CHAPTER 4. REPRESENTATION OF A PLANE IN A DRAWING

4.1 Ways of Specifying a Plane

The position of a plane on a drawing may be specified in one of the following ways (Fig.4.1):

a) by the projection of three points not lying on one line;

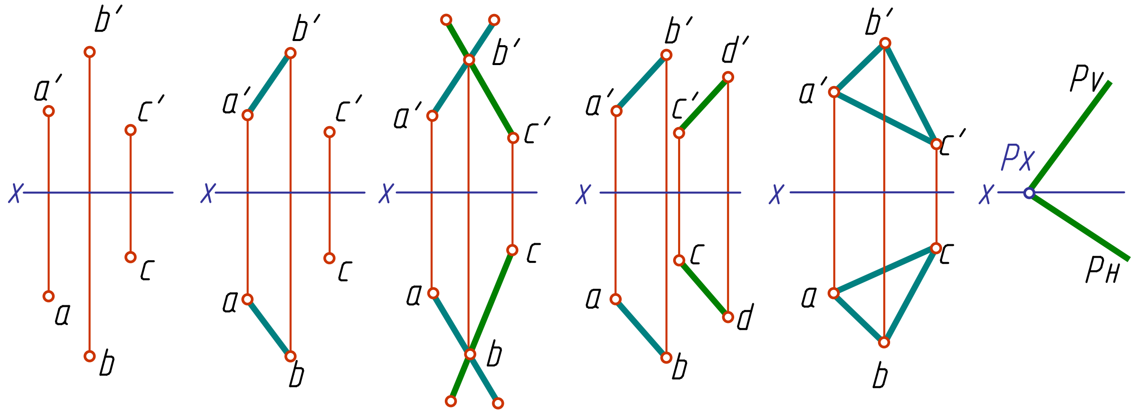
b) by a line and a point projection not lying on one line;

c) by projections of two intersecting lines;

d) by projections of two parallel lines;

e) by any plane figure projection;

f) by the plane traces.



a b c d e f

Fig.4.1

It is possible to move from one way of specifying a plane to another. For example, passing a line through the *A* and *B* points (Fig.4.1, a) we move from specifying the plane by three points to specifying it by a point and a line (Fig.4.1, b), etc.

In particular cases a plane can be represented more visually by means of its intersection lines with the projection planes.

The intersection lines of a plane with the projection planes are called traces of the plane (Fig.4.2):

P_V - the frontal trace of the plane *P*;
P_H - the horizontal trace of the plane *P*;
P_W - the profile trace of the plane *P*.

The points of intersection of a plane with the axes are called the vanishing points of traces (Fig.4.2)

Construct the like traces of two lines lying in a plane to obtain a trace of the plane.

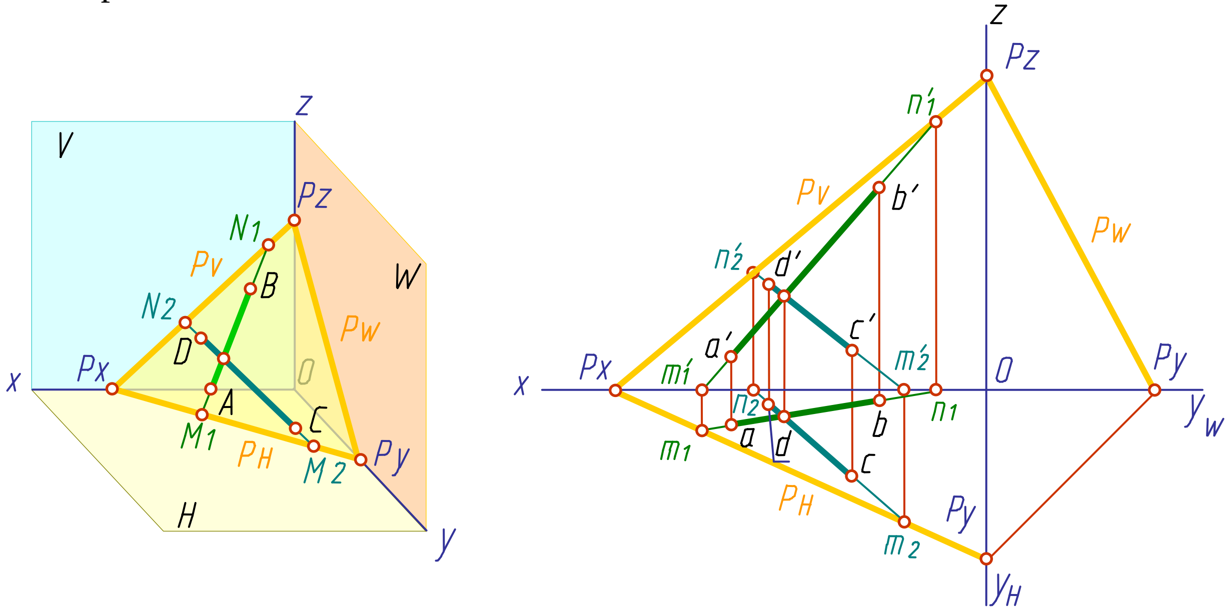


Fig.4.2

4.2 The Position of a Plane Relative to the Projection Planes

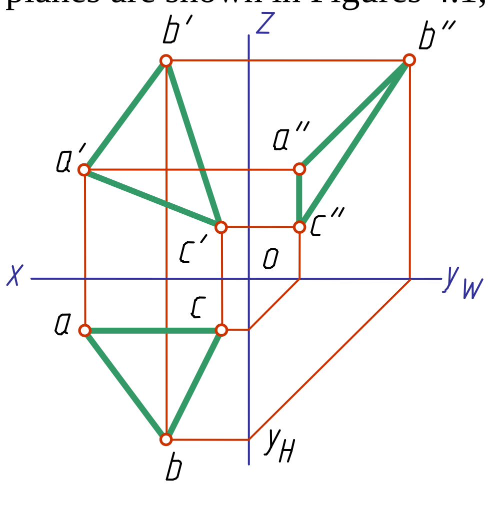
A plane may have the following positions relative to the projection planes:

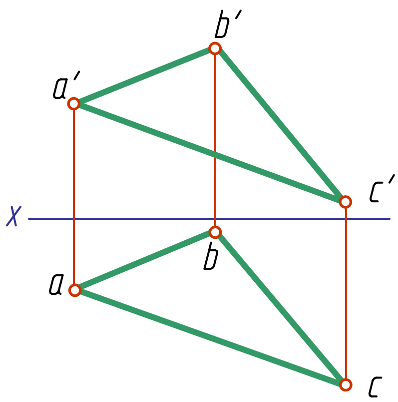
Inclined to all projection planes;

Perpendicular to the projection plane;

Parallel to the projection plane.

A plane which is not perpendicular or parallel to any of the projection planes is called an oblique plane (a plane of general position). The oblique planes are shown in Figures 4.1, 4.2 and 4.3.





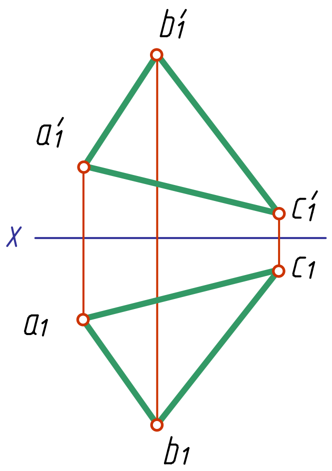


Fig. 4.3 Fig. 4.4 Fig. 4.5

A plane which comes higher, the further it gets from a viewer, is referred to as an ascending plane (Fig.4.4). A plane which comes lower, the further it gets from a viewer, is called a descending one (Fig. 4.5).

It is possible to distinguish representations of an ascending and a descending plane after analysis of the triangle the plane is specified by. The drawing of an ascending plane (Fig.4.4) shows that both of the *ABC* triangle projections (the plan *abc* and the elevation *a′b′c′*) are designated clockwise. However, the $A_1B_1C_1$ projections which specify the descending plane in Fig.4.5, are designated in counterpart ways - the plan $a_1b_1c_1$ is designated counter-clockwise, the elevation $a'_1b'_1c'_1$ - clockwise.

The Planes of Particular Position. The planes perpendicular or parallel to the projection planes are called the planes of particular position.

