

Практика

Уравнения в полных дифференциалах

$$P(x, y)dx + Q(x, y)dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$u = \int P(x, y)dx + C(y) \text{ и } u'_y = Q(x, y)$$

1. $(e^x + y + \sin y)dx + (x + e^y + x \cos y)dy = 0$
2. $(xy + \sin y)dx + (0.5x^2 + x \cos y)dy = 0$
3. $(y + x \ln y)dx + \left(\frac{x^2}{2y} + x + 1\right)dy = 0$

Практика

2. Линейные уравнения

$$y' + P(x)y = Q(x)$$

$$\text{Замена } y = uv \begin{cases} u' + P(x)u = 0, \\ v'u = Q(x) \end{cases} \quad u' = \frac{du}{dx}, v' = \frac{dv}{dx}$$

1. $xy' - y = x^2 \cos x$
2. $y' \sin x - y \cos x = 1$
3. $y' + 2y = 4x$
4. $xy' - \frac{y}{x+1} = x, y(1) = 0$

3. Уравнения Бернулли

$$y' + P(x)y = Q(x)y^m$$

$$\text{Замена } z = \frac{1}{y^{m-1}} \frac{z'}{m-1} + P(x)z = Q(x)$$

1. $y' + \frac{y}{x} = x^2 y^4$
2. $y' - \frac{2xy}{1+x^2} = \frac{4\sqrt{y}}{\sqrt{1+x^2}} \operatorname{arctg} x$
3. $y' - y \operatorname{tg} x = -y^{-2} \sin x$

Дома

1. $2(x+y)dy + (3x+3y-1)dx = 0, y(0) = 2$
2. $(x+y+2)dx + (2x+2y-1)dy = 0$
3. $\left(\frac{y}{y^2+x^2} - y\right)dx + \left(y^2 - x - \frac{x}{x^2+y^2}\right)dy = 0$
 1. $(\ln y - 5y^2 \sin 5x)dx + \left(\frac{x}{y} + 2y \cos 5x\right)dy = 0, y(0) = e$

5. $e^y dx + (xe^y - 2y)dy = 0$

6. $y' + \frac{1-2x}{x^2}y = 1$

7. $(1+x^2)y' - 2xy = (1+x^2)^2$

8. $xy' + y - e^x = 0, y(a) = b$

9. $y' + 2xy = 2x^3y^3$

10. $y' + \frac{y}{1+x} + y^2 = 0$