MINISTRY OF EDUCATION AND SCIENCE OF THE RUSSIAN FEDERATION

«National Research Tomsk Polytechnic University»

CLAIM
Director of EC
Zakharova A.A.
"01 ' 09, 2014.

Department of Informatics and systems design

Fund evaluation tools

INPUT/CURRENT EVALUATION/INTERMEDIATE CERTIFICATION

on discipline: "**Methods of optimization**»

Developed in accordance with the Federal state educational standard and the work program by Reizlin V.I.,

approved "10" 06, 2014.

Direction of preparation: 09.04.01 (230100)

"Computer Science and Engineering".

Course 1, Semester 1.

Allocation of study time:

Lectures 16 h

Laboratory 32 h.

Independent work 60 h.

Total: 108 h.

Date of creation: 01.09.2014

Preface

1. Assignment: In accordance with the requirements of the Federal state educational standard for he evaluation of students to meet their academic achievements phased requirements of the relevant basic educational program (BEP) to create a fund of assessment tools (FAT) for input and evaluation of The current, intermediate and final assessment of students. FAT is an integral part of the normative-methodical providing of quality evaluation system of The BEP, is a member of The BEP in General and teaching methods (in particular, The work program) of the appropriate discipline.
2. Fund performance monitoring tools developed on the basis of the work program of the discipline of optimization methods in line with The BEP "Informatics and computer engineering (master's degree).
3. An examination of the composition of The Expert Commission:

Assoc. Prof. Demin A., Chairman of EC, Assoc.prof. Gorbunov V. M., Associate Professor Pogrebnoy A.V.
Expert opinion: Fund of assessment tools meet the requirements of The basic educational program and The of The Federal State educational standards of higher education.

The Chairman of the Commission:

Associate Professor A.Y. Demin 01.09. 2014

4. Considered and approved at the meeting of the IPS, Protocol No. 1 from 01.09. 2014

Head of Department Sonkin M.A. 01.09. 2014

5. Developers:

Associate Professor Reizlin V.I. 01.09. 2014

Passport to evaluation of The learning outcomes (competencies) for The discipline

Year range: 2014
Direction of preparation: 09.03.01 (230100) Computer Science and Engineering

Educational module (academic discipline)	Competence Of The GEF	The results of learning OOP TPU	Decomposition		
			Knowledge	Skills	Ownership experience
Methods of optimization	Is able to improve and develop Their intellectual and cultural level (OK-1); capable of independent learning new research methods, a change of scientific and research-and-production profile of his career (OK-2); able to acquire through information technology and use of new knowledge and skills, including in new areas of knowledge not directly related to work (OK-6) to apply The advanced methods of research and professional tasks based on knowledge of world trends in The development of computer science and information technology (PC-1); choose methods and develop algorithms for solving problems of control and automation (PC-5)	**R1** Put and solve The problem of industrial analysis associated with The creation and processing of materials using modelling of objects and processes chemical technology petrochemical and biotechnology **R3** To set and deal with innovative engineering tasks associated with	Optimization methods and making design decisions; The basic Theory of The numerical methods for continuous improvement; implementation of The optimization algorithms using computers. MaThematical methods of analysis of The simplest systems	Put The optimization problem and develop an algorithm for its solution; use optimization techniques to solve scientific and engineering problems. Use applications programming for solving	The methods of making design decisions; basic methods for solving continuous optimization problems; skills development and debugging of programs for solving continuous optimization.

		creating hardware and software information and automated systems, using analytical methods and complex models.	in science, economics and technology.	optimization problems	

Bank of evaluation tools

1. Incoming control of knowledge

The laboratory work № 1 is the entrance control of knowledge.
To complete this lab work requires knowledge and skill in disciplines: programming, programming technology.
Required skills for IDEs.

Entrance control includes the following items:

1. What is The integrated development environment (IDE).
2. What is the effectiveness of the program over time?
3. What is a user-friendly interface?
4. Methods of tabulation.

2. The list of issues of The current control of knowledge

1. Definition of a global minimum.
2. Definition of a local minimum.
3. Definition of tasks of unconditional optimization.
4. Conditional optimization task.
5. Definition of The level line.
6. Definition of The Lagrange function.
7. Fixed point of a classical problem on conditional extremum.
8. Definition of The task of linear programming (LP).
9. Total task form LP.
10. Canonical form task LP.
11. Determination of The feasible set of LP problems in vector-matrix form. It's name?
12. View the machine zero.
13. What are the groups of optimization techniques?
14. Determination methods of the descent.
15. Definition of an acceptable direction.
16. Convergence of Th What is the effectiveness of the program over time e method to define the optimization.
17. Define linear convergence rate.
18. Define convergence with superline speed.
19. Define convergence speed of order p.
20. To formulate criteria for the breakpoint.
21. What are the constants in The stop criteria?
22. Definition of a unimodal function.
23. Identify localization minimum cut by Fibonacci.
24. The golden section method.
25. Regularization methods dimensional search.
26. The principle model scheme of gradient methods.
27. Terms of convergence circuit diagram model of gradient methods.
28. The search direction is chosen as the gradient methods?

29. Iterative gradient method schema.
30. Modification of The gradient method. Their properties.
31. Optimal gradient method.
32. Optimal trajectory characteristics of gradient method.
33. Newton's Method.
34. The Newton-Raphson Method.
35. What is meant by conversion method in the method of penalty functions?
36. What are the problems of application of barrier methods?
37. Penalty functions. Their properties.
38. Penalty functions for restrictions inequalities.
39. Penalty functions for restrictions inequalities.

3. The list of intermediate certification (questions for The exam and tickets)

1. Mathematical model of object and its properties.
2. The notion of optimality criteria and objectives.
3. The main tasks of optimization.
4. The 1-dimensional optimization. General search method. Unimodal function. A method of dividing the interval in half.
5. The Unimodal function. The method of "golden section".
6. The Sven method to search for the segment containing the minimum point.
7. The 1-dimensional optimization. The Newton-Raphson Method
8. The 1-dimensional optimization. Kuazi Newton method.
9. Multidimensional optimization. Relief is function. The wise of the descent.
10. Method of ravines. Random search.
11. Multidimensional optimization. A gradient method. Method of steepest descent.
12. Multidimensional optimization. Newton's Method.
13. Multidimensional optimization. Markvardt Method.
14. The tasks with constraints. Search the optimum type constraints in problems with PARS. The method of Lagrange multipliers uncertain.
15. The search for the optimum in problems with constraints.
Methods of penalty and barrier functions.
16. The search for the optimum in problems with constraints.
Method of factors.
17. Linear programming. Setting targets. Basic (canonical) problem and bringing it to an arbitrary problem.
18. Linear programming. Converting the primary task to the main problem with the LP inequality constraints (Form A).
19. Linear programming. Geometrical solution of two-dimensional problems.
20. The main Theorem on the LP.

Examination (examples)

TICKET # 1
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Tasks with constraints. Search The optimum type constraints in problems with PARS. The method of Lagrange multipliers uncertain.

2. The ration for food animals consists of two types of feeds (I) and (II). One kilo of poop I worth $ 9 and contains: 3 units. fat, 4 g. protein 1 g. nitrates. One kg of feed (II) costs $ 4 and contains: 1 PC. fat, 3 u. protein 2 g. nitrates.
You want to make The most expensive diet for fat not less than 3 g, proteins not less than 6 units, nitrates up to 3 units.

TICKET # 2
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Search the optimum in problems with constraints. Methods of penalty and barrier functions.

For The production of two kinds of products a and b company uses three kinds of raw materials. The total number of raw materials and The consumption of raw materials on one product are listed in The table below:

View raw	Consumption of raw materials on one product, kg		The total number of raw material, kg
	(A)	In	
(I)	12	4	300
(II)	4	4	120
(III)	3	12	252

2. Profits from The sales of one product a and b are respectively 30 and 40 rubles.
Such a plan should be drawn up of production in which The profits from The sale of products is most cost-effective.

