Electrical Conduction
Electrical Properties of Solids: Ohm's Law

Ohm's law

$$V = IR$$

I - current (C/s) or (A) (time rate of charge)

V- applied voltage (V)

R- resistance (Ω) or (V/A)

current density

$$J = \sigma E$$

J - current density (A/m^2)

$J = I / A$

resistivity

$$\rho = \frac{RA}{l}$$

ρ - resistivity ($\Omega \cdot$m)

l - distance (m)

A- cross-section area (m^2)

electric field intensity

$$E = \frac{V}{l}$$

E - electric field intensity (V/m)

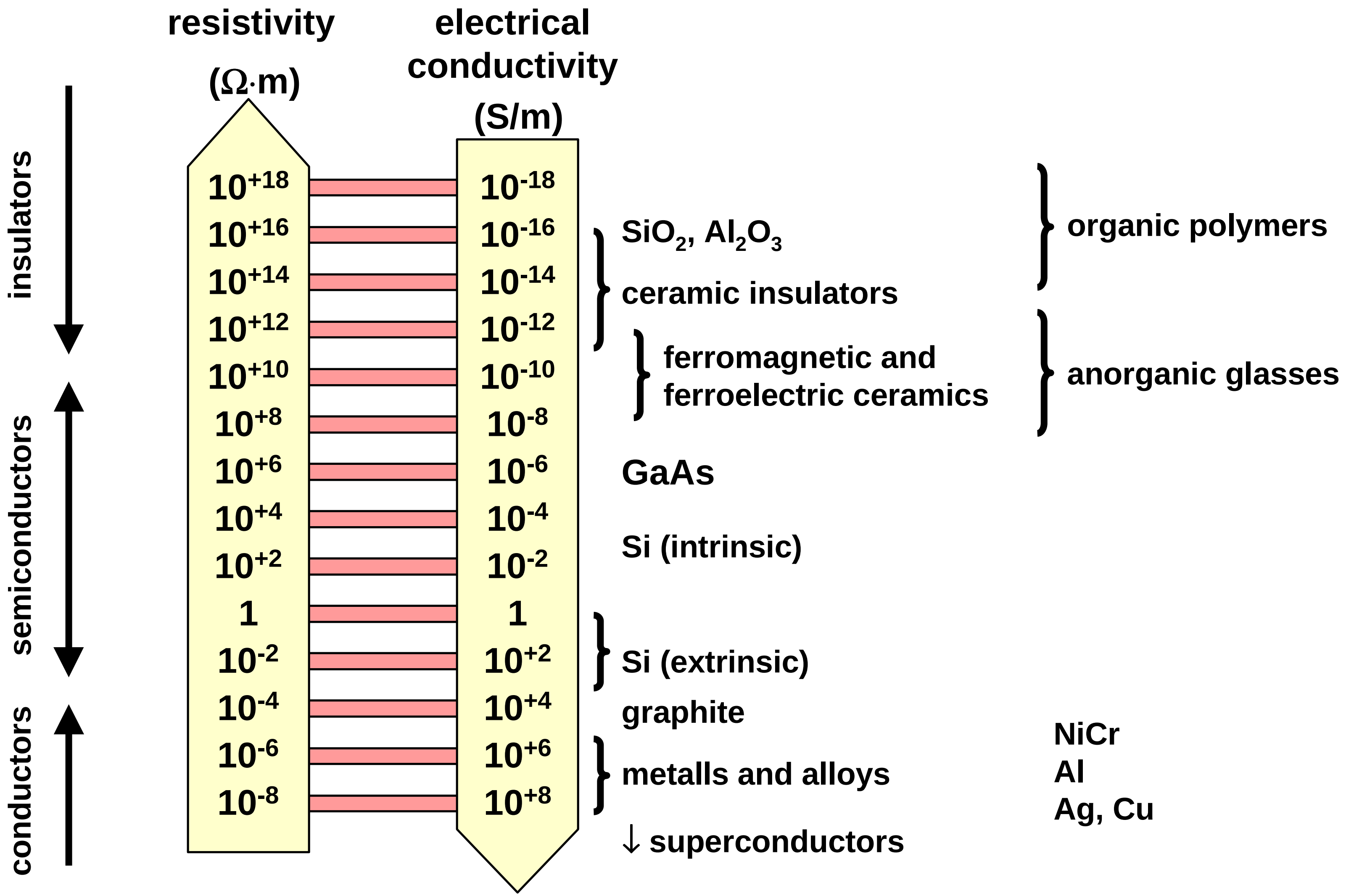
electrical conductivity

$$\sigma = \frac{1}{\rho}$$

σ - conductivity ($\Omega \cdot$m)$^{-1}$ or (S/m)

source: [Cal 99 / 606]

Electrical Conduction
Electrical Properties of Solids: Resistivity and Conductivity



resistivity ($\Omega\cdot$m)	electrical conductivity (S/m)
10^{+18}	10^{-18}
10^{+16}	10^{-16}
10^{+14}	10^{-14}
10^{+12}	10^{-12}
10^{+10}	10^{-10}
10^{+8}	10^{-8}
10^{+6}	10^{-6}
10^{+4}	10^{-4}
10^{+2}	10^{-2}
1	1
10^{-2}	10^{+2}
10^{-4}	10^{+4}
10^{-6}	10^{+6}
10^{-8}	10^{+8}

SiO_2, Al_2O_3

ceramic insulators

ferromagnetic and ferroelectric ceramics

GaAs

Si (intrinsic)

Si (extrinsic)

graphite

metalls and alloys

↓ superconductors

organic polymers

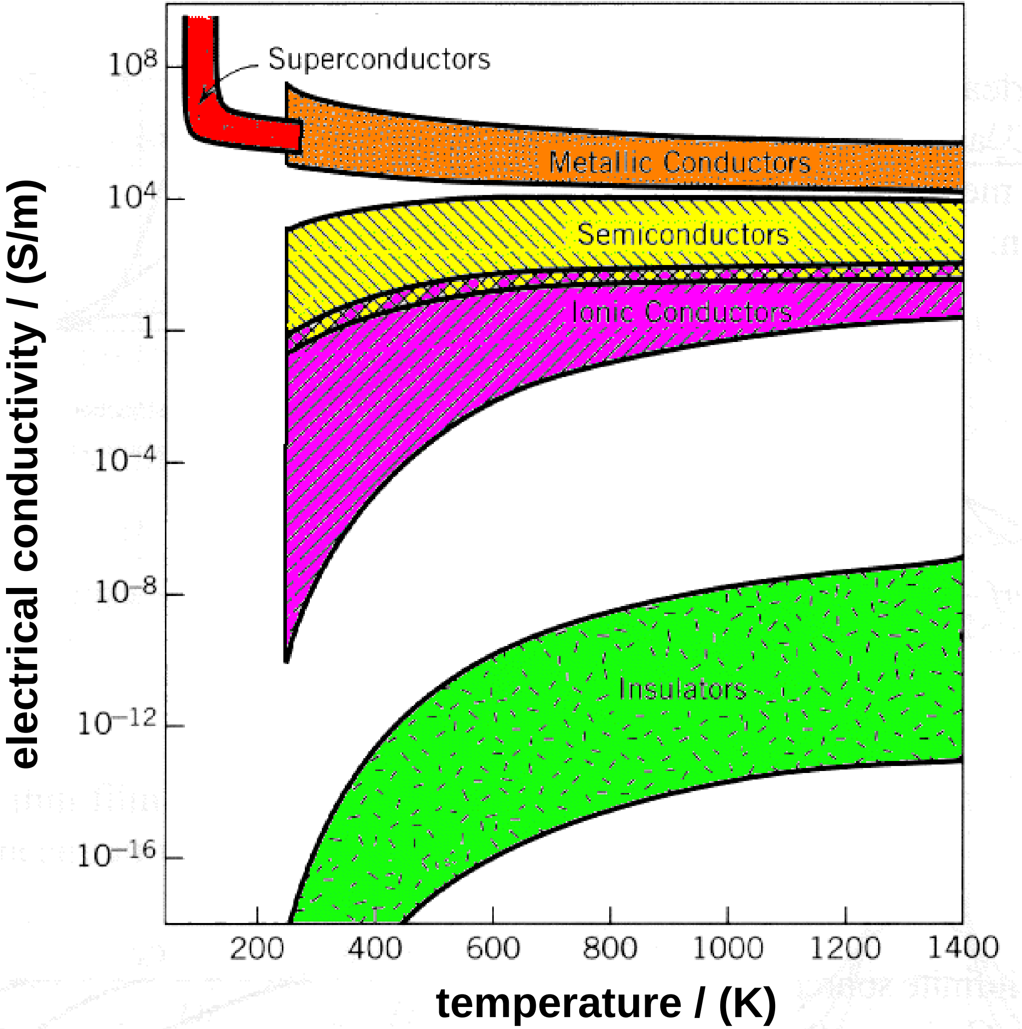
anorganic glasses

NiCr
Al
Ag, Cu

source: [Schau 90 / 219]

Electrical Conduction

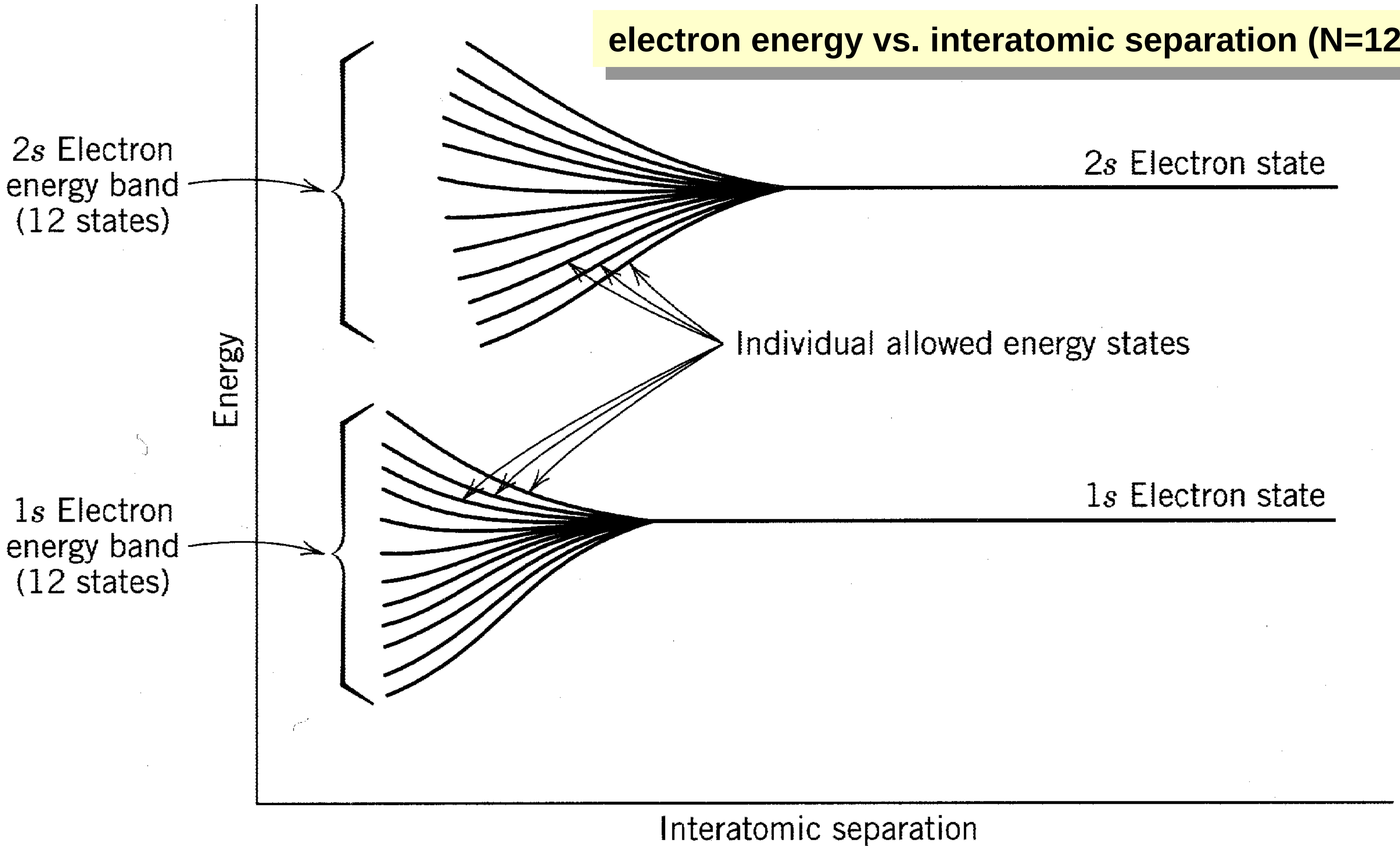
Electrical Properties of Solids: Electrical Conductivity vs. Temperature



source: [Chi 97 / 190]

