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Numerical method of Control Object identification and models' robustness property maintenance

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Abstract. In this paper the Real Interpolation Method is offered for the solution of systems' Identification problem. The Real Interpolation Method allows to create a sufficiently cost-effective algorithmic basis and to solve all complex of Identification problems. It is based on use of a special case of the Laplace transformation, when the complex variable s =δ + jω is degenerated to real value δ.

Introduction

At creation and maintenance of Automatic-Control System it is necessary to solve an obtaining problem of the system dynamic elements mathematical description. The problem body refers to definition of Control Object mathematical model. This part of a problem is solved by two ways - analytical and experimental [1]. In the first case the model is searched on the basis of those laws which present the processes in the Object. In the second case, experimental way solves an identification problem on the basis of the experiences with the Object. Each of these possibilities has the advantages and restrictions. The main advantage of the analytical solution variant consists in possibility of obtaining enough exact structure of Control Object model. However, numerical values of factors of these models can have considerable errors. Really, at the analytical approach it is impossible to consider individual properties of specific objects samples. In this sense the experimental method of identification appears preferable as in the Object's reaction all features of observed Object are shown.

In practical work the best results in procedure of Objects models' obtaining are attained at use of both approaches: analytical calculations allow generating model structure, and values of factors are defined according to experiment. In these terms the experimental stage of work is necessary and for its realization corresponding tools are required: test signals units, the Control Object output signal registration hardware, the application software allowing computing Control Object model on input and output signals. All listed components are known and separately are successfully put into practice. At the same time it is obvious that it is expedient to have these components in the integrated device for the effective solution of the problem [2,3]. It should be independent and mobile allowing carrying out procedure of model obtaining (i.e. the identification) direct on Control Object workplaces [4].

Devices of similar destination - the Identifiers oriented on "a wide user" - are absent on the market. At the same time their significance can be high. It is necessary to note some of their possibilities. Firstly, this is their direct use for experimental definition of Control Objects mathematical descriptions at creation of new Automatic-Control Systems elements. Secondly, they can be applied at monitoring of Control Objects condition and systems. Results of observations allow noting changes of controllable parameters, to estimate trends of approach to gradual failures. In the third, such device allows to gain the objective information on a current Control Object's condition, on its mathematical description and to adjust changeable parameters of Regulators on this basis. Obviously, other applications of the Identifier are possible also.

In the capacity of a mathematical basis for Identifier construction it is possible to accept one of assemblage of known problem solution methods [5,6,7]. However, rationality demands dictate the conditions and they need to be considered. One of the main conditions consists in use of algorithms which provide computing operations small volume. By this criterion the frequency methods are not the best because of necessity of allocation of Fourier images real and imaginary parts [8]. Laplace images are characterized by the same deficiency due to complex variable presence. From these positions it is necessary to pay attention to the Real Interpolation Method [9,10]. In the present work the solution of the identification problem on the base of the Real Interpolation Method is stated. For understanding of the approach short data on bases of the method and its application in identification problems are resulted. Necessity of the models robustness maintenance in the conditions of Identifier application should be defined as the obligatory demand shown to the solution of the problem. The reason is that non-robust solutions are inadmissible because of serious consequences.

Bases of the Real Interpolation Method

Real Interpolation Method is based on application of integrated transformation form

$$F(\delta)=\int_0^\infty f(t)e^{-\delta t} \quad , \ \delta\in[C\,,\infty[\ \ , \ C\geq 0, \tag{1}$$

where f(t) - pre-image of image-function $F(\delta)$ in time domain; t and δ - variable accordingly in domains of pre-images and images; C- parameter defining convergence condition of the integral.

Formula (1) is usually named direct δ - transformation. It puts function pre-image f(t) in correspondence to real function - image $F(\delta)$. Formula (Eq.1) can be considered as a special case of the direct Laplace transformation at substitution of real δ variable on complex variable $s=\delta+j\omega$. It is the fundamental feature which ensures appreciable reducing of computational burden during organization of numerical information processing procedures – more than 2 times in comparison with frequency method [1,9].

The basic advantages of such approaches are following:

- numerical methods developed for the considered class of functions are applicable for action with real functions;
- it is possible to find $F(\delta)$ not only with the help of time functions, but also with help of Laplace images, replacing a complex variable s on real δ;
- the mathematical models have graphic representations: that considerably increase obviousness of them;
- models $F(\delta)$ do not contain imaginary and complex variables, predetermining significant profitability of analytical and especially numerical operations in comparison with models on the basis of Laplace and Fourier transformations.