TICKET # 3
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

4. Multidimensional optimization. Relief is function. The wise of The descent.

2. There are two types of feed (I) and (II), containing nutrients (vitamins) S_1, S_2 and S_3. The contents of The number of units of nutrients in 1 kg of each type of feed and The necessary minimum nutrients are shown in The table.

Nutrient (vitamin)	Necessary minimum nutrients	The number of units of nutrients in 1 kg of food	
		(I)	(II)
S_1	9	3	1
S_2	8	1	2
S_3	12	1	6

The cost of 1 kg of food (I) and (II) respectively is equal to 4 and 6.
A diet that has minimum cost, that The content of each type of nutrients would be not less than a specified limit.

TICKET # 4
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Setting objectives. The main task and's (canonical) bringing to it an arbitrary task.
2. For The production of two kinds of products of P_1 and p_2 use four types of resources S_1, S_2, S_3 and S_4. Resource stocks, The number of units of The resources spent on the production of a unit of production, are given in The table below:

Type of resource	The number of units of The resources spent on the production of a unit of production		Resource reserve
	$(P)_1$	R_2	
S_1	1	3	18
S_2	2	1	16
S_3	0	1	5
S_4	3	0	21

Profit from unit P_1 and p_2, 2 and 3 respectively.
Such a plan should be drawn up of production in which The profits from The sales of The products will be at its highest.

TICKET # 5
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. 1-dimensional optimization. General search method. Unimodal function. A method of dividing The interval in half.
2. The ration for food animals consists of two types of feeds (I) and (II). One kilo of poop I costs 80 USD. and contains: 1 PC. fat, 3 u. protein 1 g. carbohydrates, 2 PCs. nitrates. One kg of feed (II) costs 10 $. contains: 3 units. fat, 1 PC. protein 8 g. carbohydrates, 4 units. nitrates.

You want to make The most expensive diet for fat not less than 6 g, proteins not less than 9 g, carbohydrates at least 8 units, nitrates up to 16 units.

TICKET # 6
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. One-dimensional optimization. Unimodal function. Golden section search.

2. On the trading floor to expose for sale goods T1 and T2. Working time does not exceed a sellers 1100 hours and square trading Hall to take does not exceed 700 m^2. Each commodity unit powered brings profit, respectively, 50 and 80 den. u. Resource cost rates per unit of sold goods are listed in The table below:

Resources	T1	T2
Operating time, h	3	2
Area, m^2	1	2

Find The best structure of turnover (**The smaller** The number of units of The item, The better) for profit no less 28500 den. u.

TICKET # 7
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Basic conOption tasks to The main task of The LP with restrictions-inequalities (form a).

2. Spinning Mill for The production of 2 types of yarn uses three types of raw materials-pure wool, nylon and acrylic.

The type of raw materials	Rates of consumption of raw materials in 1 t of yarns		The number of raw materials
	Type 1	View 2	
Wool	0.5	0.2	583
Nylon	0.1	0.4	626
Acrylic	0.4	0.2	523
Profit from sales of yarn	1100	900	

You want to plan The production of yarns in order to maximize total profits.

TICKET # 8
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Basic conOption tasks to The main task of The LP with restrictions-inequalities (form a).

2. Tea-packing factory produces tea grades a and b by mixing 3 ingredient: Indian, Georgian and Krasnodar teas. You want to plan The production of tea, can profit.

Ingredients	Usage rate (t/t)		Stocks (tonnes)
Indian tea	0.5	0.2	600
Georgian tea	0.2	0.6	870
Krasnodar tea	0.3	0.2	430
Profit from The sale of 1 t products	320	290	

TICKET No. 9
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Geometrical solution of two-dimensional problems. The main Theorem on the LP.
2. Solve The problem or make sure its equivalent:

$$\begin{cases} 0 \le x_1 \le 1 \\ 0 \le x_2 \le 2 \\ 0 \le x_1 + x_2 \le 3 \\ -1 \le x_1 - x_2 \le 0 \end{cases}$$

$$f(x_1, x_2) = x_1 + x_2 \to \max$$

TICKET # 10
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. MaThematical model of object and its properties.

2. The company has a diet containing at least 20 units of proteins, carbohydrates, 10 units 30 units 30 units of fat and vitamins. As The cheapest way to achieve this is shown in table 1 kg price products available?

	Bread	Dried fish
Proteins	2	10
Carbohydrates	10	5
Fats	1	2
Vitamins	1	10
Price	12	32

TICKET # 11
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1-dimensional optimization. The Newton-Raphson Method.

For The production of two kinds of products a, b uses three different kinds of raw materials. Each species of raw material can be used in a quantity not greater respectively 180, 210 and 244 kg. Rates of each of The types of raw materials per unit of output of each species are given in The table.

View raw	Rates of raw materials (kg) per unit of production	
	(A)	In
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents.)	10	14

Define a roadmap that is its maximum value.

TICKET # 12
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. One-dimensional optimization. Kvazin′ûtonovskij method.

2. For The production of two types of P1, P2 is used three different types of raw materials. Each species of raw material can be used in The volume not greater, respectively, 220 and 310 244 kg. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production	
	The product P1	The product P2
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents).	5	7

You want to define a roadmap that provides The maximum output in value terms.

TICKET # 13
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Multidimensional optimization. Relief is function. The wise of The descent.
2. Solve The problem or make sure its equivalent:

$$\begin{cases} x_1 + x_2 \le 3, \\ x_1 - x_2 \le 1, \end{cases}$$

$$4x_1 + 2x_2 \to \max.$$

TICKET # 14
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Method of ravines. Random search.
2. Solve The problem or make sure its equivalent:

$$f = x_1 + 2x_2 \to \max$$
$$\begin{cases} x_1 - x_2 \le 1 \\ x_1 - 2x_2 \le 1 \end{cases}$$
$$x_1 \ge 0, x_2 \ge 0$$

TICKET # 15
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)
1. Multidimensional optimization. A gradient method. Method of steepest descent.
2. Solve The problem or make sure its equivalent:

$$f = x_1 + 2x_2 \to \max$$

$$\begin{cases} 3x_1 - 2x_2 \le 6 \\ -x_1 + 2x_2 \le 4 \\ 3x_1 + 2x_2 \le 12 \\ x_1 \ge 0,\ x_2 \ge 0. \end{cases}$$

TICKET # 16
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1) Dimensional optimization. Newton's Method.
2) Solve The problem or make sure its equivalent:

$$f = 2x_1 + x_2 \to \max$$

$$\begin{cases} -x_1 + x_2 \le 2 \\ x_1 + 2x_2 \le 7 \\ 4x_1 - 3x_2 \le 6 \\ x_1 \ge 0, x_2 \ge 0 \end{cases}$$

TICKET # 17
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. search for The optimum in problems with constraints. Method of factors.
2. solve The problem or make sure its equivalent:

$$f = 7x_1 + 5x_2 \to \min$$

$$\begin{cases} x_1 + x_2 \geq 3 \\ x_1 + 5x_2 \geq 5 \\ 2x_1 + x_2 \geq 4 \end{cases}$$

$$x_1 \geq 0,\ x_2 \geq 0.$$

TICKET # 18
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

3. The search bar Svenna contains The minimum.
4. Solve The problem or make sure its equivalent:

$$f = 7x_1 + 5x_2 \to \min$$

$$\begin{cases} x_1 + x_2 \geq 3 \\ x_1 + 5x_2 \geq 5 \\ 2x_1 + x_2 \geq 4 \end{cases}$$

$$x_1 \geq 0,\ x_2 \geq 0.$$

TICKET # 19
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Tasks with constraints. Search The optimum type constraints in problems with PARS. The method of Lagrange multipliers uncertain.
2. The company produces two products (A, B), for each of Them takes some processing time for all 4 devices I, II, III, IV.

Kind of product	Processing time				Profit $
	(I)	(II)	(III)	(IV)	
(A)	1	3	1	2	300
(B)	6	1	3	3	600

Time limit work on devices respectively, 79, 33, 43 and 50 hours. Determine what products and quantities should be made. The market for each product is unlimited. Consider The task of maximising profits.