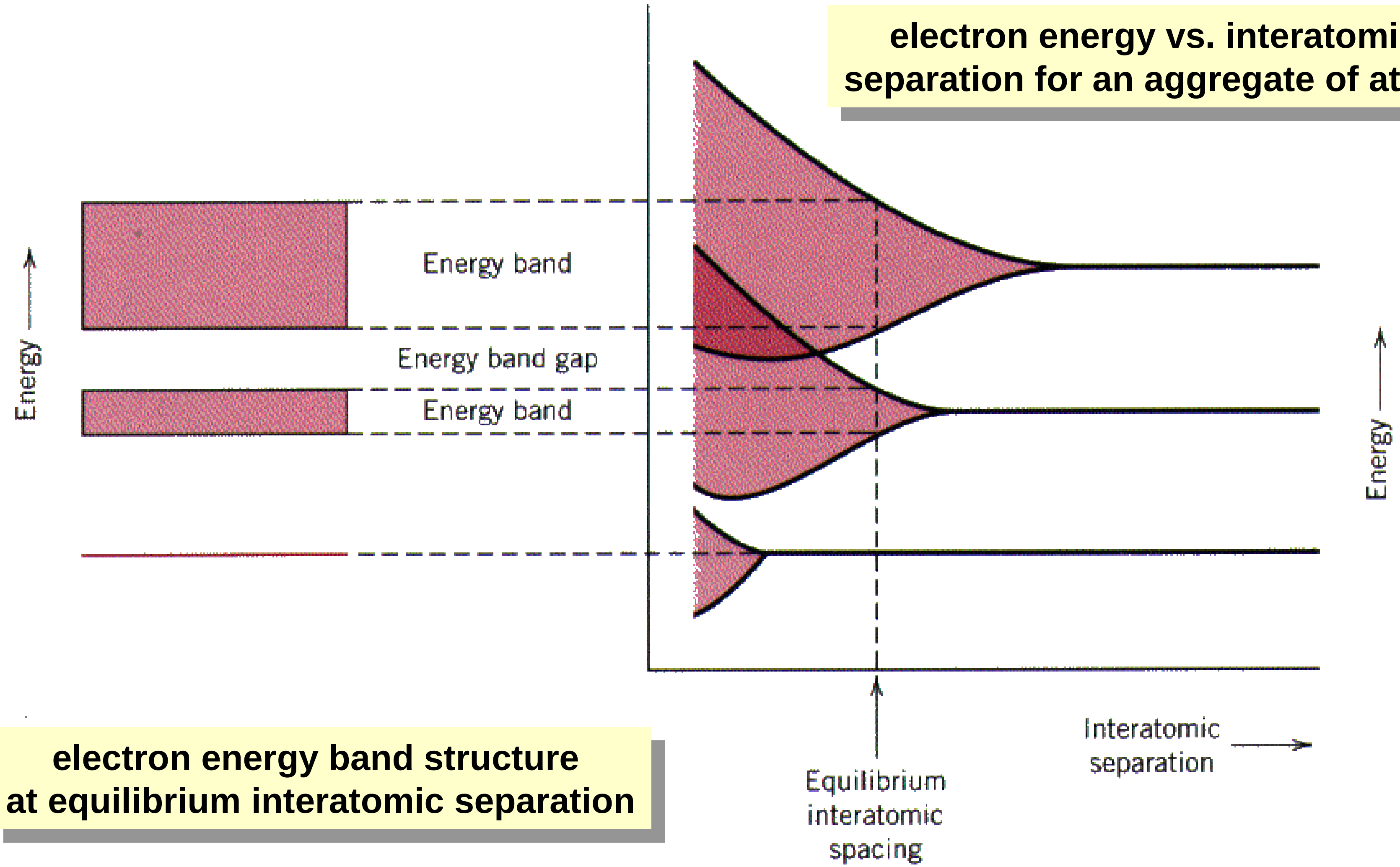
Electrical Conduction
Energy Band Structure in Solids

electron energy vs. interatomic separation (N=12)



source: [Cal 99 / 609]

Electrical Conduction
Energy Band Structure in Solids

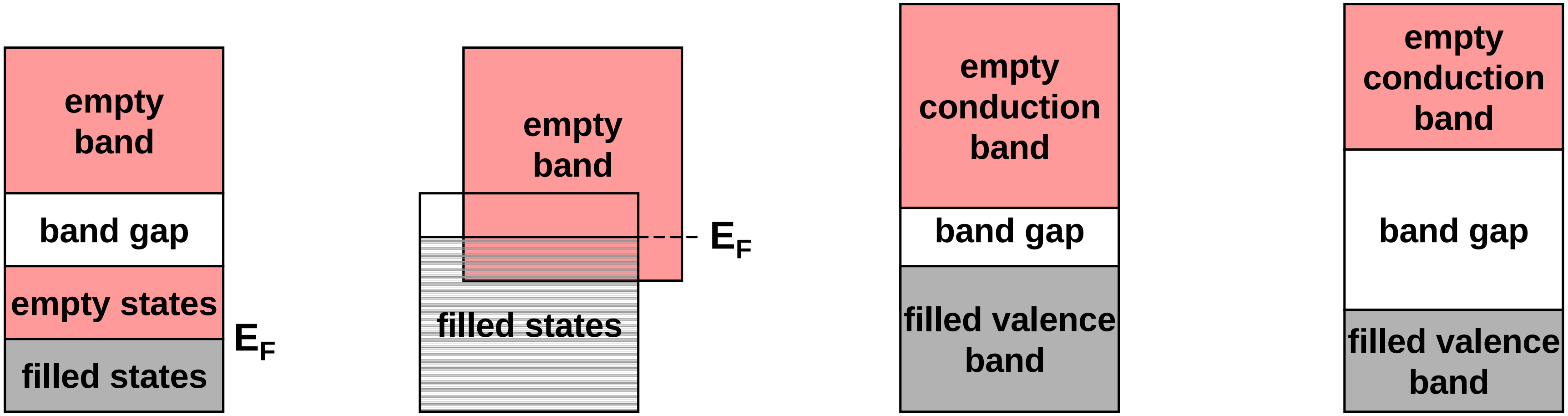


electron energy vs. interatomic separation for an aggregate of atoms

electron energy band structure at equilibrium interatomic separation

source: [Cal 99 / 609]

Electrical Conduction
Energy Band Structures in Solids at 0 K



metals:
available and filled states in the same band
(Cu, Au, Ag)

metals:
overlap between filled valence band and empty conduction band
(Al, Mg)

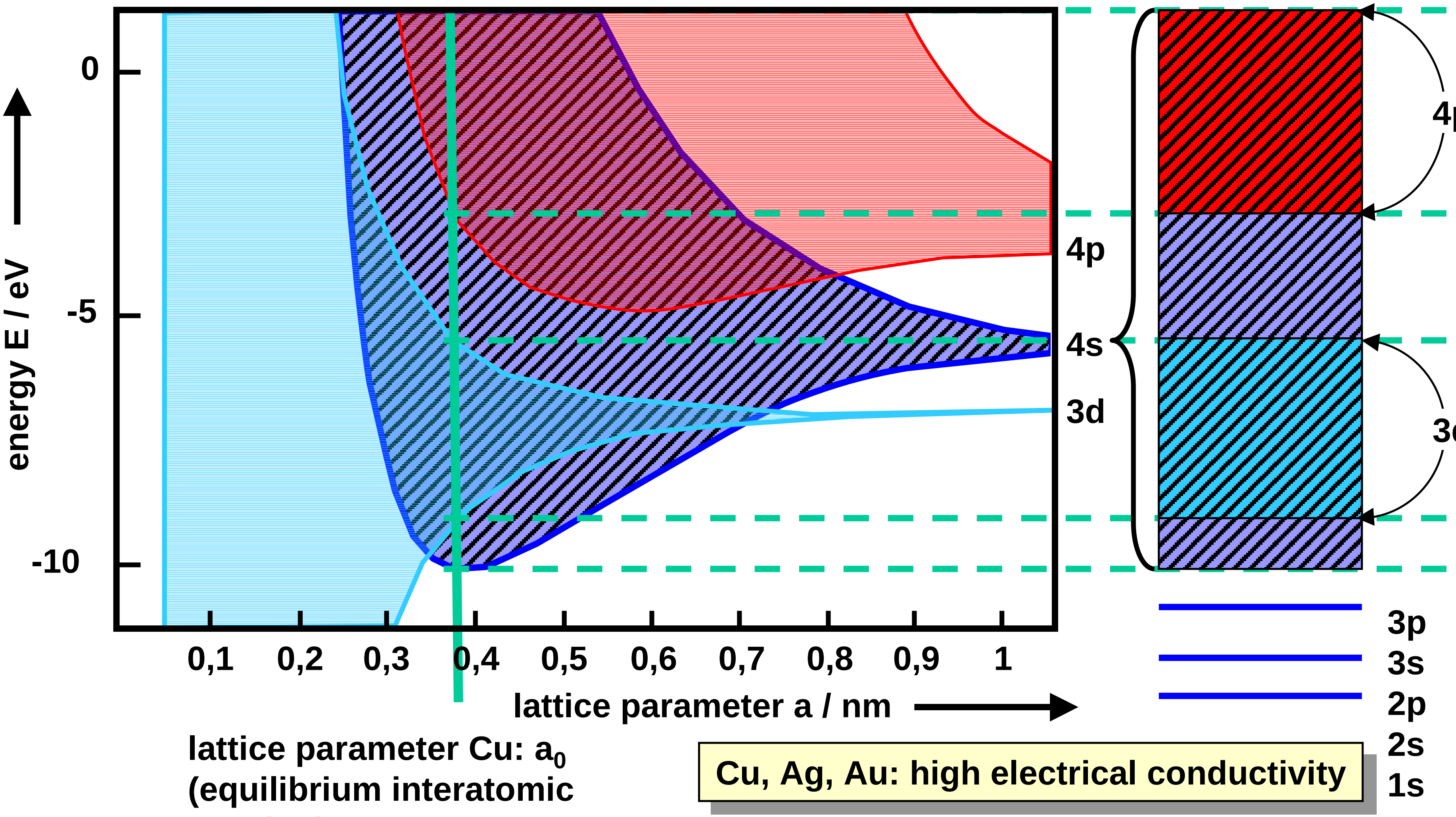
semiconductors:
filled valence band separated from empty conduction band by a narrow band gap (< 2 eV)

insulators:
filled valence band separated from empty conduction band by a large band gap (> 2 eV)