It is necessary to realize transfer from continuous functions - images $F(\delta)$ to their discrete analogs for application of real functions in numerical calculations. The base of such transfer is the interpolation method, which allows to restore function values in any intervening point on a given system of function values in points. In this connection the concept of the Numerical Characteristic [1] is introduced as values assemblage of function $F(\delta)$ in Interpolating Points δ_i, i=1,2,...,η: $\{F(\delta)\}_\eta = \{F(\delta_1),...,F(\delta_\eta)\}$, where η - dimensionality of the Numerical Characteristic, i.e. number of Interpolating Points. They are described by expression

$$\delta_i = \delta_1 \cdot i\,, \tag{2}$$

where $\delta_1=\Delta / t_c$, Δ- setup parameter, t_c - control time of the Control Object.

Let us consider linear one-dimensional continuous Control Object, which have the block diagram as in Fig. 1.

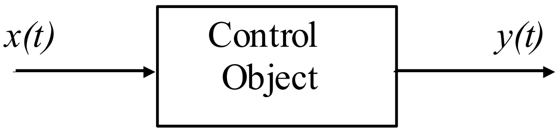

Fig.1. Block diagram of Control Object

Here x(t), y(t) - input and output signals of Control Object, which are having accordingly images $X(\delta)$ and $Y(\delta)$. Let the unit step signal 1(t)=x(t) is given on the input of the Control Object and transition function h(t)=y(t) is obtained. In this case the real transfer function can be described in the form of

$$W(\delta)=\frac{h(\delta)}{1/\delta}, \tag{3}$$

where $h(\delta)$ - the image of the transition function h(t); $1/\delta$ - the image of the unit step signal 1(t).

If the model of system is represented by transfer function looking like fractional rational function

$$W(\delta)=\frac{Y(\delta)}{X(\delta)}=\frac{b_m\delta^m+b_{m-1}\delta^{m-1}+\ldots+b_1\delta+b_0}{a_n\delta^n+a_{n-1}\delta^{n-1}+\ldots+a_1\delta+1}\quad,\ n\geq m\ , \tag{4}$$

we can write the expression for Numerical Characteristic of transfer function $\{W(\delta_i)\}_\eta$.

It is possible to find components of the Numerical Characteristic in two ways. First way is based on the usage of the prior information about transfer function of model (Eq.4) and realization of transfer from W(s) to W(δ) by formal substitution of s on δ. Second way, which is more interesting and attractive, is based on usage of the information about time dynamic characteristic h(t) of control objects. Following expression defines components of Numerical Characteristic in consideration of (Eq.1) and (Eq.3) [1,9]:

$$W(\delta_i)=\delta_i\int_0^\infty h(t)\cdot e^{-\delta_i\cdot t}dt\approx\ \delta_i\sum_{j=0}^{N}h(t)\cdot e^{-\delta_i\cdot t}\cdot\Delta\ t,\quad i=1,\ldots\eta\ ,\ \eta=n+m+1\,. \tag{5}$$

The number of integration steps N is about 50÷150 usually [1].

The identification problem is reduced to parametric identification of transfer function of control plant. It is necessary to define transfer function parameters, a_k, $k=1,\ldots,n$ and b_j , j=0,... ,m via known signals x(t) and h(t) on predetermined degrees m and n of numerator and denominator polynomials of transfer function (Eq.4). Model of control plant is found according to (Eq.5) in the form of Numerical Characteristic $\{W(\delta_i)\}_\eta$, $i=\overline{1,\eta}$ for this problem solution. Thereupon, coefficients of real transfer function (Eq.4) are finding regarding Numerical Characteristic $\{W(\delta_i)\}_\eta$ by solving of linear algebraic equations system. This system of equations is defined with the respect to unknown coefficients a_k and b_j by interpolation of transfer function (Eq.4):

$$b_0+b_1\delta_1+\ldots+b_m\delta_i^m-a_1W(\delta_i)\ -\ldots-a_nW(\delta_i)\delta_i^n=W(\delta_i)\,,\ i=\overline{1,\eta}\ . \tag{6}$$

Vector of the right part (Eq.6) is defined from expression (Eq.5) by experimental data. It is possible to use standard computing algorithms for the solution of the system (Eq.6). Following conditions must be fulfilled for existence and determination of unique solution: η=n+m+1 and Interpolating Points δ_i, $i=\overline{1,\eta}$ (Eq.2) must be different.