A *plane perpendicular to a projection plane is called a projecting plane.*

1. The horizontal projecting plane *P(ABCD)* ⊥ *H*

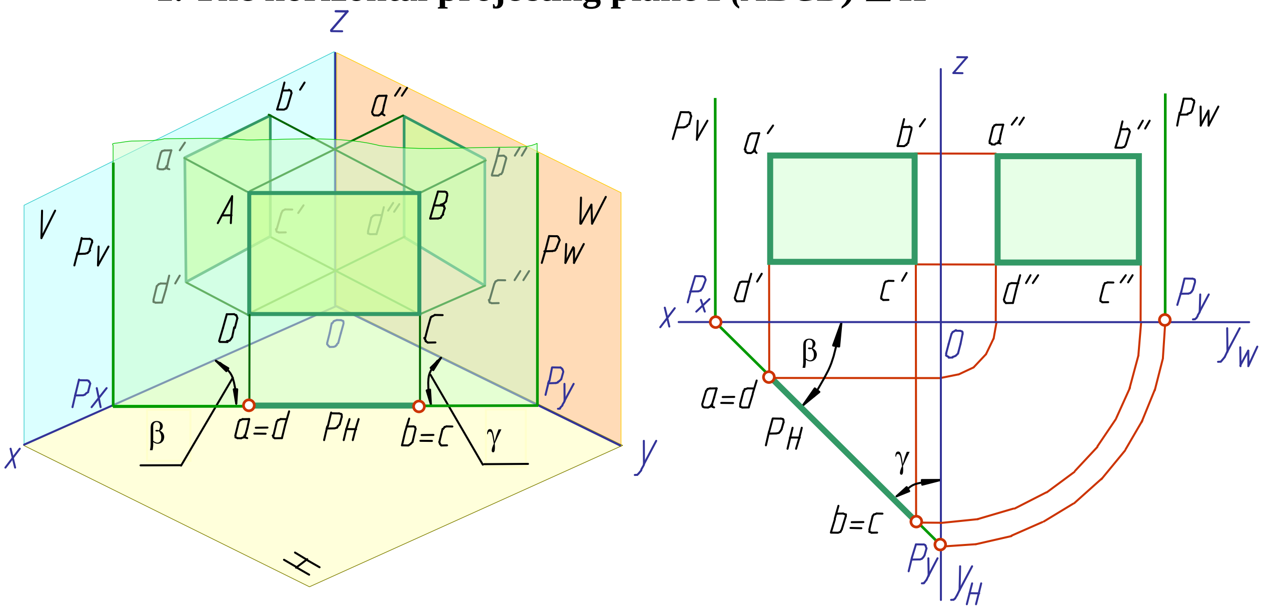


Fig.4.6

2. The vertical projecting plane *Q(ABCD)* ⊥ *V*

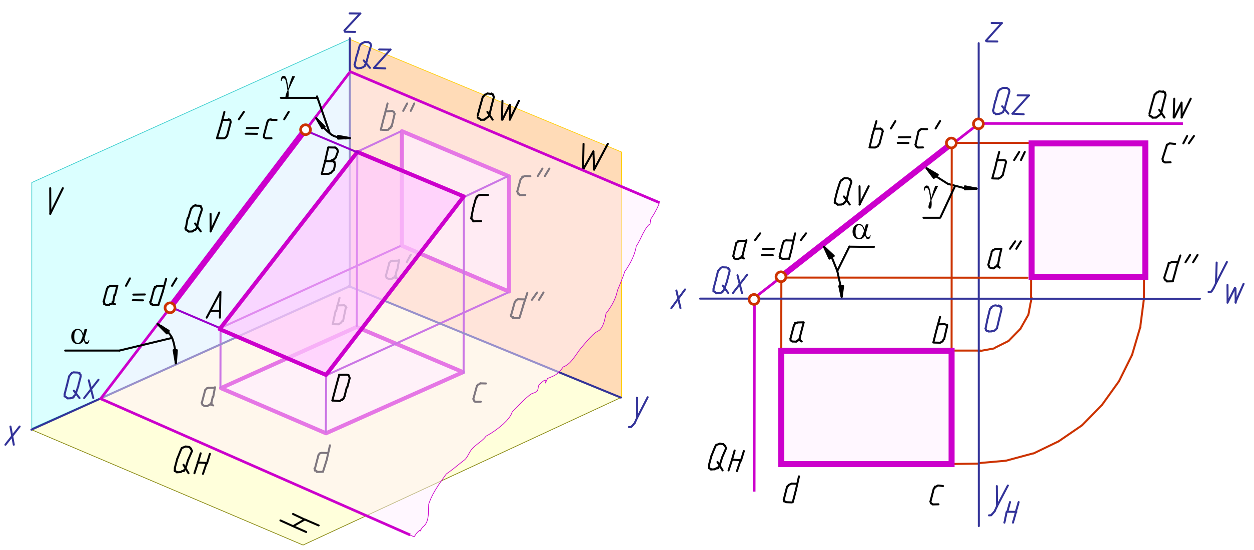


Fig.4.7

3. The profile projecting plane *T(ABCD) ⊥ W*

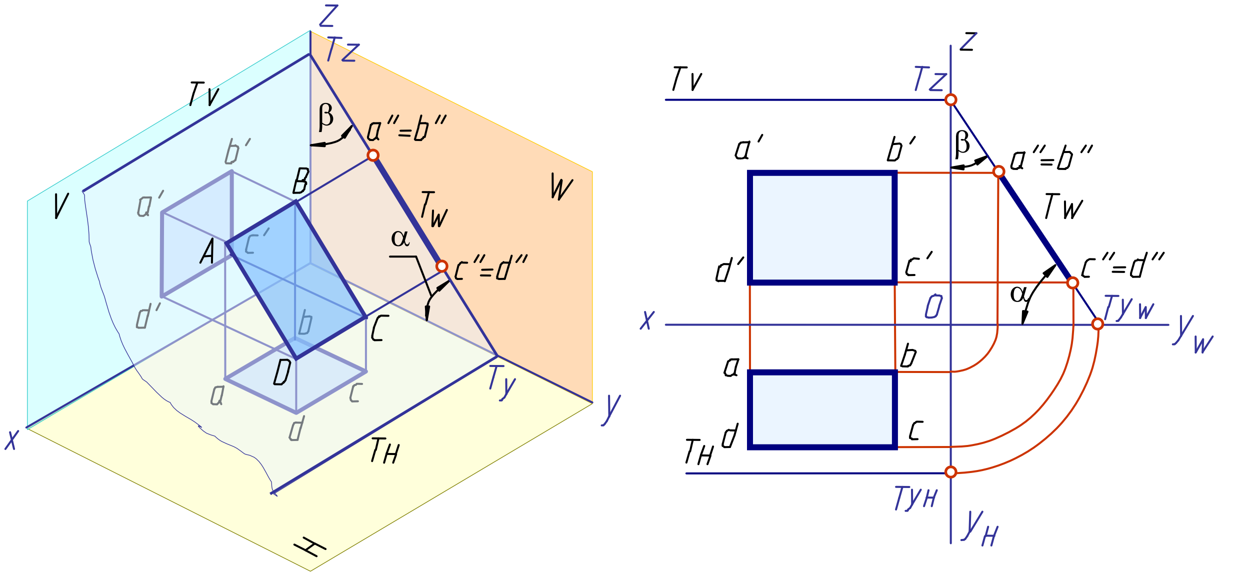


Fig.4.8

A plane projects to a perpendicular projection plane as a straight line. This projection can also be considered as a trace of the plane.

There is an important property of the projecting planes, called a collective one: if a point, a line or a figure are contained in a plane perpendicular to the projection plane, their projections on the above plane coincide with the trace of the projecting plane.

The planes parallel to the projection planes are called the planes of level (the level planes). The level planes are perpendicular to two projection planes simultaneously (double projecting planes).

