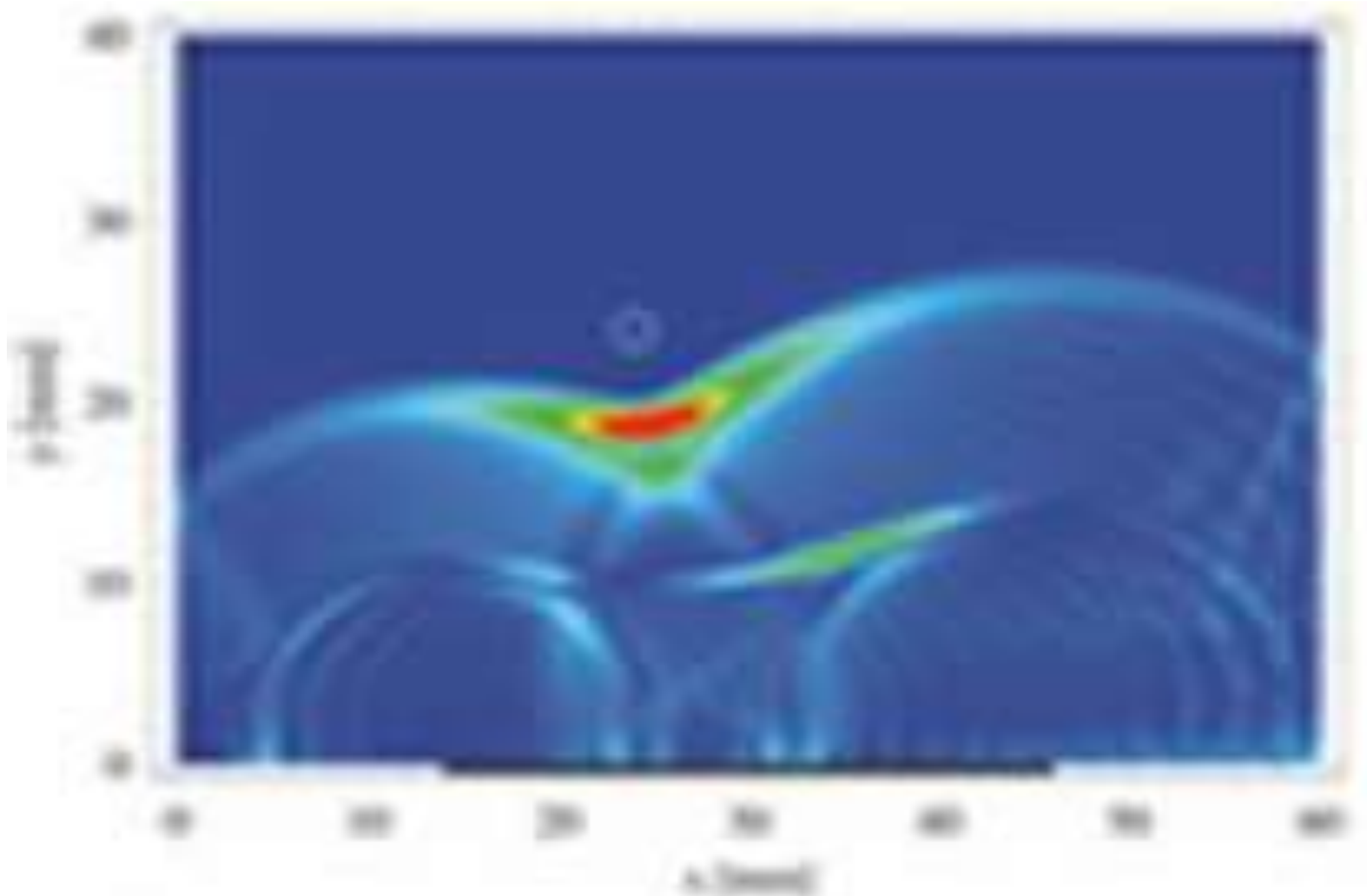
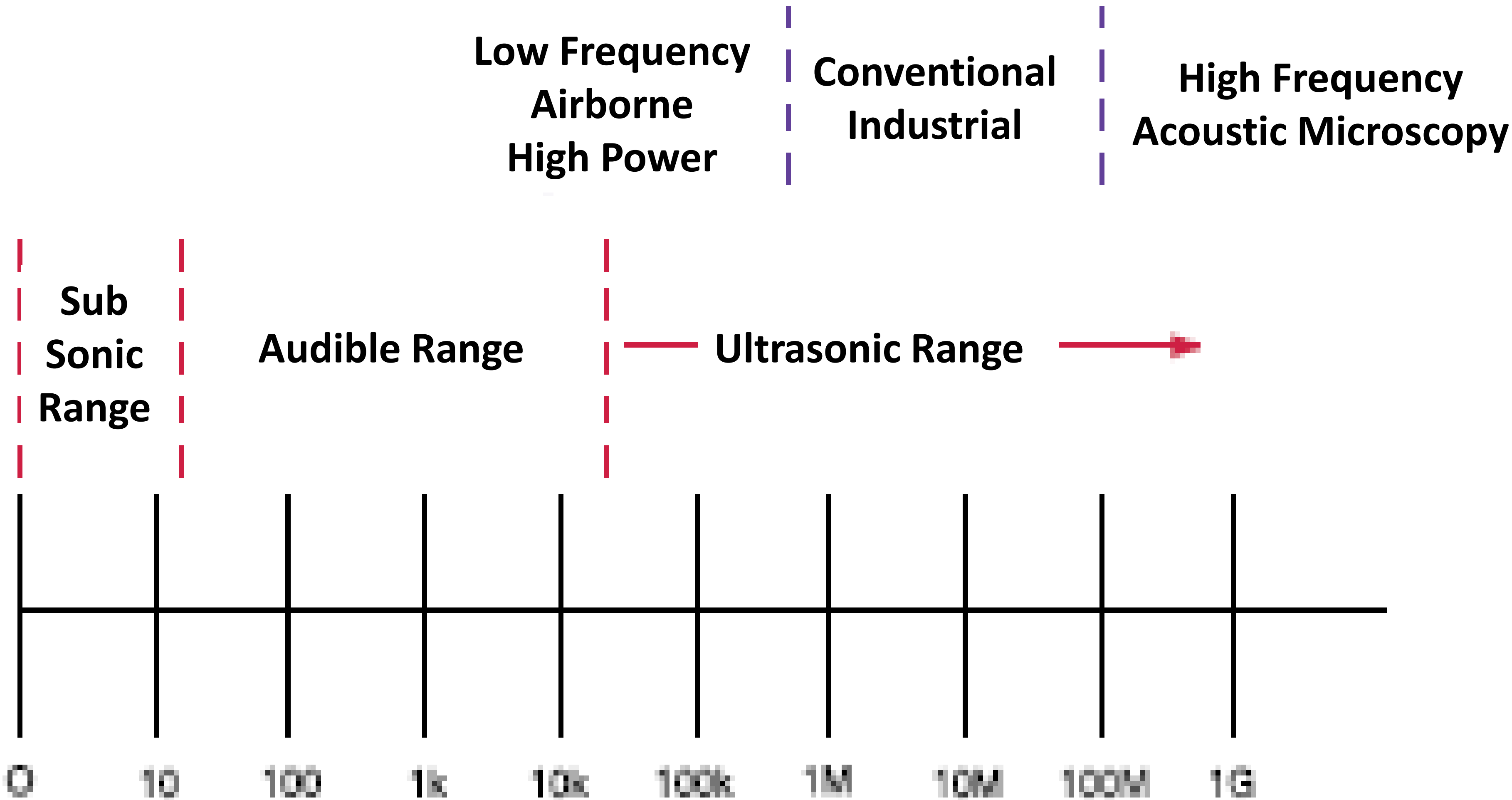
NDT&E Methods: UT



NDT&E Methods: UT

6.	**NDT&E: Introduction to Methods**
6.1.	Ultrasonic Testing: Basics of Elasto-Dynamics
6.2.	Principles of Measurement
6.3.	The Pulse-Echo Method
6.4.	UT-Systems: Transducer, Instrument, Manipulator
6.5.	Current Developments
6.6.	Case Studies by Movies

NDT&E Methods: UT

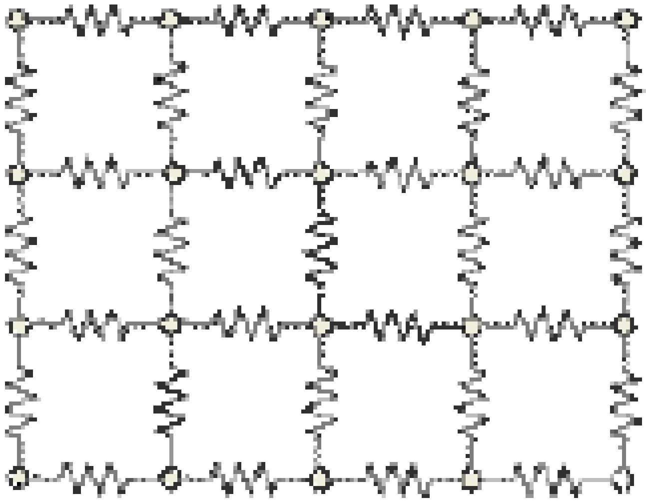


UT BASICS

In the 1870s:

