CHAPTER 3. THE POINT AND THE STRAIGHT LINE

To obtain a clear understanding of the all external and internal forms of the components and their joints as well as to be capable of solving other problems, it is usually necessary to have three or more views of each detail. That is why there can be three or more projection planes.

Into the system of the planes *H* and *V*, let us introduce one more plane, perpendicular to them. This plane is called the profile projection plane and is denoted by letter *W* (Fig.3.1). The plane *W* intersects the planes *H* and *V* along the lines *y* and *z* (axes of projection). The intersection point of all axes is called origin of co-ordinates and is designated by letter *O* (the first letter of the Latin word "*origo*" which means "*origin*"). The axes *x, y, z* are mutually perpendicular.

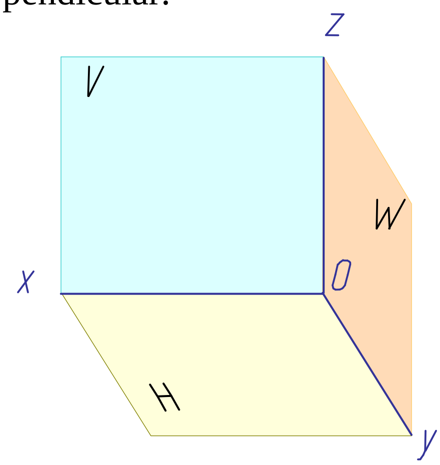


Fig.3.1

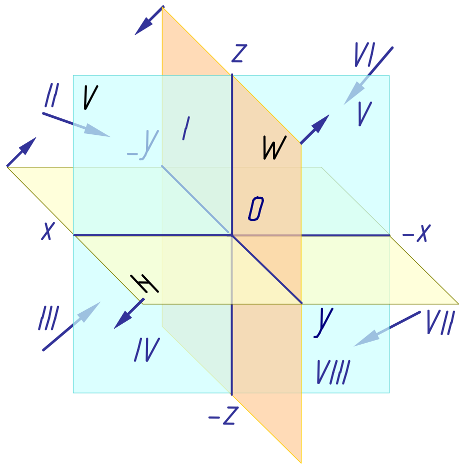


Fig.3.2

The three mutually perpendicular planes divide space into eight parts, eight octants, Fig.3.2 (from Latin "*octo*" - "eight"). Most of the Continental countries adopted the right-handed system, the so-called "European system of projection positioning". The axis *x* is directed to the left from the origin of co-ordinates, *y* - forward (to us), *z* - upward. The opposite directions of the co-ordinate axes are considered negative.

3.1. The Point Drawing

Rectangular projections of a point (in the *H*, *V* and *W* planes) are always obtained as the bases of perpendiculars, dropped from the given point onto each of projection planes:

a – horizontal; *a′* – frontal; *a″* – profile.

Representation of a point in the system of the planes *H, V* and *W* shown in Fig.3.3, is too complex and, therefore, is inconvenient for drawing. Let us convert it so that the horizontal and profile planes will coincide with the frontal projection plane, forming one drawing plane (Fig.3.4).

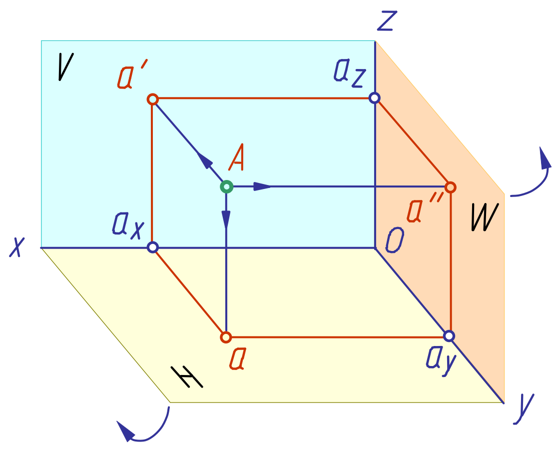


Fig.3.3

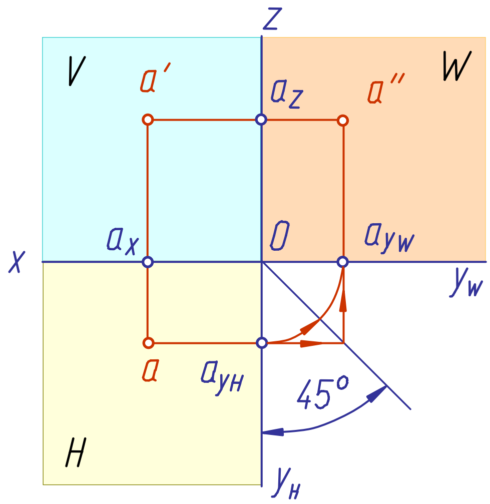


Fig.3.4

Such a conversion can be realized by the rotation of the plane *H* around the axis *x* down at the angle of 90°, and of the plane *W* to the right around the axis *z*, at the angle of 90°. As a result of the above coincidence obtain the drawing known as "epure of Monge" or "orthographic drawing of Monge" (from French "*epure*" - drawing, projection).