1. The horizontal plane *P(ABCD) // H*

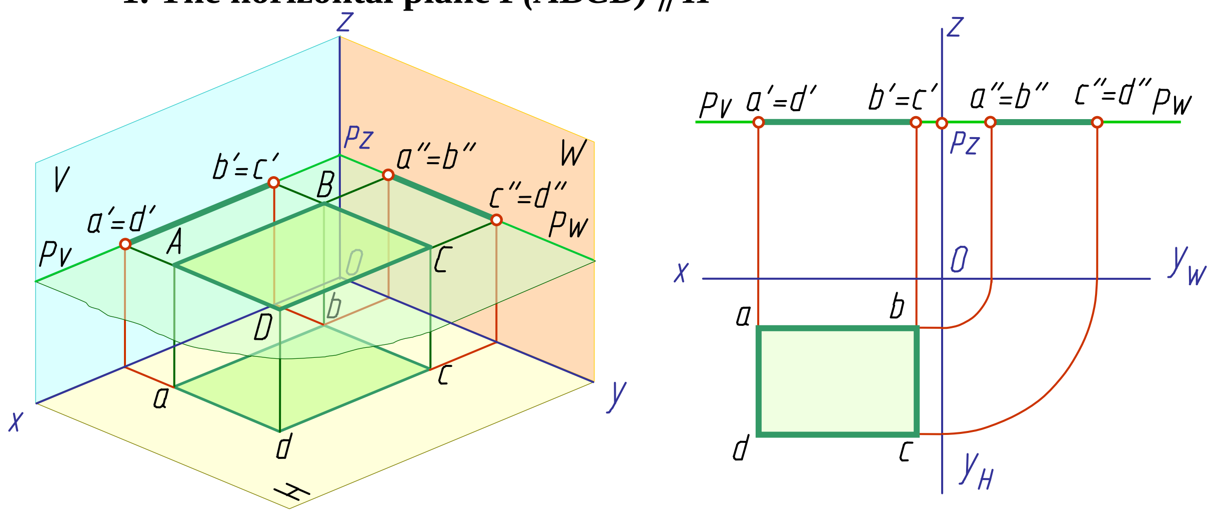


Fig.4.9

2. The frontal plane *Q(ABCD)* // *V*

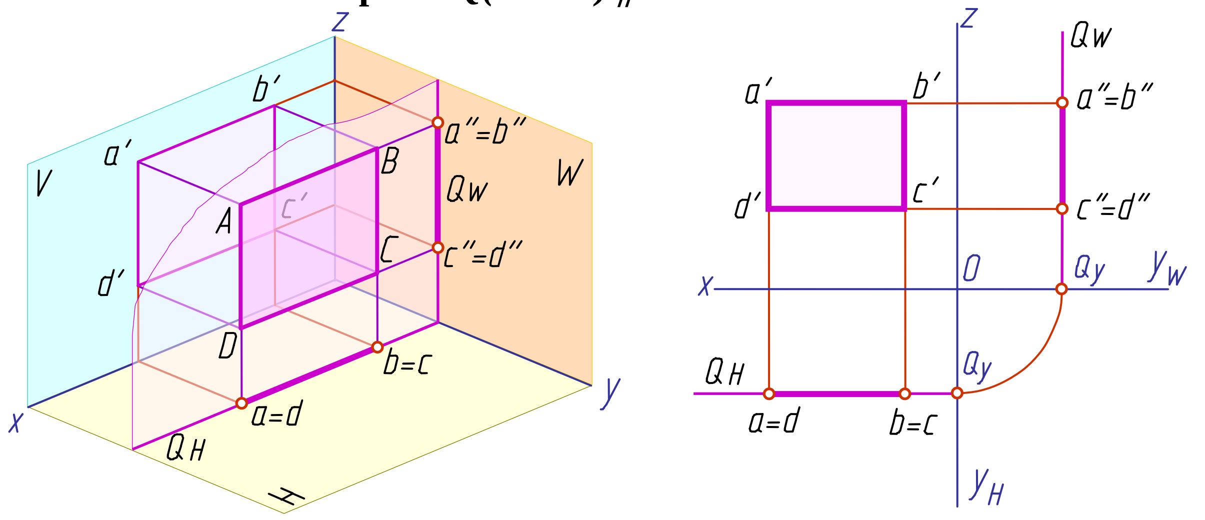


Fig.4.10

3. The profile plane *T(ABCD)* // *W*

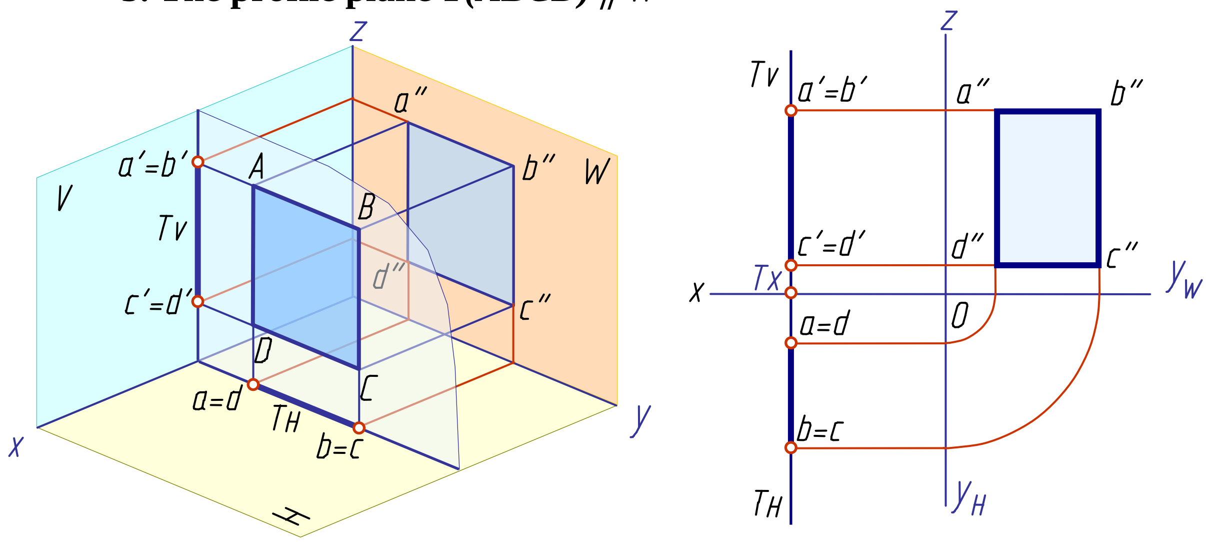


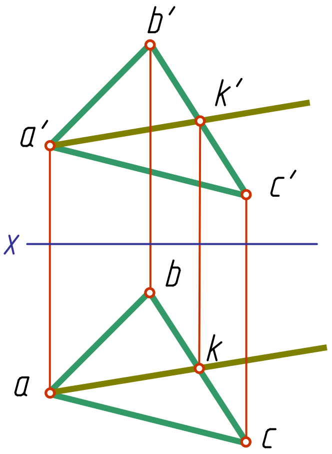
Fig.4.11

Any line or figure contained in a level plane parallel to a projection plane, projects to the last plane in true shape.

4.3 The Point and the Line in the Plane

The following problems are considered to be the principal ones among those being solved in the plane: drawing a line in a plane; constructing a point on a plane; constructing a lacking projection of a point contained in a plane; checking of a point belonging to a plane.

Solution of the above problems is based on well-known geometric principles: a line belongs to a plane, if it passes through two points belonging to the plane; or if it passes through one point of the above plane and is parallel to a line contained in the plane.



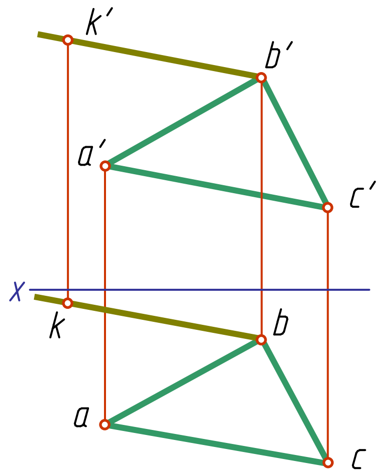


Fig.4.12 Fig.4.13

Construction of a straight line in a plane

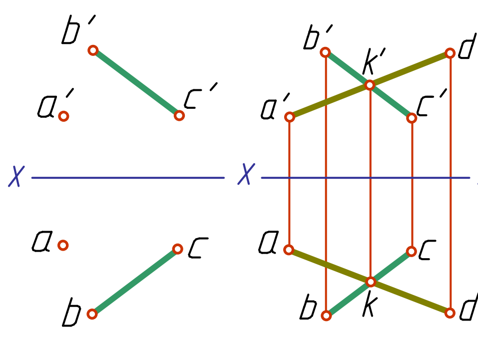
To construct a straight line in a plane (Fig 4.12) it is necessary to specify two points contained in this plane, say, the points *A* and *1*, and draw the line *A1* through them (*a1* and *a′1′*).

Fig.4.13 – The line *B1* belongs to the plane of the triangle *ABC* as it passes through the vertex *B* and is parallel to the side *AC* (*b′1′* // *a′c′* and *b1*// *ac*).

Construction of a point in a plane

A point belongs to a plane if it lies on a line contained in this plane.

To construct a point in a plane draw an auxiliary line in it and specify a point on this line.



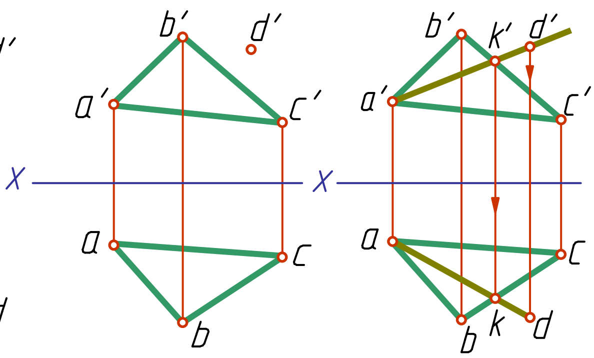


Fig.4.14 Fig.4.15

Projections of the auxiliary line *A1* (*a1* and *a′1′*), belonging to the plane specified by the projections of the point *A* (*a* and *a′*) and the line *BC* (*bc* and *b′c′*) (Fig.4.14), are drawn in the above plane. Mark down on it the projections *d* and *d′* of the point *D*, belonging to this plane.