TICKET # 20
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Search for The optimum in problems with constraints. Methods of penalty and barrier functions.
2. Solve graphical method linear programming problem:

Set The target function

$F(x) = 3x_1+2\cdot x_2$,

for which you want to find The maximum under The following restrictions:

$-2\cdot x_1-x_2\geq -4$;
$3\cdot x_1-2\cdot x_2\geq -6$;
$x_1+x_2\geq 3$;
$x_1\leq 3$; $x_2\leq 4$;
$x_1\geq 0$; $x_2\geq 0$;

TICKET # 21
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Geometrical solution of two-dimensional problems. The main Theorem on the LP.

2. Solve The problem or make sure its equivalent:

$$f = 2x_1 + x_2 \to \max$$

$$\begin{cases} -x_1 + x_2 \le 2 \\ x_1 + 2x_2 \le 7 \\ 4x_1 - 3x_2 \le 6 \\ x_1 \ge 0, x_2 \ge 0 \end{cases}$$

TICKET # 22
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. The notion of optimality criteria and objectives. The main tasks of optimization.
2. on a sewing factory for The production of two types of fabric may be used in three articles. Rates of consumption of all tissue products for tailoring a product listed in The table. It also shows The available factory total number of tissues of each product and The price of one item of this type:

Item No. fabric	The consumption rate of The tissue (m) on one product type		The total number of fabrics (m)
	1	2	
(I)	2	1	120
(II)	3	2	210
(III)	-	4	330
Price per garment (conventional cents.)	4	7	

Determine how many items of each type must be factory, to The cost of production was highest.

TICKET # 23
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Tasks with constraints. Search The optimum type constraints in problems with PARS. The method of Lagrange multipliers uncertain.

2. The company produces two products and uses three types of main equipment: turning, milling and grinding. The amount of time required to produce a unit of production for each of The types of equipment are given in The table. It is specified in The General Fund of working time each type of equipment, as well as profit from sales of one product of this type.

Type equipment	Staff time (Stanko-h) per unit of product type		General Fund operating time (Stanko-h)
	1	2	
Milling equipment	1	1	130
Turning	2	3	300
Grinding	1	-	240
Profits from The sales of one product (conventional cents.)	7	8	

Determine The volume of each of The products in which The gross profit from The sale is The maximum.

TICKET # 24
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Setting objectives. The main task and's (canonical) bringing to it an arbitrary task.

2. For The production of tables and cabinets furniture factory uses The necessary resources. Resource cost rates for a product of this type, The profits from The sales of one product and The total number of available resources of each type are listed in The table below:

Resources	Cost rates per product		The total number of resources
	table	closet	
Wood: 1 views	0.2	0.1	40
2 views	0.1	0.3	60
Labour input (person-h)	1.2	1.5	371.4
Profit from The sale of 1 products (conventional cents.)	6	8	

Determine how many tables and cabinets are factory should produce in order to profit from Their sale was possible.

TICKET # 25
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. One-dimensional optimization. The Newton-Raphson Method.

2. For The production of two types of a, b is used three different types of raw materials. Each species of raw material can be used in The volume, not more 260 respectively, 210 and 236 kg. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production	
	product and	The product in
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents).	10	14

You want to define a roadmap that provides The maximum output in value terms.

TICKET # 26
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. One-dimensional optimization. Kvazin′ûtonovskij method.

2. Cake factory for The production of two types of caramel as well, uses three types of basic raw materials: sugar, molasses, fruit puree. Rates of consumption of each type of raw material for production of 1T caramel-table. It also refers to The total number of each type of raw material, which can be used in a factory, and provides a profit from sales of this type of caramel 1T.

View raw	Raw material consumption rate (t) for 1 ton of caramel		The total number of raw materials (t)
	(A)	In	
Granulated sugar	0.8	0.5	800
Glucose syrup	0.8	0.4	600
Fruit puree	-	0.1	120
Profit from sales of 1T products (conventional cents.)	108	112	

Find The production plan, providing The caramel maximum profit from its implementation.

TICKET # 27
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Multidimensional optimization. Newton's Method.

2. For The production of 2 types of products (A), (B) The use of turning, milling, welding, and grinding equipment. Time for processing of one item for each type of equipment are listed in The table. It is specified in The General Fund of working time, as well as profit from The sale of 1 St of each kind of product:

Type of equipment	Time (Stanko-h) processing 1-go type products		General Fund operating time (h)
	(A)	(B)	
Milling equipment	2	4	120
Turning	1	8	280
Welding	7	4	240
Grinding	4	6	360
Profit	10	14	

You need to determine how many items of each type should make The company to profit from Their sale was possible.

TICKET # 28
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Multidimensional optimization. Markvardta Method.

2. Factory produces two kinds of fabrics. Subsistence resources The following factories: 700 units. production equipment, 800 units. raw materials and 870 units. The electricity consumption per unit tissue which is presented in The table below:

Resources	Fabric	
	1	2
Equipment	2	3
Raw materials	1	4
Electricity	3	4

Gains from The disposal of one metre of fabric 1 equals 8 conventional cents, fabric 2-32 conventional cents. How to make a tissue of each species, to profit from The sales was The greatest?

TICKET # 29
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Search for The optimum in problems with constraints. Methods of penalty and barrier functions.

2. To issue The two types of products require raw materials costs, working time and equipment. The source data are given in The table.

Type of resource	Resource cost rates per unit of production		Availability of resources
	1	2	
Raw materials	2	1	280
Working time	1	1	200
Equipment	1	1	250
Profit per unit of production	4	7	

Find The best roadmap for maximum profit.

TICKET # 30
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Search for The optimum in problems with constraints. Method of factors.

2. The production of two kinds of products A, B uses three different kinds of raw materials. Raw material resources are limited. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production		The total number of resource
	(A)	In	
(I)	15	30	720
(II)	6	4	192
(III)	5	3	180
Unit price (USD)	9	10	

(C)leave The production plan, in which The total value of all of The company's products is The maximum.

TICKET # 31
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. The method of search of Svenna contains The minimum.

2. To process parts and machines are used in I, II and III. The table lists The standard processing time machine related items, The unit selling price of parts (in LCY) and The time limit of The machine:

	Details	Standard time		Before. time machine
Machine tools	(A)	In		
	(I)	0.2	0.1	40
	(II)	0.6	0.4	60
	(III)	0.2	0.4	30
	Price	10	16	

Solve The problem of determining The optimal production programme, maximising The cost of production.

TICKET # 32
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Method of ravines. Random search.

2. To issue The two types of products require raw materials costs, working time and equipment. The source data are given in The table.

Type of resource	Resource cost rates per unit of production		Availability of resources
	1	2	
Raw materials	3	5	92
Working time	20	14	420
Equipment	10	14	300
Profit per unit of production	30	25	

Find The best roadmap for maximum profit.

TICKET # 33
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Multidimensional optimization. A gradient method. Method of steepest descent.
2. The company produces two products and uses three types of main equipment: turning, milling and grinding. The amount of time required to produce a unit of production for each of The types of equipment are given in The table. It is specified in The General Fund of working time each type of equipment, as well as profit from sales of one product of this type.

Type equipment	Staff time (Stanko-h) per unit of product type		General Fund operating time (Stanko-h)
	1	2	
Milling equipment	10	1	170
Turning	2	3	300
Grinding	1	2	340
Profits from The sales of one product (conventional cents.)	8	1	

Determine The volume of each of The products in which The gross profit from The sale is The maximum.

TICKET # 34
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Multidimensional optimization. Newton's Method.
2. For The production of two kinds of products of P_1 and p_2 use four types of resources S_1, S_2, S_3 and S_4. Resource stocks, The number of units of The resources spent on the production of a unit of production, are given in The table below:

Type of resource	The number of units of The resources spent on the production of a unit of production		Resource reserve
	$(P)_1$	R_2	
S_1	1	3	18
S_2	2	1	16
S_3	0	1	5
S_4	3	0	21

Profit from unit P_1 and p_2, 2 and 3 respectively.
Such a plan should be drawn up of production in which The profits from The sales of The products will be at its highest.

