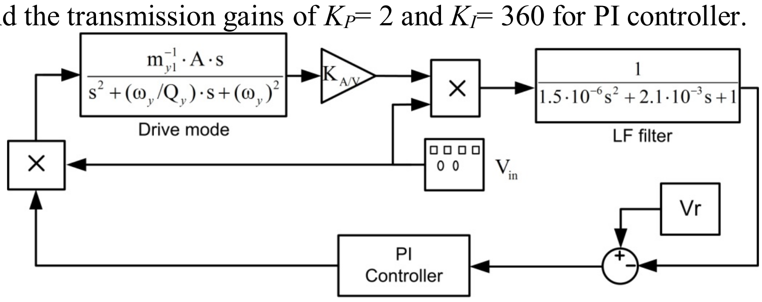

Figure 4. Numerical model of the amplitude control system in the drive mode.

The output signal of the low-frequency filter is depicted in Figure 5. We also ascertain setting times of 50 ms. This signal describes the amplitude value for drive-mode oscillations.

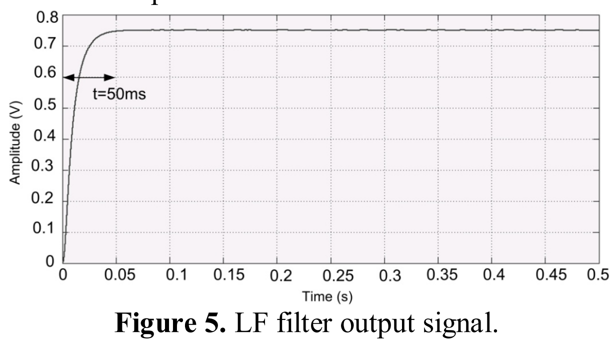

Figure 5. LF filter output signal.

Drive-mode oscillations with stabilized amplitude are shown in Figure 6. After the transient process, the output amplitude is constant and corresponds to the given reference signal.

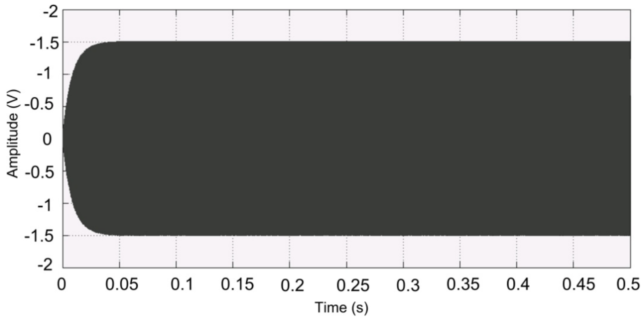

Figure 6. Drive mode oscillations with stabilized amplitude.

4. Conclusions

We proposed a theoretical analysis and a method of parameter determination for the amplitude control system in the drive mode. Modeling showed that the proposed control system stabilized the amplitude of drive-mode oscillations at a level corresponding to that of the reference signal.

Acknowledgments
This work was performed in National Research Tomsk Polytechnic University. The authors would like to acknowledge the financial support of the Ministry of Science and higher education of the Russian Federation (Agreement № 14.578.21.0232, unique identifier RFMEFI57817X0232).

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