Having used the orthographic drawing we have lost the spatial picture of the projection planes and point positioning. But epure ensures representational precision and convenience of dimensioning, at a considerable simplicity of drawing.

For simplicity, epure of Monge as well as projection drawings based on the method of Monge are called drawings (or complex drawings) in this book.

Horizontal and frontal projections of a point (a and a') are situated on one perpendicular to the x-axis - on the connection line aa'; frontal and profile projections (a' and a'') are situated on one perpendicular to the z-axis - on the connection line $a'a''$.

Construction of a profile point projection, given its frontal and horizontal projections, is shown in Fig.3.4. For the construction one may use either a circular arc passed from the point O, or bisectrix of the angle $y_H O y_W$. The first of the methods is more preferable as it is more precise.

Thus, in a complex drawing consisting of three orthogonal point projections:
two projections are situated on one connection line;
connection lines are perpendicular to the projection axes;

two projections of a point specify the locus of its third projection;

two projections of a point specify its locus in space.

The location of a point in space is specified by means of its three coordinates (abscissa x, ordinate y, applicate z), i.e. of three figures showing the distance from this point to the co-ordinate projection planes. Recording of the point co-ordinates is made in the following form: $A(x,y,z)$.

In regards to the projection planes a point may have a general (the point *A*) or particular (the points *B* and *C*) position (Fig.3.5). If a point is located in a projection plane, its two projections lie on the projection axes (the point *B*). One of the co-ordinates of such a point is equal to zero. A point belonging to two projection planes simultaneously (the point *C*) lies on a projection axis. Its two projections coincide, the third one merges with the point *O*, origin of co-ordinates. In this case two of its co-ordinates are equal to zero. A point belonging to three projection planes is situated in the origin of co-ordinates.

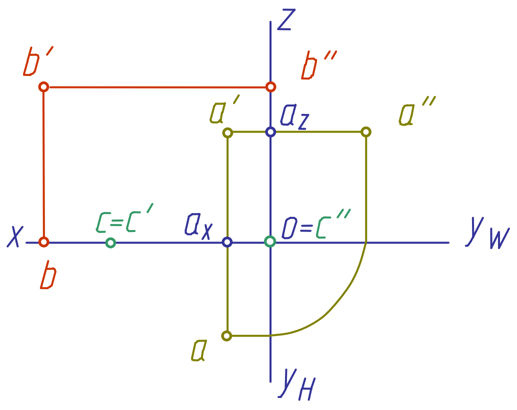


Fig. 3.5

Therefore, the sizes of the connection lines segments in a drawing specify the numerical distance from a projection point to a projection plane. The segment $a_x a$ shows the distance (depth) from the point to the frontal projection plane, the segment $a_x a'$ - the distance (height) from the point to the horizontal projection plane, and the segment $a' a_2$ - the distance to the profile projection plane.

3.2 Mutual Positions of Two Points. Terms of Visibility in a Drawing

Consider a model drawing shown in Fig.3.6. Projections of some points coincide as they are situated on one projecting line. For example, images *a* and *b* of the vertices *A* and *B* on the horizontal projection merged into one point as they lie on one horizontal projecting line. On the frontal projection representations c and d of the vertices *C* and *D* merged into one point as they lie on one frontal projecting line.

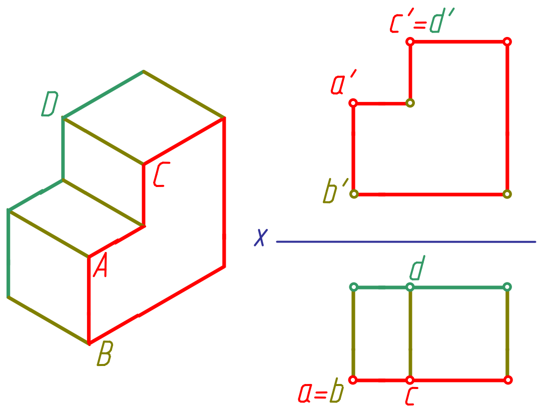


Fig. 3.6

The points lying on one projecting line are called competitive points. *A* and *B* are horizontal competitive points, *C* and *D* - frontal competitive points, etc.