Construction of a lacking point projection

The plane on Fig.4.15 is specified by the triangle *ABC* (*abc* and *a′b′c′*).

The point *D* belonging to this plane is specified by *d′*. Find the horizontal projection of the point *D*. It may be found by a construction of an auxiliary line belonging to the plane and passing through the point *D*. To do this draw the frontal projection of the line *A1*, construct its horizontal projection *a1* and mark off on it the desired horizontal projection *d* of the point.

Testing if a point belongs to a plane

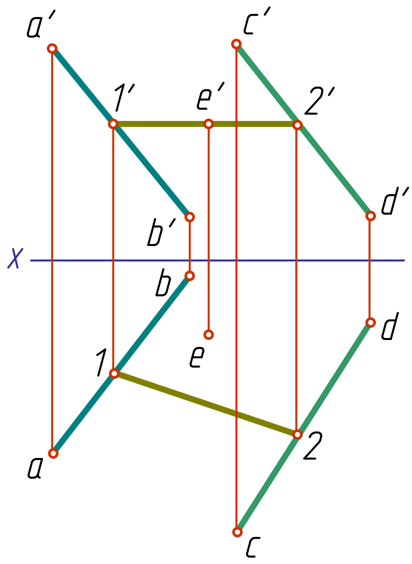


Fig. 4.16

Use an auxiliary line included in a plane to check whether the point belongs to this plane. The plane on Fig.4.16 is specified by the parallel lines *AB* and *CD*, the point - by the projections *e* and *e′*. Draw the projections of the auxiliary line to pass through one of the point projections. For example, the frontal projection of the auxiliary line *1′-2′* passes through the frontal projection of the point *e′*. Construct the horizontal projection of the line *1-2*. It is obvious from the drawing that the horizontal projection *e* of the point does not belong to it. Thus, the point *E* does not belong to the plane.

4.4 The Principal Lines of the Plane

There are a lot of lines belonging to a plane. Those of them which have a special or particular position should be distinguished. They are: *H* parallels or horizontal lines, *V* parallels or frontal or vertical lines, profile lines and lines of maximum inclination or the steepest lines. The above lines are referred to as the principal lines of the plane.

H parallels or horizontal lines are lines lying in a given plane and parallel to the horizontal plane of projection (Fig.4.17).

The frontal projection a’1’ of the horizontal line is parallel to the *x* axis, the profile one - to the *y* axis.

V parallels or frontal or vertical lines are lines lying in a given plane and parallel to the vertical plane of projection (Fig.4.18). The horizontal projection *c1* of the frontal line is parallel to the *x* axis, the profile one - to the *z* axis.

Profile lines are lines lying in a given plane and parallel to the profile plane of projection. The horizontal projection *b1* of the profile line is parallel to the *y* axis, the frontal one - to the *z* axis (Fig.4.19).

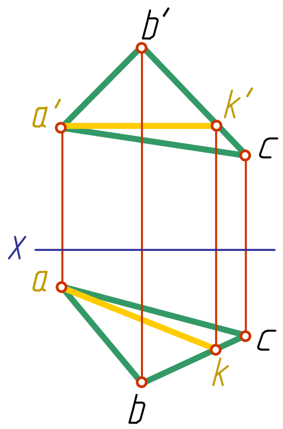


Fig.4.17

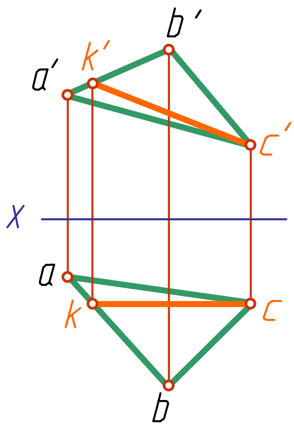


Fig.4.18

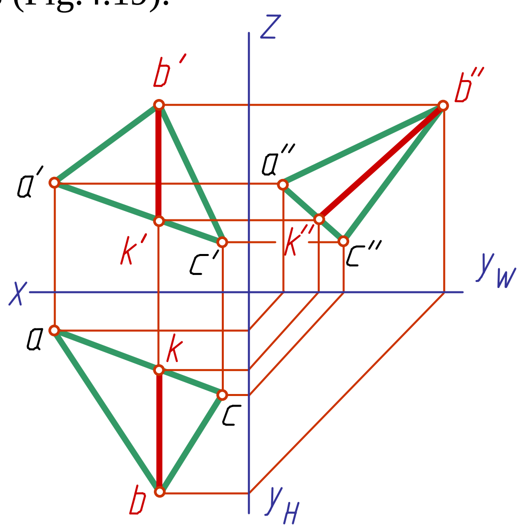


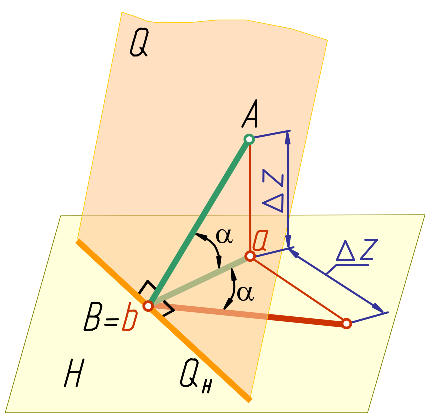
Fig.4.19

The lines considered above are the lines of minimum inclination to the planes of projections.

Among three lines of maximum inclination or the steepest lines let us mark out the inclination line to the horizontal plane. It is called the steep line.

The steep line is a line lying in a given plane and perpendicular to its horizontal trace or to its *H* parallel (Fig.4.20). Having constructed the steepest line in a drawing, one may determine the size of the dihedron between the given and projection planes. This angle is equal to the linear angle between the steepest line and its projection on the plane.

Use the method of a right triangle to determine an angle of inclination.



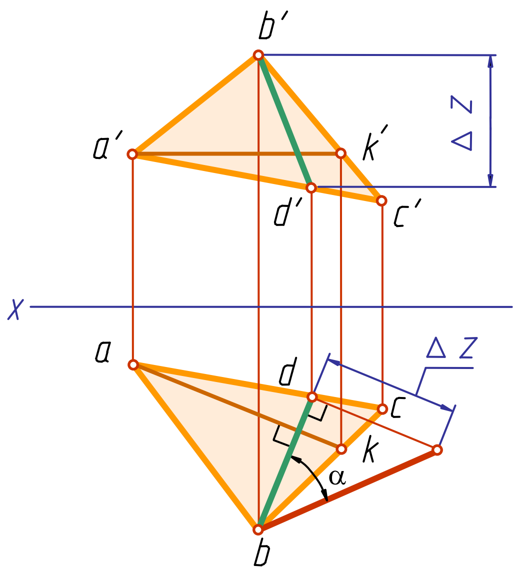


Fig.4.20 Fig.4.21

4.5 The Relative Positions of a Line and a Plane

The relative positions of a line and a plane are determined by the quantity of points belonging both to the plane and to the line:

a) if a line and a plane have two common points, the line belongs to the plane;

b) if a line and a plane have one common point, the line intersects the plane;

c) if the point of intersection of a line and a plane is at infinity, the line and the plane are parallel.

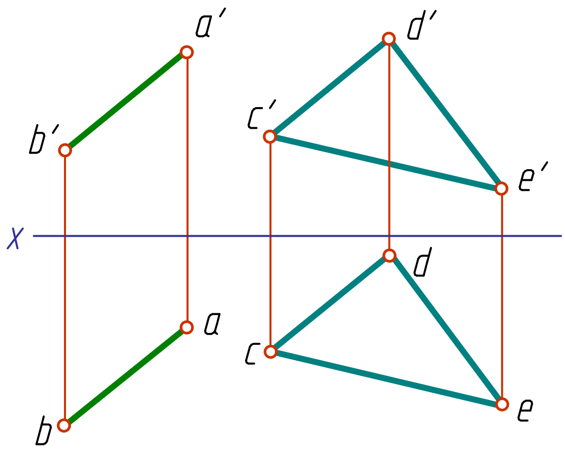


Fig. 4.22

The problems in determining the relative positions of different geometric figures are called positional problems.

A line is parallel to a plane if it is parallel to any line contained in this plane. To construct such a line, specify a line in the plane and draw the required one parallel to it.

Through point *A* (Fig.4.22) draw the line *AB* parallel to the plane *P*, which is specified by the triangle *CDE*. To do this through the frontal projection *a′* of the point *A* draw the frontal projection *ab* of the required line parallel to the frontal projection of any line contained in the plane, say, the line *CD* (*a'b'* // *c'd'*). Parallel to *cd* through the horizontal projection *a* of the point *A* pass the horizontal projection *ab* of the desired line *AB* (*ab* // *cd*). The line *AB* is parallel to the plane *P* specified by the triangle *CDE*.