The electric properties of a solid material are a consequence of it’s electron band structure:
the arrangement of the outermost electron bands and the way in which they are filled with electrons.

source: [Schau 93 / 77]

Electrical Conduction
Energy Band Structures in Solids: Energy States in Copper



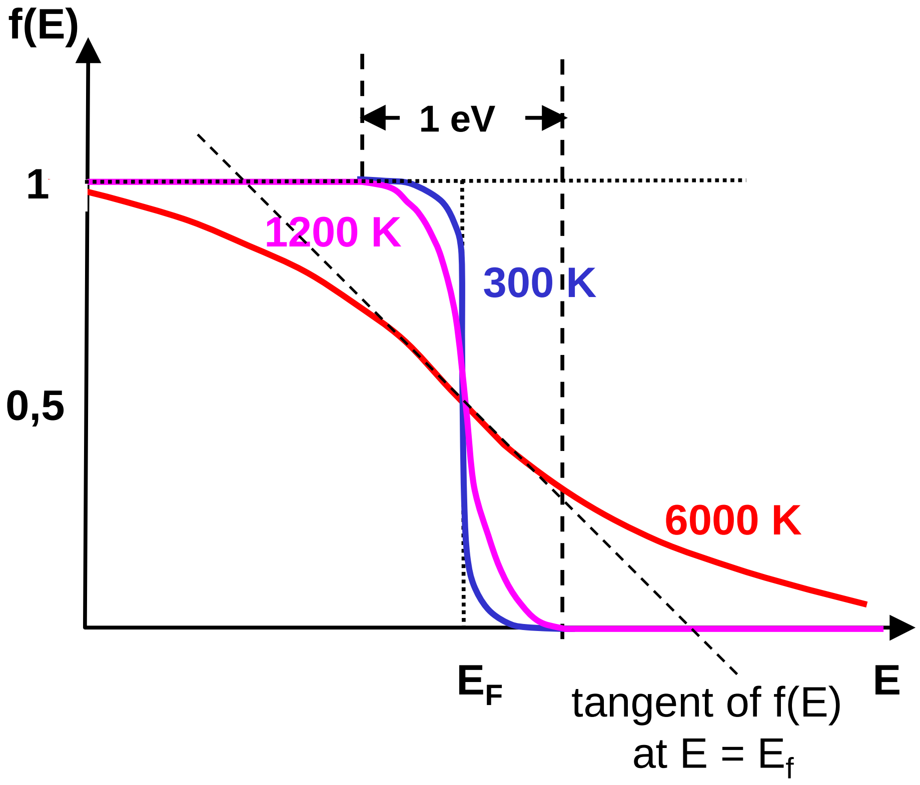
lattice parameter Cu: a_0 (equilibrium interatomic spacing)

Cu, Ag, Au: high electrical conductivity

source: [Wij 67 / 43]

Electrical Conduction
Fermi-Distribution f(E) at different Temperatures

Fermi energy E_f : the energy corresponding to the highest filled state at 0K



Fermi-Distribution of electron energy states (Fermi-Dirac-Statistic)

$$f(E,T) = \frac{1}{1+e^{\frac{E-E_F}{kT}}}$$

Boltzmann's constant
k = 1,38 × 10^{-23} J/atom·K

For E ≥ (E_f + 3kT),
Fermi-Distribution *f(E,T)* →
Boltzmann-Distribution *$f_B(E,T)$*

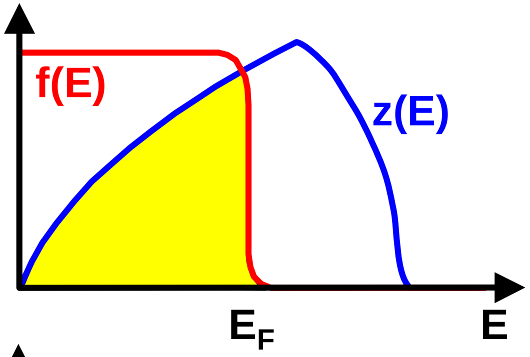
$$f(E,T) \approx f_B(E,T) = e^{-\frac{E-E_F}{kT}}$$

source: [Heime / 3.32]

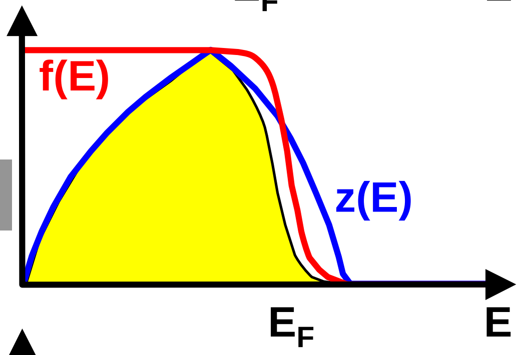
Electrical Conduction
Energy Band Structures in Solids

metals

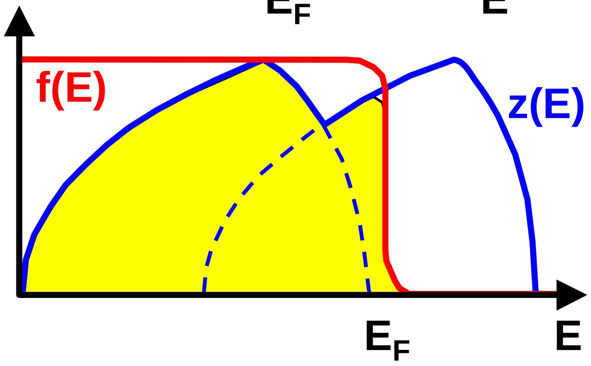
mono-valent



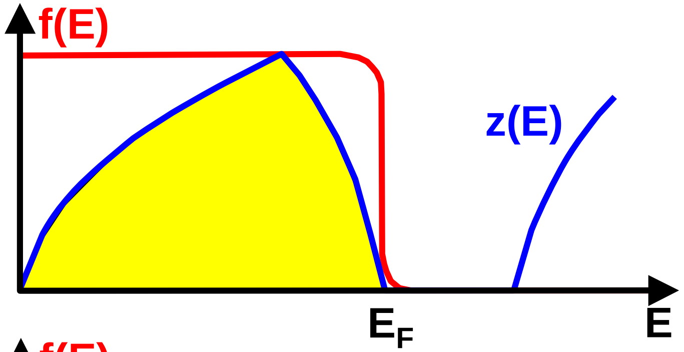
divalent



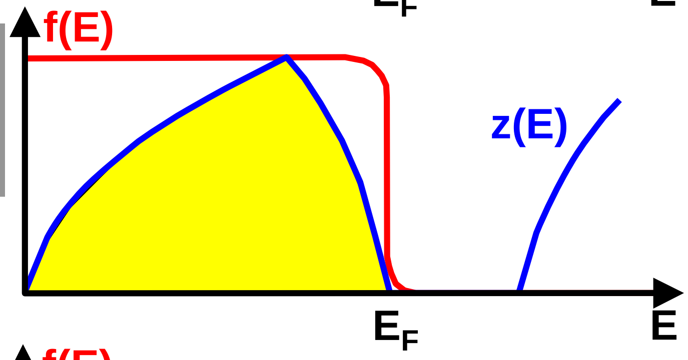
2 over-lapping bands



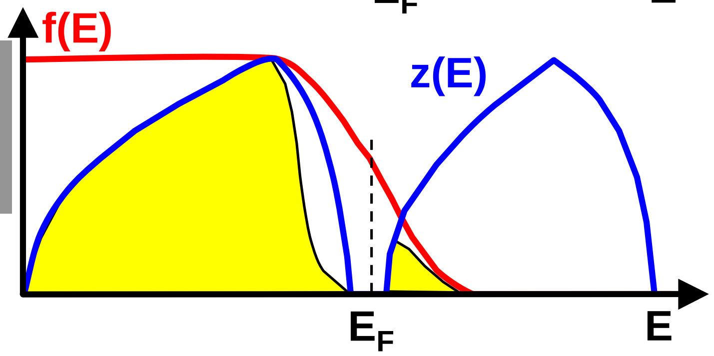
insulator



semi-conductor T = 0 K



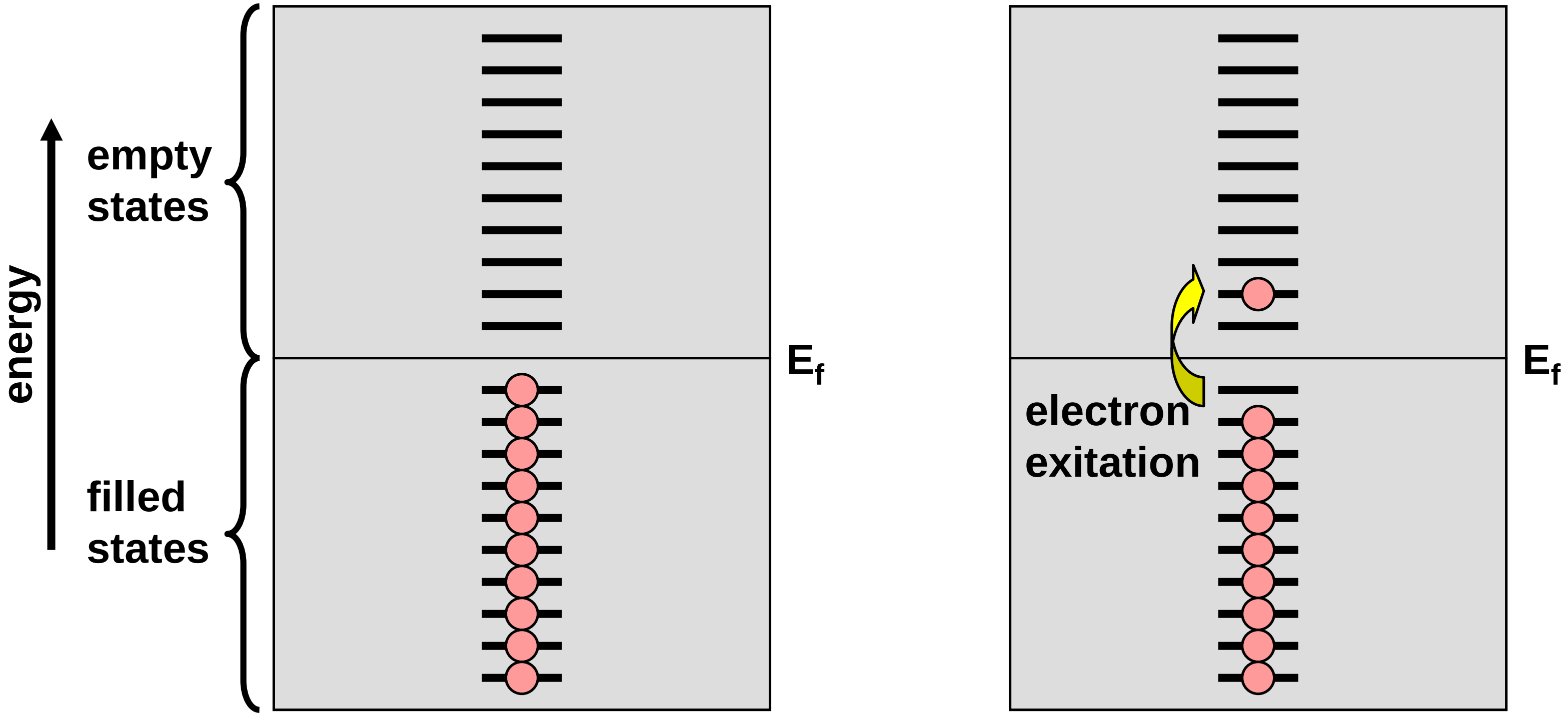
semi-conductor T >> 0 K



source: [Heime 3-34ff]