It is obvious that if two points were situated on one projecting line, one of them covers the other. How to determine which of them is visible and which is invisible one?

Of two horizontal competitive points in the horizontal plane that one is visible which is situated higher in space.

The frontal projection (Fig.3.7) shows that the *z*-co-ordinate of the point *A* is bigger than that of the point *B*. Hence, the point A is located higher than *B* and being projected onto the horizontal projection plane it covers the point *B*. On the horizontal projection *A* is visible, *B* - invisible. On the frontal one both of the points are visible. Of two frontal competitive points in the frontal plane that one is visible which is situated closer to a viewer facing the frontal projection plane (Fig.3.8).

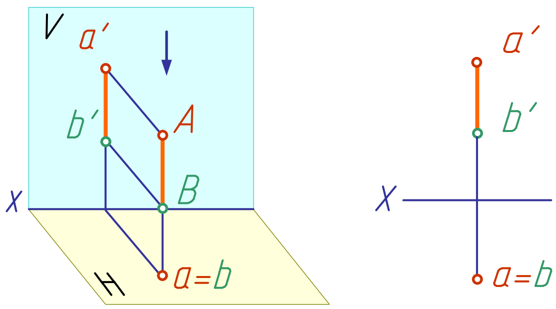


Fig. 3.7

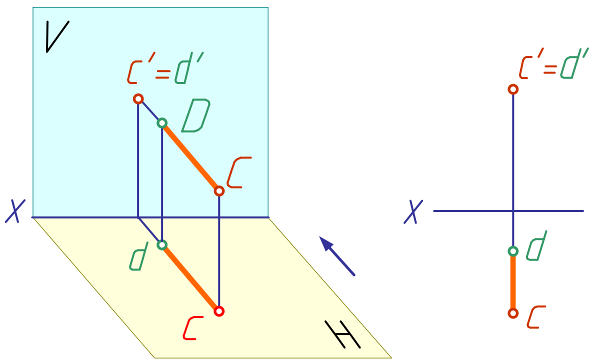


Fig. 3.8

The horizontal projection helps to determine which of the points is located closer to the viewer. For example, comparing the horizontal projections of the points *D* and *C* one can conclude that the point *C* is visible on the plane *V*, and *D* - is invisible.

Likewise determine the visible point of two profile competitive ones in the projection plane *W* - that one is visible which is situated closer to the left side.

So, when projections of the like points in a drawing do not coincide, or only one projection pair coincides, these points do not coincide in space and are located a certain distance from each other.

3.3 Drawing of a Line-Segment. Straight Lines of a Particular Position

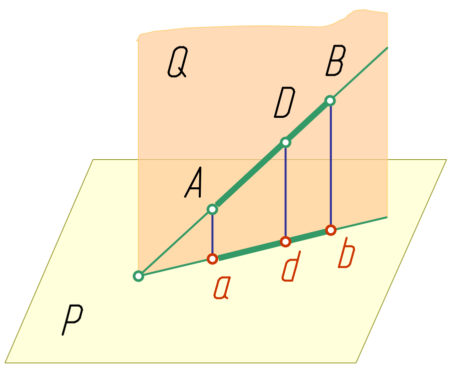


Fig. 3.9

Fig.3.9 presents the line-segment *AB* and its orthogonal projecting onto the plane *P*. Let us consider orthogonal projecting of the segment AB subject to the properties of parallel projection. The projecting lines *Aa* and *Bb* passed from the points *A* and *B* produce the projecting plane *Q*. The cutting line of the planes *Q* and *P* passes through the projections *a* and *b* of the points *A* and *B* on the projection plane *P*. This very line is the only projection of the above line in the projection plane *P*. Fig.3.10 presents a visual picture of the segment *AB* projecting onto two projection planes in the system *H*, *V*, Fig.3.11 presents a drawing.