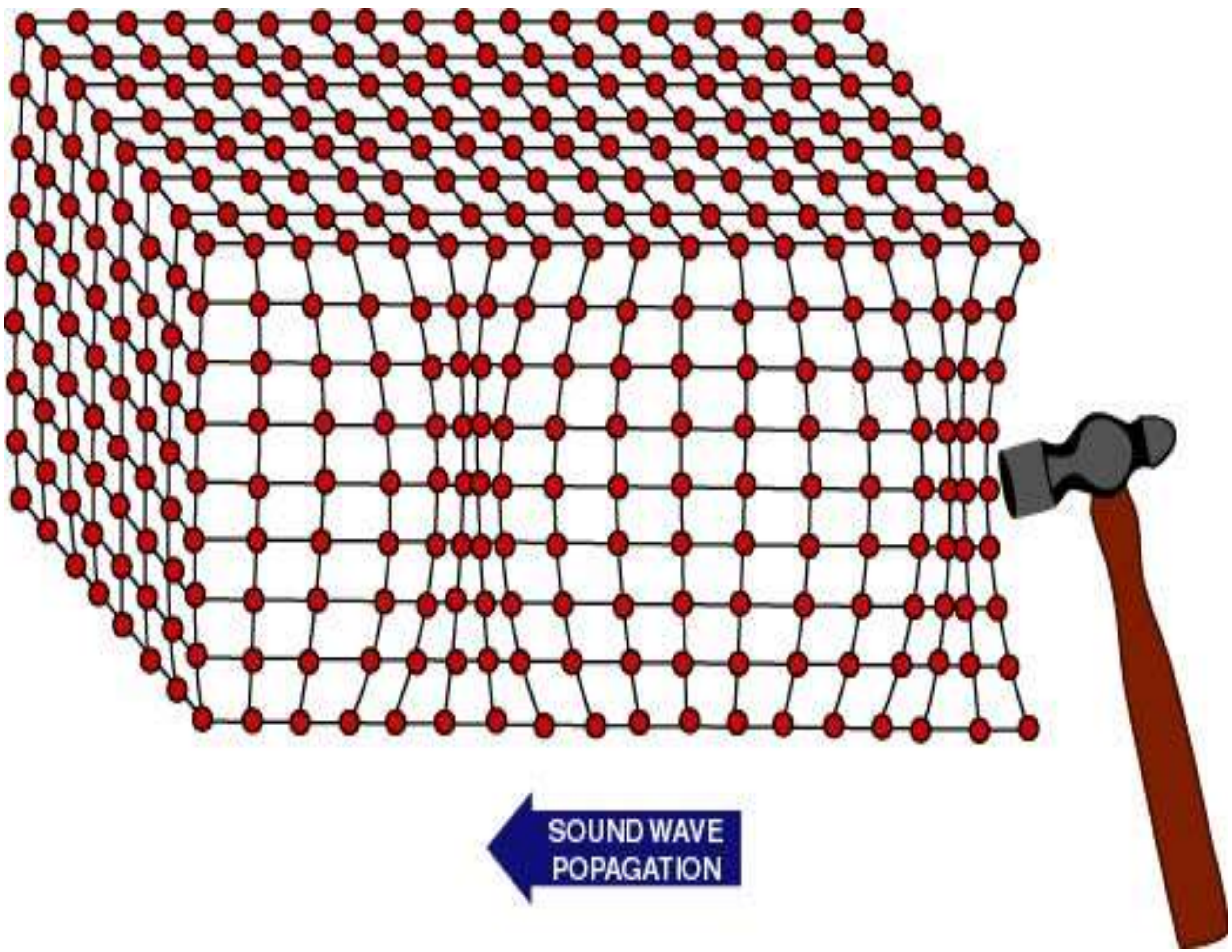
Lord Rayleigh: Theory of Sound

Nature and Properties of Sound Result from Mechanical Vibrations in Solids, Liquids, and Gases



Spring Model of Vibrating Particle Masses

NDT&E Methods: UT



Sound Generation by mechanical Impact

NDT&E Methods: UT

Constitutive Equations of Sound Waves

In physics and engineering,
a constitutive equation or constitutive relation is
a relation between two physical quantities that is specific to a material,
and approximates the response of that material to external stimuli,
usually as applied fields or forces.
They are combined with other equations governing physical laws
to solve physical problems;

for example in fluid mechanics the flow of a fluid in a pipe,
in solid state physics the response of a crystal to an electric field,
or in structural analysis, the connection between applied stresses or forces
to strains or deformations.

Some constitutive equations are simply phenomenological;
others are derived from first principles
(From WIKIPEDIA)

Stress and Strain - The roots for elastic waves

The stress-strain constitutive relation for linear materials
is commonly known as **Hooke's law.**
In its simplest form, the law defines
the spring constant (or elasticity constant) *k* in a scalar equation,
stating the tensile/compressive force is proportional
to the extended (or contracted) displacement *x*:

$$F_i = -kx_i$$

meaning the material responds linearly.

Equivalently, in terms of
the stress σ, Young's modulus *E*, and strain ε (dimensionless):

$$\sigma = E\epsilon$$

Stress and Strain

The roots for elastic waves

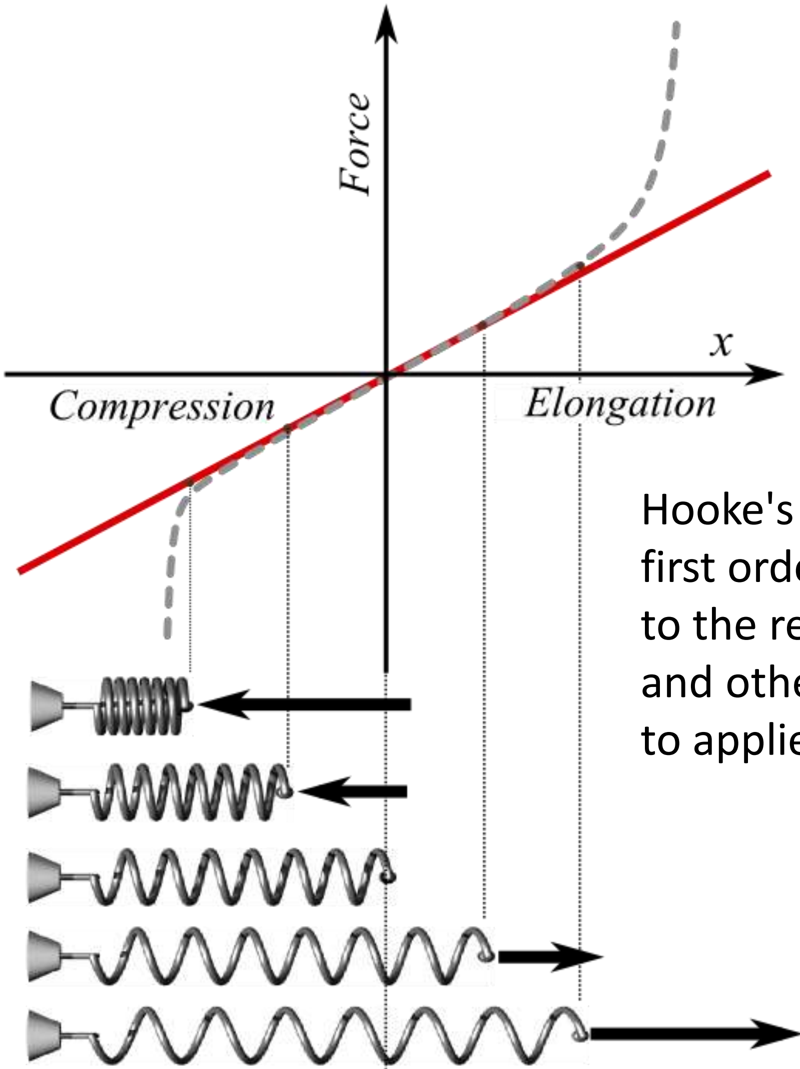
In general, forces which deform solids can be normal to a surface of the material (normal forces), or tangential (shear forces), this can be described mathematically using the stress tensor:

$$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl} \rightleftharpoons \epsilon_{ij} = S_{ijkl}\,\sigma_{kl}$$

where *C* is the elasticity tensor
and *S* is the compliance tensor

NDT&E Methods: UT – the spring model

Constitutive Equations of Sound Waves



Hooke's law is only a first order linear approximation to the real response of springs and other elastic bodies to applied forces

Hook's Law:

$$F = -k\Delta x$$

k: stiffness
(positive constant)

Simplifications:

