CHAPTER 5. SURFACES

5.1 Determining and Specifying Surfaces in a Drawing.

Classification

In descriptive geometry surfaces are referred to as a set of consecutive locations of a moving line. This method of a surface formation is called the *kinematic* method.

A line (a curved or a straight one) moving in space and producing a surface is called a *generatrix* **(**a generating line). As a rule, this line moves along another one, called a *directrix* (a directional line).

Except mentioned above kinematic method, the surface may be specified by: the *analytical* method, i.e. presented in a mathematical expression; and the *frame* method which is used for specifying complex surfaces subjected to no rules.

In this case, to specify a surface, a number of its parallel sections (frame) is required which may be referred to as the locations of a variable generatrix. This method is used in lorry body manufacturing, in aircraft industry, shipbuilding, etc.

Method of specifying a surface by a frame, for instance, by intersection lines of a surface with level planes, is applied in topography, mining, road making. Projections of a level line on a projection plane with the corresponding marks represent a relief landscape map. A surface referred to as the earth one, is called *topographic* surface.

To specify a surface in a complex drawing it is necessary to present in it only those elements of a surface which give the opportunity to construct each of its points. A collection of these elements is called surface determinant. Surface determinant consists of two parts: a *geometric part*, including constant geometric elements (points, lines), which form the surface; and an *algorithm part*, specifying the principle of generating line motion, the nature of its form modification.

When a surface Ω is projected on the projection plane P in a parallel way, projecting lines tangential to Ω, form a cylindrical surface (Fig. 5.1). These projecting lines contact the surface Ω in certain points, forming the line M which is called a level line.

Projection of the level line M on the plane P-m_p, is called the surface outline.

To simplify understanding of a drawing, the draughtsmen represent not only outline of the surface but also its most significant lines and points.

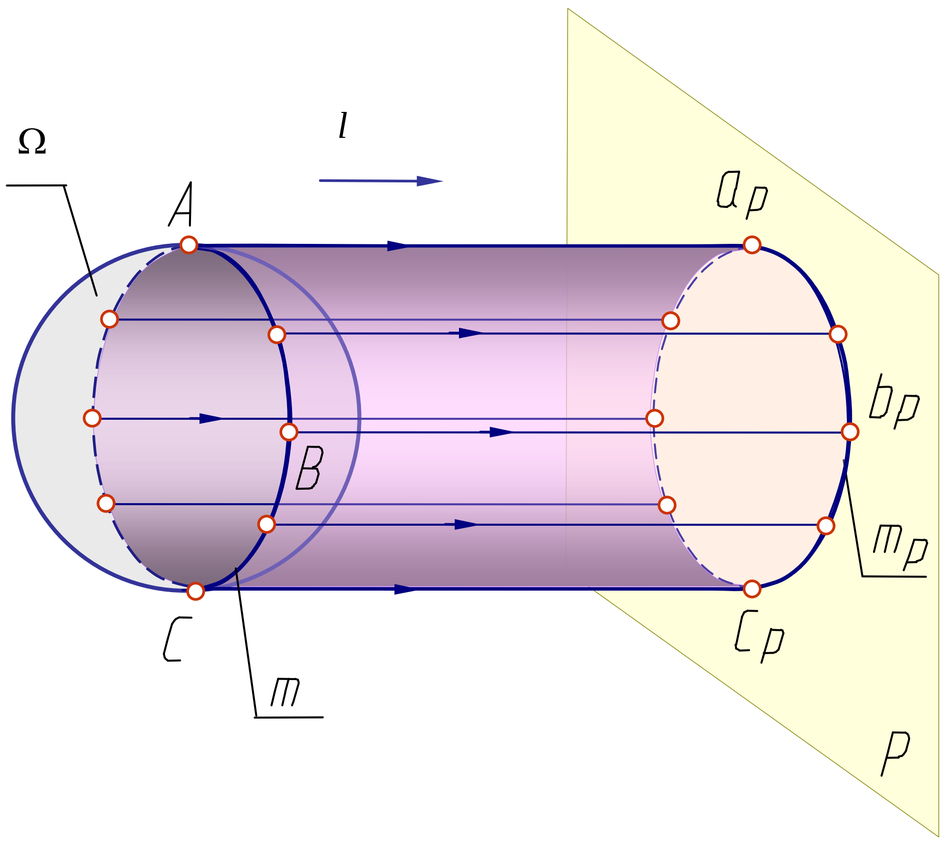


Fig. 5.1

Classification of the Surfaces

There is a great quantity of different surfaces. But some of them are considered to be the most significant. Classification of them depends on generatrix form, also on the form, number and location of directrices:

1. Regular and nonregular surfaces;
2. Ruled surfaces (formed by a travel of a straight line) and nonruled (curve-lined) surfaces;
3. Developable surfaces (or torses) and nondevelopable ones;
4. Surfaces with generatrix of a constant form and of a variable form;
5. Surfaces with translational, rotary and helical motion of generatrix.

5.2 A Point and a Line on the Surface

A point belongs to a surface when it belongs to a line of the surface.

A line belongs to a surface when it passes through the points of the surface. A straight line belongs to a surface when it passes through two points belonging to the surface.

Hence, if a point belongs to a surface, its projections belong to the like projections of the surface line.

To construct the points lying on a surface, use the prime lines of this surface.

5.3 Polyhedral Surfaces and Polyhedrons.

A Polyhedron Cut by a Plane

A polyhedral surface is a surface formed by a travel of a linear generating line along a polygonal directrix. Polyhedral surfaces are divided into two kinds: pyramidal (Fig. 5.2) and prismatic (Fig. 5.3) surfaces.

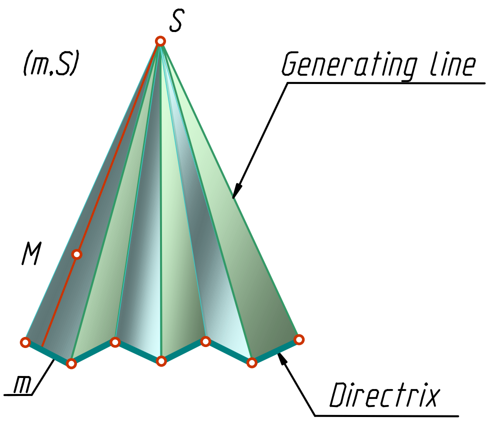


Fig. 5.2

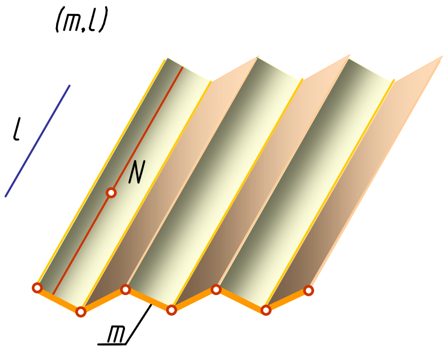


Fig. 5.3

A *pyramidal* surface is a surface obtained by a travel of a linear generatrix along a polygonal directrix. Note: All generating lines pass through a certain fixed point *S*.

A *prismatic* surface is a surface obtained by a travel of a linear generatrix along a polygonal directrix. Note: All generating lines are parallel to a certain given direction *l*.

The points *M* and *N* belong to a pyramidal and prismatic surfaces respectively, as they belong to the straight lines contained in these surfaces.

A part of space bounded in all directions by a surface is called a *body*.

A body bounded by plane polygons is called a *polyhedron*. Among all polyhedrons only prisms and pyramids are considered in this textbook.

Prism is a polyhedron with the bases being equal mutually parallel faces and the sides being parallelograms. If the edges of the sides are perpendicular to the base, the prism is a right prism.

To specify a prism it is necessary to specify its base and a lateral edge (Fig.5.4).

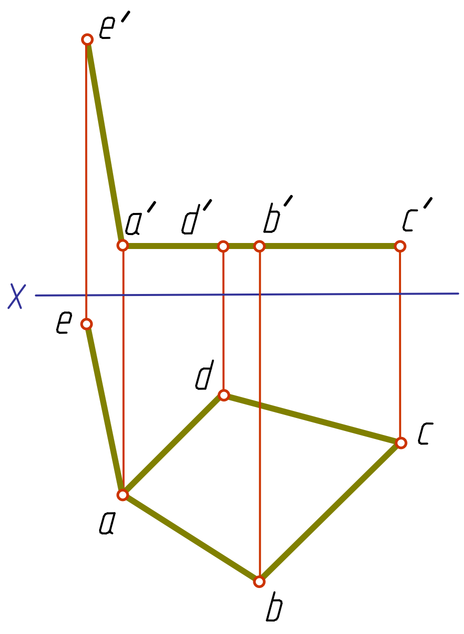


Fig. 5.4

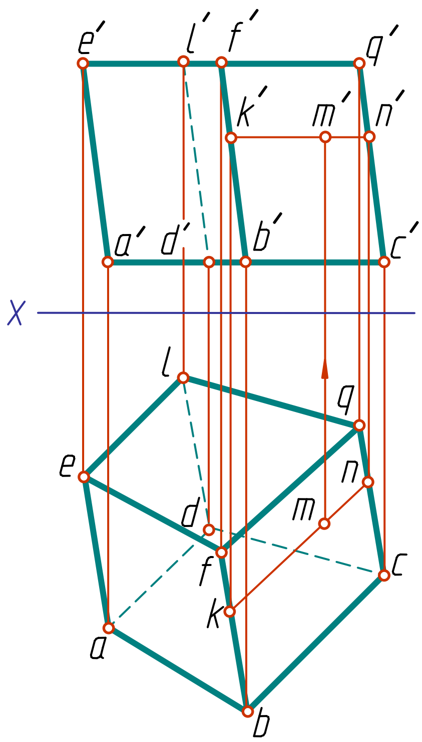


Fig. 5.5

Construct the edges *DL*, *BF* and *CQ* parallel and equal in length to the given edge *AE* to determine the second base and, hence, all the prism faces (Fig. 5.5).