TICKET # 35
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Multidimensional optimization. Markvardta Method.
2. Spinning Mill for The production of 2 types of yarn uses three types of raw materials-pure wool, nylon and acrylic.

The type of raw materials	Rates of consumption of raw materials in 1 t of yarns		The number of raw materials
	Type 1	View 2	
Wool	0.5	0,1	583
Nylon	0.1	0.4	626
Acrylic	0.4	0.2	523
Profit from sales of yarn	1100	900	

You want to plan The production of yarns in order to maximize total profits.

TICKET No. 36
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Tasks with constraints. Search The optimum type constraints in problems with PARS. The method of Lagrange multipliers uncertain.

2. solve The problem or make sure its equivalent:

$$\begin{cases} x_1 - x_2 \le 1, \\ x_1 + 2x_2 \le 1, \end{cases}$$
$$2x_1 + x_2 \to \max$$

TICKET # 43
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

3. Multidimensional optimization. Relief is function. The wise of The descent.

4. There are two types of feed (I) and (II), containing nutrients (vitamins) S_1, S_2 and S_3. The contents of The number of units of nutrients in 1 kg of each type of feed and The necessary minimum nutrients are shown in The table.

Nutrient (vitamin)	Necessary minimum nutrients	The number of units of nutrients in 1 kg of food	
		(I)	(II)
S_1	9	3	1
S_2	8	1	2
S_3	12	1	6

The cost of 1 kg of food (I) and (II) respectively is equal to 4 and 6.
A diet that has minimum cost, that The content of each type of nutrients would be not less than a specified limit.

TICKET # 44
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

3. Linear programming. Setting objectives. The main task and's (canonical) bringing to it an arbitrary task.
4. For The production of two kinds of products of P_1 and p_2 use four types of resources S_1, S_2, S_3 and S_4. Resource stocks, The number of units of The resources spent on the production of a unit of production, are given in The table below:

Type of resource	The number of units of The resources spent on the production of a unit of production		Resource reserve
	$(P)_1$	R_2	
S_1	1	3	18
S_2	2	1	16
S_3	0	1	5
S_4	3	0	21

Profit from unit P_1 and p_2, 2 and 3 respectively.
Such a plan should be drawn up of production in which The profits from The sales of The products will be at its highest.

TICKET # 45
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

3. 1-dimensional optimization. General search method. Unimodal function. A method of dividing The interval in half.
4. The ration for food animals consists of two types of feeds (I) and (II). One kilo of poop I costs 80 USD. and contains: 1 PC. fat, 3 u. protein 1 g. carbohydrates, 2 PCs. nitrates. One kg of feed (II) costs 10 $. contains: 3 units. fat, 1 PC. protein 8 g. carbohydrates, 4 units. nitrates.

You want to make The most expensive diet for fat not less than 6 g, proteins not less than 9 g, carbohydrates at least 8 units, nitrates up to 16 units.

TICKET # 46
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. One-dimensional optimization. Unimodal function. Golden section search.

2. On the trading floor to expose for sale goods T1 and T2. Working time does not exceed a sellers 1100 hours and square trading Hall to take does not exceed 700 m^2. Each commodity unit powered brings profit, respectively, 50 and 80 den. u. Resource cost rates per unit of sold goods are listed in The table below:

Resources	T1	T2
Operating time, h	3	2
Area, m^2	1	2

Find The best structure of turnover (The smaller The number of units of The item, The better) for profit no less 28500 den. u.

TICKET # 47
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Basic conOption tasks to The main task of The LP with restrictions-inequalities (form a).

2. Spinning Mill for The production of 2 types of yarn uses three types of raw materials-pure wool, nylon and acrylic.

The type of raw materials	Rates of consumption of raw materials in 1 t of yarns		The number of raw materials
	Type 1	View 2	
Wool	0.5	0.2	583
Nylon	0.1	0.4	626
Acrylic	0.4	0.2	523
Profit from sales of yarn	1100	900	

You want to plan The production of yarns in order to maximize total profits.

TICKET # 48
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. Linear programming. Basic conOption tasks to The main task of The LP with restrictions-inequalities (form a).

2. Tea-packing factory produces tea grades a and b by mixing 3 ingredient: Indian, Georgian and Krasnodar teas. You want to plan The production of tea, can profit.

Ingredients	Usage rate (t/t)		Stocks (tonnes)
Indian tea	0.5	0.2	600
Georgian tea	0.2	0.6	870
Krasnodar tea	0.3	0.2	430
Profit from The sale of 1 t products	320	290	

TICKET # 49
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

5. Linear programming. Geometrical solution of two-dimensional problems. The main Theorem on the LP.
6. Solve The problem or make sure its equivalent:

$$f = x_1 + x_2 \to \min$$

$$\begin{cases} 0 \le x_1 \le 1 \\ 0 \le x_2 \le 2 \\ 0 \le x_1 + x_2 \le 3 \\ -1 \le x_1 - x_2 \le 0 \end{cases}$$

TICKET # 50
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. MaThematical model of object and its properties.

2. The company has a diet containing at least 20 units of proteins, carbohydrates, 10 units 30 units 30 units of fat and vitamins. As The cheapest way to achieve this is shown in table 1 kg price products available?

	Bread	Dried fish
Proteins	2	10
Carbohydrates	10	5
Fats	1	2
Vitamins	1	10
Price	12	32

TICKET No. 51
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1-dimensional optimization. The Newton-Raphson Method.

For The production of two kinds of products a, b uses three different kinds of raw materials. Each species of raw material can be used in a quantity not greater respectively 180, 210 and 244 kg. Rates of each of The types of raw materials per unit of output of each species are given in The table.

View raw	Rates of raw materials (kg) per unit of production	
	(A)	In
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents.)	10	14

Define a roadmap that is its maximum value.

TICKET # 52
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

1. One-dimensional optimization. Kvazin′ûtonovskij method.

2. For The production of two types of P1, P2 is used three different types of raw materials. Each species of raw material can be used in The volume not greater, respectively, 220 and 310 244 kg. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production	
	The product P1	The product P2
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents).	5	7

You want to define a roadmap that provides The maximum output in value terms.

TICKET # 53
on the discipline
"Methods of optimization»
(INSTITUTE of CYBERNETICS, 1-St course)

2. The multidimensional optimisation. Relief is function. The wise of The descent.
2. Solve The problem or make sure its equivalent:

$$\begin{cases} x_1 + x_2 \le 3, \\ x_1 - x_2 \le 1, \end{cases}$$

$$4x_1 + 2x_2 \to \max.$$

TICKET No. 54
on the discipline
"Methods of optimization»
(Institute of Cybernetics, 1-St course)

3. Method of ravines. Random search.
4. Solve The problem or make sure its equivalent:

$$f = x_1 + 2x_2 \to \max$$
$$\begin{cases} x_1 - x_2 \le 1 \\ x_1 - 2x_2 \le 1 \end{cases}$$
$$x_1 \ge 0,\ x_2 \ge 0$$

4. The list of jobs to The experiments

Individual tasks to The experiments (options)

No. 1.

Calculate and print a table of three functions, ***y***, ***z***, ***w*** .

The argument ***x*** is changed from x_0 to $x_{(k)}$ with step ***h***. Function of ***y*** is said to converge near The amount you want to take it until its next member becomes less than The specified modulo small positive ***(e)*** .

Consider The task in several different ***(e)*** .

The table presented in The following form:

x	***y***	***z***	***w***
–	–	–	–
–	–	–	–
…	…	…	…
–	–	–	–

Align The data in a table by using The features of cout.width (), cout.precicion().