1-dimensional

Linear

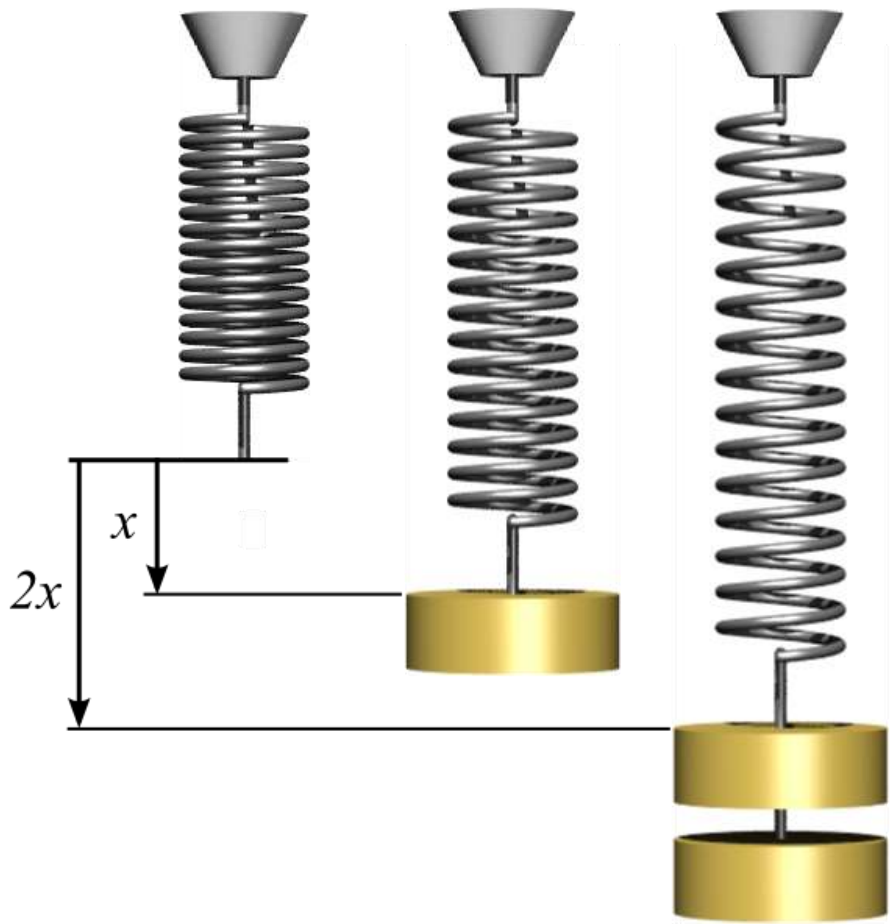
Newton's Law:

$$F = m\frac{d^2x}{dt^2}$$

m: mass

NDT&E Methods: UT – the spring model

Constitutive Equations of Sound Waves



Harmonic Oscillator:

$$F = m\frac{d^2x}{dt^2} = -kx$$

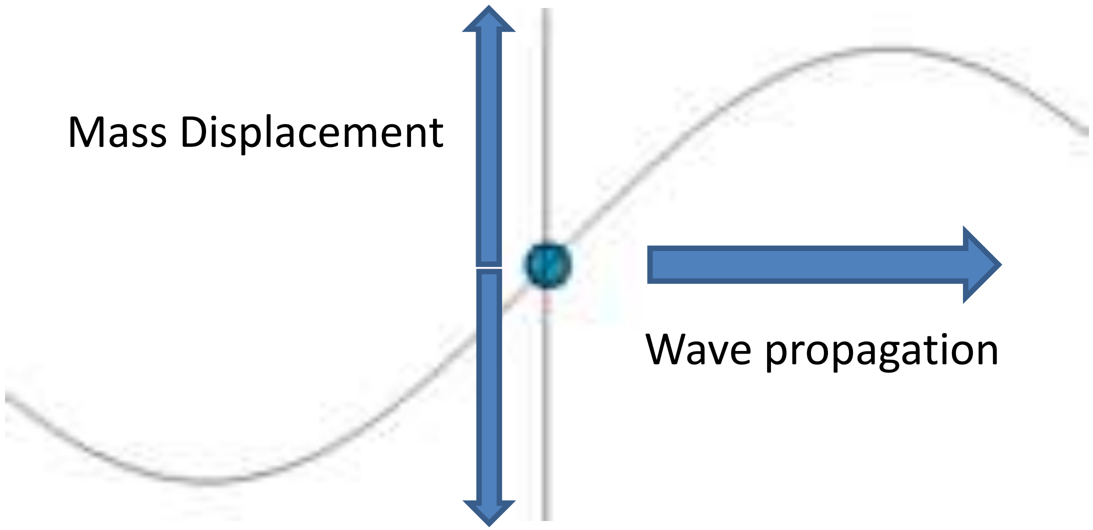
Solving this differential equation, we find that the motion is described by the function:

$$x(t) = A\cos(\omega t + \phi),$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}.$$

NDT&E Methods: UT – the spring model

Constitutive Equations of Sound Waves



It is already a
- ***1-dimensional***
- ***undamped***
- ***harmonic wave***

$$x(t) = A\cos(\omega t + \phi),$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}.$$

NDT&E Methods: Elastic Waves in Solids
Constitutive Equations of Sound Waves

ASSUMING:

Linear Hook's Law, 1-dimensional case, isotropic material

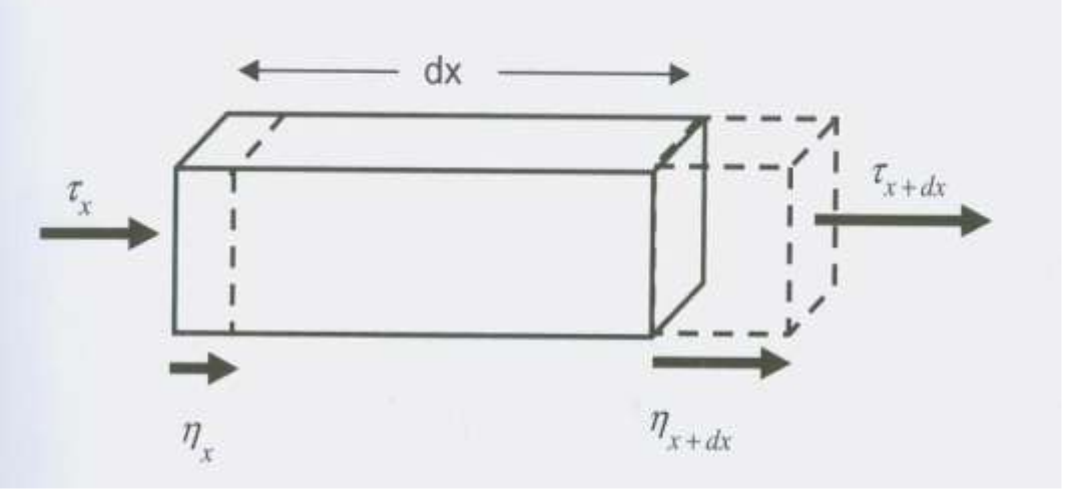
$$F = E\Delta L \rightarrow \Delta L = \varepsilon L \rightarrow \tau = \frac{F}{L} = \varepsilon E$$

*Hook' Law: Stress = E * Strain*

$$\therefore \tau = E\varepsilon$$

A Little Experiment

The stress created by the hammer blow, causes one side of the rod to be displaced by an amount η_x and h$_{x+dx}$ on the right due to the wave taking time to travel along *dx* and to the elastic properties of the material



NDT&E Methods: Elastic Waves in Solids
Constitutive Equations of Sound Waves

- The force on the left face is given as (using the definition of stress):

$$F_{left} = A\tau_x = AE\varepsilon = AE\left.\frac{\partial\eta}{\partial x}\right|_x$$

- While the force on the right side is:

$$F_{right} = A\tau_{x+dx} = AE\varepsilon = AE\left.\frac{\partial\eta}{\partial x}\right|_{x+dx}$$

- The net force is the difference between the left and right sides and of course is equal to the mass times the acceleration of the segment.

$$\Delta F = F_{right} - F_{left} = m\frac{\partial^2\eta}{\partial t^2}$$

- Or, more explicitly

$$AE\left(\left.\frac{\partial\eta}{\partial x}\right|_{x+dx} - \left.\frac{\partial\eta}{\partial x}\right|_x\right) = \rho A dx\frac{\partial^2\eta}{\partial t^2}$$

NDT&E Methods: Elastic Waves in Solids
Constitutive Equations of Sound Waves

• Rearranging the previous equation we have:

$$E\left(\frac{\left.\frac{\partial\eta}{\partial x}\right|_{x+dx}-\left.\frac{\partial\eta}{\partial x}\right|_{x}}{dx}\right)=\rho\frac{\partial^2\eta}{\partial t^2}$$

• This represents a 1-dimensional wave equation for the propagation of a longitudinal wave through an elastic homogeneous medium as a function of position and time.