Electrical Conduction
Conduction in Terms of Band and Atomic Bonding Models

metal



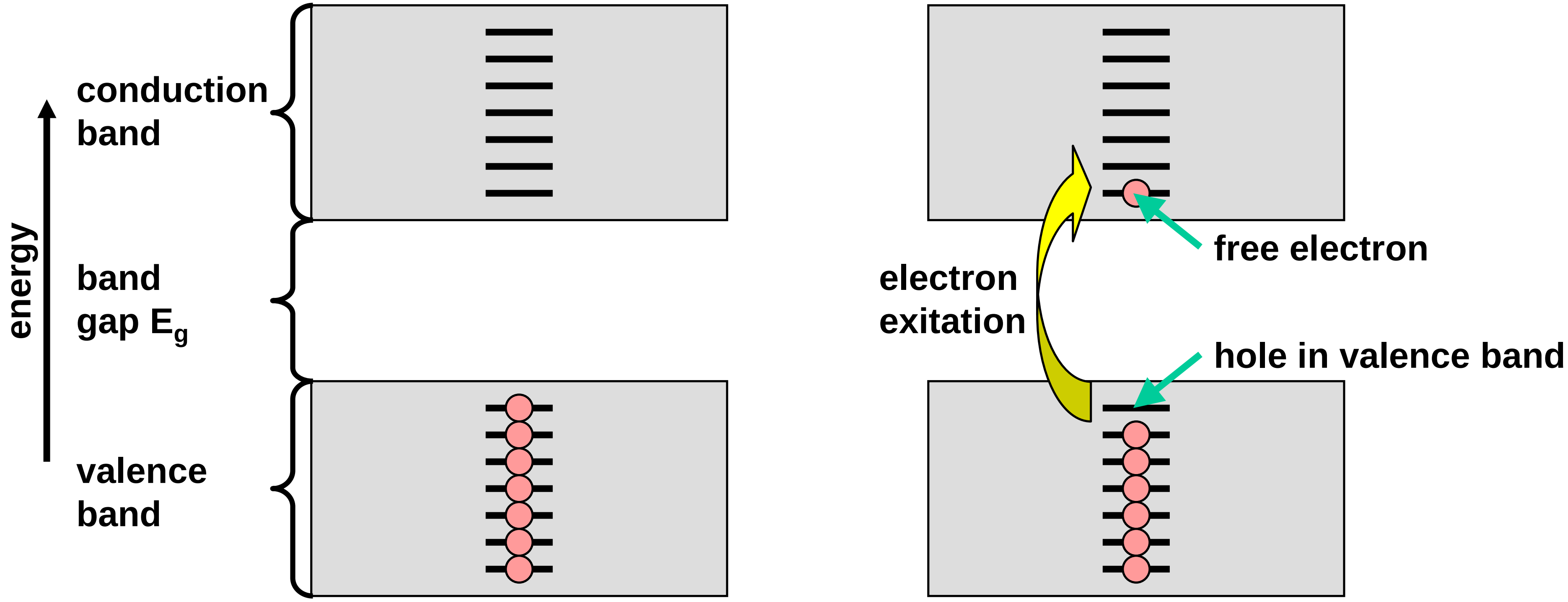
occupancy of electron states before an electron excitation

occupancy of electron states after an electron excitation

source: [Cal 99 / 609]

Electrical Conduction
Conduction in Terms of Band and Atomic Bonding Models

insulator or semiconductor



occupancy of electron states before an electron excitation

occupancy of electron states after an electron excitation

source: [Cal 99 / 609]

Electrical Conduction
Electron Drift Velocity and Electron Mobility

A current reaches a constant value while an electric field is applied

→ “frictional forces” counter the acceleration from the external field

→ scattering of electrons by imperfections in the crystal lattice and the thermal vibrations of atoms

→ cause an electron to lose kinetic energy and to change its motion direction

To describe the extent of scattering:

1. The drift velocity of an electron v_d : $\boxed{v_d = \mu_e E}$

the average electron velocity in the direction of the force imposed by the applied field.

2. Electron mobility μ_e (m^2/ V·s):

an indication of the frequency of scattering events.

conductivity $\boxed{\sigma = n|e|\mu_e}$

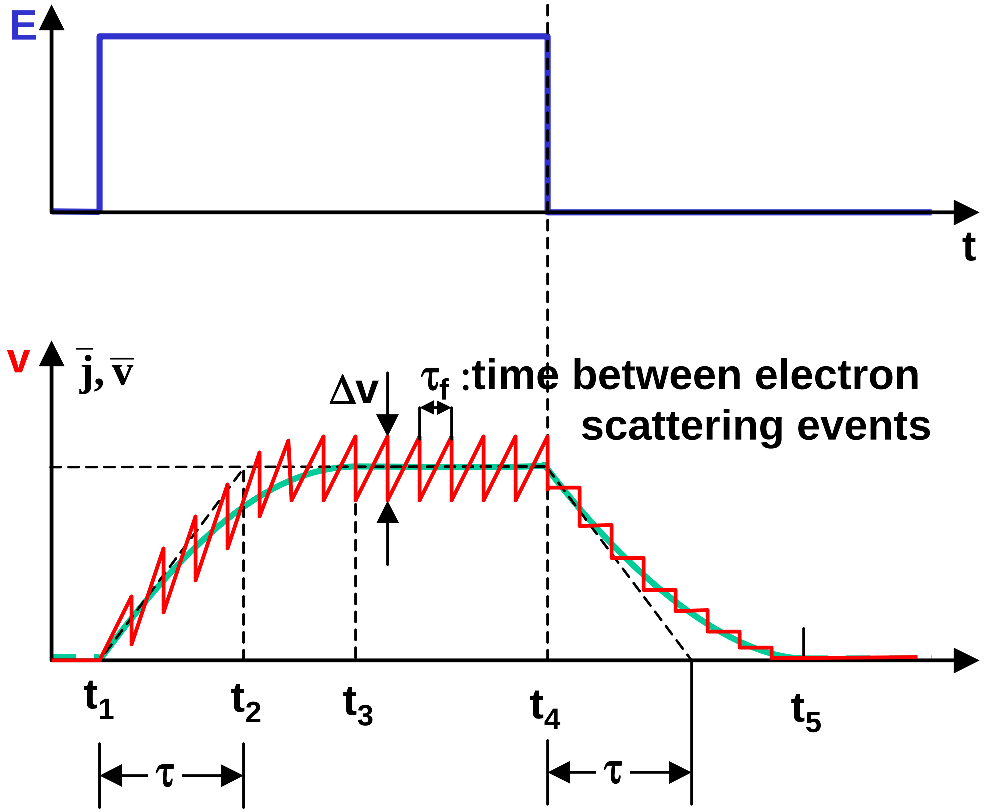
n **- the number of free or conducting electrons per unit volume**

$|e|$= 1,6×10^{-19} C

source: [Cal 99 / 613]

Electrical Conduction
Electron Drift Velocity and Electron Mobility

electron drift velocity at E > 0 (t_1 to t_4)



- **acceleration of an electron with an effective mass m***

$$m^* \cdot \frac{dv}{dt} = e_0 \cdot E$$

$$\Delta v = \int_0^{\tau_f} \frac{e_0}{m^*} \cdot E \cdot dt = \frac{e_0}{m^*} \cdot E \cdot \tau_f$$

- **mean drift velocity $\bar{v}$ of an electron**

$$\bar{v} = \frac{\tau}{\tau_f} \cdot \Delta v = \tau \cdot \frac{e_0}{m^*} \cdot E$$

- **mobility μ of an electron**

$$\mu = \frac{\bar{v}}{E} = \tau \cdot \frac{e_0}{m^*}$$

- **current density j**

$$j = \sigma \cdot E = \frac{e_0}{m^*} \cdot \tau \cdot e_0 \cdot n \cdot E$$

source: IWE

Electrical Conduction
Electron Concentration and Mobility as f(T)

material	concentration of charge carriers	mobility of charge carriers
metalls	n = const	$\mu_n \sim T^{-a}$
semiconductors	n ~ $e^{\frac{-E_g}{2kT}}$*	$\mu_n \sim T^{-a}$
Insulators	n ~ $e^{\frac{-E_g}{2kT}}$	$\mu_n \sim T^{-a}$ or $\mu_n \sim \mathbf{e}^{-\frac{\mathbf{A}}{\mathbf{T}}}$
	N_{ion} = const	$\mu_{ion} \sim \mathbf{e}^{-\frac{\mathbf{B}}{\mathbf{T}}}$

* band gap $E_g \leq 100$ kT at 25 °C (kT = 0,025 eV at 25 °C)

source: [Mü 93 / 158]

Electrical Conduction
Electrical properties of Metals

values at room temperature

metal	resistivity ρ $10^{-6}\Omega cm$	charge carrier mobility μ* $cm^2V^{-1}s^{-1}$	scattering time τ* $10^{-14}s$	Lorenz-number L $10^{-8}V^2K^{-2}$
Ag	1,62	66	3,7	2,31
Cu	1,68	44	2,5	2,28
Au	2,22	48	2,7	2,38
Al	2,73	13	0,7	2,22
Na	4,74	50	2,8	2,23
W	5,39	9,2	0,5	2,39
Zn	6,12	7,8 (+)	0,4	2,37
Cd	7,72	8,7 (+)	0,5	2,54
Fe	9,71	3,8	0,2	2,39
Pt	10,5	8,9	0,3	2,57
Sn	12,2	3,5	0,4	2,62
Pb	20,8	2,0 (+)	0,3	2,49