1.

$$y = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...;$$

$$z = \operatorname{arctg} x;$$

$$w = y - z;$$

$$x_0 = -0.5, \quad x_k = 0.5, \quad h = 0.1.$$

2.

$$y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...;$$

$$z = e^{-x};$$

$$w = y - z;$$

$$x_0 = -1, \quad x_k = 1, \quad h = 0.2.$$

3.

$$y = 2x - \frac{2^3 x^3}{3!} + \frac{2^5 x^5}{5!} - \frac{2^7 x^7}{7!} + ...;$$

$$z = \sin 2x;$$

$$w = y - z;$$

$$x_0 = -\frac{\pi}{4}, \quad x_k = \frac{\pi}{4}, \quad h = \frac{\pi}{20}.$$

4.

$$y = 1 - \frac{2^2 x^2}{2!} + \frac{2^4 x^4}{4!} - \frac{2^6 x^6}{6!} + \ldots;$$
$$z = \cos 2x;$$
$$w = y - z;$$
$$x_0 = 0, \quad x_k = \frac{\pi}{2}, \quad h = 0.05\pi.$$

5.

$$y = 2x + \frac{2^3 x^3}{3!} + \frac{2^5 x^5}{5!} + \frac{2^7 x^7}{7!} + \ldots;$$
$$z = \operatorname{sh} 2x;$$
$$w = y - z;$$
$$x_0 = -0.5, \quad x_k = 0.5, \quad h = 0.1.$$

6.

$$y = 1 + \frac{2^2 x^2}{2!} + \frac{2^4 x^4}{4!} + \frac{2^6 x^6}{6!} + \ldots;$$
$$z = \operatorname{ch} 2x;$$
$$w = y - z;$$
$$x_0 = -0.5, \quad x_k = 0.5, \quad h = 0.1.$$

7.

$$y = -\left(2x + \frac{2^2 x^2}{2} + \frac{2^3 x^3}{3} + \frac{2^4 x^4}{4} + \ldots\right);$$
$$z = \ln(1 - 2x);$$
$$w = y - z;$$
$$x_0 = -0.25, \quad x_k = 0.25, \quad h = 0.05.$$

8.

$$y = 2x - \frac{2^2 x^2}{2} + \frac{2^3 x^3}{3} - \frac{2^4 x^4}{4} + \ldots;$$
$$z = \ln(1 + 2x);$$
$$w = y - z;$$
$$x_0 = -0.25, \quad x_k = 0.25, \quad h = 0.05.$$

9.

$$y = x^2 + \frac{x^4}{2} + \frac{x^6}{3} + \frac{x^8}{4} + ...;$$

$$z = \ln\frac{1}{1-x^2};$$

$$w = y - z;$$

$$x_0 = -0.5, \quad x_k = 0.5, \quad h = 0.1.$$

10.

$$y = 2x - \frac{2^3 x^3}{3} + \frac{2^5 x^5}{5} + \frac{2^7 x^7}{7} + ...;$$

$$z = \operatorname{arctg} 2x;$$

$$w = y - z;$$

$$x_0 = -0.25, \quad x_k = 0.25, \quad h = 0.05.$$

Laboratory work № 2, 3 (The objective function the same)
Find The minimum point x* of The function f(x) on the interval [a,b] with The accuracy e=10^{-5} and The minimum value f(x*).

Apply The techniques
Lab # 2:

1. General search;
2. Divide in half;
3. The golden section;

Count The number of iterations and The number of calculations functions .
Compare The results.

Lab # 3:
Find The minimum point x* of The function f(x) on the interval [a,b] with The accuracy e=10^{-5} and The minimum value f(x*). (select The starting value in The interval [a,b]) in The following ways:

1. Newton-Raphson;
2. Kvazin′ûton (with an approximation to The derivative).

Count The number of iterations and The number of calculations functions $f(x)$, $f'(x)$, $f''(x)$.
Compare The results.

1. $f(x) = x^2 - 2x + e^{-x}$, $[1;1,5]$
2. $f(x) = tg\ x - 2\sin x$, $[0;\pi/4]$
3. $f(x) = \sqrt{1+x^2} + e^{-2x}$, $[0;1]$
4. $f(x) = x^4 + 4x^2 - 32x + 1$, $[1,5;2]$
5. $f(x) = \frac{1}{7}x^7 - x^3 + \frac{1}{2}x^2 - x$, $[1;1,5]$
6. $f(x) = x^3 - 3\sin x$, $[0,5;1]$
7. $f(x) = 5x^2 - 8x^{5/4} - 20x$, $[3;3,5]$
8. $f(x) = \frac{1}{3}x^3 - 5x + x\ln x$, $[1,5;2]$
9. $f(x) = x\sin x + 2\cos x$, $[-5;-4]$
10. $f(x) = x^4 + 8x^3 - 6x^2 - 72x + 90$, $[1,5;2]$
11. $f(x) = x^6 + 3x^2 + 6x - 1$, $[-1;0]$
12. $f(x) = 10x\ln x - \frac{x^2}{2}$, $[0,5;1]$
13. $f(x) = x^2 + 2\left(x\lg\frac{x}{e} - 2\right)$, $[1,5;2]$
14. $f(x) = \frac{2x}{\ln 2} - 2x^2$, $[3,5;4,5]$
15. $f(x) = e^x - \frac{1}{3}x^3 + 2x$, $[-1,5;-1]$
16. $f(x) = x^4 + 2x^2 + 4x + 1$, $[-1;0]$
17. $f(x) = x^5 - 5x^3 + 10x^2 - 5x$, $[-3;-2]$
18. $f(x) = x^2 + 3x(\ln x - 1)$, $[0,5;1]$
19. $f(x) = x^2 - 2x - 2\cos x$, $[0,5;1]$
20. $f(x) = (x+1)^4 - 2x^2$, $[-3;-2]$
21. $f(x) = 3(5-x)^{4/3} + 2x^2$, $[1,5;2]$
22. $f(x) = -x^3 + 3(1+x)[\ln(1+x) - 1]$, $[-0,5;0,5]$
23. $f(x) = 2 + x^2 + x^{2/3} - \ln(1+x^{2/3}) - 2x\,arctg\,x^{1/3}$, $[0,5;1]$
24. $f(x) = x - \ln x$, $[0,1;2]$
25. $f(x) = x^2 - \sin x$, $[0;\pi/2]$
26. $f(x) = x^4 + x^2 + x + 1$, $[-1;2]$

27. $f(x) = \sqrt{1 + x^2} + e^{-2x}, \ [0;1]$

28. $f(x) = e^x + \frac{1}{x}, \ [0,1;2]$

29. $f(x) = (x-4)^2 + \ln x, \ [3;5]$

30. $f(x) = x^4 + e^{-x}, \ [0;1]$

Lab No. 4

Multidimensional absolute optimization

Find The minimum of The function f(x) with precision $e=10^{-5}$.
A) method of gradient descent.
B) MarkvardtMethod.

Compare The methods, for which a number of iterations of The computation of The function and its derivative.

№ вар.	Функция
1.	$f(x) = x_1^2 + 2x_2^2 + e^{x_1^2+x_2^2} - x_1 + 2x_2.$
2.	$f(x) = \sqrt{x_1^2 + x_2^2 + 1} + 0.5x_1 - 0.5x_2.$
3.	$f(x) = x_1^4 + 2x_2^4 + x_1^2x_2^2 + 2x_1 + x_2.$
4.	$f(x) = x_1^2 + 3x_2^2 + \cos(x_1 + x_2).$
5.	$f(x) = \sqrt{1 + 2x_1^2 + x_2^2} + e^{x_1^2+2x_2^2} - x_1 - x_2.$
6.	$f(x) = x_1 + 5x_2 + e^{x_1^2+x_2^2}.$
7.	$f(x) = x_1^4 + x_2^4 + \sqrt{2 + x_1^2 + x_2^2} - 2x_1 + 3x_2.$
8.	$f(x) = 2x_1^2 + 3x_2^2 - \sin(\frac{x_1 - x_2}{2}) + x_2.$
9.	$f(x) = \ln(1 + 3x_1^2 + 5x_2^2 + \cos(x_1 - x_2)).$
10.	$f(x) = x_1^2 + e^{x_1^2+x_2^2} + 4x_1 + 3x_2.$
11.	$f(x) = x_1 + 2x_2 + 4\sqrt{x_1^2 + x_2^2 + 1}.$
12.	$f(x) = 2x_1 - 5x_2 + e^{x_1^2+0.5x_2^2}.$
13.	$f(x) = 2\sqrt{3 + x_1^2 + 2x_2^2 + 3x_3^2} - x_1 - x_3.$
14.	$f(x) = x_1^2 + 2x_2^2 + x_1^2x_2^2 + 2x_3 - x_2 + e^{x_2^2+x_3^2}.$
15.	$f(x) = 4\sqrt{1 + x_1^2 + x_2^2 + 3x_3^2} + x_1 - 2x_2.$
16.	$f(x) = 2x_1^4 + x_2^4 + x_1^2x_2^2 + x_3^4 + x_1^2x_3^2 + x_1 + x_2.$
17.	$f(x) = x_1^2 + 5x_2^2 + 2x_3^2 + \cos(x_1 - x_2 + x_3).$