$$\frac{\partial^2\eta}{\partial x^2}=\frac{\rho}{E}\frac{\partial^2\eta}{\partial t^2}$$

• With *v* representing the wave speed through the medium and comparing this to the standard form of the wave equation we have

$$\frac{\partial^2\eta}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2\eta}{\partial t^2}=\frac{\rho}{E}\frac{\partial^2\eta}{\partial t^2}$$

Remember: $$m\frac{d^2x}{dt^2}=-kx$$

NDT&E Methods: Elastic Waves in Solids
Constitutive Equations of Sound Waves

• We've determined the speed of a sound wave in a 1-dimensional medium by applying Newton's laws of to a small segment of material and we find that that the speed of sound depends further on its density, ρ and the elastic properties of the medium, or equivalently in terms of the compressibility, K, through

$$v = \sqrt{\frac{E}{\rho}} = \frac{1}{\sqrt{K\rho}}$$

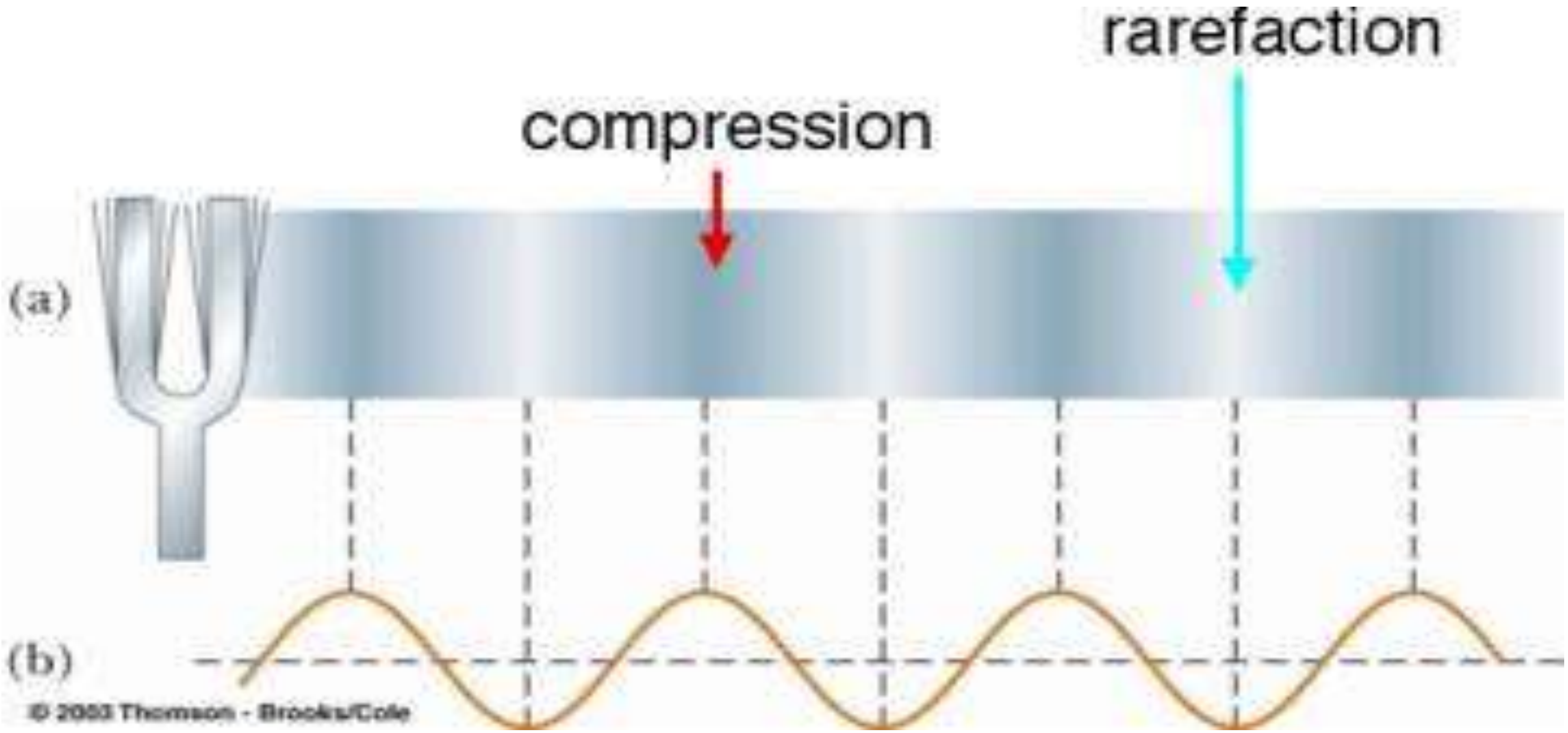
• The compressibility is defined as the fractional change in volume of material per unit increase in pressure

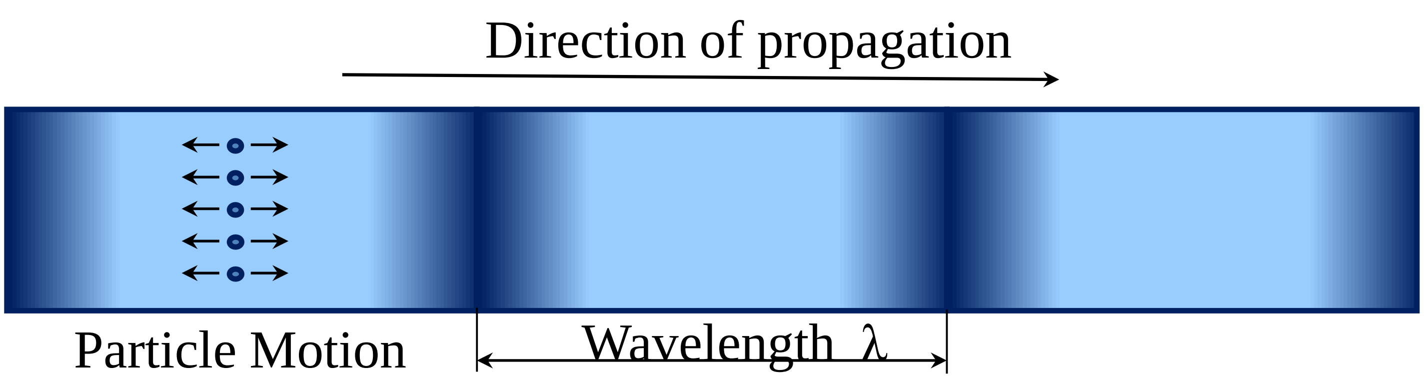
$$K = -\frac{1}{V}\frac{dV}{dP}$$

• Thus a large compressibility, K, means that it is easier to squeeze something.

• Again, E is called the elastic (or Young's) modulus of the material and is a measure of the stiffness of the material.

NDT&E Methods: UT – Waves in Liquids





NDT&E Methods: UT – Waves in Liquids

An acoustic wave is a traveling pressure disturbance that produces alternating

- compressions
- rarefactions (expansions) of the propagation medium

The compressions and rarefactions displace incremental volumes of the medium and the wave propagates via transfer of momentum among incremental volumes

Each incremental volume of the medium undergoes small oscillations about its original position but does not travel with the pressure disturbance

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NDT&E Methods: UT – Waves in Liquids

A **pressure plane wave**, $p\,(x,t)$, propagating along one spatial dimension, *x*, through a homogeneous, non-attenuating fluid medium can be formulated starting from **Euler's equation** and **the equation of continuity** :

$$\frac{\partial}{\partial x}p(x,t)+\rho_o\frac{\partial}{\partial t}u(x,t)=0$$

$$\frac{\partial}{\partial t}p(x,t)+\frac{1}{\kappa}\frac{\partial}{\partial x}u(x,t)=0$$

ρ_o is the undisturbed mass density of the medium

κ is the compressibility of the medium (*i.e.*, the fractional change in volume per unit pressure in units of Pa^{-1})

$u(x,t)$ is the particle velocity produced by the wave

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NDT&E Methods: UT – Waves in Liquids
Constitutive Equations of Sound Waves

$$e^{i\pi}+1=0$$

Euler's identity

is an equality found in mathematics
that has described as
"the most beautiful equation."
It is a special case of a foundational equation
in complex arithmetic called Euler's Formula

In Fluid Dynamics, the constitutive equation is a Euler equation

NDT&E Methods: Elastic Waves in Solids
Constitutive Equations of Sound Waves

In fluid dynamics,

the **Euler equations** are a set of quasilinear hyperbolic equations governing adiabatic and inviscid flow.