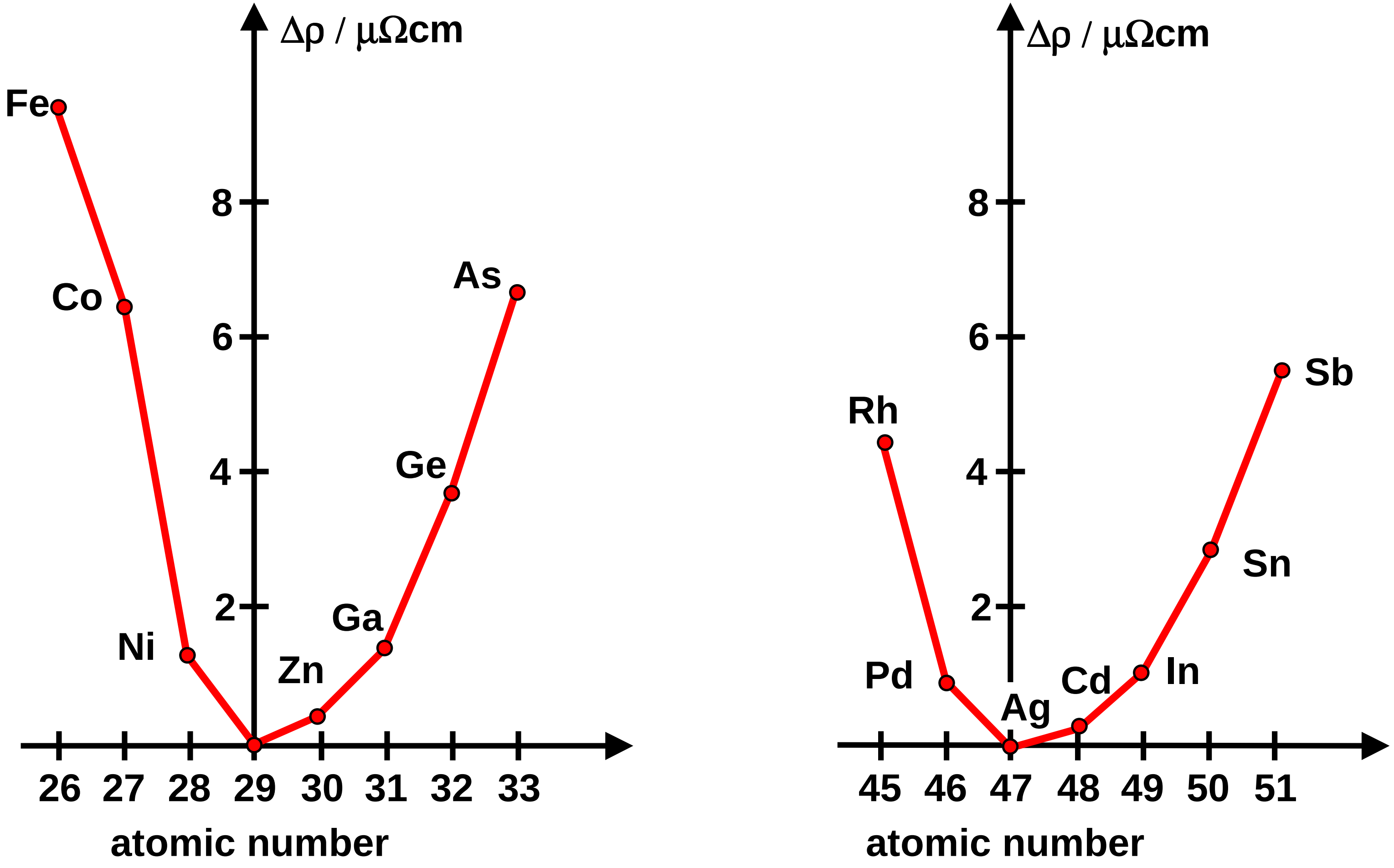
(+) hole conduction

*** calculated using σ-values, electron-concentrations according to the number of valence electrons s and effective mass m*=m**

source: [Arl 89 / 188]

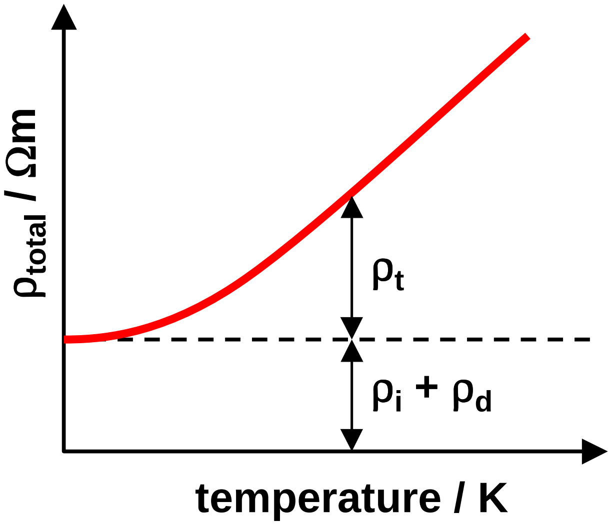
Electrical Conduction
Influence of different Impurity Elements (1 at%)

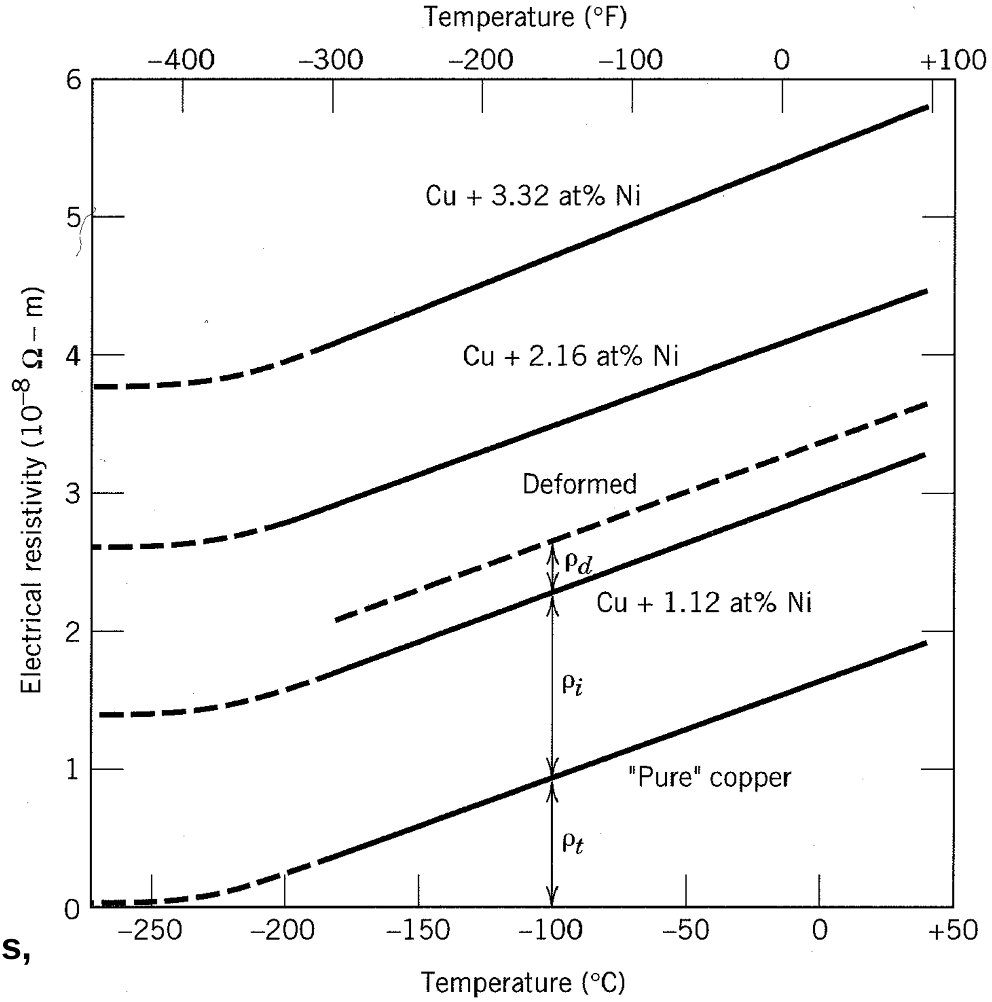
different impurity elements in Cu



source: [Mü 87 / 79]

Electrical Conduction





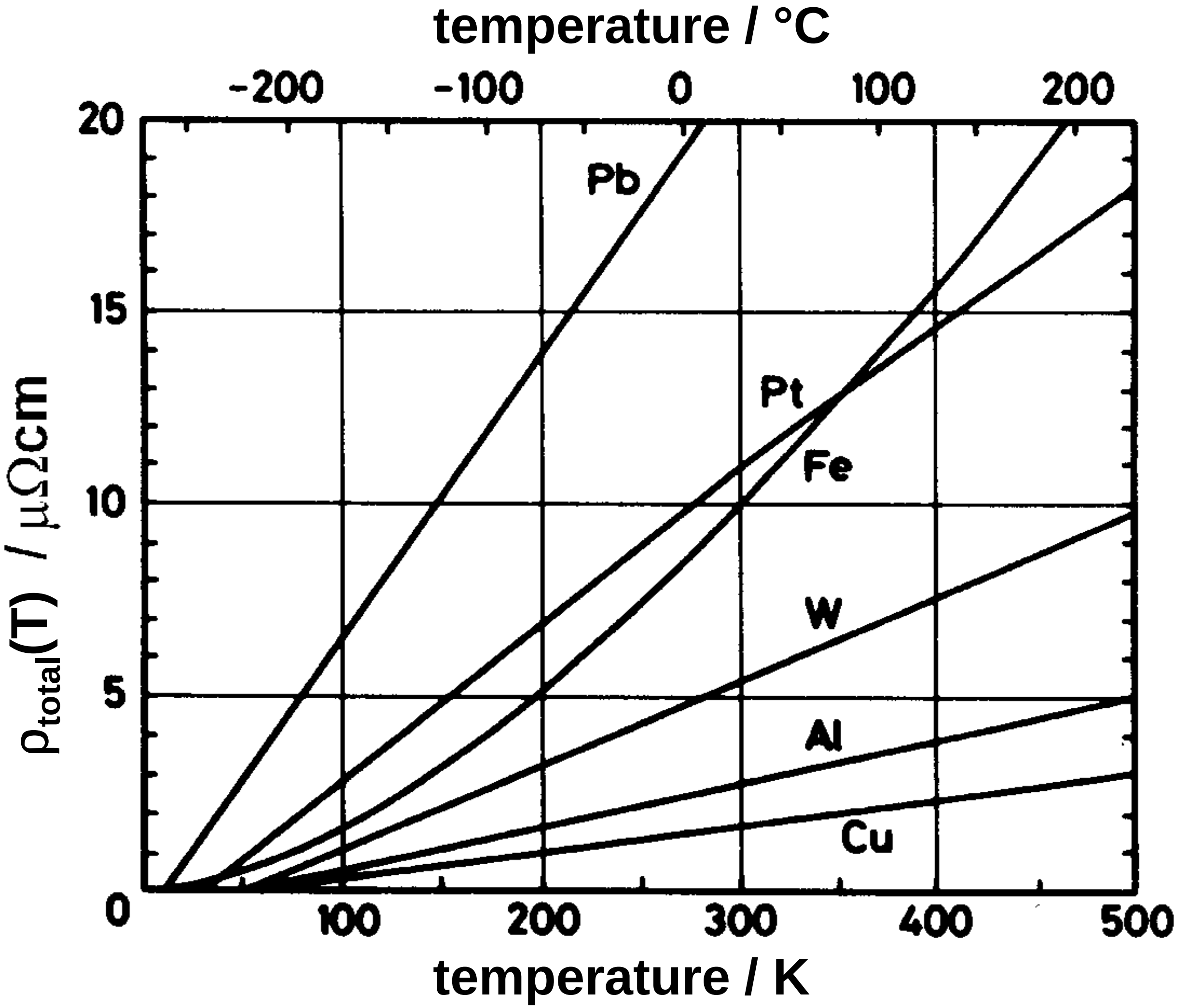
Matthiessen´s rule

$$\rho_{total} = \rho_t + \rho_i + \rho_d$$

ρ_i , ρ_d: **temperature independent parameters: impurities, dislocations, grain boundaries, secondary phases**

source: [Schau 90 / 96, Tip 94 / 1355]