18.	$f(x) = e^{x_1^2 + x_2^2} + \ln(4 + x_2^2 + 2x_3^2).$
19.	$f(x) = x_1 + x_2 - 5x_3 + e^{x_1^2 + 2x_2^2 + x_3^2}.$
20.	$f(x) = x_1^4 + x_2^4 + x_1^2 x_2^2 + \sqrt{5 + x_2^2 + 2x_3^2} + x_1 + x_3.$
21.	$f(x) = 2x_1^2 + x_2^2 + 4x_3^2 - 2\sin\frac{x_1 + x_2 - x_3}{2}.$
22.	$f(x) = 2\sqrt{x_1^2 + 3x_2^2 + 3} + x_2^2 x_3^2 - x_1 - x_2.$
23.	$f(x) = x_1 - x_2 + x_2^2 + x_3^2 + e^{x_1^2 + x_2^2 + x_3^2}.$
24.	$f(x) = x_1 + x_2 + x_3 + 3\sqrt{x_1^2 + x_3^2 + 1} + e^{x_1^2 + x_2^2}.$
25.	$f(x) = \sqrt{x_2^2 + x_3^2 + 3} + x_1^2 + x_2^2 + \sin(x_1 + x_2).$
26.	$f(x) = x_1 + 10x_2 - 3x_3 + e^{x_1^2 + x_2^2 + x_3^2}.$
27.	$f(x) = e^{x_1^2} + (x_1 + x_2 + x_3)^2.$
28.	$f(x) = 3x_1^2 + x_2^2 + 2x_3^2 - 2\cos\frac{x_1 - x_2 + x_3}{2}.$
29.	$f(x) = 2x_1^2 + 3x_2^2 - \sin(x_1 + 2x_2).$
30.	$f(x) = 3x_1^2 + x_2^2 + e^{x_1^2 + x_3^2} - x_2 + 3x_3.$

Lab # 5

Multivariate conditional optimization

Find The minimum of The function $f(x)$ with precision $\varepsilon = 10^{-4}$:
A) Method of penalty functions or methods of The barrier functions.
B) factors method.

Compare The methods, for which a number of iterations, The number of calculation functions, etc.

№	Function
1.	$f(x) = 10x_1 - x_2,$ $x_1^2 - 4x_1 + x_2^2 + 3 \le 0,$ $x_1^2 + 4x_2^2 - 4x_2 - 4 \le 0.$
2.	$f(x) = x_1^2 - 3x_1 + x_2^2,$ $x_1^2 - 2x_2 \le 0,$ $-x_1 + x_2 \le 0.$
3.	$f(x) = x_1^2 + 9x_2^2 - 12x_1 - 36x_2,$ $-1 \le x_1 \le 4, \;\; 1 \le x_2 \le 2.$
4.	$f(x) = 2x_1 + x_2,$ $(x_1 - 4)^2 + (x_2 - 2)^2 \le 1.$
5.	$f(x) = 2\sqrt{1 + x_1^2 + 2x_2^2} + x_1 + x_2,$ $5 \le x_1 \le 8, \;\; 1 \le x_2 \le 10.$
6.	$f(x) = x_1^2 - 8x_1 + x_2^2,$ $x_1 + (x_2 - 4)^2 \le 9.$
7.	$f(x) = x_1^2 + x_2^2 + x_3^2 + x_1 + x_2 + x_3,$ $x_1 + x_2 - x_3 = 3.$
8.	$f(x) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 6x_2 - 2x_3,$ $2x_1 + x_3 = 2.$
9.	$f(x) = (x_1 - 2)^4 + (x_2 - 1)^4,$ $2x_1 + x_2 \le 2.$
10.	$f(x) = x_1^2 - x_2,$ $2x_1 - 2x_2 \le 1.$
11.	$f(x) = x_1^2 + x_2^2 - 20x_1 - 30x_2,$ $2x_1 + 3x_2 \le 13,$ $2x_1 + x_2 \le 10.$

12.	$f(x)=x_1^2+x_2^2-10x_1-15x_2,$ $5x_1+13x_2\le 51,$ $15x_1+7x_2\le 107.$
13.	$f(x)=x_1^2+x_2^2-5x_1-4x_2,$ $2x_1+3x_2\le 6.$
14.	$f(x)=x_1^2+x_2^2-5x_1-10x_2,$ $9x_1+8x_2\le 72,$ $x_1+2x_2\le 10.$
15.	$f(x)=x_1^2-2x_1-2x_2,$ $2x_1+3x_2\le 6,$ $2x_1+x_2\le 4.$
16.	$f(x)=x_2^2+2x_1-2x_2+x_3,$ $x_1+3x_2+2x_3\le 6,$ $3x_1+x_2+x_3\le 2.$
17.	$f(x)=x_2^2-x_1-2x_2,$ $2x_1^2+2x_2^2\le 6.$
18.	$f(x)=x_1^2+x_2^2-6x_1-3x_2,$ $x_1^2+x_2^2\le 9.$
19.	$f(x)=x_1^2+x_2^2-3x_2,$ $-2x_1+x_2^2\le 0,$ $x_1-2x_2\le 0.$
20.	$f(x)=x_1^2-6x_1+x_2^2,$ $x_1^2+(x_2+4)^2\le 9.$
21.	$f(x)=2x_1+x_2,$ $(x_1-4)^2+(x_2-2)^2\le 1.$
22.	$f(x)=x_2^2-x_1-2x_2,$ $2x_1^2+2x_2^2\le 6.$
23.	$f(x)=2x_1^2+3x_2^2-6x_1+8x_2,$ $5x_1+12x_2\le 50,$ $15x_1+9x_2\le 100.$
24.	$f(x)=5x_1-2x_2,$ $(x_1-3)^2+(2x_2-7)^2\le 2.$

Lab # 6
Linear programming
Job options

Option 1

For The production of tables and cabinets furniture factory uses The necessary resources. Resource cost rates for a product of this type, revenue from sales and The total number of available resources of each type are described in The following table:

Resources	Resource cost rates per product		The total number of resources
	table	closet	
I kind of wood (cubic m)	0.2	0.1	40
Wood (I)(I) The form (cubic meters)	0.1	0.3	60
Effort (person-hours)	1.2	1.5	371.4
Profits from The sales of one product (thousand rubl.)	600	800	

Determine how many tables and cabinets are factory it is necessary to make a profit from The sale was The maximum.

Option 2

For The release of The two types of products require raw materials costs, working time and equipment. The source data are given in The table.

Type of resource	Resource cost rates per unit of production		Availability of resources
	1	2	
Raw materials	3	5	60
Working time	22	14	400
Equipment	10	14	128
Profit per unit of production	30	25	

Find The best roadmap for maximum profit.

Option 3

For The production of two kinds of products (A), (B) The use of turning, milling and welding and grinding equipment. Time for processing of one item for each type of equipment are listed in The table. It is specified in The General Fund of working time each type of equipment used, as well as profit from sales of one product of this type.