To construct a lacking projection point lying on a polyhedron face draw a straight line through the above point. E.g. Given: horizontal projection of the point M belonging to the face *BCQF*. To construct its frontal projection draw the line *KN* through the given point. The point m′ is a desired point belonging to the projection *k*′*n*′.

Pyramid is a polyhedron (Fig. 5.6), one face of which is an arbitrary polygon *ABCD* taken for the base, the other faces (lateral) are the triangles with the common vertex *S* being called the vertex of pyramid.

To specify a pyramid it is necessary to specify its base and vertex. To construct projections of a point on a pyramid surface pass a line through this point like it was shown above (Fig. 5.5) for a prism.

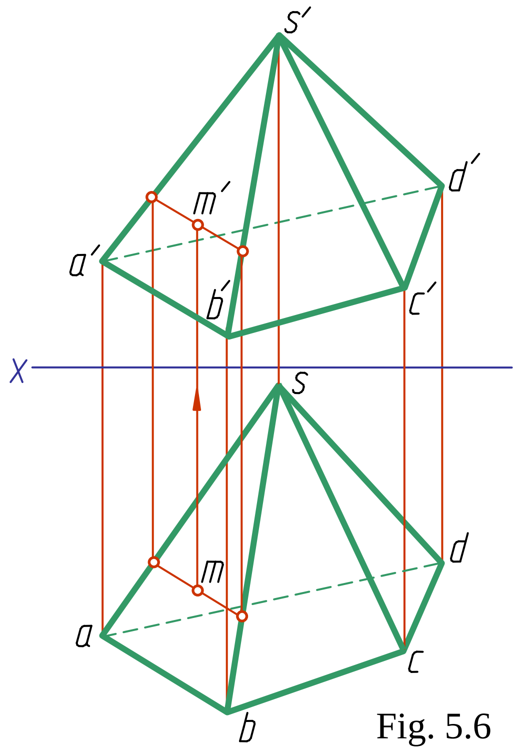


Fig. 5.6

A Polyhedron Cut by a Plane

When polyhedral surfaces are cut by planes we obtain polygons in the section, whose vertices are determined as the points of intersection of the polyhedron edges with a cutting plane.

A polygon obtained by cutting may be determined in two ways:

1. Its vertices may be found in the points of intersection of straight lines (the edges) with a cutting plane;

2. Its sides may be distinguished as the lines of intersection of polyhedron planes (the faces) with a cutting plane.

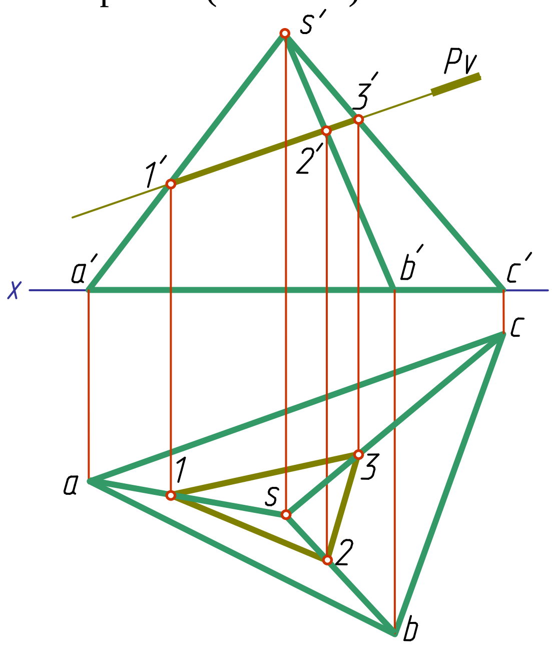


Fig. 5.7

Consider an example of construction of a pyramid cutting by a frontal projecting plane *P* (Fig. 5.7).

The cutting plane is a frontal projecting plane, therefore, all lines lying in this plane, including the section figure, coincide with the frontal trace P_V of the plane *P*. Thus, the intersection of the frontal projections of the pyramid edges with the trace P_V yields the frontal projection of the section figure *1′-2′-3′*. Find the horizontal projections of the points *1, 2, 3* by means of the connection lines on the horizontal projections of the corresponding edges.

A Pyramid with a Notch

As an example of drawing a polyhedron cutting by a few planes consider construction of a pyramid with a notch produced by three planes (*P*, *R* and *T*) (Fig. 5.8).

The plane *P*, parallel to the horizontal projection plane, intersects the pyramid surface along a pentagon *1-2-3-K-6*. The pentagon sides are parallel to the sides projections of the pyramid base on the horizontal projection plane. After the horizontal projection of the pentagon has been constructed, denote the points *4* and *5*.

The frontal projecting plane *R* cuts the pyramid along the pentagon *1-2-7-8-9*. To find the horizontal projections of the points *8* and *9* pass through them additional generatrices *SM* and *SN*. First do it on the frontal projection - *s′m′* and *s′n′*, then on the horizontal projection - *sm* and *sn*.

The frontal projecting plane *T* cuts the pyramid in pentagon *4-5-10-9-8*. Having constructed the horizontal notch projection, draw its profile projection.

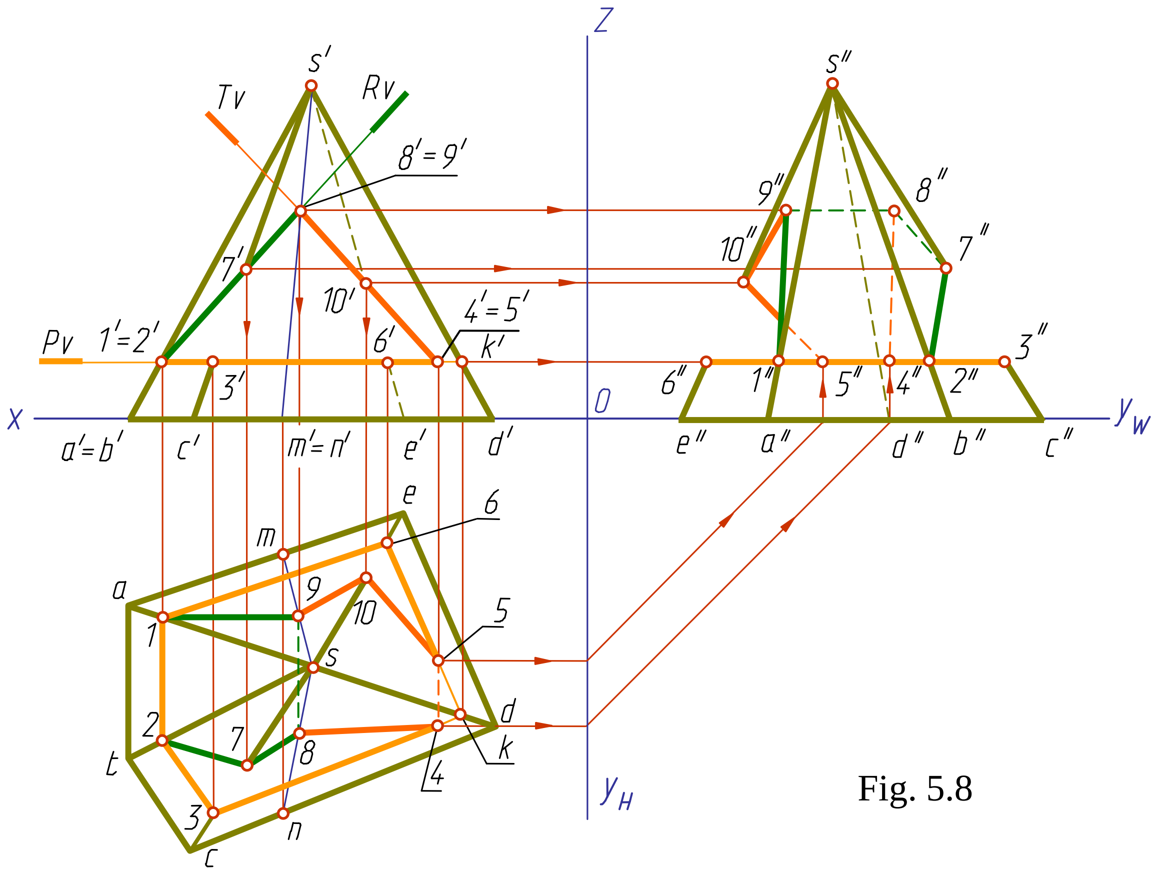


Fig. 5.8

5.4 Conical and Cylindrical Surfaces. Torses

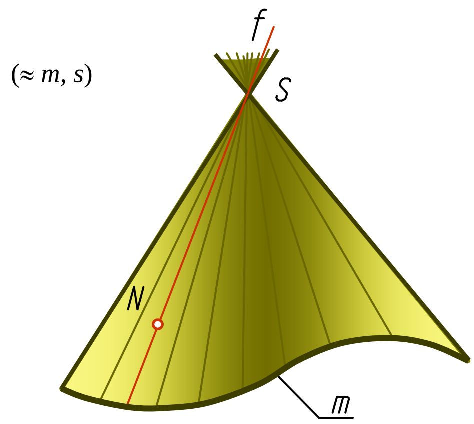


Fig. 5.9

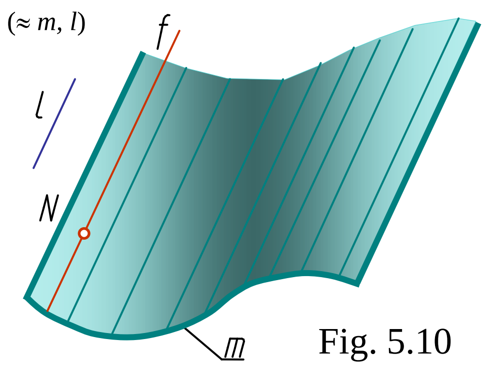


Fig. 5.10

The conical surface is produced by the motion of a linear generating line along a curved directrix. At that, the generatrix passes some fixed point *S*, referred to as a vertex (Fig. 5.9).

The cylindrical surface is produced by parallel to a given straight line *l* motion of a linear generating line along a curved directrix (Fig. 5.10).