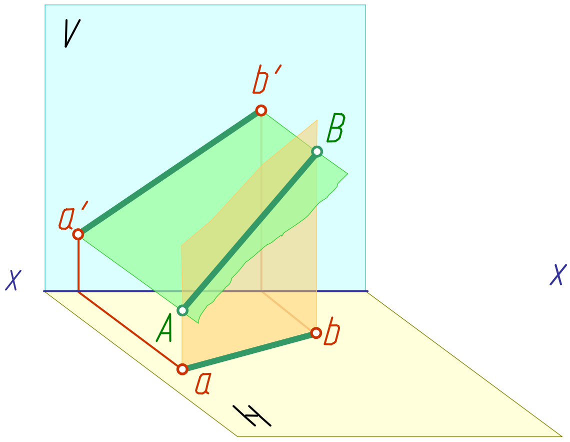


Fig.3.10

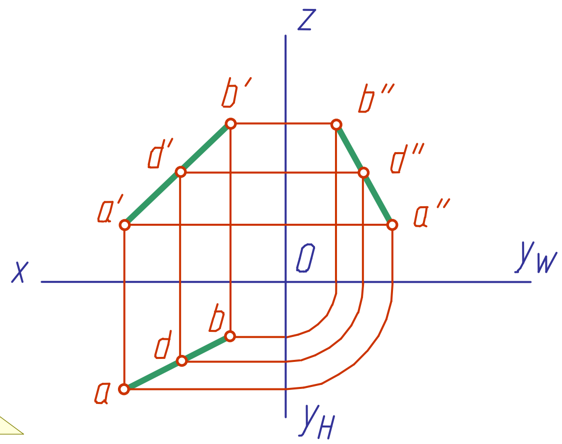


Fig.3.11

If a point belongs to a line, its projection belongs to the line projection. For example, the point *D* (Fig.3.9) belongs to the line *AB*, so, its projections belong to the line projections (Fig.3.11).

A line can have different positions relative to the projection planes:

- parallel to neither of the projection planes *H*, *V*, *W*;
- parallel to one of the projection planes (the line may as well belong to this plane);
- parallel to two of the projection planes, that is, perpendicular to the third one.

A line parallel to neither of the projection planes is referred to as the line of general position (Fig.3.9 through 3.11).

A line parallel to one of the projection planes or to two (that is, perpendicular to the third one) is referred to as the line of a particular position.

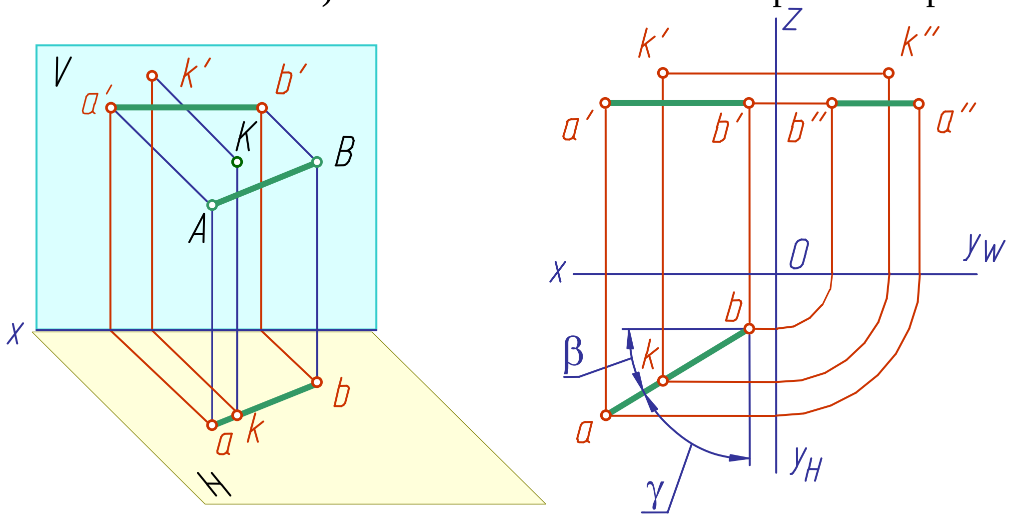


Fig.3.12

Fig.3.12-3.14 show the visual pictures and drawings of the lines of a particular position - the lines parallel to the projection planes. Such lines are called level lines.

a) The line *AB* is parallel to the plane *H* (it is called a horizontal line). The frontal projection *a′b′* of the line is parallel to the axis; the profile projection *a″b″* is parallel to the axis y_w; the length of the segment horizontal projection is equal to the length of the segment proper *ab*=*AB*; the angle β, contained by the horizontal projection and projection axis *x*, is equal to the inclination angle of the line to the frontal projection plane; the angle γ, contained by the horizontal projection and the axis y_H, is equal to the inclination angle of the line to the profile projection plane.