They are named after Leonhard Euler (St. Peterburg)

The equations represent Cauchy equations of
- conservation of mass (continuity), and
- balance of momentum and energy,

and can be seen as particular Navier–Stokes equations with zero viscosity and zero thermal conductivity

NDT&E Methods: UT – Waves in Liquids

Euler's equation, which can be derived starting from Newton's second law of motion:

$$\frac{\partial}{\partial x} p(x,t) + \rho_o \frac{\partial}{\partial t} u(x,t) = 0$$

Equation of continuity, which can be derived by writing a mass balance for an inc remental volume of the medium:

$$\frac{\partial}{\partial t} p(x,t) + \frac{1}{\kappa} \frac{\partial}{\partial x} u(x,t) = 0$$

Acoustic wave equation is obtained, combining both equations :

$$\frac{\partial^2}{\partial x^2} p(x,t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} p(x,t) = 0$$

$c = 1 / \sqrt{\rho_o \kappa}$ **is the speed of sound**

A monochromatic plane wave solution is:

$$p(x,t) = P\cos(\omega t - kx)$$

P is the amplitude of the wave
$\omega = 2\pi f$ is the radian frequency
$k = 2\pi/\lambda$ is the wave number

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NDT&E Methods: UT

The strength of an ultrasound wave can also be characterized by its intensity, *I*, which is the average power per unit cross-sectional area evaluated over a surface perpendicular to the propagation direction. For acoustic plane waves, the intensity is related to the pressure amplitude by:

$$I = \frac{E}{S\Delta t}$$

$$I(\mathrm{W/m^2}) = \frac{P^2}{2\rho_0 c}$$

P is the pressure amplitude of the wave;
ρ_o is the undisturbed mass density of the medium;
c is the speed of sound

Diagnostic imaging is typically performed using peak pressures in the range 0.1 – 4.0 MPa

When the acoustic intensity I_{dB} is expressed in *decibels*, dB:

$$I_{dB}(\mathrm{dB}) = 10\log\left(I/I_{ref}\right)$$

I_{ref} is the reference intensity

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NDT&E Methods: Elastic Waves in Solids
Constitutive Equations of Sound Waves

However,

The World,
Material Properties and Applied Stresses
are
3-Dimensional

Makes it much more difficult

NDT&E Methods: UT – Plane Body Waves

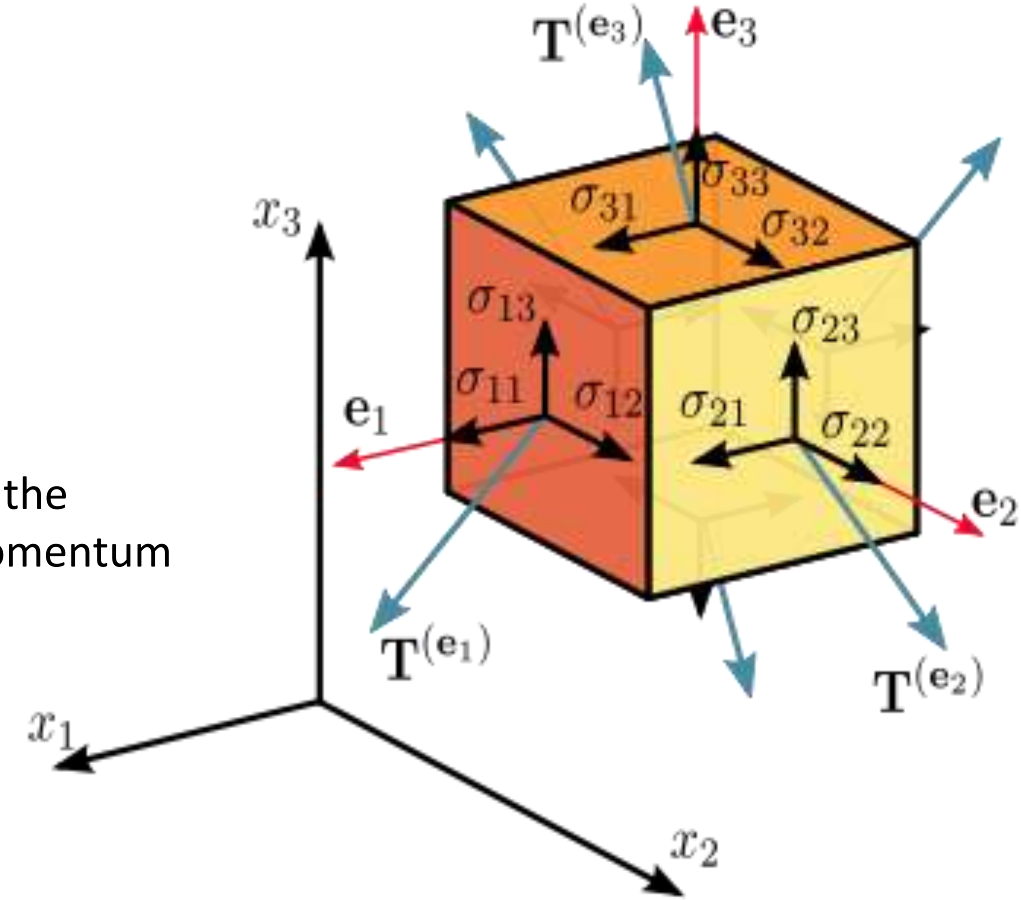
The 3-D Constitutive Equation for an infinite medium containing a linear, elastic, homogeneous isotropic material (Hook's Law)

Cauchy STRESS-Tensor Σ:

$$\begin{matrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{matrix}$$

The Stress tensor is symmetric due to the conservation of linear and angular momentum

$$\sigma_{xy} = \sigma_{yx}$$
$$\sigma_{xz} = \sigma_{zx}$$
$$\sigma_{yz} = \sigma_{zy}$$



NDT&E Methods: UT – Plane Body Waves

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Euler–Cauchy stress principle

$$\sigma_{xy} = \sigma_{yx}$$
$$\sigma_{xz} = \sigma_{zx}$$
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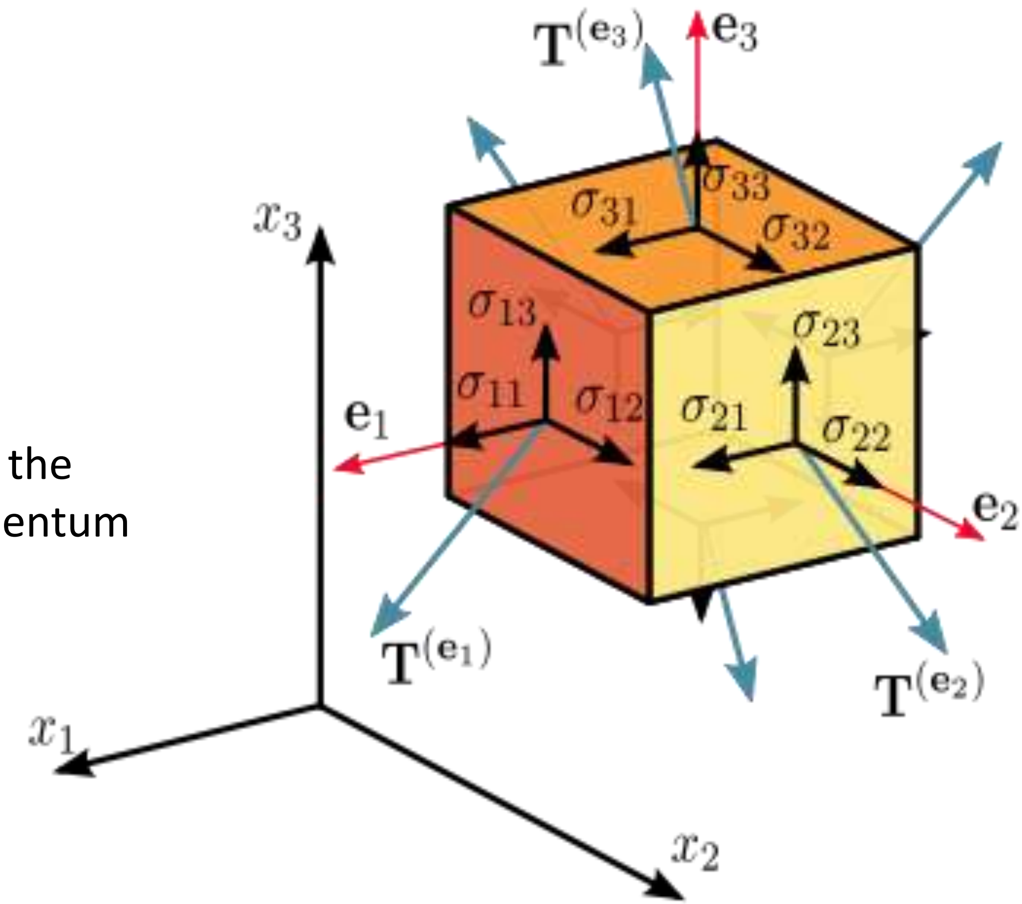


Deformation and Strain

We distinguish

Displacement & Deformation

Deformation:

An alteration of shape, as by pressure or stress.