Construction of the intersection point of a line and a plane.

The problem of the construction of intersection point of a line and a plane is widely used in descriptive geometry. It is fundamental for the problems of: the intersection of two planes; of a plane and a surface; a line and a surface; and on the mutual intersection of surfaces.

To construct the point of intersection of a line and a plane means to find a point belonging to both, a given line and a plane. Graphically this is a point of intersection of the straight line and a line contained in the plane.

The plane has a projecting position.

If a plane has a projecting position (for example, it is perpendicular to the frontal plane of projections - Fig.4.23), the frontal projection of the cutting point must belong to both the frontal trace of the plane and the frontal projection of the line, i.e. to be in the point of their intersection. That is why, first the frontal projection of the point K is determined (the cutting point of the line *AB* and the frontal projecting plane *Q*(*ΔCDE*), and then its horizontal projection.

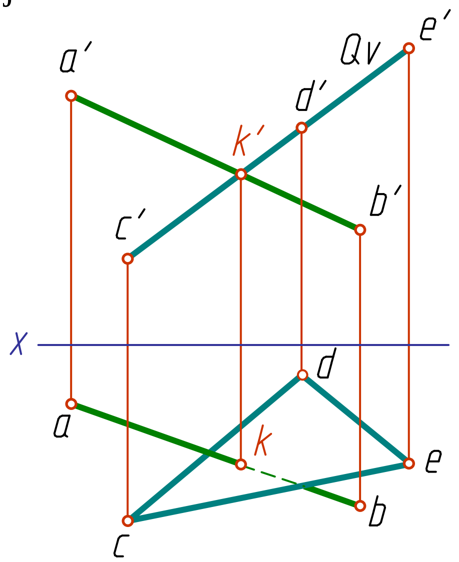


Fig.4.23

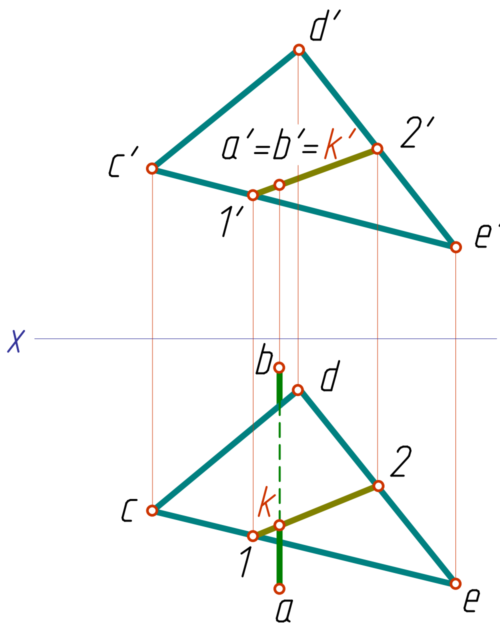


Fig.4.24

The line has a projecting position.

Fig.4.24 shows the oblique plane *P* (*ΔCDE*) and the frontal projecting line *AB*, cutting the plane at the point *K*. The frontal projection of the point (the point *k*) coincides with the points *a′* and *b′*. Draw through the point *K* in the plane *P* a straight line (say, *1-2*) to obtain the horizontal projection of the intersection point. First construct its frontal projection, then - the horizontal one. The point *K* is the point of intersection of the lines *AB* and *1-2*. It means that *K* belongs to both, the line *AB* and the plane *P* and, therefore, is the point of their intersection.

The line and the plane have a general position.

In this case a line lying in the plane and intersecting the given line may be obtained as a line of intersection of an auxiliary plane passed through the line with the given plane (Fig.4.25, 4.26).

To determine the point of intersection of a straight line and a plane proceed as follows:

Pass an arbitrary auxiliary plane *P* through the line *AB* (the simplest way is to pass a projecting plane);

Find the line *MN* of intersection of the given [*Q* (*ΔCDE*)] and auxiliary (*P*) planes;

As the lines *AB* and *MN* lie in one plane *P*, the point of their intersection (*K*) yields the desired point.

Determine the relative visibility of the line *AB* and the plane *Q*.

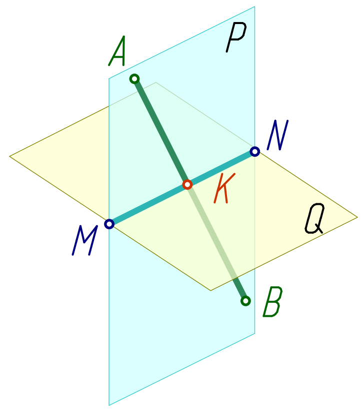


Fig.4.25

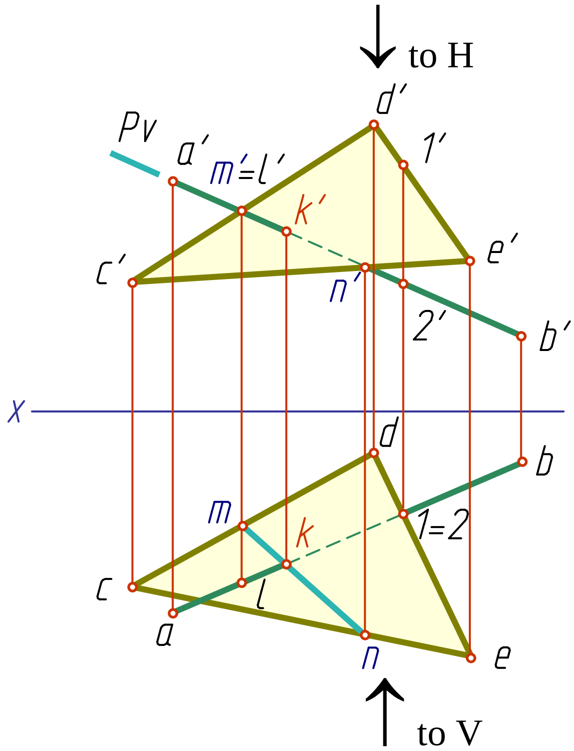


Fig.4.26

To determine the visible sections of the line *AB* analyse the position of the points on the skew lines (the competitive points).

The points *M* and *L* are situated on the skew lines *AB* and *CD*: *M*∈*CD*, *L*∈*AB*. Their frontal projections *m′* and *l′* coincide. The horizontal projection shows that, if the *V* plane is viewed in the direction of the arrow, the point *L* (projection *l*) is situated in front of the point *M* (projection *m*), i.e. being projected on the frontal plane it covers the point *M*. Therefore, the line *AB* to the left of the point *K* is situated in front of the triangle *CDE*, and it is visible on the frontal projection. The triangle *CDE* covers the line *AB* to the right of the point *K* up to the point *N*, that is why the segment *k′n′* is shown as an invisible one.

The invisible part of the horizontal projection of the line *AB* is determined by analysis of the position of the points *1* and *2* (*1*∈*DE*, *2*∈*AB*), belonging to the skew lines *AB* and *DE*. The frontal projection shows that, if the *H* plane is viewed in the direction of the arrow, the point *1*, which lies above the point *2*, is visible first. On the horizontal projection the point *1* covers the point *2*. In this section the line *AB* is covered by the triangle *DEF* as far as the intersection point *K* (the projection section *k2*).

4.6 Mutual Positions of the Planes

A general case of the mutual positions of planes is their intersection. In the particular case when the intersection line is at infinity, the planes become parallel. The parallel planes coincide when the distance between them is shortened to zero.

The parallel planes

The planes are considered to be parallel if two intersecting lines of one plane are relatively parallel to two intersecting lines of the other (Fig.4.27)

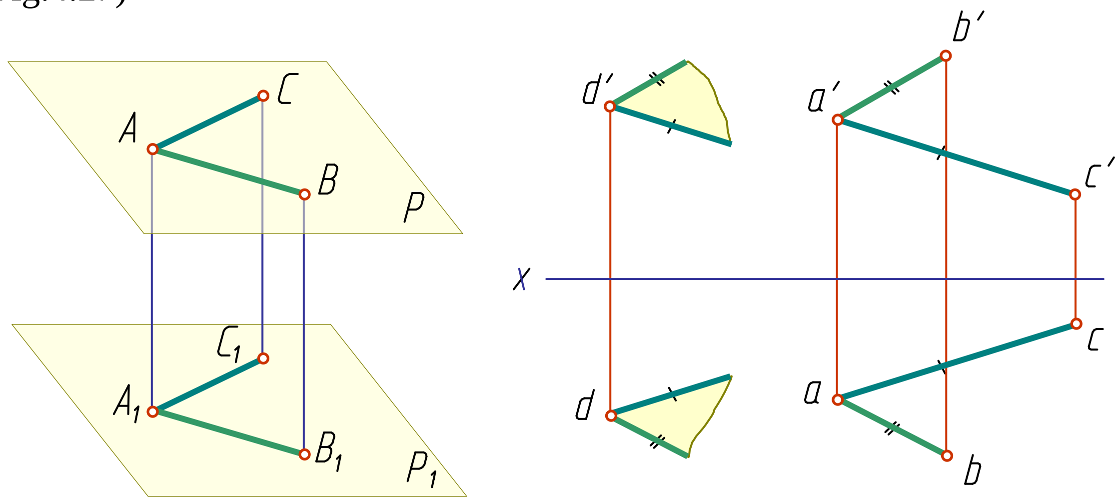


Fig.4.27

To pass through the point *D* a plane parallel to a given plane (*ΔABC*), draw two lines through the point, parallel to any two lines contained in the given plane, say, the triangle sides.