The point *N* belongs to the given surfaces as it belongs to their generatrix *B*.

A conical surface is considered to be distinguished in a drawing when a directrix (by form and position) and a vertex are specified. Depending on the directrix form the conical surface may be closed or not. A body bounded by a conical surface and a plane is called a cone. If the base of a cone is a circle, the cone may be circular.

A cylindrical surface is considered to

be distinguished if a directrix (by form and position) and a generatrix (by position) are specified. To draw a cylindrical surface It is advisable to take as a directrix a line of intersection of this surface with a projection plane or another plane parallel to it.

Cylindrical surface may also be closed or not. A body bounded by a cylindrical closed surface and two parallel planes is called a cylinder. Cylindrical surfaces are distinguished by the view of its normal section, i.e. a curve obtained by cutting the surface with a plane perpendicular to its generating lines.

Torse

Torse (a surface with a cuspidal edge) is obtained by the motion of a linear generating line tangential in all its positions to a space curve, referred to as a cuspidal edge (*tors* (Fr.) - twisted, swirled).

A cuspidal edge is a directrix of a torse. A torse consists of two sheets separated by a cuspidal edge (Fig. 5.11).

If a cuspidal edge turns into a point, the torse surface turns into a conical surface. In case the cuspidal edge turns into a point at infinity, the torse surface becomes a cylindrical one.

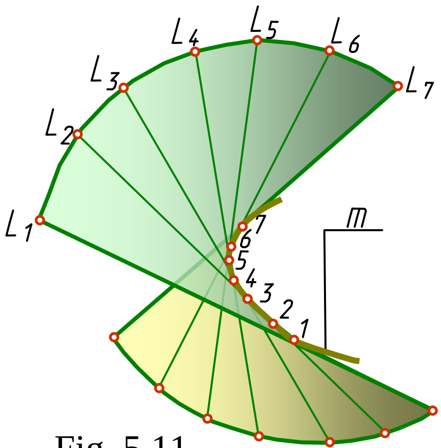


Fig. 5.11

5.5 Rotation Surfaces. Rotation Surface Cut by a Plane

Rotation surface is a surface described by a curve (or a straight line), rotating on its axis (Fig. 5.12). This surface is represented in a drawing by specifying its generatrix and rotation axis.

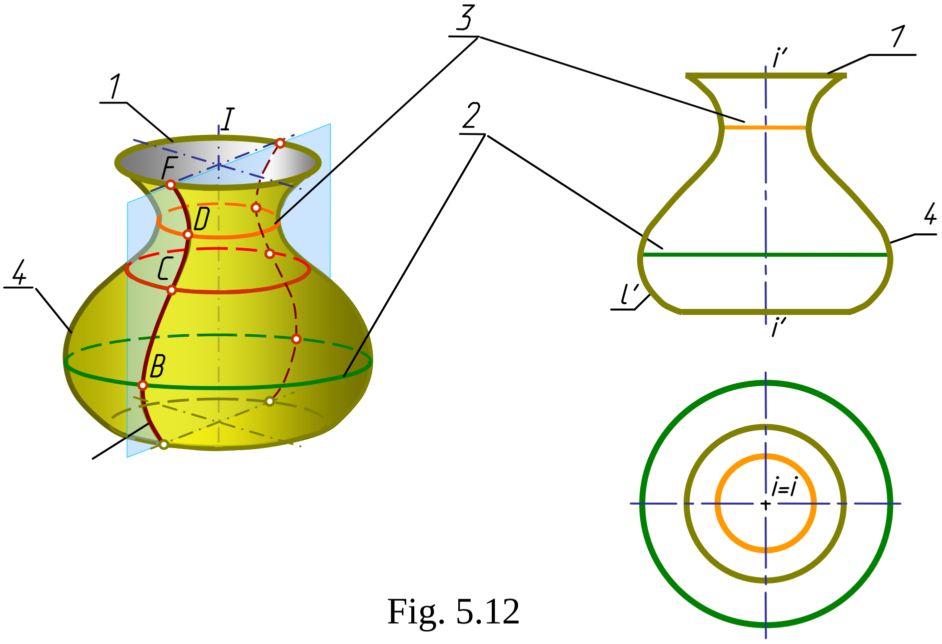


Fig. 5.12

Each point of a generatrix describes, rotating, a circle with the centre on the axis. These circles (say, circle *1*) are called parallels. The largest of them (circle *2*) is called an equator, the smallest (circle *3*) - a gorge circle.

The curves obtained by cutting a rotation body by the planes, passing through the axis, are called meridians. Meridian *4*, parallel to the frontal projection plane, is referred to as the principal meridian. All meridians are equal to each other.

In a drawing the axis of a surface rotation is positioned perpendicular to one of the projection planes, say, horizontal one. Then all parallels are projected on this plane in true size, and the equator and gorge circle determine a horizontal outline of the surface. A meridian located in the frontal plane, that is the principal meridian *4*, is considered to be a frontal outline.

The points on a rotation surface are constructed by means of parallels (i.e. the circles on the surface).

Let us consider some bodies and rotation surfaces.

1. Surfaces obtained by rotation of a straight line:

a) *cylinder* of rotation - this is a surface produced by rotation of the line *L* round the axis *I* parallel to it (Fig. 5.13);

b) *cone* of rotation - this is a surface produced by rotation of the line *L* round the axis *I* intersecting it (Fig. 5.14);

c) *one sheet hyperboloid* of rotation - this is a surface produced by rotation of the line *L* round the skew axis *I* (Fig. 5.15).

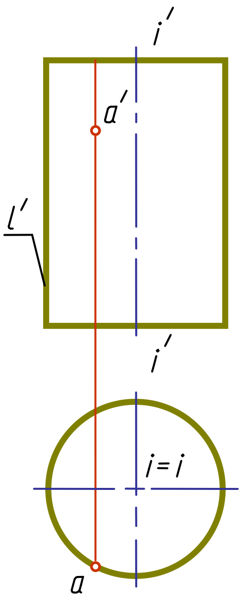


Fig. 5.13

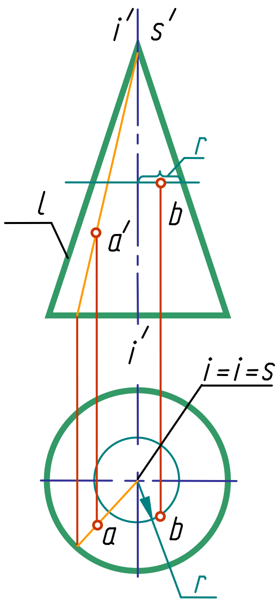


Fig. 5.14

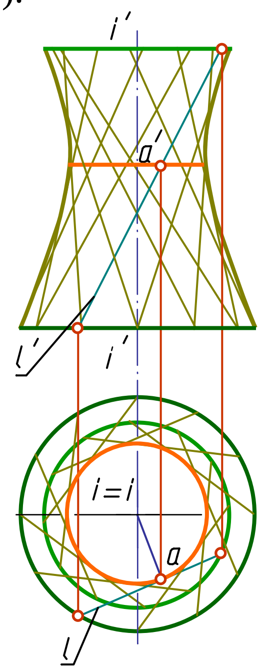


Fig. 5.15

The point *A*, lying on a perpendicular to the rotation axis and generating line, describes the smallest circle, which is the gorge of hyperboloid. One sheet hyperboloid may also be obtained by rotation of hyperbola on its conjugate axis.

2. Surfaces obtained by rotation of a circle round a fixed axis:

a) *sphere* - this is a surface produced by rotation of a circle round its diameter (Fig. 5.16);

b) *torus* - this is a surface produced by rotation of a circle round the axis I lying in the plane of this circle but not passing through its centre (Fig. 5.17 through 5.20). If the axis of rotation passes beyond a circle, the surface is called an open torus or a ring-torus (Fig. 5.17); if the axis is tangential to a circle, it is a closed torus (Fig.5.18); if the axis intersects a circle - it is self-intersecting torus (Fig. 5.19, 5.20).

The torus presented by Fig. 5.19 is also called an apple-torus, the one from Fig. 5.20 - a lemon-torus.

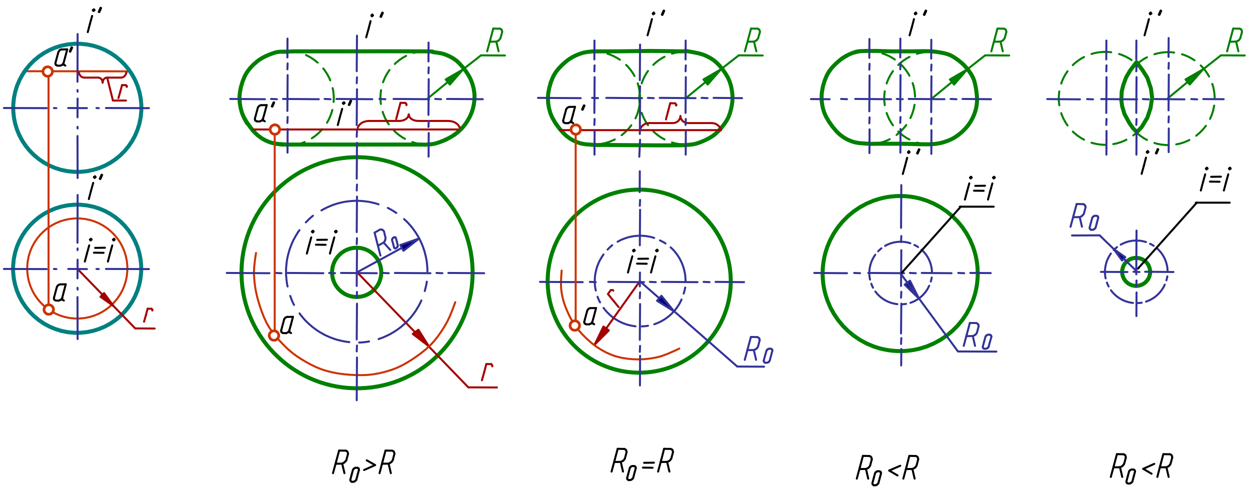
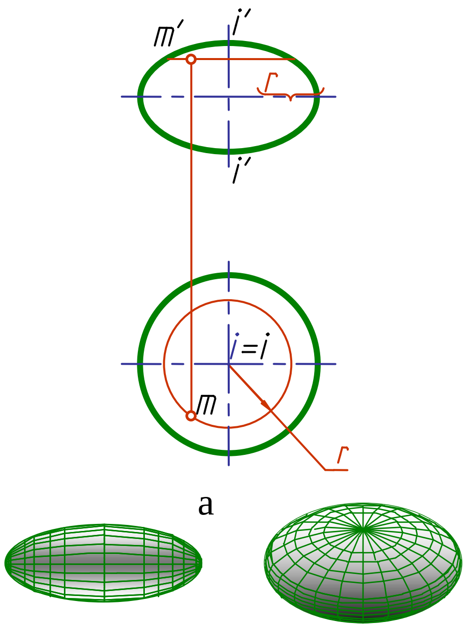