Type of equipment	Time (machine-hours) on the processing of a single product type		General Fund of working time of equipment (h)
	(A)	(B)	
Milling equipment	2	4	120
Turning	1	8	280
Welding	7	4	240
Grinding	4	6	360
Profits from The sales (LCY)	100	140	

Determine how many items of each type should make The company to profit from Their sale was possible.

Option 4

For The release of The two types of products require raw materials costs, working time and equipment. The source data are given in The table.

Type of resource	Resource cost rates per unit of production		Availability of resources
	1	2	
Raw materials	4	3	200
Working time	1	2	80
Equipment	2	2	130
Profit per unit of production	60	80	

Find The best roadmap for maximum profit.

Option 5

For machining and machine tools used in The I, II and III. The table lists The standard processing time machine related items, The unit selling price of parts (in LCY), The value of The 1 h of machine and machine operating time limit:

Details	Standard time		The cost of	Time machine
Machine tools	(A)	In		
(I)	0.2	0.1	30	40
(II)	0.6	0.3	10	60
(III)	0.2	0.1	20	30
Price	10	16		

Solve The problem of determining The optimal production programme, maximising The cost of production.

Option 6

The confectionery factory for The production of two types of caramel (A), (B) uses three types of basic raw materials: sugar, molasses and fruit puree. Rates of consumption of each type of raw material for production of 1T caramel are given in The table. It also refers to The total number of each type of raw material, which can be used in a factory, and provides a profit from sales of this type of caramel 1T.

View raw	Raw material consumption rate (t) for 1 ton of caramel		The total number of raw material (t)
	(A)	(B)	
Sugar	0.8	0.5	800
Glucose syrup	0.4	0.4	600
Fruit puree	-	0.1	120
Profit from sales of product 1 k (USD)	108	112	

Find The production plan, providing The caramel maximum profit from its implementation.

Option 7

The garment factory for The production of two types of fabric may be used in three articles. Rates of consumption of fabrics for tailoring a product listed in The table. It also shows The total number of plant tissues of each product and The price of one item of this type.

Marking of fabric	The consumption rate of The tissue (m) on one product type		The total number of fabrics (m)
	1	2	
(I)	1	-	180
(II)	-	1	210
(III)	4	2	800
Price per unit (USD)	9	6	

Determine how many items of each type must be factory, to The cost of production was highest.

Option 8

The company produces two products and uses three types of main equipment: turning, milling and grinding. The amount of time required to produce a unit of production for each of The types of equipment are given in The table. It also shows The common fund of working time each type of equipment, as well as profit from sales of one product of this type. Determine The volume of each of The products in which The gross profit from The sale is The maximum.

Type of equipment	Time (machine-hours) per unit of product type		The share of working hours (Stanko-hours)
	1	2	
Turning	1	3	300
Milling equipment	2	1	70
Grinding	1	-	340
Profits from The sales (LCY)	2	1	

Option 9

The Distributor plans to organize The sale of The two kinds of goods (A), (B) using only 2 types of resources: working time sellers of 840 h area sales area of 180 sq.m. The planned regulations are cost of These resources per unit of product (A), (B) and profit from Their sale, which are listed in The table below:

You want to determine The optimal structure of trade, which provides commercial enterprise to maximum profits. Standard costs are listed in The table below:

Indicators	Product		The total number of resources
	(A)	(B)	
Time consumption per unit (h)	0.6	0.8	840
Use square trading Hall on the unit (sq. m)	0.1	0.2	180

Profit from sale of unit of The commodity (USD)	5	8	

Option 10

The production of two kinds of products A, B uses three different kinds of raw materials. Raw material resources are limited. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production		The total number of resource
	(A)	In	
(I)	4	2	180
(II)	3	1	210
(III)	1	2	244
Unit price (USD)	10	14	

(C)leave The production plan, in which The total value of all of The company's products is The maximum.

Option 11

The company produces two kinds of products a, b, for each of which requires a certain processing time on all four devices I, II, III, IV.

Kind of product	Processing time, h				Profit $
	(I)	(II)	(III)	(IV)	
(A)	1	3	1	2	3
In	6	1	3	3	6

The maximum working time on devices, respectively, 84, 42, 21, 42 hours. Determine what products and quantities should be made.

Option 12

The production of two kinds of products A, B uses three different kinds of raw materials. Raw material resources are limited. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production		The total number of resource
	(A)	In	
(I)	18	15	360
(II)	6	4	192
(III)	5	3	180
Unit price (USD)	9	10	

(C)leave The production plan, in which The total value of all of The company's products is The maximum.

Option 13

For The release of The two types of products require raw materials costs, working time and equipment. The source data are given in The table.

Type of resource	Resource cost rates per unit of production		Availability of resources
	1	2	
Raw materials	2	1	280
Working time	1	1	80
Equipment	1	-	250
Profit per unit of production	4	7	

Find The best roadmap for maximum profit.

Option 14

The production of three kinds of products A, B uses three different kinds of raw materials. Raw material resources are limited. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production		The total number of resource
	(A)	In	
(I)	1	3	250
(II)	2	1	100
(III)	1	2	150
Unit price (USD)	40	50	

(C)leave The production plan, in which The total value of all of The company's products is The maximum.

Option 15

At The shoe factory can produce two types of shoes: men's and women's. For each pair of masculine and feminine respectively require glue 22 and 20 g, skin 4 and 2 DM^2. The cost of men's and women's shoes all work respectively equal to 200 and 300 rub. Stocks of glue is 3T, and skin-4000 m^2. Find a plan for production of footwear, in which The cost of production will be maximal.

Option 16

Central heating elements manufacturer manufactures two models of radiators and in production resulting from The restrictions on the amount of labour force and The number of steel sheets, of which produced radiators.

Model radiator	(A)	(B)
The required amount of labor, person-hours	0.5	1.5
The required amount of steel sheet, m^2	4	2
Profit from selling a single radiator, $	5	5

Solve this problem with maximizing profits as The target function.

Option 17

Parts (A), (B) can be processed in three machines (I, II, III). The table lists The standard processing time machine related items, The cost of one hour of machine and machine working time limit.

Machine tools	Standard processing time		The cost of 1:0 work	Time machine
	(A)	(B)		
(I)	0.3	0.1	30	50
(II)	0.5	0.2	20	60
(III)	0.4	0.5	15	40

Determine The optimal production programme, which ensures The maximum total issue of details.

Option 18

The factory produces two types of tissues. Subsistence resources The following factories: 700 units. production equipment, 800 units. raw material and 600 units. The electricity consumption per unit tissue which is presented in The table below:

Resources	Fabric	
	1	2
Equipment	2	3
Raw materials	1	4
Electricity	3	4

Gains from The disposal of one metre of fabric 1 is equal to 8 min. den. UN., fabric 2.7 conventional cents. and tissue 3.6 min. den. u. .. How to make a tissue of each species, to profit from The sales was The greatest?

Option 19

For 2 types of products (A), (B) The use of turning, milling, welding, and grinding equipment. Time for processing of one item for each type of equipment are listed in The table. It is specified in The General Fund of working time, as well as profit from The sale of 1 St of each kind of product:

Type of equipment	Time (Stanko-h) processing 1-go type products		General Fund operating time (h)
	(A)	(B)	
Milling equipment	2	4	120
Turning	1	8	280
Welding	7	4	240
Grinding	4	6	360
Profit	10	14	

You need to determine how many items of each type should make The company to profit from Their sale was possible.

Option 20

The production of city milk plant is milk and yogurt, packaged in packages. On the production of milk and yogurt to 1 k, respectively, 1010 and 1050 kg of milk. The hours spent on bottling milk and kefir comprise 1T 0.18 and 0.19 machine-hours. Just for The production of dairy products plant can use 136000 kg of milk. Basic equipment can be occupied during car-21.4 hours. Profit from sales of milk and kefir 1T respectively equal to 30, 32 min. den. u. The plant has a daily production of at least 100 tons of milk, packaged in packages. On the production of yogurt, There are no restrictions.

You need to determine what products and quantities should be daily manufacturing plant to profit from its realization was a maximum.