$|ab| = |AB|;\quad (a'b') \,//\, (Ox);\quad (a''b'') \,//\, (Oy_w);$
$(AB\hat{\ }V) = (ab\hat{\ }Ox) = \beta;\quad (AB\hat{\ }W) = (ab\hat{\ }Oy_H) = \gamma.$

b) The line *CD* is parallel to the plane *V* (it is a frontal line) - Fig.3.13

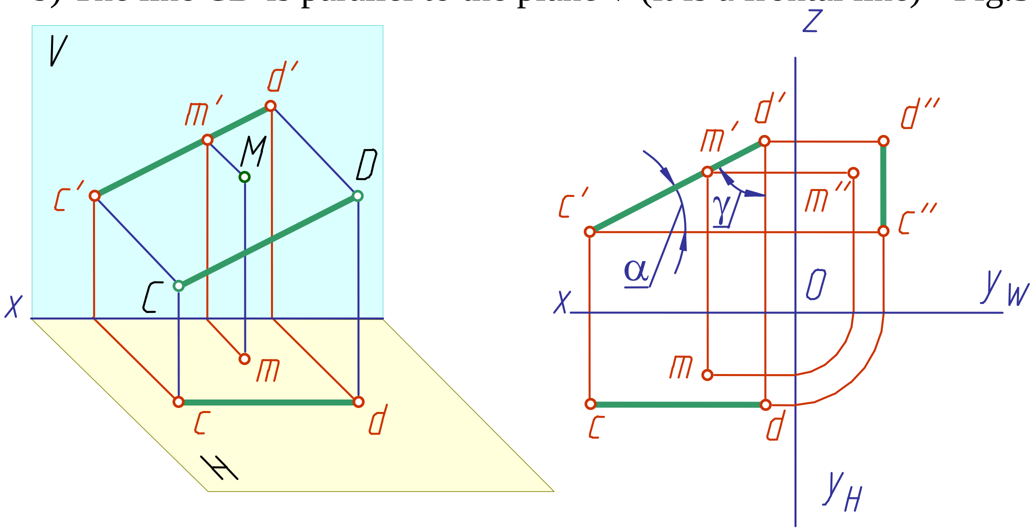


Fig.3.13

The horizontal projection *cd* of the line is parallel to the axis *x*; the profile projection *c″d″* is parallel to the axis *z*; the length of the segment frontal projection is equal to the length of the segment proper *c′d′*=*CD*; the angle α, contained by the frontal projection and projection axis *x*, is equal to the inclination angle of the line to the horizontal projection plane; the angle γ, contained by the frontal projection and the axis *z*, is equal to the inclination angle of the line to the profile projection plane.

$|c'd'| = |CD|;\quad (cd) \,//\, (Ox);\quad (c''d'') \,//\, (Oz);$
$(CD\hat{\ }H) = (c'd'\hat{\ }Ox) = \alpha;\quad (CD\hat{\ }W) = (c''d''\hat{\ }Oz) = \gamma.$

c) The line *EF* is parallel to the plane *W* (it is a profile line).

The horizontal projection *ef* of the line is parallel to the axis y_H; the frontal projection *e′f ′* is parallel to the axis *z*; the length of the segment profile projection is equal to the length of the segment proper $e''f''=EF$; the angles α and β, contained by the profile projection and projection axes y_w and z, is equal to the inclination angles of the line to the horizontal and frontal projection planes respectively.