Electrical Conduction
Electrical Resistivity of Metals as a Function of Temperature $\rho_t = f(T)$



temperature dependence of resistivity ρ_{total}

$\rho_{total}(T) = \rho_{total}(0\ °C) \cdot (1 + \alpha T)$

temperature T in °C

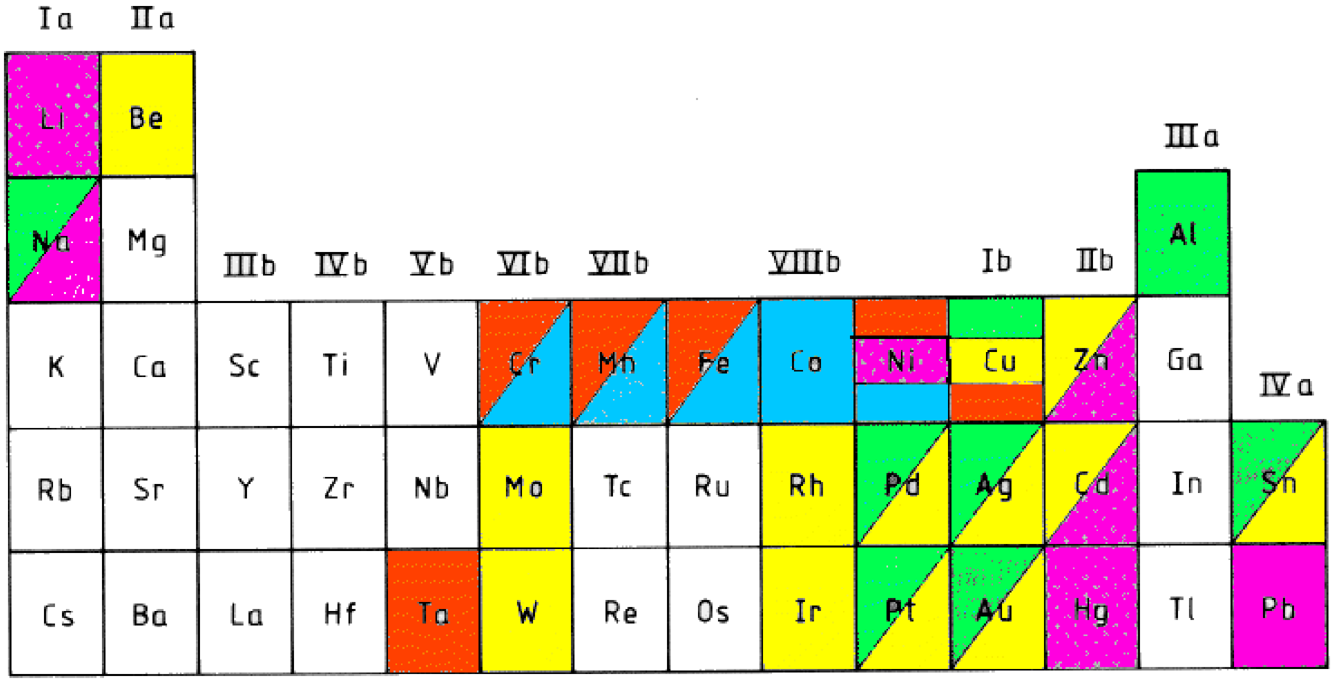
source: [Mü 93 / 78]]

Electrical Conduction
Electrical Properties of Metals

		ρ [μΩcm]	d	$\rho \cdot d$ [μΩcm]	TK_ρ [% / K]	λ [W / cm K]
I a	Na	4,2	0,97	4,1		1,4
	K	6,2	0,86	5,3		0,9
I b	Cu	1,7	8,9	15	0,43	4,0
	Ag	1,6	10,5	17	0,41	4,1
	Au	2,2	19,3	45	0,40	3,1
II a	Mg	4,5	1,7	7,7	0,41	1,4
	Ca	3,9	1,5	5,9	0,42	
II b	Zn	5,9	7,2	43	0,42	1,1
	Cd	6,8	8,6	59	0,42	1,0
	Hg	97	13,5	1310	0,08	0,08
III a	Al	2,7	2,7	7,3	0,43	2,3
IV a	Sn	12	7,3	88	0,43	0,7
	Pb	21	11,3	237	0,35	0,4
VIII b	Fe	9,7	7,9	77	0,65	0,7
	Co	6,2	8,9	55	0,60	0,7
	Ni	6,8	8,9	61	0,69	0,9
V b / VI b	Ta	13	16,6	216	0,38	0,5
	Cr	14	7,2	100	0,30	0,7
	Mo	5,2	10,2	53	0,40	1,4
	W	5,5	19,3	106	0,40	1,6
VIII b	Rh	4,5	12,5	57	0,42	0,9
	Pd	9,8	12,0	118	0,38	0,7
	Pt	9,8	21,4	210	0,39	0,7

source: [Mü 93 / 77]

Electrical Conduction
Application of different Metals



Ia	IIa	IIIb	IVb	Vb	VIb	VIIb	VIIIb			Ib	IIb	IIIa	IVa
Li 🟪	Be 🟨												
Na 🟩🟪	Mg											Al 🟩	
K	Ca	Sc	Ti	V	Cr 🟥🟦	Mn 🟥🟦	Fe 🟥🟦	Co 🟦	Ni 🟥🟪🟦	Cu 🟩🟨🟥	Zn 🟨🟪	Ga	
Rb	Sr	Y	Zr	Nb	Mo 🟨	Tc	Ru	Rh 🟨	Pd 🟩🟨	Ag 🟩🟨	Cd 🟨🟪	In	Sn 🟩🟨
Cs	Ba	La	Hf	Ta 🟥	W 🟨	Re	Os	Ir 🟨	Pt 🟩🟨	Au 🟩🟨	Hg 🟪	Tl	Pb 🟪

🟨 **contact materials**

🟥 **resistors**

🟦 **magnetic materials**

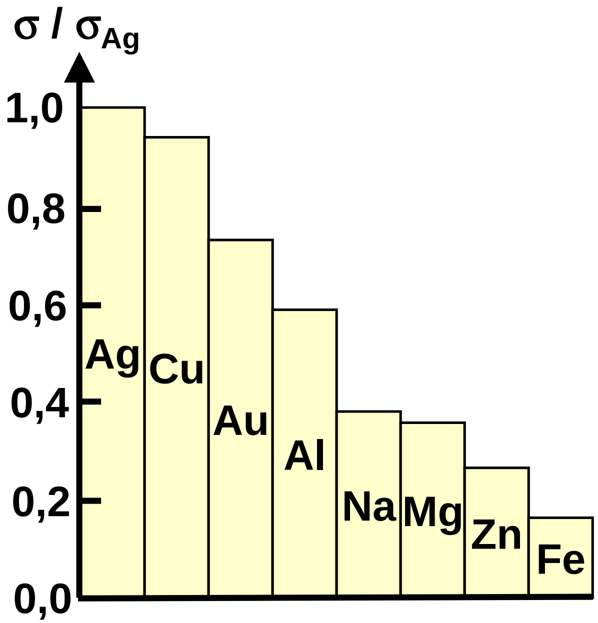
🟩 **conductors**

🟪 **current sources**

source: [Mü 87 / 98]

Electrical Conduction
Application of different Metals and Alloys

conductors	**3-dimensional**	**coils**		Cu
		cables	**communications technology**	Cu
			energy-technology	Al, Cu, (Na)
		overhead line		Al/St, Cu, Al
		waveguide		Ag
	planar	**integrated circuits**		Al, Al/Cu, Au
		thin film technology		Al, Au, Ag, Cu
		thick film technology		Au, Ag, Pd, Pt
		circuit boards		Cu, Sn



source: [Mü 87 / 99]

Electrical Conduction
Alloys for Precision-Resistors

material	**alloy elements / wt.%** Mn	Ni	Al	**maximum operation temperature / °C**	**ρ* / µΩcm**	**TK_ρ * / K^{-1}**	**thermal voltage vs. copper * ** / µV/K**
CuMn12Ni2	**12**	**2**	**-**	**140**	**43**	**$\pm 1\cdot10^{-5}$**	**- 0,4**
CuNi20Mn10	**10**	**20**	**-**	**300**	**49**	**$\pm 2\cdot10^{-5}$**	**- 10**
CuNi44	**1**	**44**	**-**	**600**	**49**	**$+ 4\cdot10^{-4}$ $- 8\cdot10^{-4}$**	**- 40**
CuMn2Al	**2**	**-**	**0,8**	**200**	**12**	**$4\cdot10^{-4}$**	**+ 0,1**
CuNi30Mn	**3**	**30**	**-**	**500**	**40**	**$1\cdot10^{-4}$**	**- 25**
CuMn12NiAl	**12**	**5**	**1,2**	**500**	**40**	**$\sim 10^{-5}$**	**- 2**

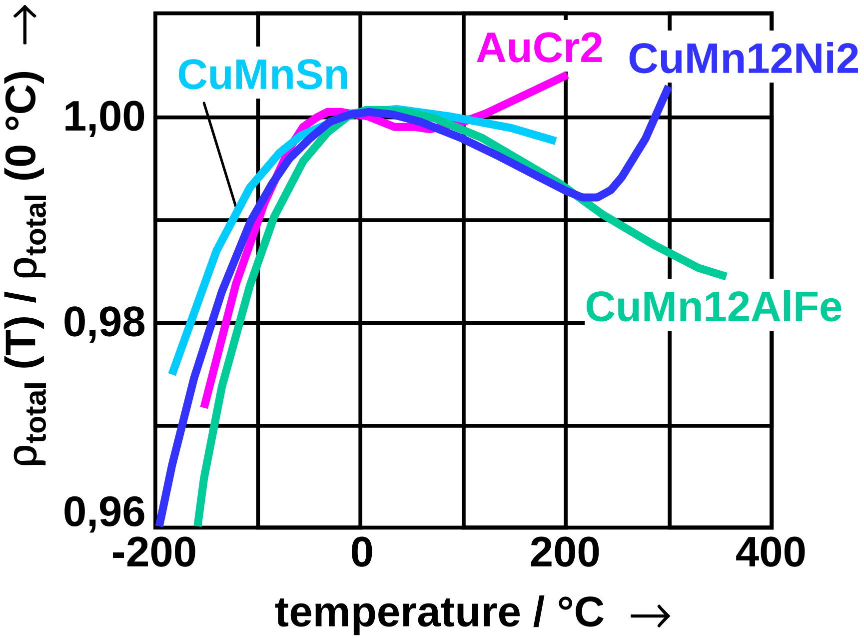
*** T = 20 °C** **** Seebeck-coefficient**