Fig. 5.16 Fig. 5.17 Fig. 5.18 Fig. 5.19 Fig.5.20

3. *Rotation surfaces obtained by the curves of the second order:*



b c

Fig.5.21

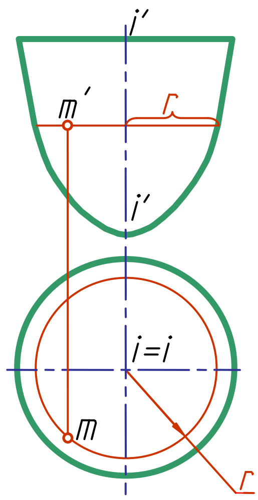


Fig.5.22

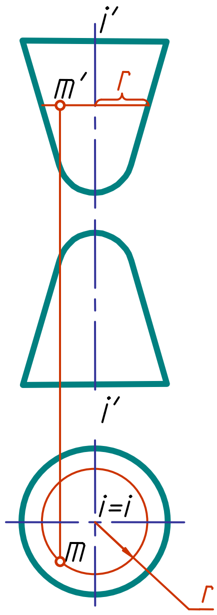


Fig.5.23

a) *ellipsoid* of rotation - this is a surface produced by rotation of an ellipse on its axis (Fig.5.21). Surface obtained by rotation on its major axis is called an oblong ellipsoid of rotation, on its minor axis - an oblate one;

b) *paraboloid* of rotation - this is a surface produced by rotation of a parabola on its axis (Fig.5.22);

c) two sheet *hyperboloid* of rotation - this is a surface produced by rotation of a hyperbola on its axis (Fig.5.23).

Drawing a Projection of an Intersection Line of a Cylinder Cut by a Plane

When a cylinder of rotation is cut by a plane parallel to the rotation axis, a pair of straight lines (generatrices, Fig.5.24) appears in the section. If a section plane is perpendicular to the axis of rotation, the cutting results in a circle (Fig. 5.25). Generally, when a cutting plane is inclined to the rotation axis of a cylinder, an ellipse is obtained by cutting (Fig.5.26).

Let us consider an example of drawing an intersection line of a cylinder and a frontal projecting plane *Q*, when an ellipse is obtained in the section (Fig.5.27).

The frontal projection of the section line, in this case, coincides with the frontal trace of the plane Q_V , the horizontal one - with the horizontal projection of the cylinder surface (a circle). The profile projection of the line is constructed by two projections available, the horizontal and frontal ones.

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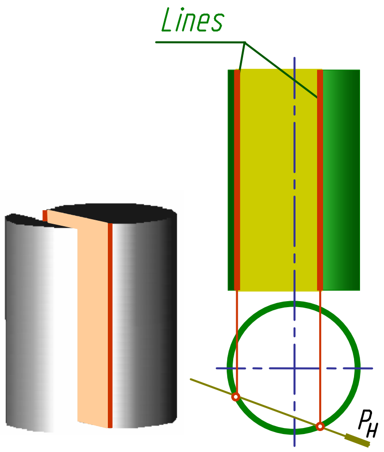


Fig.5.24

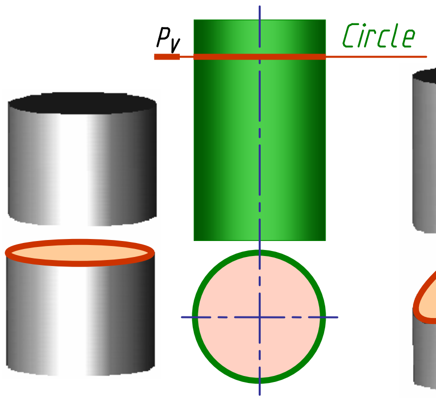


Fig.5.25

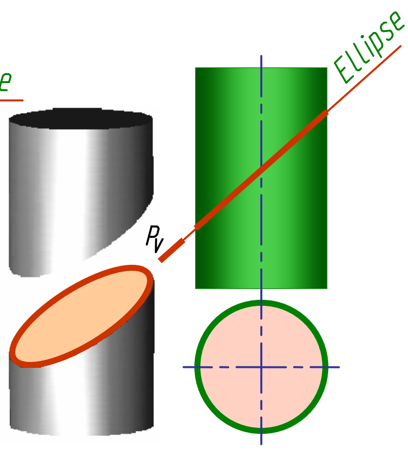


Fig.5.26

On the whole, drawing a line of intersection of a surface with a plane consists in distinguishing the common points, belonging to both, a cutting plane and a surface.

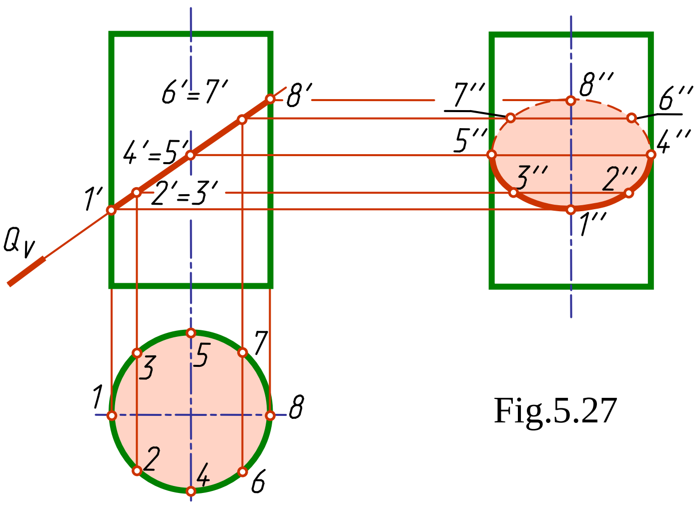


Fig.5.27

The method of auxiliary cutting planes is usually applied to find the above points:

1. Pass an auxiliary plane;
2. Construct the intersection lines of this plane with the surface and with the given plane;
3. Determine the intersection points of thus obtained lines.

The auxiliary planes should be drawn so, that they cut the surface along the prime (most simple) lines.

Determining the points of intersection start with determining the characteristic or control points. They are:

1. Upper and lower points;
2. A left and a right points;

3. Points of visibility bounds;
4. Characteristic points of a given intersection line (for an ellipse - points of major and minor axes).

To make the construction of the intersection line more accurate it is necessary to draw additional (passing) points as well.

In the example above points *1* and *8* are the lower and the upper points. For the horizontal and frontal projections point *1* is a left point, point *8* is a right one. For the profile projections points *4* and *5* are the points of visibility bounds: the points of the line of intersection located lower than *4* and *5* are visible, all the rest are invisible.

Points *2*, *3* and *6*, *7* - are additional, used for the drawing accuracy. The profile projection of the section figure is an ellipse, the minor axis of which is the segment *1*-*8*, the major one - *4*-*5*.

Drawing the Intersection Lines Projections of a Cone Cut by a Plane

Depending on the direction of a cutting plane, different lines, called the lines of conical sections, may be obtained in the section of a rotation cone.

If a cutting plane passes through a vertex of a cone, we get in its section a pair of generating lines (triangle) (Fig.5.28, a). As a result of intersection of a cone with a plane perpendicular to the cone axis, a circle is obtained (Fig.5.28, b). If a cutting plane is inclined to the rotation axis of a cone and does not pass through its vertex, an ellipse, parabola or hyperbola may be obtained in the section (Fig.5.28, c, d, e) - it depends on the size of inclination angle of the cutting plane.

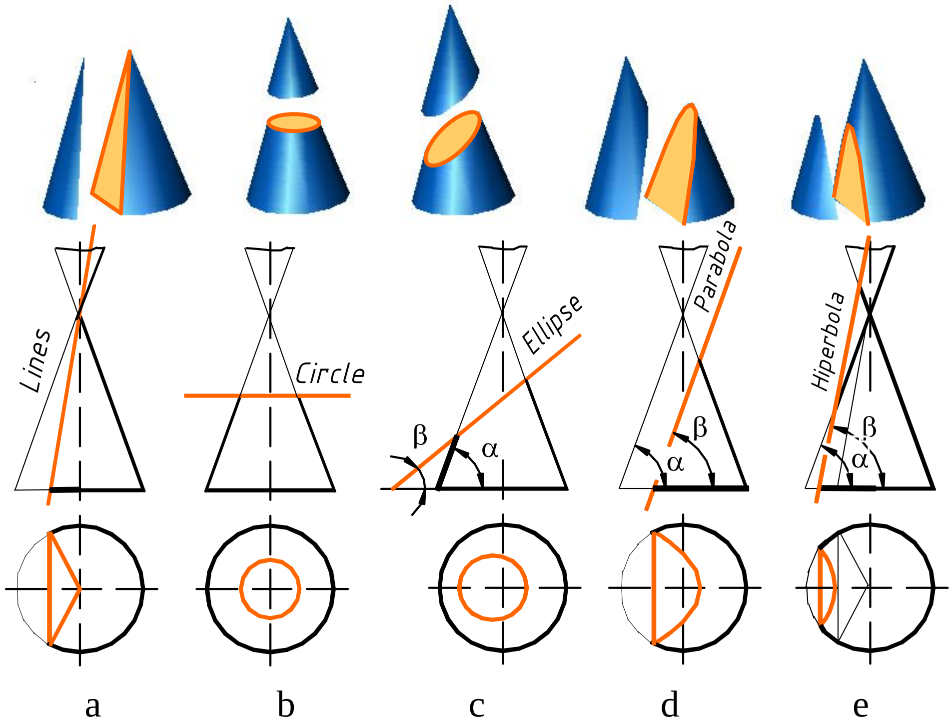


Fig.5.28

An ellipse is obtained when the inclination angle β is less than the inclination angle α of the cone generatrix to its base ($\beta < \alpha$), that is when a plane cuts all generating lines of a given cone (Fig.5.28, c).