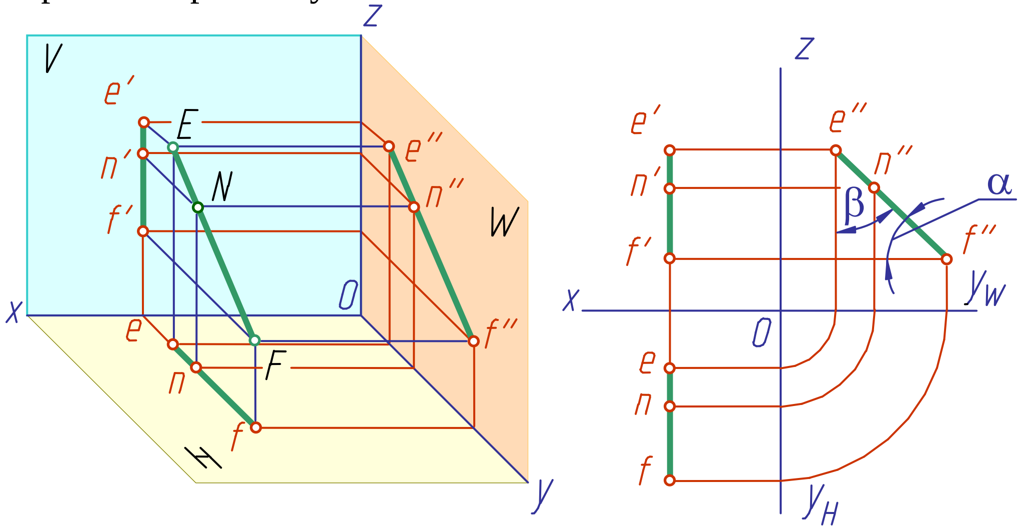


Fig.3.14

$|e''f''| = |EF|$; $(ef) \parallel (Oy_H)$; $(e'f') = (Oz)$;

$(EF \wedge H) = (e''f'' \wedge Oy_w) = \alpha$; $(EF \wedge V) = (e''f'' \wedge Oz) = \beta$.

Thus, each level line projects in true size onto that projection plane to which it is parallel. The angles contained by this line and two other planes, also project on the above plane in true size.

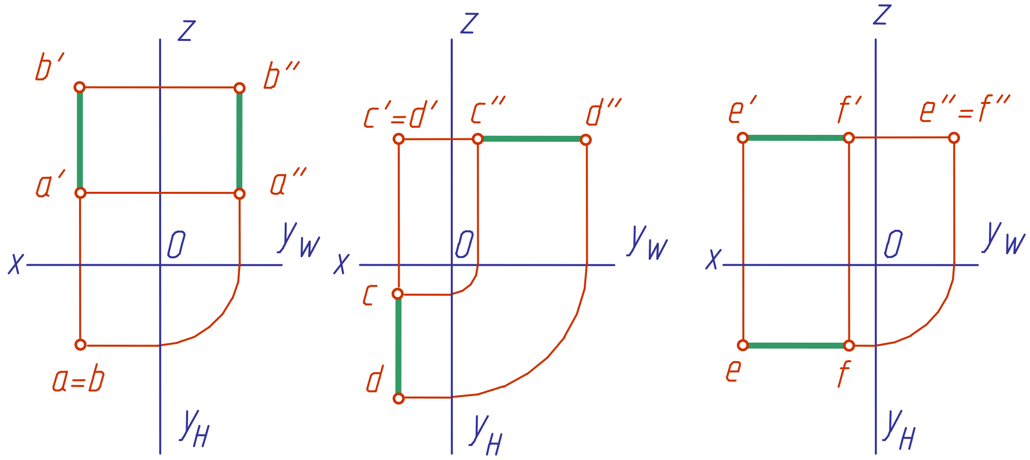


Fig.3.15

Fig.3.15 shows drawings of the straight lines perpendicular to the projection planes. These lines are called projecting lines.

a) The line AB is perpendicular to the plane H (horizontal projecting line), its projection $a'b'$ is perpendicular to the axis x, projection $a''b''$ is perpendicular to the axis y, projections a and b coincide.

(AB) $\perp$ *H;* *(AB) // V;* *(AB) // W;*

ab – point; $|a'b'| = |a''b''| = |AB|$; $(a'b') \perp (Ox)$; $(a''b'') \perp (Oy_W)$.

b) The line *CD* is perpendicular to the plane *V* (frontal projecting line), its projection *cd* is perpendicular to the axis *x*, projection *c″d″* is perpendicular to the axis *z*, projections *c′* and *d′* coincide.

(CD) $\perp$ *V;* *(CD) // H;* *(CD) // W;*

c′d′ – point; $|cd| = |c''d''| = |CD|$; $(cd) \perp (Ox)$; $(c''d'') \perp (Oz)$.

c) The line *EF* is perpendicular to the plane *W* (profile projecting line), its projection *ef* is perpendicular to the axis y_H, projection *e′f′* is perpendicular to the axis *z*, projections *e″* and *f″* coincide.

(EF) $\perp$ *W;* *(EF) // H;* *(EF) // V;*

e″f″ – point; $|ef| = |e'f'| = |EF|$; $(ef) \perp (Oy_H)$; $(e'f') \perp (Oz)$.

The drawing proves that the projecting line is also a level line as it is parallel at the same time to two other projection planes.

So, projecting lines are projected on two projection planes in true shape, i.e. in true size, and on the third plane they are projected as a point.