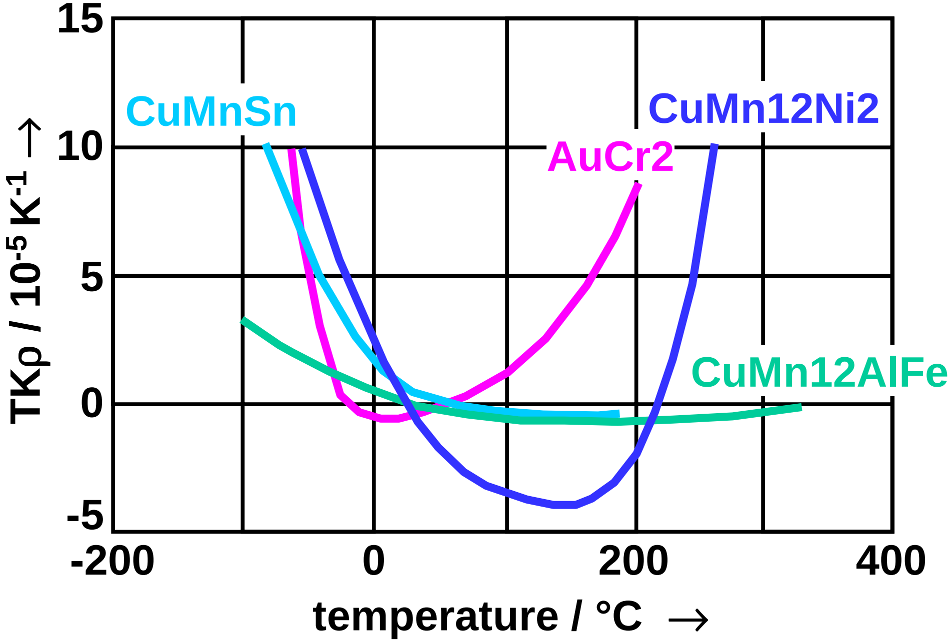
<u>choice criteria</u>: high resistivity ρ, long term stability, well defined and very low TK_ρ ,
small thermal voltage vs. copper ⇒ alloys

source: [Mü 87 / 102)

Electrical Conduction

ρ and TKρ of Alloys for Precision-Resistors





source: [Mü 93 / 103]

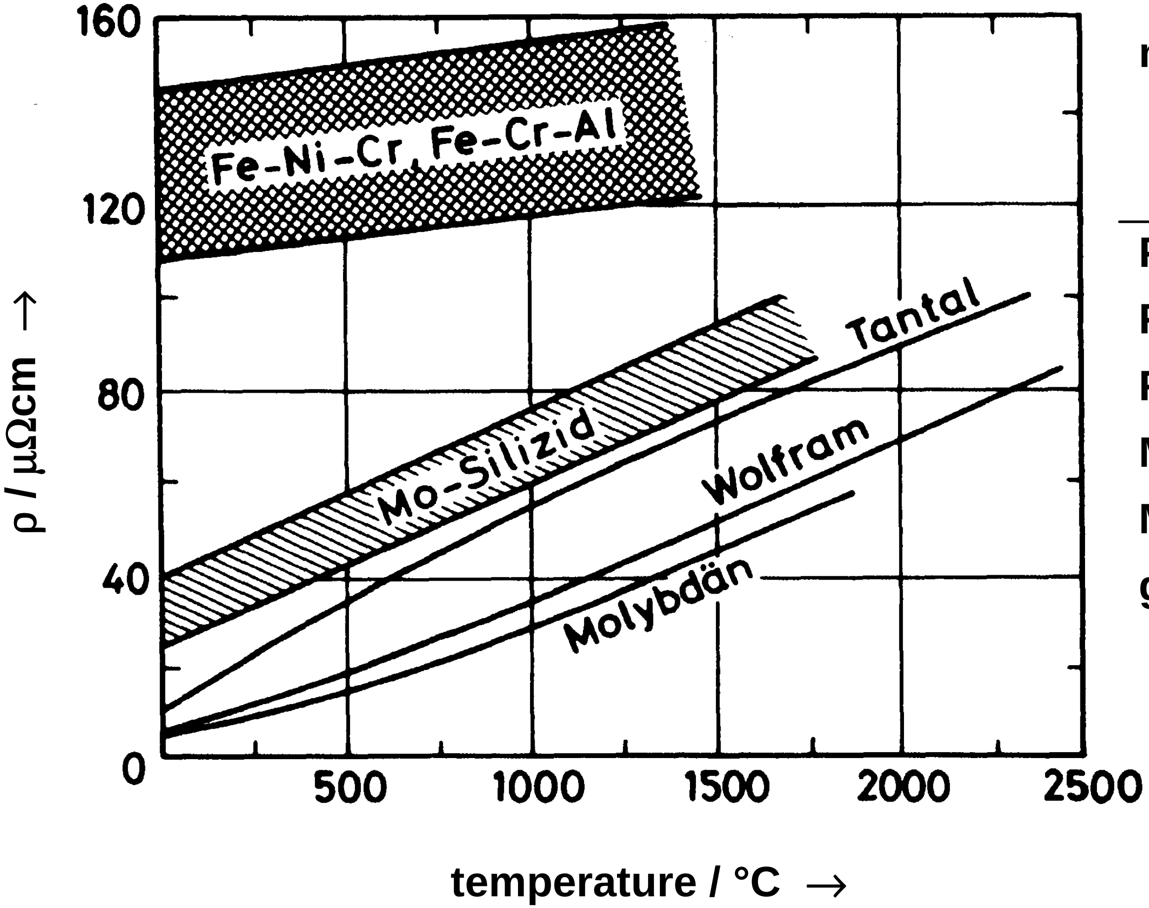
Electrical Conduction
Alloys for Furnace Heating Elements

Alloys	alloy elements / wt.%				structure	ρ	maximum operation temperature	coating
	Fe	Ni	Cr	Al		/ μΩcm	/°C	
NiCr 80 20	-	80	20	-	kfz	112	1200	Cr_2O_3
NiCr 60 15	25	60	15	-		113	1150	
NiCr 30 20	50	30	20	-		104	1100	
CrNi 25 20	55	20	25	-		95	1050	
CrAl 25 5	70	-	25	5	krz	144	1300	Al_2O_3
CrAl 20 5	75	-	20	5		137	1200	

choice criteria: high melting point, formation of protective coating

source: [Mü 87 / 103]

Electrical Conduction
Resitivity of Heating Elements as f(T)



material	operation temperature T_{max} / °C	protective coating
Pt	1000	none*
Fe Ni Cr	1200	Cr_2O_3
Fe Cr Al	1300	Al_2O_3
$MoSi_2$	1600	SiO_2
Mo, W, Ta	1700	in gas (H_2)
graphite	3000	in gas (H_2)

source: [Mü 93 / 106]

Electrical Conduction
Metals and Alloys for Sensor Applications

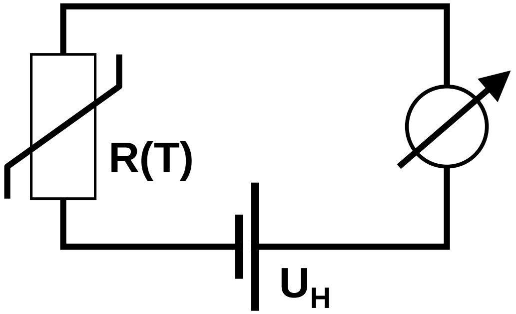
Sensor Applications

temperature T

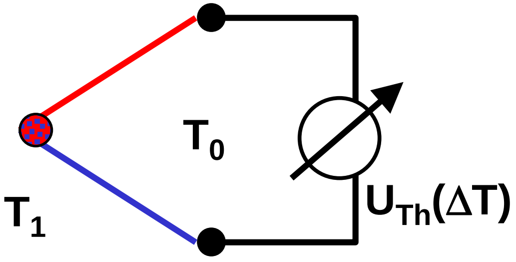
force F, strain ε

operating voltage U_H

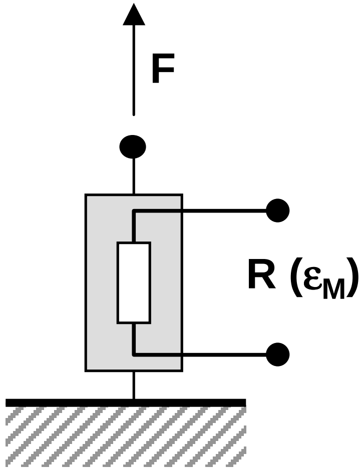
known temperature T_0



resistive thermometer



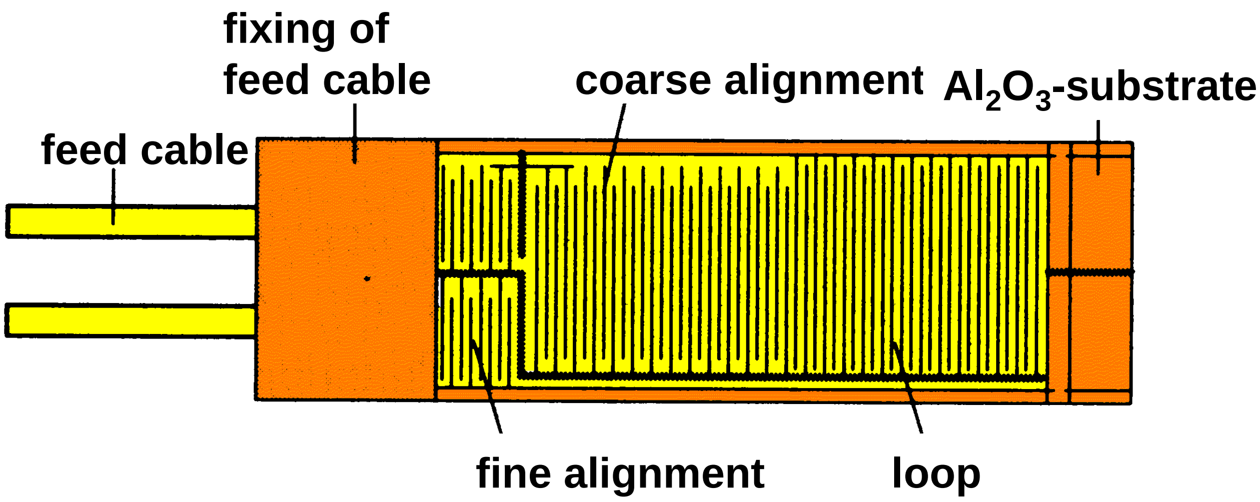
thermocouple

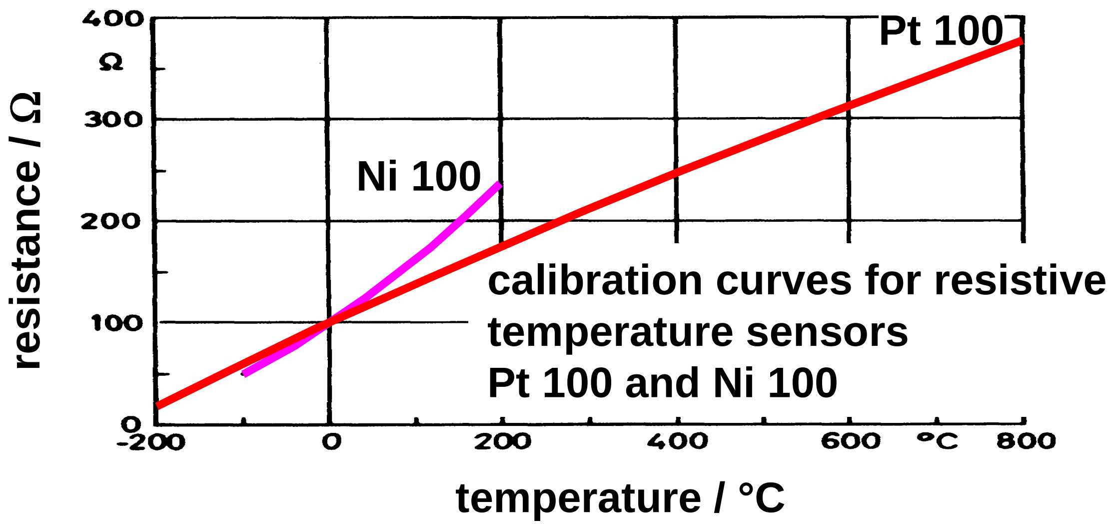


strain gauge

source: [Mü 87 / 103]