Displacement:

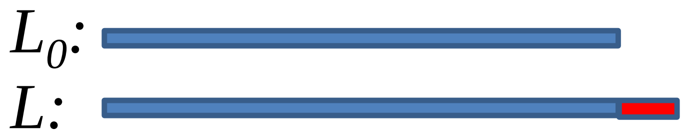
A vector or the magnitude of a vector from the initial position to a subsequent position assumed by a body

NDT&E Methods: UT – Waves in Solids
Constitutive Equations of Sound Waves

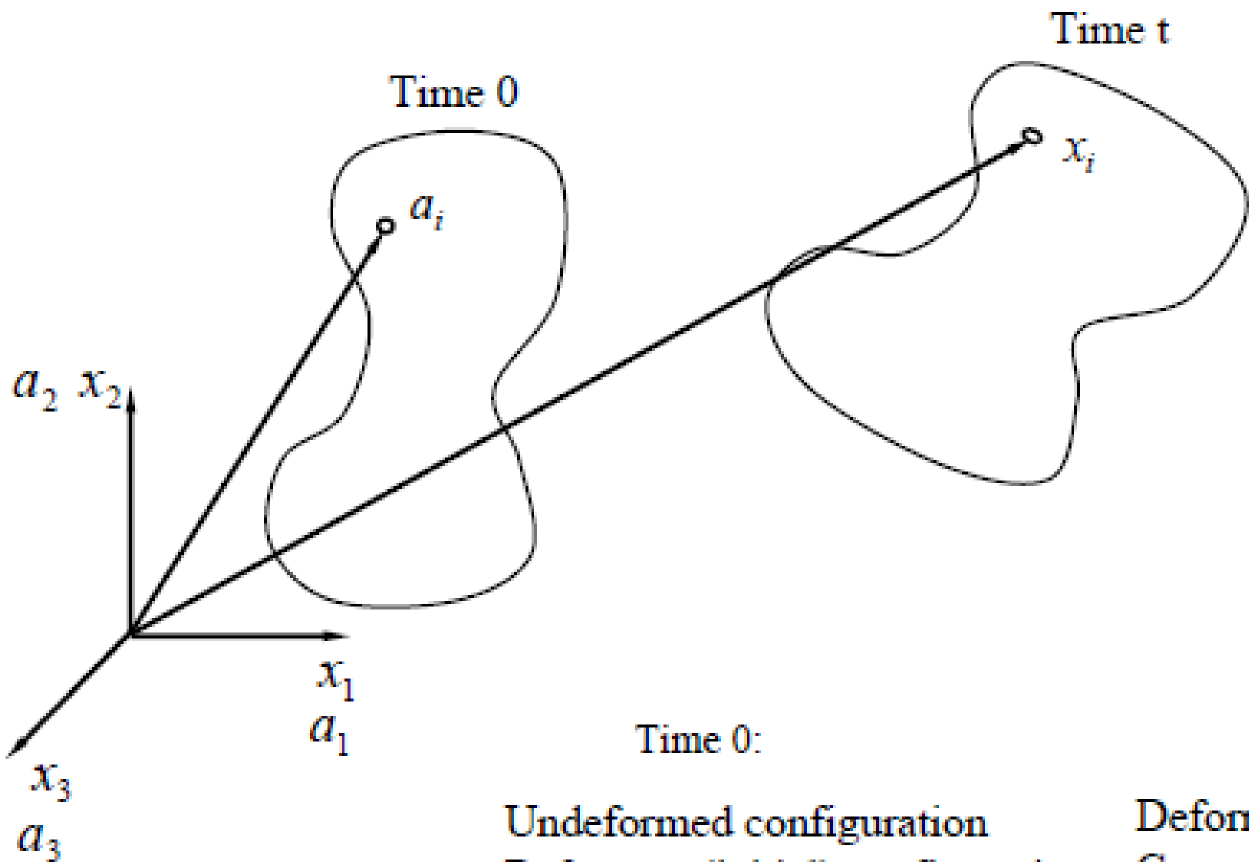
Strain characterizes a deformation

Example:

$$\text{1D strain } \varepsilon_1 = \frac{L - L_0}{L_0}$$



NDT&E Methods: Strain & Stress
Kinematics of Continuous Body



Time 0:

Undeformed configuration
Reference (initial) configuration
Material configuration

Time t:

Deformed configuration
Current configuration
Spatial configuration

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

THERE ARE TWO POSSIBLE DESCRIPTIONS:

LAGRANGIAN: $$x_i = x_i(a_1, a_2, a_3, t)$$

The motion is described by the material coordinate and time

EULERIAN: $$a_i = a_i(x_1, x_2, x_3, t)$$

The motion is described by the spatial coordinate and time

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

Lagrangian

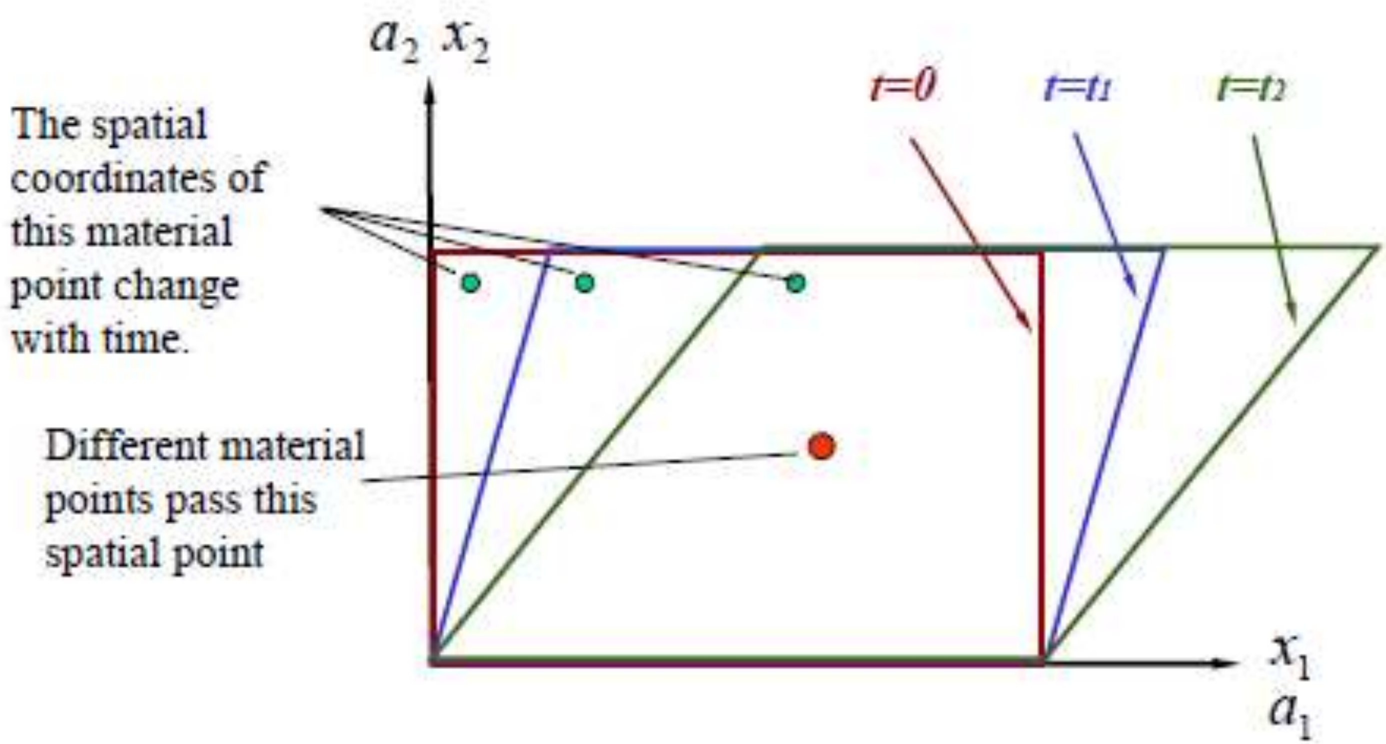
$$x_i = x_i(a_1, a_2, a_3, t)$$

(Tacking a material point)