In case the angles α and β are equal, i.e. a cutting plane is parallel to one of the generatrices of the cone, a parabola is obtained in the section. Here the cutting plane intersects all the generating lines except one to which it is parallel (Fig.5.28, d). If a cutting plane is inclined at an angle which changes in the following limits - $90° \geqslant \beta > \alpha$, a hyperbola is obtained in the section. The cutting plane here is parallel to two generating lines of the cone. The obtained hyperbola has two branches as the conical surface is of two sheets (Fig.5.28, e).

It is well-known that a point belongs to a surface if it belongs to any line of this surface. The prime lines of a cone are straight (generating) lines and circles. So, if a problem statement requires to find the horizontal projections of the points *A* and *B*, belonging to a cone surface, it is necessary to pass one of the prime lines through those points.

Find the horizontal projection of the point A by means of a generating line. To do that pass an auxiliary frontal projecting plane $P(P_V)$ through the point *A* and the cone vertex *S*. This plane intersects the cone along two generatrices *SM* and *SN*, the frontal projections of which coincide. Construct the horizontal projections of the generating lines, pass a connection line through the point a' and determine the horizontal projection of the point. There are two variants of answers in this problem - the points a_1 and a_2 (Fig.5.29).

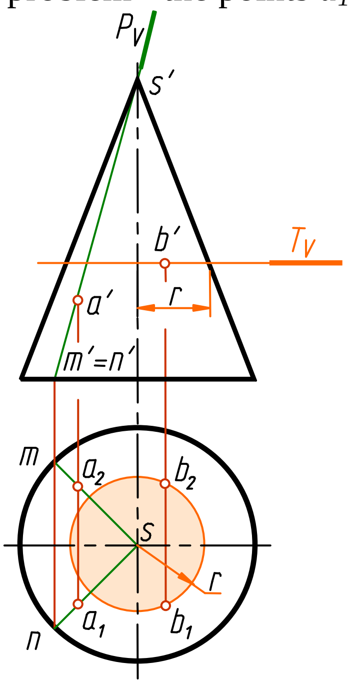


Fig.5.29

Find the horizontal projection of the point B by construction a circle on which it is located. For that pass through the point the horizontal plane $T(T_V)$. The plane intersects the cone in a circle of radius *r*. Construct the horizontal projection of this circle. Through the point *b′* pass a connection line to meet the circle. There are also two answers to this problem - the points b_1 and b_2.

Now let us consider an example of drawing the projections of an intersection line of the cone cut by the frontal projecting plane $P(P_V)$, when there is an ellipse in the section (Fig.5.30).

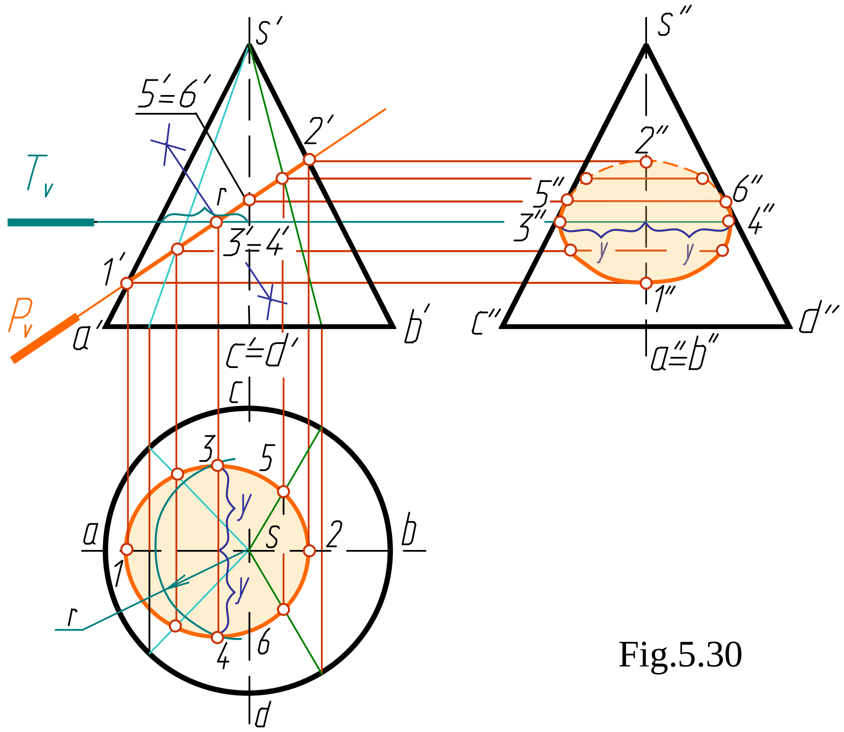


Fig.5.30

The frontal projection of the intersection line coincides with the frontal trace of the plane P_V.

To simplify the solution designate the utmost generatrices of the cone and determine the characteristic or control points.

The lower point *1* lies on the generatrix *AS*, the upper point *2* - on the generatrix *BS*. These points specify the value of the cone major axis. The minor axis is perpendicular to the major one. To find the minor axis bisect the segment *1-2*. Points *3* and *4* specify the cone minor axis. Points *5* and *6* located on the generating lines *CS* and *DS* are the points of visibility bound of the profile projection plane. The projections of points *1*, *2*, *5* and *6* are situated on the corresponding projections of the generating lines. To find the projections of points *3* and *4* pass an auxiliary cutting plane $T(T_V)$, which cuts the cone in a circle of radius *r*. The projections of the given points are located on this circle. On the horizontal projection plane the circle is projected in true size, so, if we draw the connection line, we find the horizontal projections of points *3* and *4*. Lay off the co-ordinates of points *3* and *4* from the cone axis *y* on the connection line to find the profile projections (Fig.5.30).

To make an accurate construction of an ellipse it is not enough to find the points mentioned above, hence, determine additional (arbitrary) points. Find the projections of these points in a similar fashion with points *3* and *4*, or by passing generating lines through these points. Having found the projections of all points, join them subject to visibility. All points lying on the cone surface are visible on the horizontal projection. On the profile one - only the line passing throgh points *5*, *3*, *1*, *4*, *6* are visible, the rest - are not.

Ball Surface (a Sphere)

Ball surface is a surface obtained by rotation of a circle round an axis of its diameter. A plane intersects a sphere always in a circle. This circle may be projected as:

- *a straight line* if the cutting plane is perpendicular to the projection plane;
- *a circle* of radius equal in length to distance from the axis of the sphere rotation to the outline (e.g. a circle of radius *r* (Fig.5.31) if the cutting plane is parallel to the projection plane;
- *an ellipse* if the cutting plane is not parallel to the projection plane.

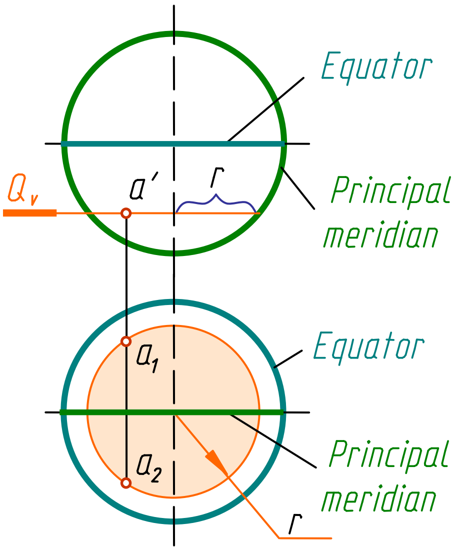


Fig. 5.31

To construct the projections of a point lying on a sphere surface it is necessary to pass through the point a cutting plane parallel to the projection plane and draw the circle on which the point is located.

Cutting a Sphere by a Plane

Intersect a sphere by the frontal projecting plane *Q*. To begin with, determine the characteristic points. Points *1* and *2* are located on the principal meridian. These points are the ends of the ellipse minor axis, also are the highest and the lowest points. Construct their horizontal and profile projections by means of the frontal projections. Points *3* and *4* are situated on a profile meridian and specify visibility on the profile projection plane.

Find the horizontal projections of the points by their profile projections. Points 5 and 6 are situated on the equator and are the points of visibility bounds on the horizontal projection. Find the profile projections of the points by their horizontal ones. Points *7* and *8* belong to the ends of the ellipse longer axis. Construct them in the following way: First find the middle point of the line-segment *1′-2′* - this is the frontal projection of the point *O′*, the circle centre of the section. Then find its horizontal projection, the point *O*. The line-segments *O′1′* and *O′2′* on the frontal projection are equal to the true size of the circle radius. On the horizontal projection the diameter of the circle is projected without shortening, hence, lay off the segments *O7* and *O8* equal in length to *O′1′*. To construct more accurate section line it is necessary to find a few additional points. Use auxiliary cutting planes for that, as shown in Fig.5.32. Then join the points thus obtained in a smooth curve subject to its visibility.