Electrical Conduction
Resistive Temperature Sensors





calibration curves for resistive temperature sensors Pt 100 and Ni 100

looped

⇨ **resistance increased**

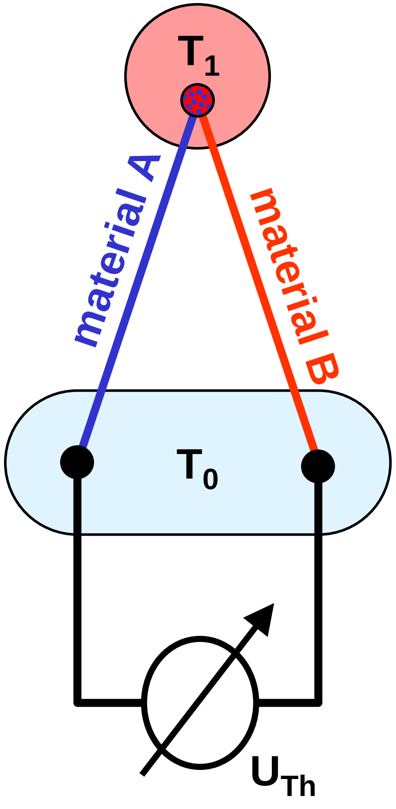
Pt 100 - 100 Ω
Pt 1000 - 1 kΩ

ρ = 9,83·10^{-6} Ωcm (0 °C)
TKρ =0,00385 K^{-1} (0...100 °C)
$T_{operation}$ -220...+800 (1200) °C

- + **cheap**
- + **easy alignment**
- – **hysteresis due to thermal expansion of Al_2O_3-substrate**
- – **adhesion of Pt on Al_2O_3 - substrate deteriorates due to thermocycling**

source: [Schau 93 / 111]

Electrical Conduction
Thermocouples



thermocouple

temperature difference generates potential difference

$$U_{Th} = \int_{T_0}^{T_1} \eta_{AB}(T)\,dT$$

Seebeck-coefficient

thermoelectric voltage

$\eta_{semiconductor} \approx 100\ldots600\ \mu V/K$

$\eta_{metal} \approx 0\ldots40\ \mu V/K$

$\eta_{AB} = \eta_A - \eta_B$

absolute Seebeck electromotive force of materials A, B

source: IWE

Electrical Conduction
Metals and Alloys for Thermocouples

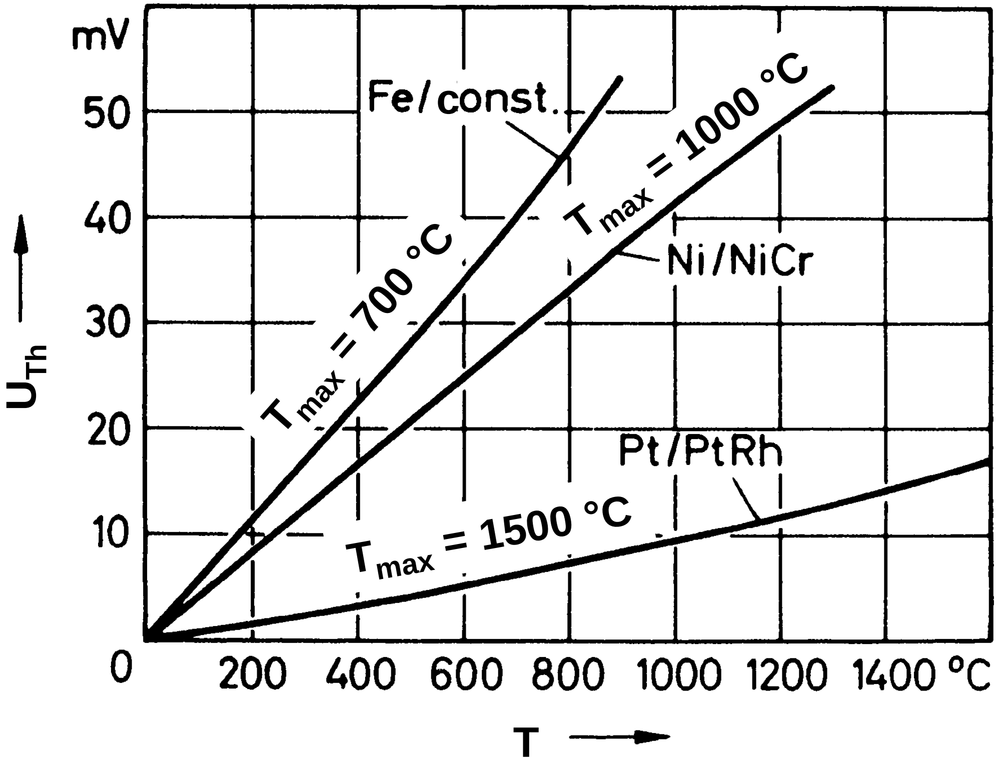
negative site	positive site	U_{Th} / mV	max. temp. / °C
constantan (55 Cu 44 Ni 1 Mn)	copper (Cu) iron (Fe)	4,25 5,37	400 700
nickel (98 Ni 2 Al)	chromnickel chromel (90 Ni 80 Cr)	4,1	1000
alumel (94,5 Ni 2,5 Mn 2Al 1 Si)			
pallaplat 32 (52 Au 46 Pd 2 Pt)	pallaplat 40 (95 Pt 5 Rh)	2,65	1300
platinum (Pt)	platinum-rhodium (90 Pt 10 Rh)	0,64	1500

U_{Th} for ΔT = 100 K

source: [Mü 93 / 109]

Electrical Conduction
Thermocouples

- $U_{Th} = f(T)$

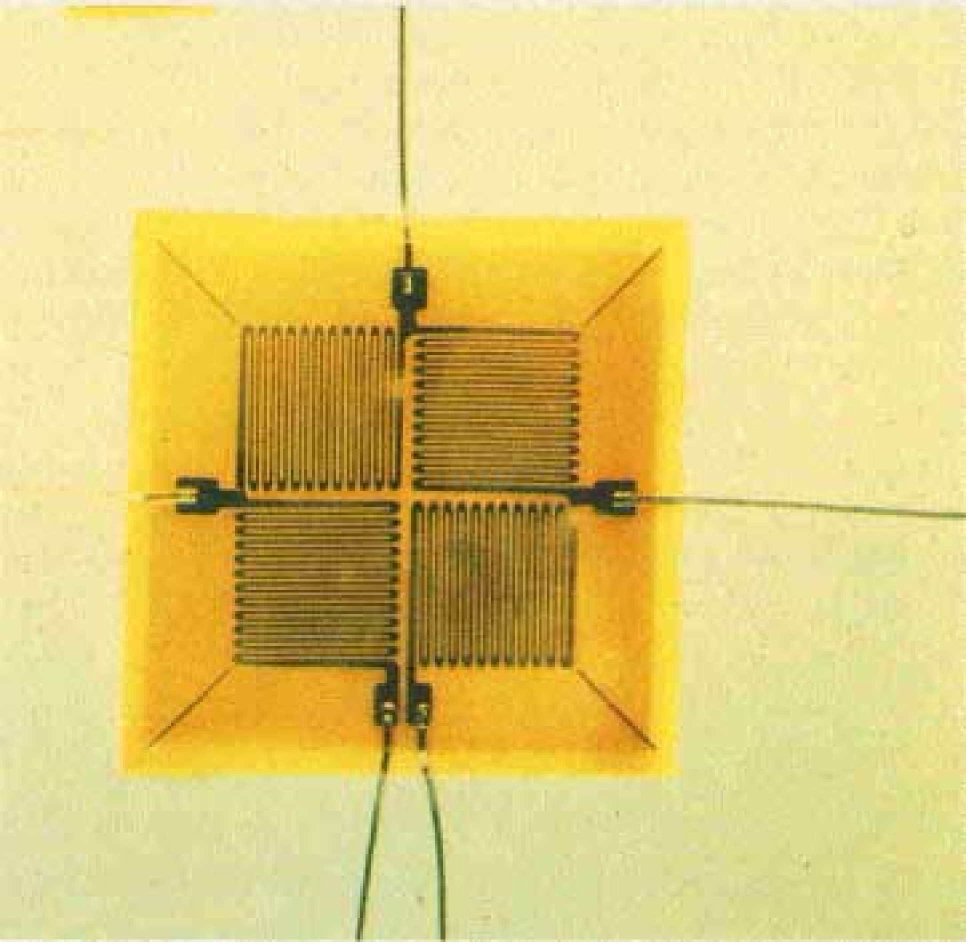


- **choice criteria**

– high U_{Th}-values
$\rightarrow$ high $\eta_{AB} = \eta_A - \eta_B$
– $U_{Th} \sim T$
– high melting point
– chemical stability at high temperatures

source: [Mü 93 / 108]

Electrical Conduction
Strain Gauges



DMS:

resitance change

due to strain / compression

$$\frac{\Delta R}{R} = K \cdot \frac{\Delta l}{l} = K \cdot \varepsilon_M$$

application:

force sensor, manometer, balance

layout:

looped arrangement

→ maximum length (l)

→ high accuracy ($K \cdot \varepsilon_M$)

source: [Tip 94 / 1353]

Electrical Conduction
Metals and Alloys for Strain Gauges

material	composition	K-factor
constantan	55 Cu 44 Ni 1 Mn	2,0
Fe-Ni-wire	65 Ni 20 Fe 15 Cr	2,5
„Iso-Elastic“-wire	52 Fe 36 Ni 8,5 Cr 3,5 Mn	3,6
Fe-wire	100 Fe	4,0

relativ resistance change:

$$\frac{\mathbf{dR}}{\mathbf{R}} = \frac{\mathbf{dl}}{\mathbf{l}} - \frac{\mathbf{dA_q}}{\mathbf{A_q}} + \frac{\mathbf{d\rho}}{\boldsymbol{\rho}}$$

using $\frac{dA_q}{A_q} = -2\upsilon \cdot \frac{dl}{l}$ **and** $\frac{\mathbf{d\rho}}{\boldsymbol{\rho}} = \mathbf{K_1} \cdot \frac{\mathbf{dl}}{\mathbf{l}} \Rightarrow$

poisson-ratio

length

area

resistivity

resistance change due to strain: $\frac{\mathbf{\Delta R}}{\mathbf{R}} = \mathbf{K} \cdot \varepsilon_M$

(K = 1 + 2ν + K_1)

source: [Mü 93 / 110]