Eulerian

$$a_i = a_i(x_1, x_2, x_3, t)$$

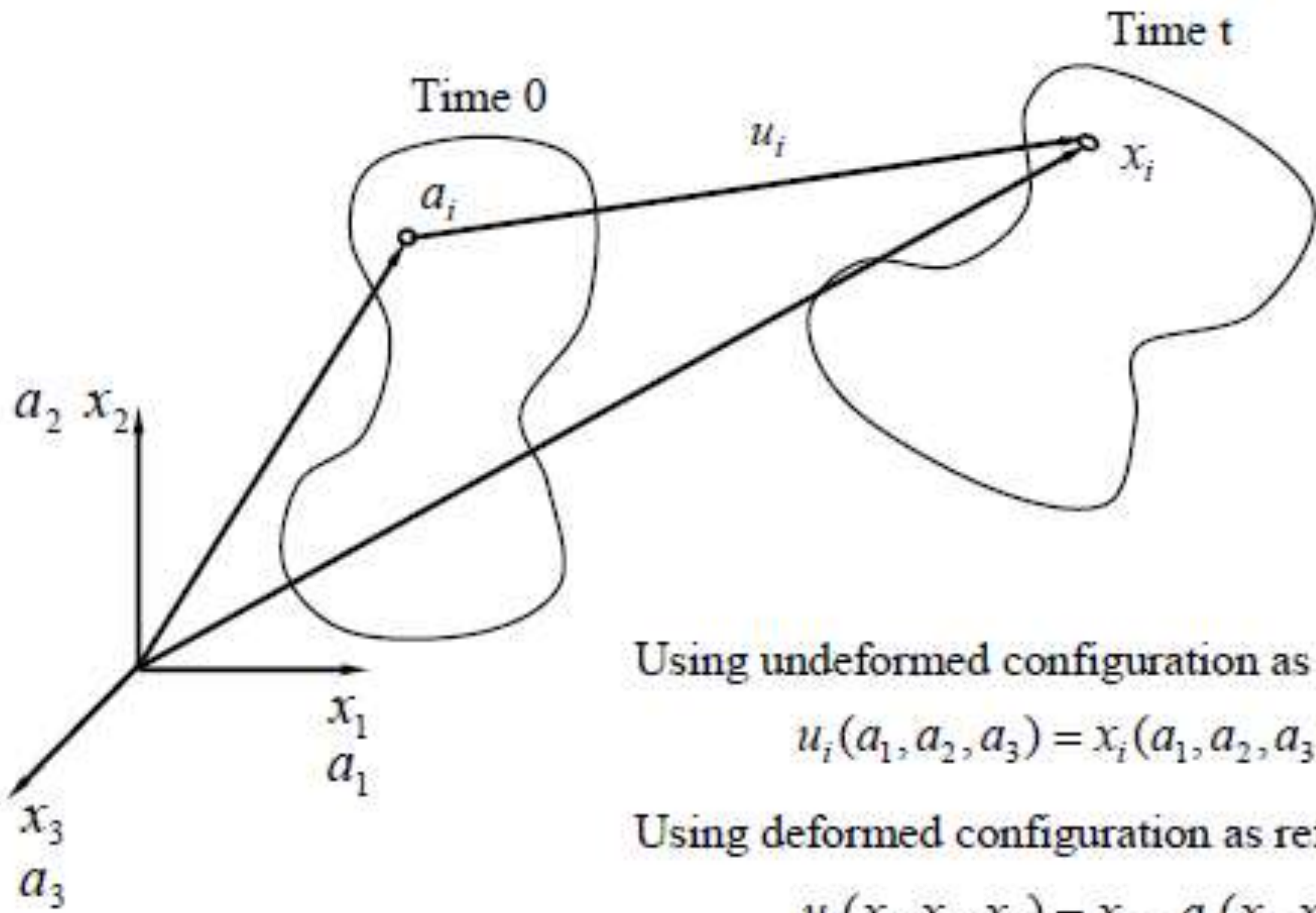
(Monitoring a spatial point)



NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

Lagrangian	Eulerian
Tracking a material point.	Tracking a spatial point.
Material point is fixed but the spatial coordinates have to be updated.	Spatial coordinates are fixed but Material points keep changing.
Good for constitutive model	Not good for constitutive model.
Solid Mechanics	Fluid Mechanics Solid Mechanics

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body



Using undeformed configuration as reference:

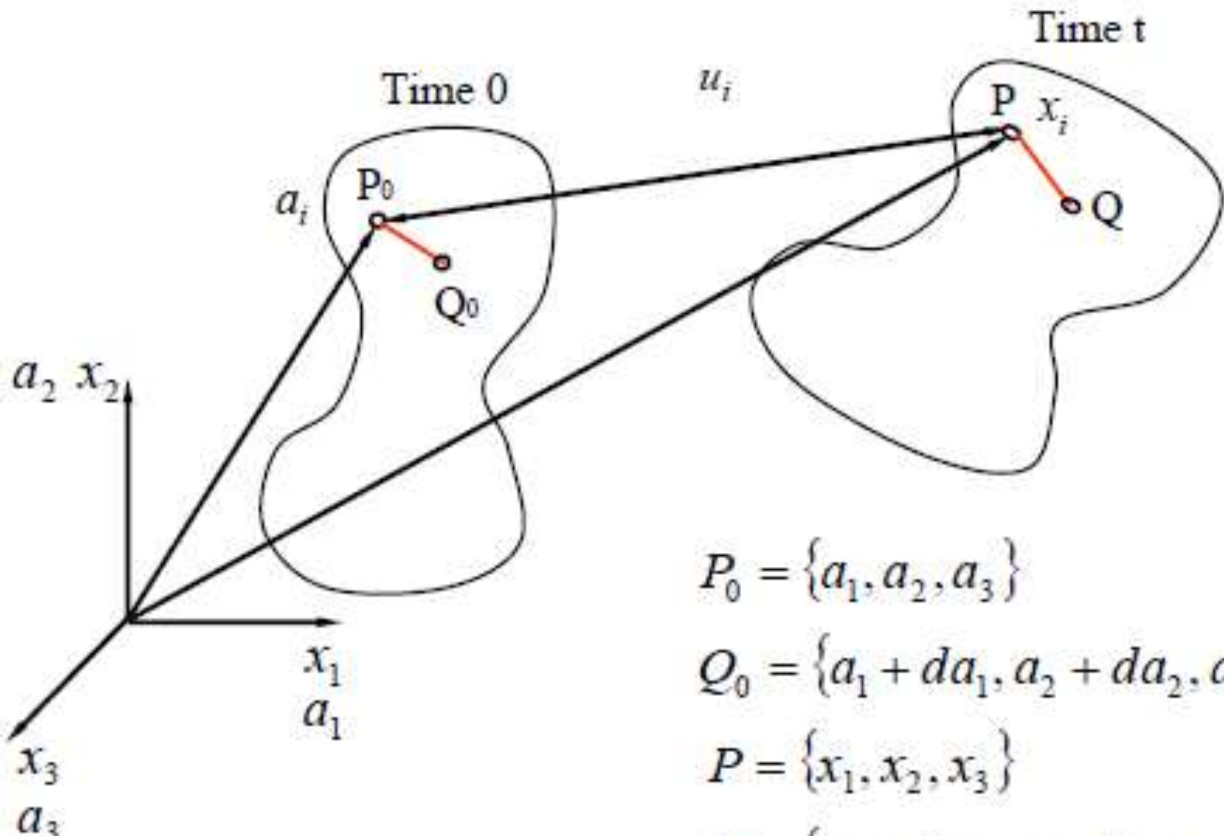
$$u_i(a_1, a_2, a_3) = x_i(a_1, a_2, a_3) - a_i$$

Using deformed configuration as reference:

$$u_i(x_1, x_2, x_3) = x_i - a_i(x_1, x_2, x_3)$$

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

Measure the deformation



$$P_0 = \{a_1, a_2, a_3\}$$

$$Q_0 = \{a_1 + da_1, a_2 + da_2, a_3 + da_3\}$$

$$P = \{x_1, x_2, x_3\}$$

$$Q = \{x_1 + dx_1, x_2 + dx_2, x_3 + dx_3\}$$

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

Strain Tensor:

$$E_{ij} = \frac{1}{2}\left(\frac{\partial x_k}{\partial a_i} \frac{\partial x_k}{\partial a_j} - \delta_{ij} \right)$$ Green Strain

$$e_{ij} = \frac{1}{2}\left(\delta_{ij} - \frac{\partial a_k}{\partial x_i} \frac{\partial a_k}{\partial x_j} \right)$$ Almansi Strain

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

Strain Tensor:

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial a_j} + \frac{\partial u_j}{\partial a_i} + \frac{\partial u_k}{\partial a_i}\frac{\partial u_k}{\partial a_j} \right)$$ Green Strain

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j} \right)$$ Almansi Strain

Applicable to both small and finite (large) deformation.

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

If $\frac{\partial u_i}{\partial a_j} << 1$ $\frac{\partial u_i}{\partial x_j} << 1$ small deformation

The quadratic term in Green strain and Almansi strain can be neglected.