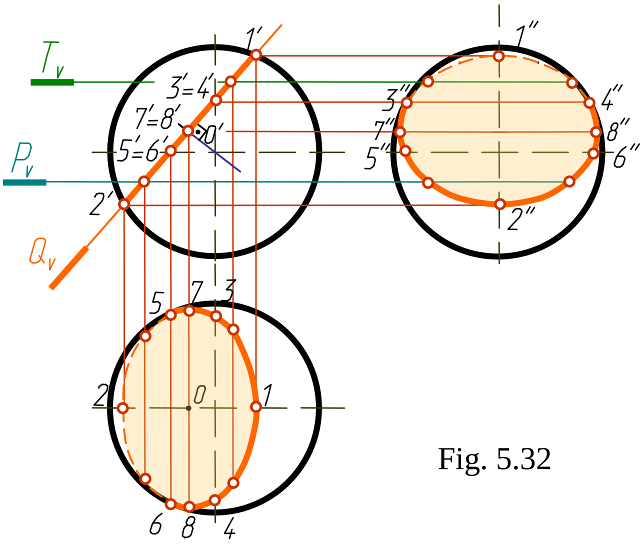


Fig. 5.32

Fig.5.32

5.6 Screw Surfaces

A screw surface is a surface described by a generatrix at its helical motion.

If a generatrix of a screw surface is a straight line the surface is referred to as *a ruled screw surface or a helicoid* (*helice (Fr.)*-a spiral, a spiral staircase). A helicoid may be right or oblique depending on the generating line being perpendicular or inclined to the helicoid axis.

A few kinds of a ruled screw surface:

1**.** *A right helicoid* is produced by the motion of the linear generatrix *l* along two directrices, one of which is the cylindrical screw line *m*, the other is its axis *i*. Note that in all its positions the line l is parallel to the plane perpendicular to the axis *I* (called a plane of parallelism). Usually one of the projection planes is taken for the plane of parallelism (Fig.5.33). The generatrix *l* of the right helicoid intersects the screw axis *I* at a right angle. The right helicoid may be referred to as one of the conoids and called a *screw conoid*.

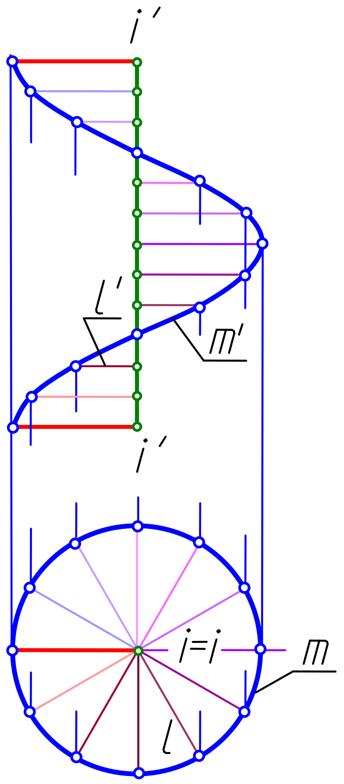


Fig.5.33

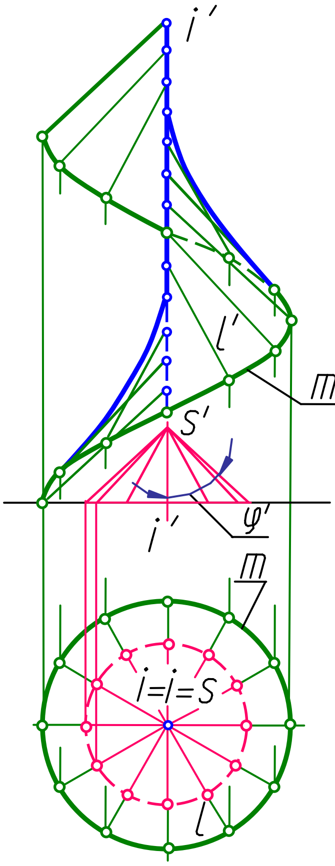


Fig.5.34

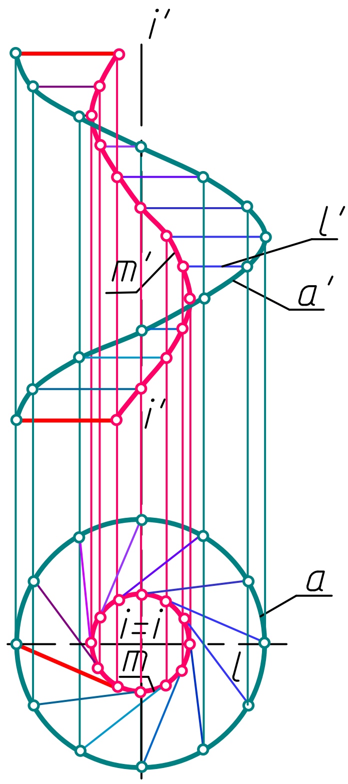


Fig.5.35

2. *An oblique helicoid* is distinguished from a right one by its generatrix *l* intersecting the helicoid axis at a constant angle α different from the right angle. In other words, the generatrix *l* of an oblique helicoid slides along two directrices, one of which is the cylindrical screw line *m*, the other - its axis *I*. Note that in all its positions the line *l* is parallel to the generating lines of a certain cone of rotation. The angle of this cone included between the generating line and the axis parallel to the helicoid axis, is equal to φ. It is called a director cone of an oblique helicoid.

Fig.5.34 shows the construction of the oblique helicoid projections. The cylindrical screw line m and the axis *I* are the directinal lines of the helicoid. They are parallel to the corresponding generating lines of the director cone.

3. *A developable helicoid* is produced by motion of the linear generatrix *l* tangential in all its positions to the cylindrical screw line *m*, the last being the helicoid cuspidal edge (Fig.5.35). The developable helicoid being a ruled surface with a cuspidal edge is considered to be one of the torses.

The surface of a developable helicoid is bounded by the cuspidal edge *m* and the line *a* obtained by intersection of helicoid surface with a surface of coaxial cylinder of a larger diameter (than the diameter of the screw line *m*).

If the generating line *l* intersects the surface axis, the helicoid is referred to as a closed one (Fig.5.33 and 5.34), if not - as an open one (Fig.5.35).

5.7 Mutual Intersection of Surfaces

The line of intersection of two surfaces is the locus of the points belonging to both surfaces.

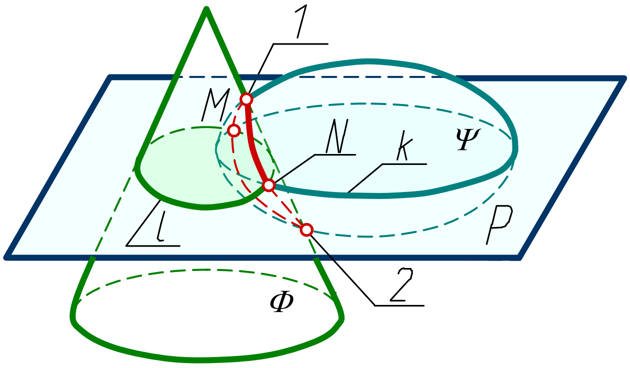


Fig. 5.36

A general method of drawing the points belonging to a curve of mutual intersection of surfaces is the method of auxiliary surfaces-mediators. This method is similar to the method of construction intersection lines of surfaces cut by planes and consists in the following:

Take some intersecting surfaces *Φ* and *Ψ* (Fig.5.36). Introduce an auxiliary plane *P* intersecting the surfaces along the lines *l* and *k* which yields the points *M* and *N* belonging to the intersection curve.

As the surfaces-mediators very often planes or ball surfaces (spheres) are used. Depending on mediators the following main methods of construction an intersection line of two surfaces are distinguished:

a) method of auxiliary cutting planes;

b) method of auxiliary spheres.

Like in construction an intersection line of surfaces with planes, to draw a line of mutual intersection of surfaces it is necessary to draw, first, control points of a curve, as these points present the bounds of intersection line, between which the passing (arbitrary) points should be determined.

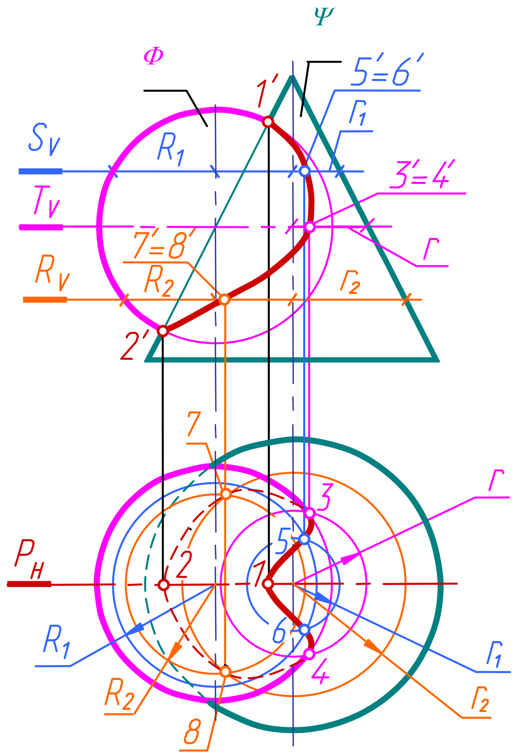
Method of Auxiliary Cutting Planes

The following example illustrates how to apply the method of auxiliary cutting planes for construction of an intersection line of a sphere with a cone of rotation (Fig.5.37).

To construct an intersection line of the given surfaces it is advisable to introduce the frontal plane *P* and a number of horizontal planes (*S*, *T*, *R*) as the auxiliary surfaces.

Start with determination of the characteristic points projections. Draw the frontal plane $P(P_H)$. This plane intersects the surfaces along the outlines. Find the frontal projections of the highest and the lowest points (*1′* and *2′*) as the points of intersection of the outlines. Pass the connection lines to determine the horizontal projections *1* and *2*.

The auxiliary horizontal planes cut the sphere and the cone in circles.



Gig. 5.37

Find the projections *3′* and *4′* of the points lying on the sphere equator with the help of horizontal plane $T(T_V)$, which passes through the centre of the sphere. The plane intersects the sphere along the equator and the cone in the circle of radius *r*. The horizontal projections intersection of the latest yields the horizontal projections *3* and *4*. The horizontal projections of points *3* and *4* are the points of visibility bounds of the intersection line on this projection. Determine the passing points (*5, 6, 7, 8*) by means of auxiliary horizontal planes $S(S_V)$ and $R(R_V)$. Join the points thus obtained in a smooth curve subject to visibility.