Intersecting planes

The line of intersection of two planes is determined by two points, each belonging to both planes; or by one point, belonging to both planes, plus a given direction of the line. In both cases the problem is to find the point common to both planes.

Intersection of two projecting planes

If the planes are of a particular position (say, a horizontal projecting one, like in Fig.4.28), the projection of the intersection line on the plane of projections to which the given planes are perpendicular (in this case, to the horizontal one) comes to be a point. The frontal projection of the line of intersection is perpendicular to the projection axis.

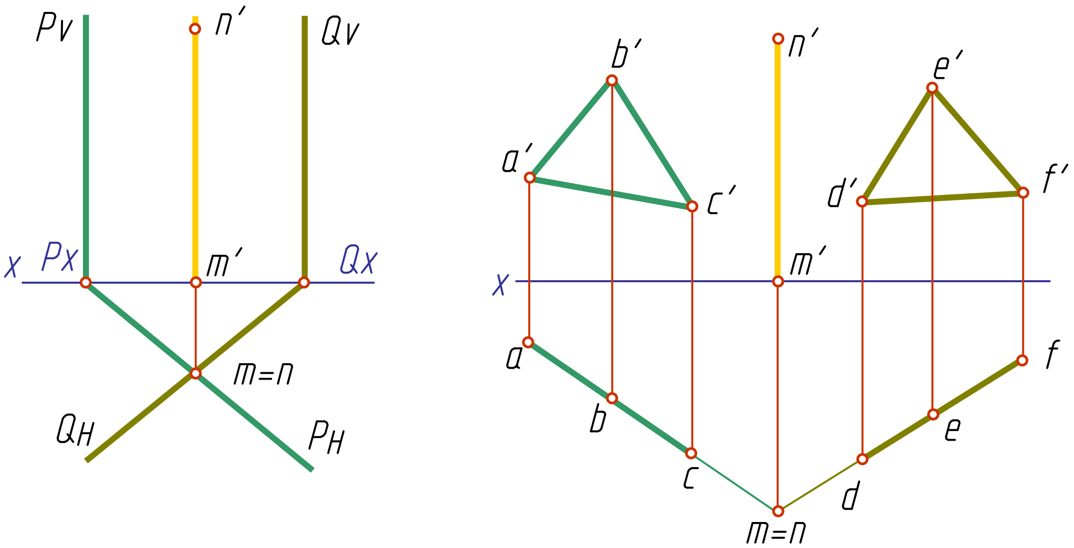
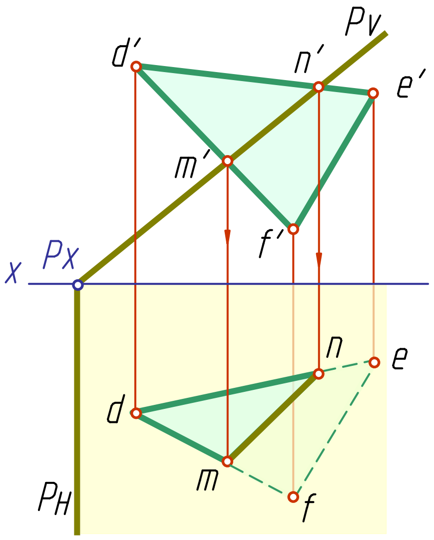


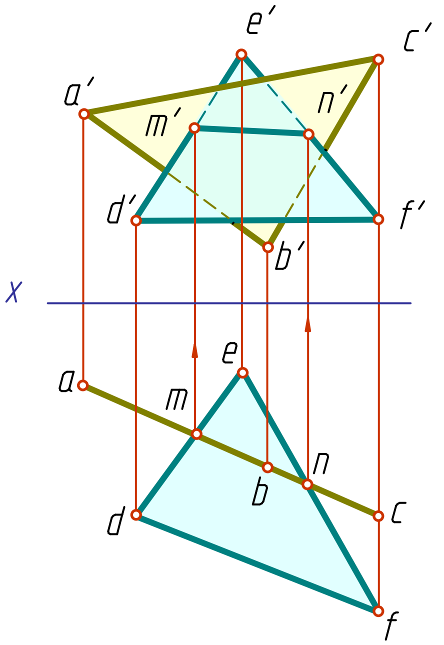
Fig.4.28

Intersection of a rojecting plane and an oblique plane

In this case, one projection of the line of intersection coincides with the projection of the projecting plane on that projection plane to which it is perpendicular. Figure 4.29 shows the construction of the projections of the intersection line of the frontal projecting plane specified by traces; Figure 4.30 - of the horizontal projecting plane specified by the triangle *ABC* with the plane of general position specified by the triangle *DEF*.

↓ to H





↑ to V

Fig.4.29 Fig.4.30

In the point of intersection of the plane P_v trace with the sides *DE* and *DF* of the triangle *DEF* on the frontal projection (Fig.4.29), find the frontal projections *m′* and *n′* of the intersection line. Drawing the connection lines find the horizontal projections of the points *M* and *N* of the intersection line.

Viewing the plane *H* in the direction of the arrow, one can see (by the plan) that a part of the triangle to the left from the cutting line *MN* (*m′n′*) is above the plane *P*, it means it is visible on the horizontal projection plane. The other part is under the plane *P*, i.e. it is invisible.

Find the intersection lines of the planes of Fig.4.30 in a similar fashion.

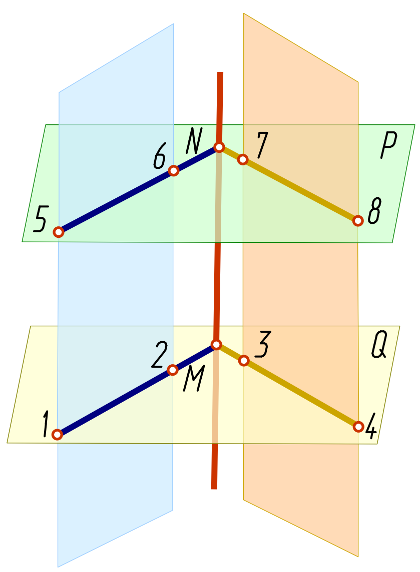
Intersection of the oblique planes

The method of drawing the intersection lines of such planes consists of the following:

Introduce an auxiliary plane (intermediary) and draw the lines of intersection of this plane with the two given ones (Fig.4.31). The intersection of the drawn lines shows the common point of the above planes. To find the other common point use another auxiliary plane.

In solving such kinds of problems, it is better to use projecting planes as intermediaries.

Fig.4.32 shows the construction of the intersection line of two triangles. The solution is as follows:



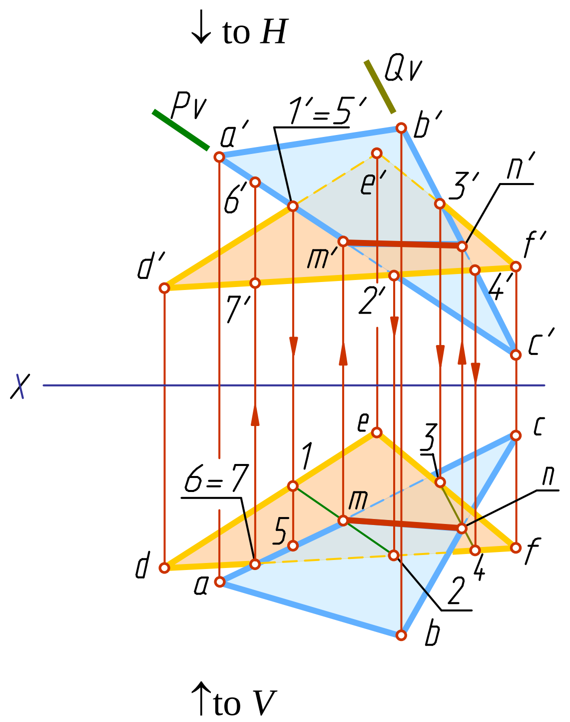


Fig.4.31 Fig.4.32

Draw two auxiliary frontal projecting planes - the plane *P* through the side *AC* and the plane *Q* through the side *BC* of the triangle *ABC*. The plane *P* cuts the triangle *DEF* along the line *1-2*. In the intersection point of the plans *1-2* and *ac* find the plan of the point *M* (*m*) of the intersection line. The plane *Q* cuts the triangle *DEF* along the line *3-4*. In the intersection point of the horizontal projections *3-4* and *bc* find the plan of the point *N* (*n*) of the intersection line. Pass the connecting lines to find the frontal projections of the above points and, therefore, of the intersection line.