3.4 Mutual Positions of a Point and a Line

A point and a line in space may have different positions relative to each other and to a projection plane.

If a point in space belongs to a line, its projections belong to the corresponding projections of the line.

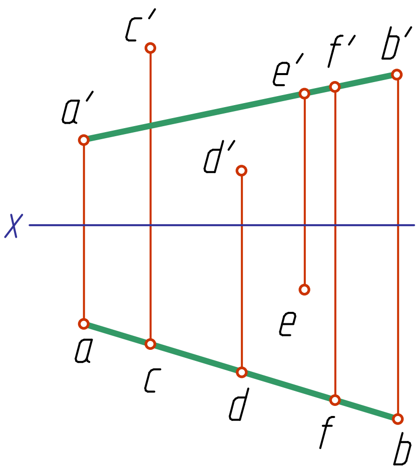


Fig. 3.16

Fig.3.12 through 3.14 illustrate the above statement.

Let us examine it again in a plane drawing (Fig.3.16).

The point *F* belongs to the line *AB* as the horizontal projection *f* of the point belongs to the line horizontal projection *ab*, and the point frontal projection *f′* belongs to the line frontal projection *a′b′*.

The point *C* is located above the line *AB*, the point *D* is situated under the line *AB*, the point *E* lies behind the line *AB*.

3.5. Traces of a Line

The trace of a line is the point at which the line intersects a projection plane. The point *M* (Fig.3.17) is a horizontal trace of the line, the point *N* - a vertical (frontal) one.

The horizontal projection m of the horizontal trace coincides with the trace proper - the point *M* (Fig.3.17, a); the frontal projection *m′* of the trace lies on the axis x. The frontal projection n' of the line vertical trace coincides with the vertical trace - the point N; the horizontal projection n lies on the same projection axis.

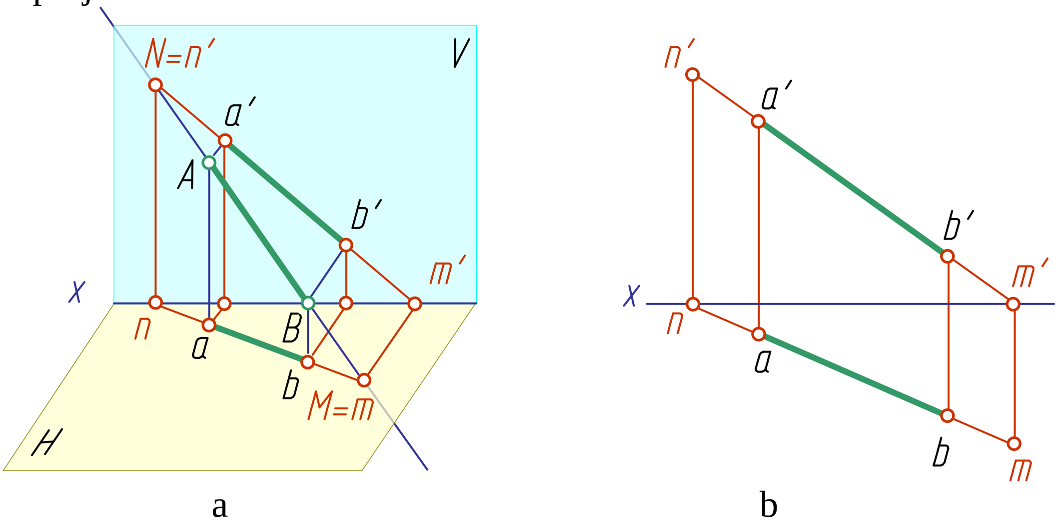


Fig.3.17

To construct the horizontal trace of a line (points m and m') in a plane drawing proceed as follows (Fig.3.17, b): prolong the vertical projection $a'b'$ of the line to intersect the x-axis at the point m'. At this point erect a perpendicular to the coordinate axis to intersect the prolongation of the horizontal projection ab. The point m thus obtained is the horizontal projection of the horizontal trace.

The vertical trace of a line (points n and n') is found in much the same manner: prolong the horizontal projection ab of the line to intersect the x-axis at the point n. At this point erect the perpendicular to the x-axis to intersect the prolongation of the frontal projection $a'b'$. The point n' is the frontal projection of the vertical trace.