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial a_j} + \frac{\partial u_j}{\partial a_i}\right) \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Also, in small deformation, the distinction between Lagrangian and Eulerian disappears.

$$E_{ij} = e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

<u>Cauchy's infinitesimal strain tensor</u>

NDT&E Methods: Strain & Stress
Kinematics of Continuous Body

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<u>Cauchy's infinitesimal strain tensor</u>

NDT&E Methods: UT – the spring model

Cauchy's infinitesimal strain tensor

$$E_{ij} = e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$e_{11} = E_{11} = \frac{\partial u_1}{\partial x_1} \qquad e_{12} = E_{12} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)$$

$$e_{22} = E_{22} = \frac{\partial u_2}{\partial x_2} \qquad e_{13} = E_{13} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right)$$

$$e_{33} = E_{33} = \frac{\partial u_3}{\partial x_3} \qquad e_{23} = E_{23} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}\right)$$

NDT&E Methods: UT – the spring model

Engineering Strains

Coordinates: x,y,z

Displacements: x,y,z

Normal Strains

$$\varepsilon_x = \frac{\partial u}{\partial x} = e_{11}$$

$$\varepsilon_y = \frac{\partial u}{\partial y} = e_{22}$$

$$\varepsilon_z = \frac{\partial u}{\partial z} = e_{33}$$

Shear Strains

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2e_{12}$$

$$\varepsilon_{xy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 2e_{23}$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 2e_{13}$$

NDT&E Methods: UT – the spring model

Integrability Condition related to Compatibility of Strain Fields:

$$\frac{\partial^2 u_1}{\partial x_2 \partial x_1} = \frac{\partial^2 u_1}{\partial x_1 \partial x_2} \qquad \frac{\partial f}{\partial x_2} = \frac{\partial g}{\partial x_1}$$

↓

Integration of Strain Fields Yields
Unique Displacement Components
Characteristic for Material Constants

NDT&E Methods: UT – Material Constants

$$\sigma_{ij} = C_{ijkl}\,\epsilon_{kl} \rightleftharpoons \epsilon_{ij} = S_{ijkl}\,\sigma_{kl}$$

The strain tensor is a field tensor – it depends on external factors.
The compliance tensor is a matter tensor –
it is a property of the material and does not change with external factors.

Let us consider the material by introducing the Lamé Constants:

Young Modulus	Shear Modulus – Poisson's Ratio
E – λ(first parameter)	**G – μ(second parameter)**

The Lamé constants are material-dependent quantities denoted by λ and μ that arise in strain-stress relationships.

NDT&E Methods: UT – Material Constants

The Constitutive Equation between Stresses and Strains
for an **Infinite Medium**
containing a **linear, homogeneous isotropic** Material
(HOOK'S LAW)

$$\sigma_{xx} = (\lambda + 2\mu)\varepsilon_{xx} + \lambda(\varepsilon_{yy} + \varepsilon_{zz})$$

$$\sigma_{yz} = \sigma_{zy} = 2\mu\varepsilon_{yz} = 2\mu\varepsilon_{zy}$$

$$\sigma_{yy} = (\lambda + 2\mu)\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{zz})$$

$$\sigma_{zx} = \sigma_{xz} = 2\mu\varepsilon_{zx} = 2\mu\varepsilon_{xz}$$

$$\sigma_{zz} = (\lambda + 2\mu)\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy})$$

$$\sigma_{xy} = \sigma_{yx} = 2\mu\varepsilon_{xy} = 2\mu\varepsilon_{yx}$$

ε_{ij}: Components of Strains; σ_{ij}: Components of Stresses

NDT&E Methods: UT – Material Constants

Lamé's Constants are related to Young's Modulus E and Poisson's Ratio ν

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = \frac{E}{2(1+\nu)}$$

Governing Differential Equation of Motion for a Continuum:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 u_x}{\partial^2 t}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho \frac{\partial^2 u_y}{\partial^2 t}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 u_z}{\partial^2 t}$$

u_i : Components of the Particle Displacement Vector

NDT&E Methods: UT – Material Constants

Some Mathematics Later:

The Three-Dimensional Wave Equation for Linear, Elastic, Homogeneous Isotropic Material

$$(\lambda+\mu)\frac{\partial \Delta}{\partial x} + \mu \nabla^2 u_x = \rho \frac{\partial^2 u_x}{\partial^2 t} \qquad (\lambda+\mu)\frac{\partial \Delta}{\partial y} + \mu \nabla^2 u_y = \rho \frac{\partial^2 u_y}{\partial^2 t}$$

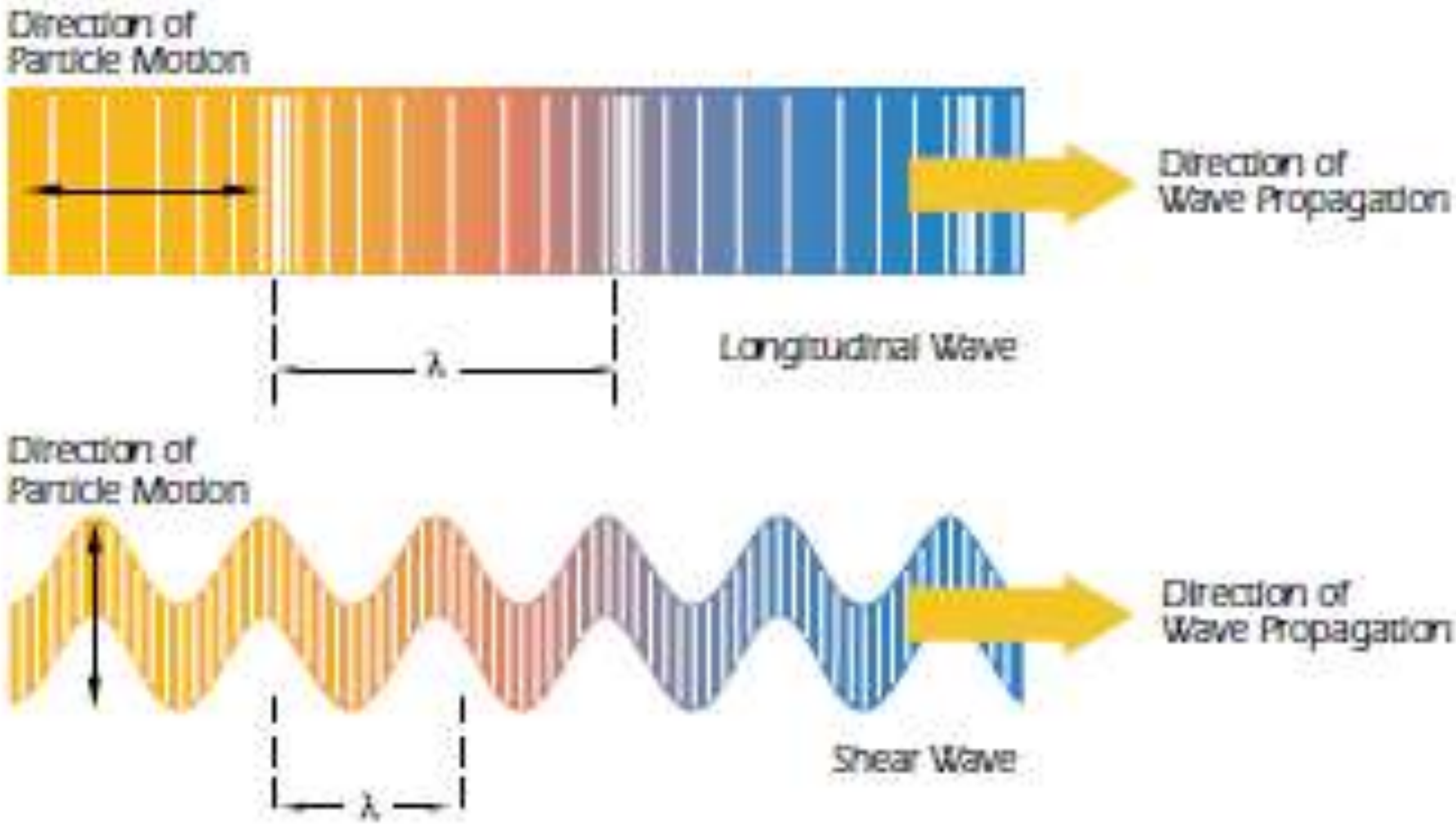
$$(\lambda+\mu)\frac{\partial \Delta}{\partial z} + \mu \nabla^2 u_z = \rho \frac{\partial^2 u_z}{\partial^2 t}$$

Volume Dilation of the Displacement Δ:

$$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

NDT&E Methods: UT – Modes & Velocities

For Isotropic Materials,
There are only two Distinct Values for Wave Velocities:



NDT&E Methods: UT

GROUP VELOCITY = **PHASE VELOCITY**

Speed of Energy Transport — **Speed of Phase Propagation**

For isotropic, homogeneous, linear materials due to linearity and causality principles.

A propagating medium is said to be dispersive if the phase velocity is a function of frequency, which is the case for attenuating materials, for example. For dispersive and anisotropic materials group and phase vectors are different

NDT&E Methods: UT

Literature

1. R. Kienzler, G. Herrmann. *On conservation laws in elastodynamics*, International Journal of Solids and Structures 41, Elsevier (2004)
2. N.S. Ottosen, M. Ristinmaa. *The Mechanics of Constitutive Modeling,* Elsevier Science (2005), ISBN 9780080446066
3. E. Henneke II, D. E. Chimenti. *Ultrasonic Wave Propagation, Nondestructive Handbook, Volume 7 Ultrasonic Testing, Chapter 2, ASNT 3rd edition*

NDT&E Methods: Elastic Waves in Solids

Constitutive Equations of Sound Waves

The strain tensor is a field tensor – it depends on external factors.

The compliance tensor is a matter tensor –

it is a property of the material and does not change with external factors.