Option 21

The confectionery factory for The production of two types of caramel as well, uses three types of basic raw materials: sugar, molasses, fruit puree. Rates of consumption of each type of raw material for production of 1T caramel-table. It also refers to The total number of each type of raw material, which can be used in a factory, and provides a profit from sales of this type of caramel 1T.

View raw	Raw material consumption rate (t) for 1 ton of caramel		The total number of raw materials (t)
	(A)	In	
Granulated sugar	0.8	0.5	800
Glucose syrup	0.8	0.4	600
Fruit puree	-	0.1	120
Income from implementation of 1T products (conventional cents.)	108	112	

Find The production plan, providing The caramel maximum profit from its implementation.

Option 22

For The production of tables and cabinets furniture factory uses The necessary resources. Resource cost rates for a product of this type, The profits from The sales of one product and The total number of available resources of each type are listed in The table below:

Resources	Cost rates per product		The total number of resources
	table	closet	
Wood: 1 views	0.2	0.1	40
2 views	0.1	0.3	60
Labour input (person-h)	1.2	1.5	371.4
Profit from The sale of 1 products (conventional cents.)	6	8	

Determine how many tables and cabinets are factory should produce in order to profit from Their sale was possible.

Option 23

The garment factory for The production of two types of fabric may be used in three articles. Rates of consumption of all tissue products for tailoring a product listed in The table. It also shows The available factory total number of tissues of each product and The price of one item of this type:

Item No. fabric	The consumption rate of The tissue (m) on one product type		The total number of fabrics (m)
	1	2	
(I)	2	1	180
(II)	3	2	210
(III)	-	4	800
Price per garment (conventional cents.)	4	7	

Determine how many items of each type must be factory, to The cost of production was highest.

Option 24

The company produces two products and uses three types of main equipment: turning, milling and grinding. The amount of time required to produce a unit of production for each of The types of equipment are given in The table. It is specified in The General Fund of working time each type of equipment, as well as profit from sales of one product of this type.

Type equipment	Staff time (Stanko-h) per unit of product type		General Fund operating time (Stanko-h)
	1	2	
Milling equipment	1	1	70
Turning	2	3	300
Grinding	1	-	340
Profits from The sales of one product (conventional cents.)	8	1	

Determine The volume of each of The products in which The gross profit from The sale is The maximum.

Option 25

To transport cargo on the two lines can be used three types of court. Performance of ships using Them on different lines is characterized by The data in The table. It also shows The total time for which each type of Court are in operation, and The minimum volumes of traffic on each of The lines.

Type of vessel	Performance of ships (million ton-miles a day) on line		The total operating time ships (d.)
	1	2	
(I)	8	11	300
(II)	6	13	300
(III)	12	4	300
The specified volume (million ton-miles)	3000	3300	-

Determine what court, for some lines and for how long should be used to ensure maximum loading of vessels with a view to a possible time of Their operation.

Option 26

In The weaving factory to manufacture two products, two types of machines are used fabrics, yarn and dyes. The table indicates The performance of machines for each type, The yarn and dyes flow rates, The price of 1 m of fabric of this article, The common fund of working time machine of each type, as well as The available funds of yarn and dye factories and limits on the possible release of The tissues of this article:

Resources	Cost rates on 1 m of cloth items		Total Qty. resources
	1	2	
Performance tools (Stanko-h): Type I	0.02	0.04	200
Type II	0.04	0.01	500
Yarn (kg)	1.0	2.0	15000
Dyes (kg)	0.03	0.025	450
Price for 1 m of fabric (conventional cents.)	5	8	—
Release fabric (m): minimum	1000	2500	—
maximum	2000	4000	—

Produce a plan for manufacturing fabrics that will be possible tissue of this product, and The total value of all tissue is maximum.

Option 27

For The production of two types of products The company uses two types of technological equipment and two kinds of raw materials. Cost of raw materials and time standards for The production of one item of each species are given in The table. It also shows The common fund of working time each group of technological equipment, The volume of each type of raw material, as well as The price of a product of this type and limits on the possible release of each of The products.

Resources	Cost rates per product type		Total count-in resources
	1	2	
Equipment performance (standard h.): 1tipa	2	4	200
2tipa	4	1	500
Raw materials (kg): 1 species	10	20	1495
2 types	30	25	4500
The price of one publishing house. (rub.)	10	20	—
Production (units): is minimal.	10	25	—
maximum	20	100	

Draw up a plan of production, which will be made The required number of items of each type, and The total value of all manufactured products There.

Option 28

In The production of 2 types of cables are 5 groups of manufacturing operations. Cost rates on a 1 km of cable of this type for each of The groups operations, profits from The sale of 1 km each cable, as well as The common fund of working time during which These operations can be performed. Data are presented in The table:

Technological operations	Standard time (h) The processing of 1 km of cable types		General Fund of The slave. time (h)
	1	2	
Drawing	1.2	1.8	7200
Application isolation	1.0	0.4	5600
Torsion elements in cable	6.4	5.6	11176
Osvincovanie	3.0	–	3600
Trial completed. and control	2.1	1.5	4200
Profit from The sale of 1 km of The cable (in conventional cents.)	1.2	0.8	

Define The roadmap of The cable, in which The gross profit from The sale of products is The maximum.

Option 29

For The production of two kinds of products a, b uses three different kinds of raw materials. Each species of raw material can be used in a quantity not greater respectively 180, 210 and 244 kg. Rates of each of The types of raw materials per unit of output of each species are given in The table.

View raw	Rates of raw materials (kg) per unit of production	
	(A)	In
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents.)	10	14

Define a roadmap that provides a maximum rate, and rate each of The raw materials used to manufacture The products. Evaluation of attributed to raw materials shall be such that The evaluation of all raw materials used was minimal, and The score of The raw material used in The production of a unit of production of this type would have been not less than its price.

Option 30

For The production of two types of a, b is used three different types of raw materials. Each species of raw material can be used in The volume, respectively, not more 180, 210 and 236 kg. Rates of each of The types of raw materials per unit of output of this type and The unit price of each species are given in The table below:

View raw	Rates of raw materials (kg) per unit of production	
	product and	The product in
(I)	4	2
(II)	3	1
(III)	1	2
Unit price (conventional cents).	10	14

You want to define a roadmap that provides The maximum output in value terms.

5. The list of topics The delayed control

FÈPO, Olympics, competitions of students ' works:

- Participation in online competitions;
- Participation in The contest of students ' scientific works;
- Participation in student's conferences.

6. Teaching equipment

Materials that define The procedure of control measures, and recommendations for The preparation of criteria, evaluation criteria, etc.:

1. Guidelines for The formation of foundations assessment tools/Tomsk, TPU, 2012. URL: http://www.enin.tpu.ru/attachments/article/692/fos.pdf
2. Recommendations for The design and use of assessment tools in The basic curriculum of higher vocational education new generation/Moscow, MOSCOW, 2014. URL: http://www.rsuh.ru/upload/main/mu/binary/recommendation% 20po% 20proektirovaniû% 20 deg. c (2) .doc
3. Modern technologies of teaching in higher vocational education/Belomestnova E.n., Dreval′ A.N., G.f. et al. Ivanov, Moscow, IZD-vo TPU, 2011. URL: http://portal.tpu.ru:7777/departments/otdel/publish/catalog/2011/departments/idno/metod/grif/idno_belomestnova_sovrem_techn_obucheniya.pdf
4. Control of educational achievements on the basis of test materials/Mikhailova N.S., Muratova E.A., Minin, M.G., Tomsk, publishing house of TPU, 2012. URL: http://portal.tpu.ru:7777/departments/otdel/publish/catalog/2012/iip/metod_2012/avtor/IP_IDNO_MIHAILOV_I_DR_MAKET.pdf

7. Examples of awarded works

Examples of awarded (The "excellent", the "good", the "satisfactory") works for each assessment method used:

- Forms of answers to questions for the exam (if it is in writing);
- Reports on laboratory work.
  (Examples of awarded works for each of the method of assessment are in The LMCS at IPS).