Analysis of the mutual visibility of the triangles on the projection planes do with the help of competitive points.

To determine visibility on the frontal projection plane compare the frontal competitive points *1* and *5* lying on the skew lines *AC* and *DE*. Their frontal projections coincide. The horizontal projection shows that on the plane *V*, viewed in the direction of the arrow, the point *5* is situated closer to a viewer and that is why it covers the point *1*. So, the segment of the line *AC* to the left of the point *M* is visible on the frontal projection plane.

To determine visibility on the horizontal projection plane compare the horizontal competitive points *6* and *7* lying on the skew lines *AC* and *DF*. Their horizontal projections coincide. The plane *H*, viewed in the direction of the arrow, shows that the point 6 and the line *AC* are situated above the point *7* and the line *DF*. So, the segment *AM* of the line *AC* is visible on the horizontal projection plane.

4.7. Method of Replacing Planes of Projection

Different methods of transformation of orthogonal projections are used to make the solution of metric and positional problems simpler. After such transformations the new projections help to solve the problem by minimal graphic means.

The method of replacing planes of projection consists in the substitution of a plane with a new one. The new plane should be perpendicular to the remaining one. The position in space of the geometric figure remains unchanged. The new plane should be positioned so that the geometric figure has a particular position to it, convenient for solving the problem.

Fig.4.33 shows a spatial drawing of the *AB* line-segment of general position and its projection on the planes *H* and *V*. Replace the plane *V* with a new vertical plane V_1 , parallel to the line-segment *AB*, to obtain a new system of two mutually perpendicular planes V_1 and *H*, relatively to each the segment *AB* has a particular position ($AB//V_1$), x_1 is a new coordinate axis. The new projection of the line-segment *AB* ($a'_1b'_1$) is equal in length to its true size, and the angle α is equal to the true size of the inclination angle between *AB* and the plane *H*.

When replacing the frontal plane of projection (Fig.4.33), *z* co-ordinates are constant, i.e. the distance between the points and the horizontal projection plane *H* remains unchanged. Therefore, to construct a new projection of the line-segment (Fig.4.34) proceed as follows:

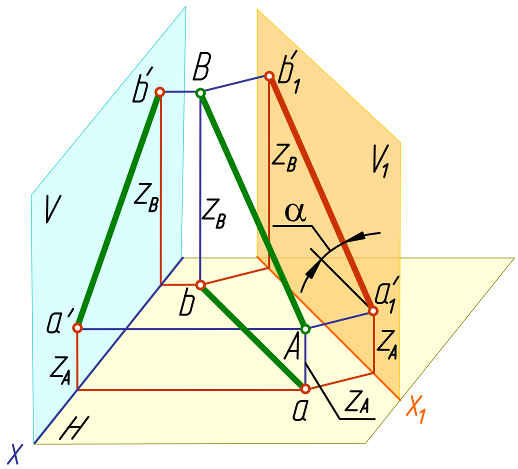


Fig.4.33

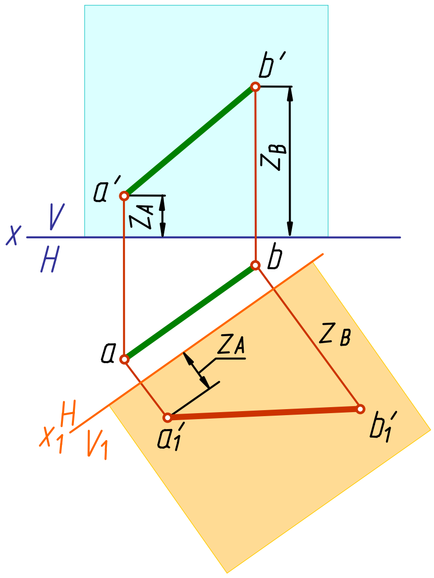


Fig.4.34

- at any distance pass the new axis x_1 parallel to the horizontal projection of the line-segment *AB*;

- through the horizontal projections *a* and *b* perpendicular to the axis x_1 pass the connection lines;

- from the point of intersection of the connection lines with the axis x_1 lay off *z*-co-ordinates of *A* and *B* points;

- connect the obtained points $a'_1b'_1$ with a straight line.

When replacing the horizontal plane *H* with a new one, *y*-co-ordinates remain unchanged, which means that the distance between the point and the frontal projection plane does not change if the point is projected on the new plane, perpendicular to the plane *V*.

Four principal problems solved by replacing the projection planes

1. Transform a line of general position into a line parallel to one of the projection planes. Such a transformation helps to determine the true size of the line-segment and its inclination angles contained by the projection planes (Fig.4.35)

To solve the problem, draw a new plane, say, V_1 (Fig.4.35), parallel to the segment. In this case the new coordinate axis passes parallel to the horizontal projection of the given line. Draw, through the horizontal projections *a* and *b* perpendicular to the new axis, the connection lines. Lay off on them *z*-co-ordinates of the points (the distance from the *x* axis to the frontal projection of the points). The new projection $a'_1b'_1$ is equal to the true size of the segment, and the angle α is equal to the inclination angle contained by the segment and the plane *H*.

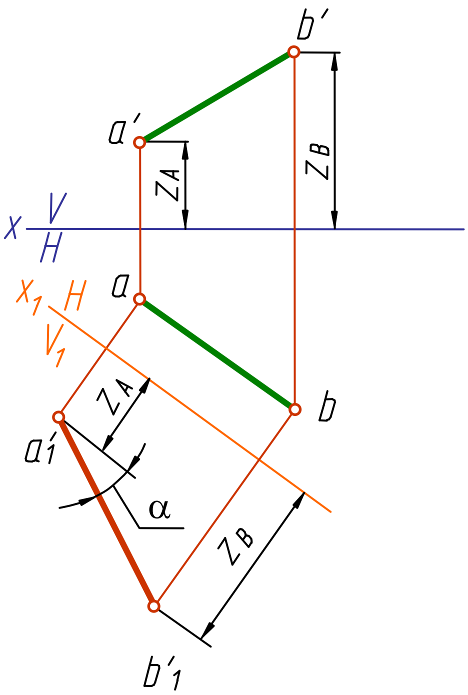


Fig.4.35

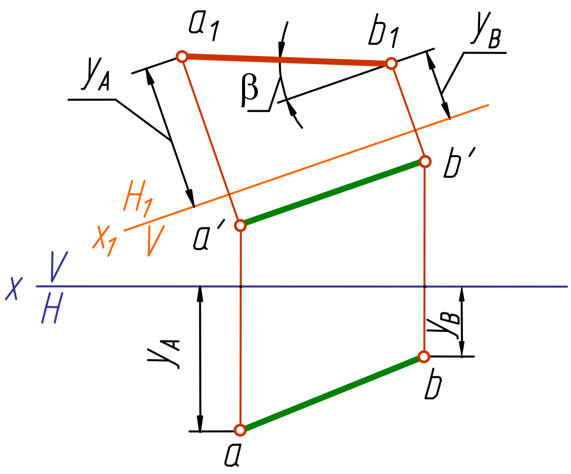


Fig.4.36

When replacing the horizontal projection with a new one, draw this plane parallel to the line-segment *AB* and determine the true size of the segment and its inclination angle with the plane *V* - the angle β (Fig.4.36).

In this case pass the coordinate axis of the new plane parallel to the frontal projection of the line *a'b'*, and take the co-ordinates *y* from the horizontal projection plane.

2. Transform a line parallel to one of the projection planes into a projecting line, i.e. position it perpendicular to the projection plane, to project the line on this plane as a point (Fig.4.37).

As the given line is parallel to the horizontal plane, to transform it into a projecting line replace the frontal plane V with a new $V_1 \perp H$, drawing the plane V_1 perpendicular to *AB*. As a result the given line is projected on the plane V_1 as a point ($a'_1=b'_1$).

To transform the general position line *AB* (Fig.4.38) into a projecting one, make two replacements, i.e. solve both problems, the first and the second ones, successively. First transform a general position line into a line parallel to a projection plane (a line of level), then the last is transformed into a projecting one.

3. Transform the oblique plane P (ΔABC) into a projecting one (Fig.4.39), i.e. positioned perpendicular to one of the projection planes.