Method of Auxiliary Spheres

This method is widely used for solution of problems on construction the intersection lines of rotation surfaces with intersecting axes.

Before studying this method let us consider a particular case of intersection of rotation surfaces, the axes of which coincide, i.e. a case of intersection of coaxial surfaces of rotation.

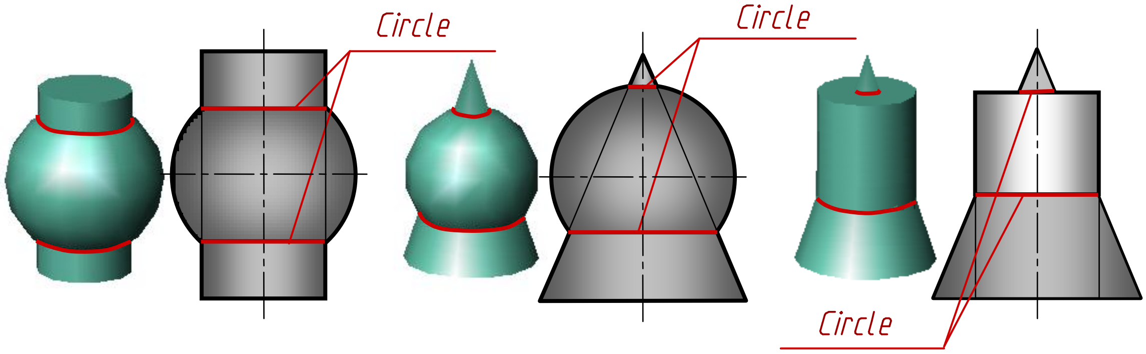


Fig.5.38

Coaxial surfaces intersect in a circle, the plane of which is perpendicular to the axis of rotation surfaces. At that, if the axis of rotation surfaces is parallel to the projection plane, the intersection line projects onto this plane as a line-segment (Fig.5.38).

This property is used for construction a line of mutual intersection of two rotation surfaces by means of auxiliary spheres. Here may be used both, concentric (constructed from one centre) and eccentric (drawn from different centres) spheres. We are going to consider application of auxiliary concentric spheres, those with a constant centre.

Note: if a plane of rotation surface axes is not parallel to the projection plane, the circles in which the surfaces intersect, are projected as ellipses and this make the problem solution more complicated. That is why the method of auxiliary spheres should be used under the following conditions:

a) intersecting surfaces are the surfaces of rotation;

b) axes of the surfaces intersect and the intersection point is taken for the centre of auxiliary spheres;

c) the plane produced by the surfaces axes (plane of symmetry) is parallel to one of the projection planes.

Using this method it is possible to construct the line of intersection of the surfaces on one projection.

Let us consider an example of drawing an intersection line of a cylinder and a cone of rotation (Fig.5.39).

The points *1, 2, 3, 4* are determined as the points of level generatrices of the surfaces belonging to the plane of axes intersection (the plane of symmetry $P(P_H)$). Find the other points by method of auxiliary spheres.

From the intersection point of the given surfaces (point *O′*) draw an auxiliary sphere of an arbitrary radius. This sphere is simultaneously coaxial to the cone and the cylinder and cuts them along the circumferences, the planes of which are perpendicular to the corresponding rotation axes. The frontal projections of those circles are line-segments. The sphere constructed intersects the cone along the circumference of diameter *AB* (*a′b′*), and the cylinder - along the circumferences *CD* (*c′d′*) and *EF* (*e′f′*). In the intersection points of the circle *AB* with the circles *CD* and *EF* obtain points *9-10* and *11-12* respectively, which belong to the intersection line.

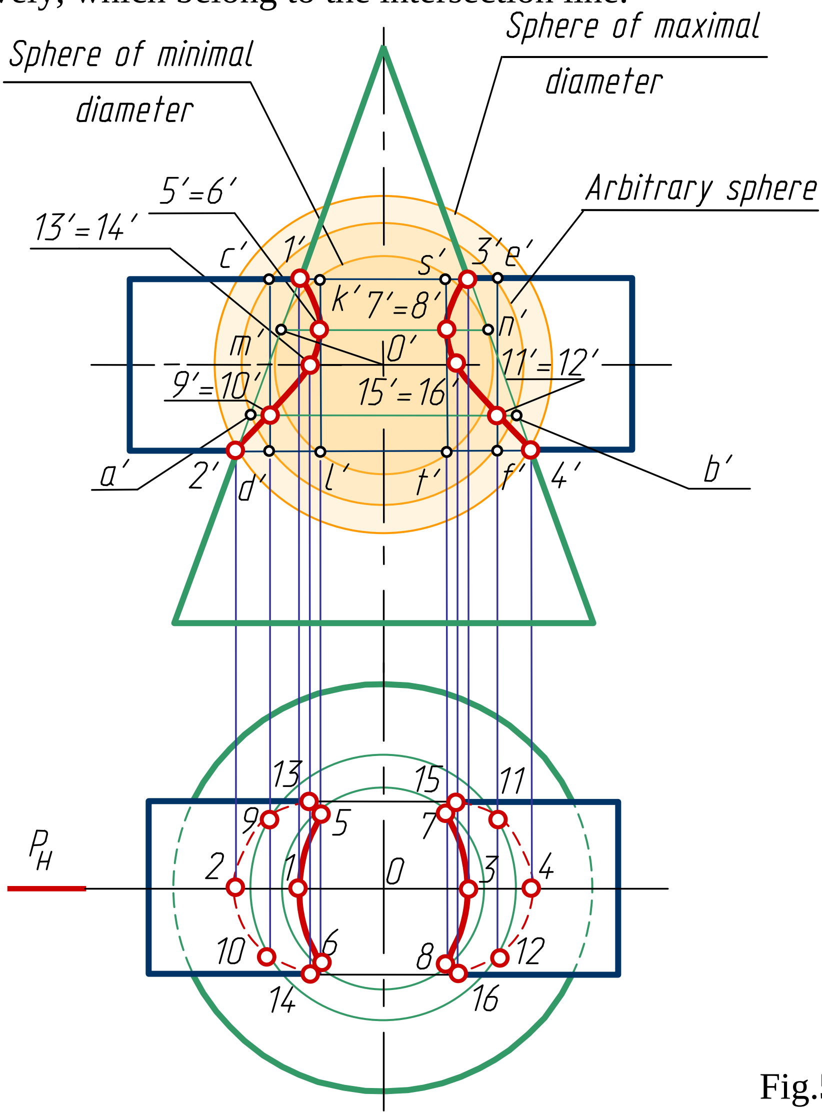


Fig.5.39

In such a way it is possible to construct a certain amount of points of the desired intersection line. Students should note that not all the spheres may be used for the problem solution. Consider the limits of the auxiliary spheres usage.

The maximal radius of a cutting sphere is equal to the distance from the centre O to the farthest intersection point of the level generatrices (from *O′* to *2′* and *4′*). The minimal cutting sphere is a sphere, which contacts one surface (the larger one) and cuts another (the smaller one). In the example above the minimal sphere contacts the cone surface in the circle *MN* (*m′n′*) and intersects the cylinder along the circumferences *KL* (*k′l′*) and *ST* (*s′t′*). Meeting each other the circles *MN* and *KL* yield the points of intersection line *5*(*5′*) and *6*(*6′*), and the circles *MN* and *ST* yield the points *7*(*7′*) and *8*(*8′*). These are the deepest points of the intersection line.

If an auxiliary sphere cuts only one given surface, this sphere is not proper for the problem solution.

$$R_{max} > R_{pass} > R_{min}$$

To construct the second projection of the intersection line one may use the circles obtained at cutting the cone by auxiliary spheres, or to draw the additional sections of the surface. Points *13-14* and *15-16* lying on the level generatrices of the cylinder, are the points of visibility bounds of the intersection line on the horizontal projection.

Possible Cases of Intersection of Curved Surfaces

There are four variants of two surfaces meeting:

1. Permeability. All generating lines of the first surface (cylinder) intersect the other surface, but not all generatrices of the second surface intersect the first one. In this case the intersection line of the surfaces decomposes into two closed curves (Fig.5.40).

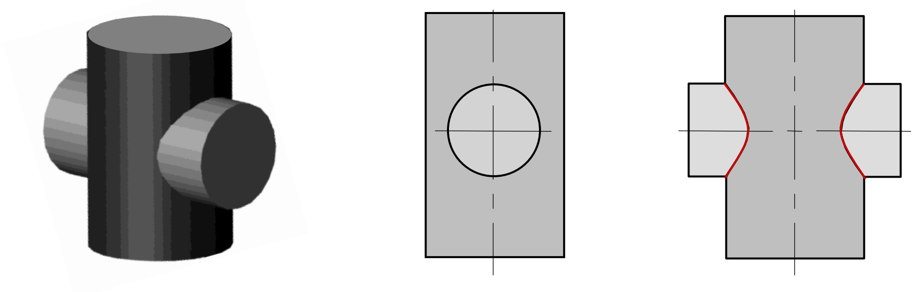


Fig.5.40

2. *Cutting-in*. Not all generatrices of both surfaces intersect each other. In this case the intersection line is one closed curve (Fig.5.41)

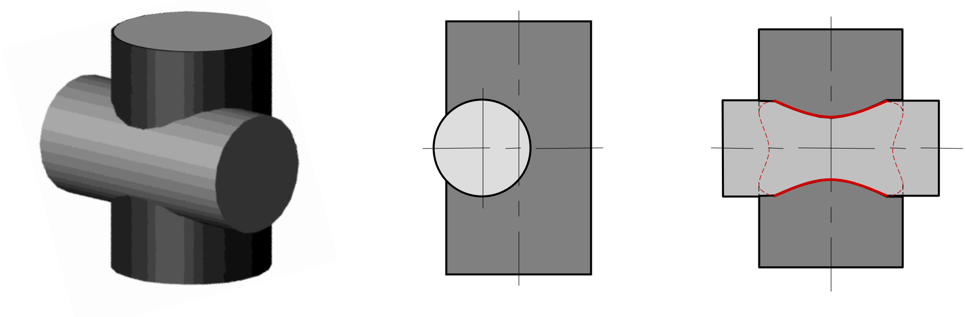


Fig.5.41

3. *Unilateral contact*. All generating lines of one surface intersect the other surface, but not all generatrices of the second surface intersect the first

one. There is a common tangent plane in one point of the surfaces (the point K, Fig.5.42). The intersection line decomposes into two closed curves meeting in the point of contact.