The obtained values a_k, b_j, k=1,...,n, j=0,...,m are coefficients of real transfer function (Eq.4). Transfer from the real transfer function W(δ) to Laplace transfer function W(s) is realized by formal substitution of δ on **s.** Transfer function W(s) which is obtained in such a way can be used for the purposes of comparing with etalon transfer function $W_e(s)$ and tuning of the system. However, the obtained function W(s) may answer the demands not always. In particular, the relative error

$$\Delta h = \max_t \left(\frac{h_{co}(t_j) - h_m(t_j)}{h_{co}(t_j)} \cdot 100\% \right), \ j = 0,1,2,...,N \quad , \tag{7}$$

where $h_{co}(t_j)$ - transient characteristic of the Control Object, $h_m(t_j)$ - transient characteristic of the Control Object model, may exceed a predetermined value Δh_0. Moreover, such situation is ordinary in most cases. In this connection it is necessary to add a procedure of minimizing the value of Δh to design equations and algorithms [9]. With this purpose a special mechanism is designed. It is based on a modification of values of interpolation points δ_i.

Robustness property

Importance of Robustness problem is determined by necessity of system parameters preservation for the given limits (Eq.7) under system elements parameters change conditions. Now methods of the systems robustness analysis are well developed. Problem of the robustness level ensuring during system design or adjustment is of the great interest [11]. Giving of robustness property to Automatic-Control System is determined by two major stages. The first is connected to finding of mathematical model received as a result of identification of Control Object. The second is the adjustment of the Automatic-Control System. Importance of the second stage is obvious in term of robustness. The role of the first stage also is considerable because it is impossible to achieve a necessary level robustness during the adjustment if the control system model will not have adequate robustness. Control object can change the parameters in beforehand known limits. Therefore, the Identifier device should act with the certain accuracy in all range of the Control Object parameters change.

Identification process analyzed above solves the first part of formulated task - finding of model which meets the exactitude demands. The second part of the task consists in the verification and the results correction for the purpose of ensuring the preset robustness level of the model. For reaching this purpose the possibility of approximate coefficients variations replacement of the real transfer function by the upper bound variations δ_η of interpolation points disposition δ_i, $i = 1,2,...$ (Eq.2) is used [1]. The typical graph $\Delta h = f(\delta_\eta)$ is shown in Fig. 2.

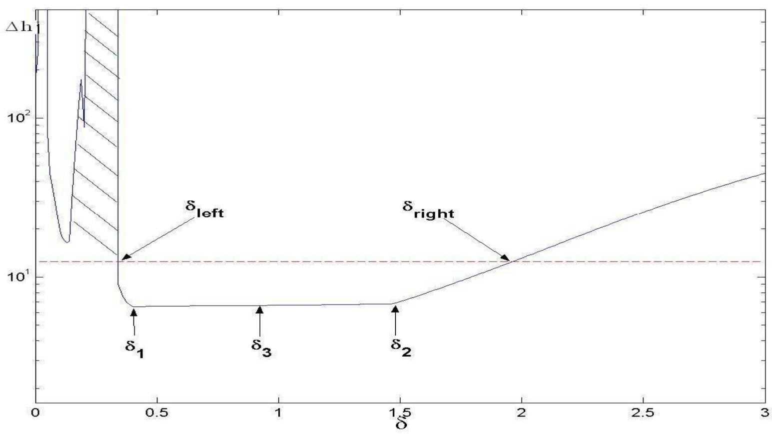

Fig. 2. The dependence $\Delta h = f(\delta_\eta)$

The left value δ_{left} and the right value δ_{rigt} admissible boundaries of variable change δ_{η} are chosen in the graph. Range of the interval $\delta_{rigt} \div \delta_{left}$ is the quantitative characteristic of robustness property. It allows to compare some variants of models of control object under robustness requirements and to find the most acceptable variant.

Range of the interval $\delta_{rigt} \div \delta_{left}$ can be corrected. The modification of the required transfer function structure of model is the simplest way. It is necessary to note that models with transfer functions of the low order have the greatest robustness. Increase of polynomials degrees of transfer function numerator and denominator results in range of interval $\delta_{rigt} \div \delta_{left}$ decreasing and simultaneous improving of the solution accuracy. Therefore, it is necessary to accept a compromise variant in practice.

Summary

Implementation of the Real Interpolation Metgod for the solution of the Identification problem is effective for the identification device creation. The Identifier on the base of the Real Interpolation Method answer the robustness requirement and, therefore, it can be used for Control tasks solution.

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