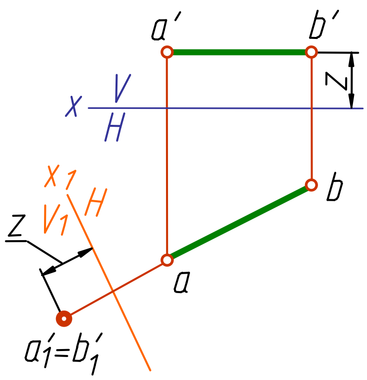


Fig.4.37

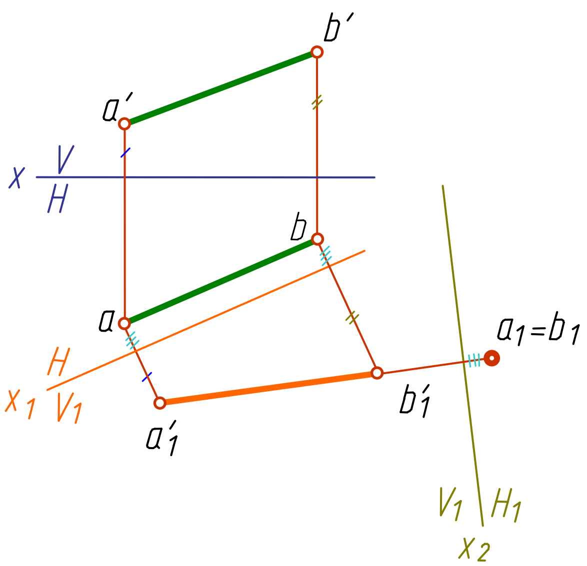


Fig.4.38

Replace, for example, the plane *V* with a new plane V_1, which is perpendicular to the plane *H* and the plane *P*. The plane V_1 is perpendicular to the plane *P* if it is drawn perpendicular to one of the lines of the plane. Let us take *H* parallel here (the line parallel to the horizontal projection plane). Draw in the plane *P* the *H* parallel *C1* and pass a new plane V_1 perpendicular to it. Pass the axis x_1 in any place perpendicular to the horizontal projection of the *H* parallel ($x_1 \perp c1$). Now construct a new frontal projection of the plane *P*. The *H* parallel is projected onto the new plane as a point ($c'_1=1'_1$), the plane *P* (*ΔABC*) - as the line $a'_1c'_1b'_1$.

To transform the plane P into a horizontal projecting plane, replace the plane *H* with a new one, perpendicular to the plane *V* and to the *V* parallel of the plane *P* (which has been drawn in this plane previously).

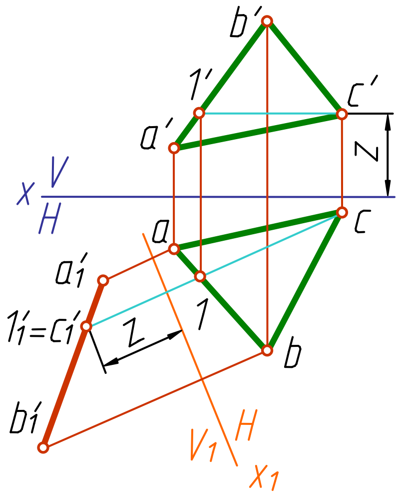


Fig. 4.39

4. Transform the plane P (ΔABC) from a projecting plane into a level plane (a plane parallel to one of the projection planes). In this case we determine the true size of the plane figure (Fig.4.40 and 4.41).

Fig. 4.40 shows the frontal projecting plane. Replace the horizontal plane *H* with a new one, positioning it perpendicular to the plane *V* and parallel to the plane *P*. Pass the new axis x_1 parallel to the frontal projection *a'b'c'*, and new connection lines - perpendicular to x_1. *Y* co-ordinates remain unchanged as the horizontal plane was changed. Carry the coordinates onto the new plane. As a result obtain a new horizontal projection of the triangle equal to the true size of *ΔABC*.

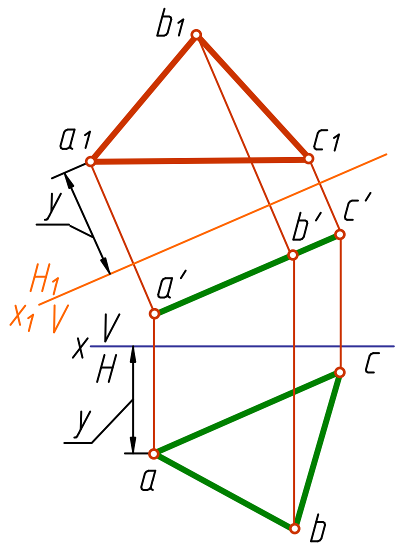


Fig.4.40

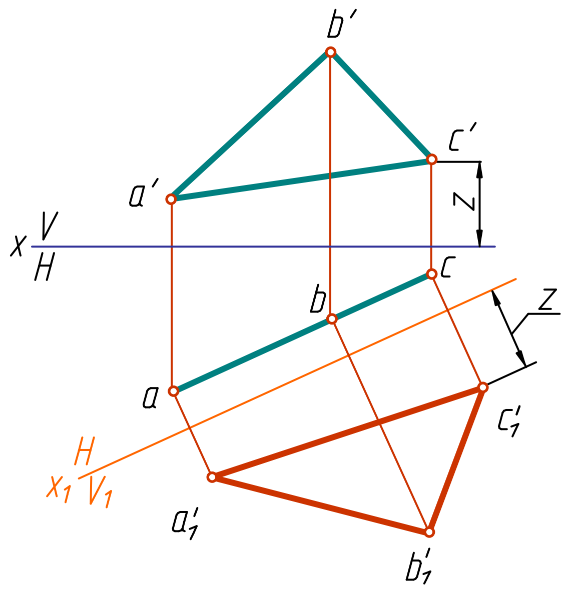


Fig.4.41

The problem is solved in a similar fashion if the plane *P* (*ΔABC*) is a horizontal projecting plane (Fig.4.41). In this case the frontal plane *V* is replaced with the new one V_1 which is drawn perpendicular to the plane *H* and parallel to the plane *P*. The axis x_1 is passed parallel to the line *abc*. With such a replacement the coordinates *z* remain unchanged, lay them off on the connection lines, from the new axis x_1.

To transform an oblique plane into a plane, parallel to one of the projection planes, two replacements are necessary (Fig.4.42); that is to solve the third and the forth problems successively.

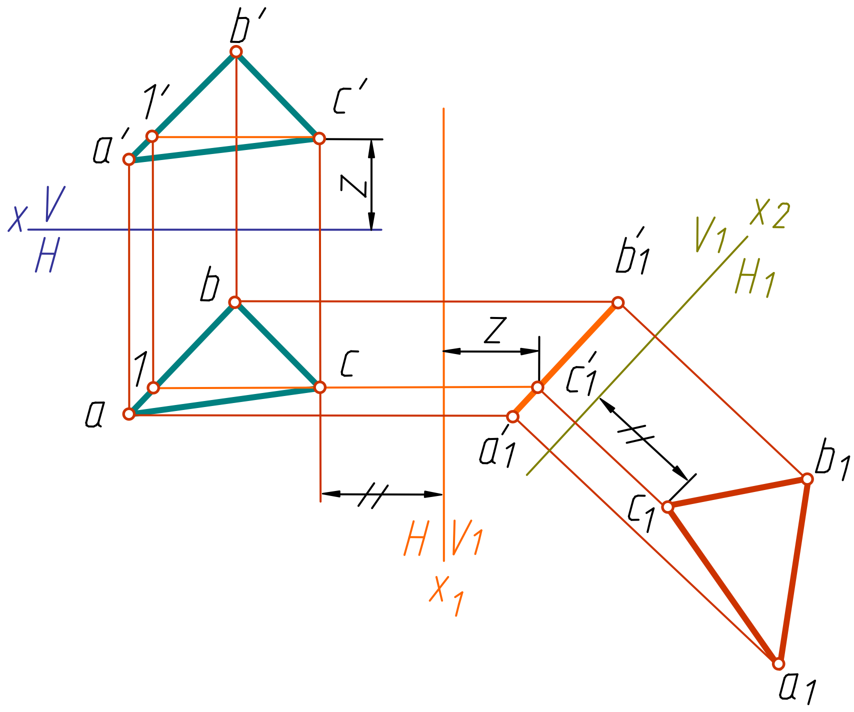


Fig.4.42

Questions to Chapter 4

1. What are the ways of specifying a plane figure?
2. What are “traces of the plane”?
3. What plane is called a projecting plane?
4. What is the level plane?
5. Under what conditions does a line belong to a plane?
6. Under what conditions does a point belong to a plane? What lines are referred to as the principal lines of the plane?
7. What are the terms of a line and a plane to be parallel?
8. How can you find the meeting point of a line and a plane?
9. What are the relative positions of the planes?
10. What determines mutual parallelism of two oblique planes in a drawing?
11. What is the way of drawing an intersection line of two planes?
12. What is the gist of the replacing planes of projection method?
13. What mutual relations must the old and new planes of projections have?
14. What actions are necessary to obtain the following transformations: of a general position line into a projecting one; of an oblique plane into a level plane?