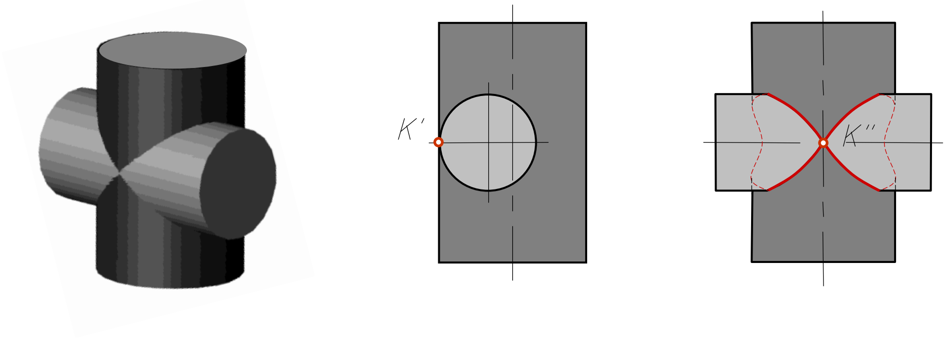


Fig.5.42

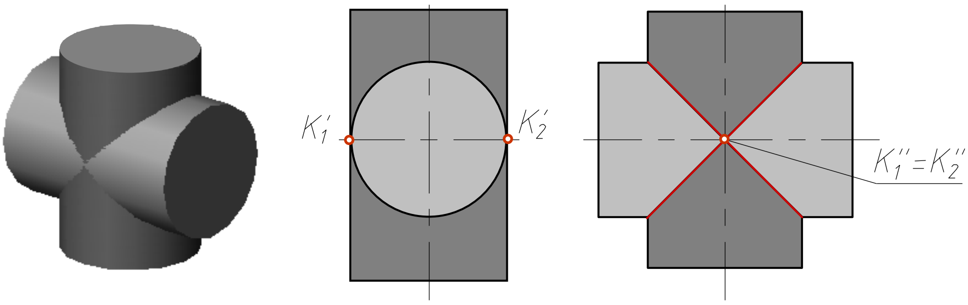


Fig.5.43

4. *Bilateral contact.* All generating lines of both surfaces intersect each other. The intersecting surfaces have two common tangent planes. In this case the intersection line decomposes into two plane curves which meet in the points of contact (Fig.5.43).

Intersection of the Surfaces of the Second Order

Generally two surfaces of the second order intersect along a spatial curve of the forth order. But in some special relative positions the surfaces of the second order may meet along the plane curves of the second order, that is the spatial curve of intersection decomposes into two plane curves.

1. If two surfaces of the second order have two common points through which two common tangent planes may be passed to them, the line of their mutual intersection decomposes into two plane curves of the second order, and the planes of the above curves pass through the straight line connecting the tangent points.

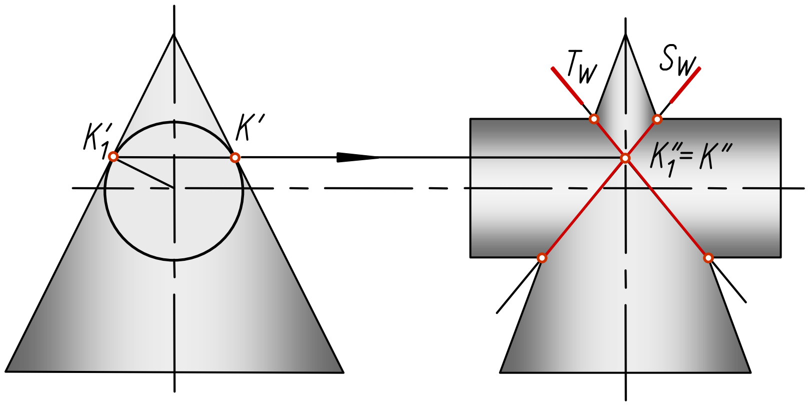


Fig.5.44

Fig.5.44 shows the construction of the intersection line of the surfaces of a right circular cylinder and an elliptic cone. At the common points *K* and *K1* these surfaces have common frontal projecting tangent planes $P(P_V)$ and $R(R_V)$. The lines of intersection (ellipses) lie in the profile projecting planes *S* and *T*, passing through the line KK_1 which connect the tangent points.

2. *Monge theorem*. If two surfaces of the second order may be inscribed into the third one or described around it, the line of their mutual intersection decomposes into two plane curves. The planes of those curves pass through a straight line connecting the intersection points of the tangent lines.

Fig.5.45, 5.46 and 5.47 present the examples of construction of the surfaces intersection lines on the basis of Monge theorem, where two cylinders, a cylinder and a cone and two cones are described around a sphere.

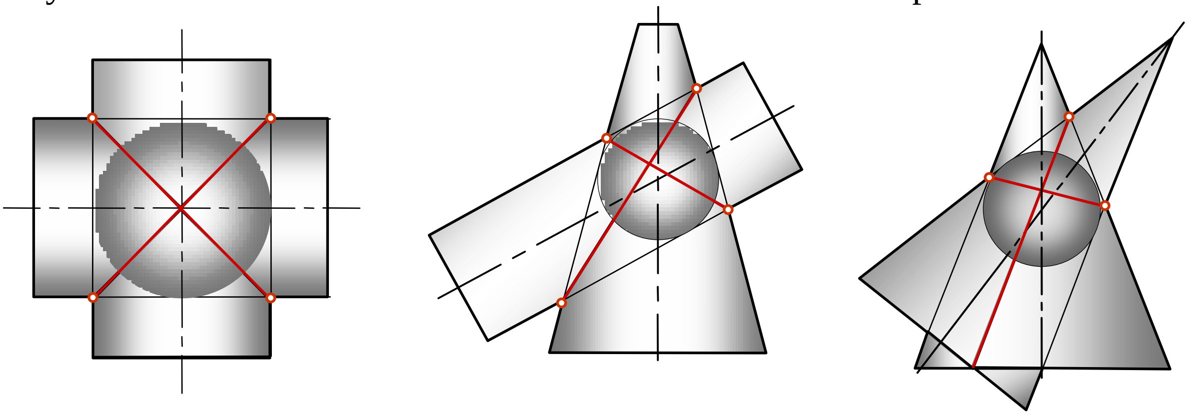


Fig.5.45 Fig.5.46 Fig.5.47

The Fig.5.48 illustrates construction of intersection line of two oblate ellipsoids of rotation which inscribed info sphere.

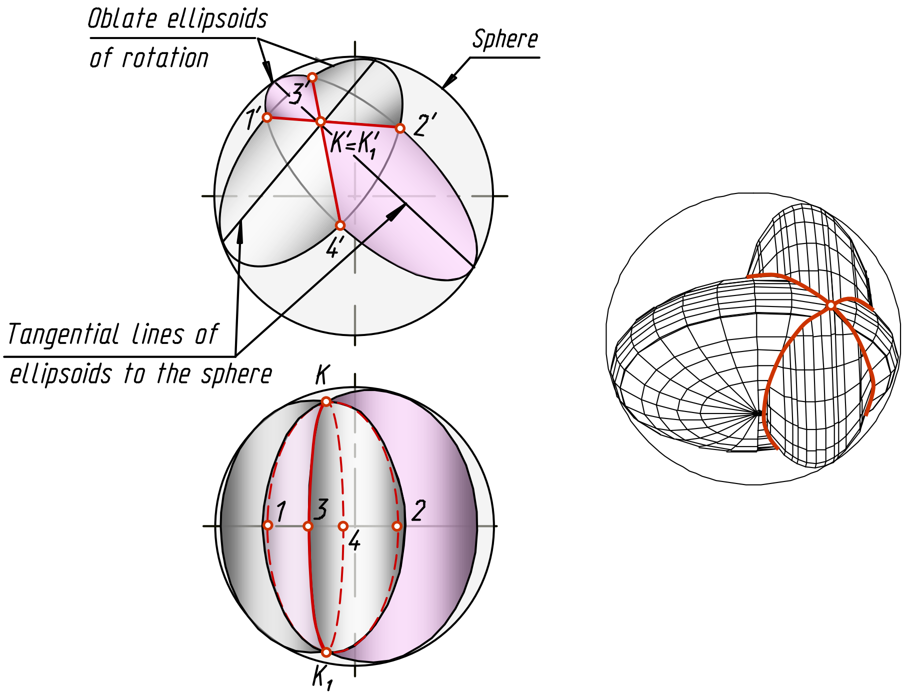


Fig.5.48

Questions to Chapter 5

1. What is “surface”?
2. What is the meaning of the expression “To specify a surface in a drawing”?
3. What surfaces are called “ruled surfaces”?
4. What is the difference between the polyhedral surfaces and polyhedrons?
5. What is the condition of a point belonging to a surface?
6. How do we obtain the surfaces of rotation?
7. What lines on a surface of rotation are referred to as parallels and meridians?
8. How is a surface of helicoid formed?
9. What lines are produced by intersection of a rotation cylinder with the planes?
10. What lines are produced by intersection of a rotation cone with the planes?
11. How to pass a plane to obtain a circle in a torus section?
12. What is the general method of drawing the intersection line of surfaces?
13. In what cases do we use projection planes, spheres as mediators for the construction of intersection lines of surfaces?
14. What points of an intersection line are referred to as control ones?
15. Give the formulation of Monge theorem and introduce the example of its application in practice.