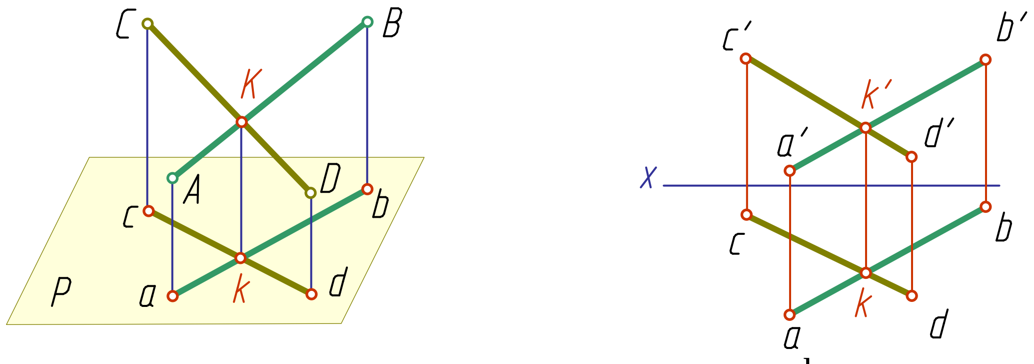
A line may intersect the profile projection plane as well, it means there may be the profile trace, too. This trace on the profile plane merges with its projection, where as, its frontal and horizontal projections lie on the axes z and y, accordingly.

3.6.The Relative Positions of Two Straight Lines

Straight lines in space may have different relative positions:

- to intersect, that is to have one common point;
- to be parallel if their intersection point is at infinity;
- to cross, that is to have no common points.

Intersecting lines. If the lines intersect, their like projections intersect and the points of intersection lie on one connection line.



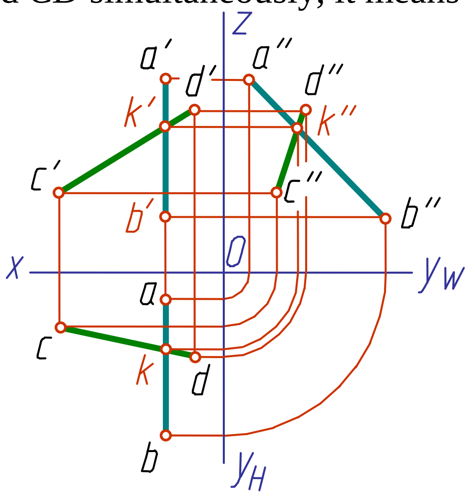
a b

Fig.3.18

Fig.3.18, a presents the visual picture of two lines *AB* and *CD*, intersecting at the point *K*; Fig.3.18, b - the drawing in the system of *H* and *V* planes.

If one of the lines in the *H* and *V* planes system is a profile line, to find out where the lines intersect, construct their profile projections.

Fig.3.19 - all projections of the point *K* (*k*, *k*′, *k*″) belong to the lines *AB* and *CD* simultaneously, it means that the lines *AB* and *CD* intersect.



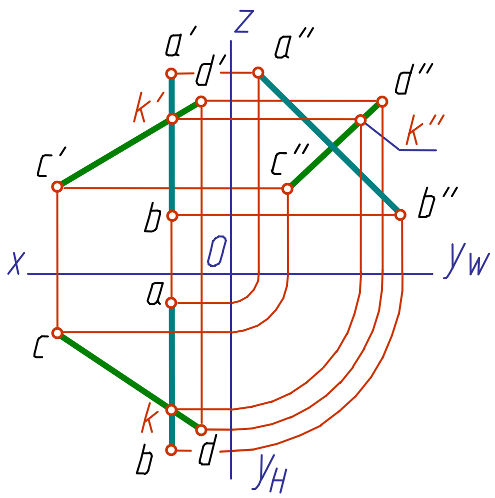


Fig.3.19 Fig.3.20

Fig.3.20 - the profile projection *k*″ of the point *K* (the *CD* line) does not belong to the profile projection *a*″*b*″, hence, the lines *AB* and *CD* do not intersect.

Parallel lines. If the lines in space are parallel, their like projections are also parallel. Fig.3.21 - the projecting planes *Q* and *R*, passed through the parallel lines *AB* and *CD*, are also parallel. They intersect the plane *P* with the parallel lines *ab* and *cd*, which are the projections of *AB* and *CD* lines. A drawing of two parallel lines of general position is shown in Fig.3.22, drawings of parallel lines of particular position - in Fig.3.23:

a) horizontal lines; b) frontal lines; c) profile lines.

Note. The lines in space are parallel when their like projections on two planes are also parallel, provided:

for the lines of general position - their like projections are parallel in the system of any two projection planes (Fig.3.22);

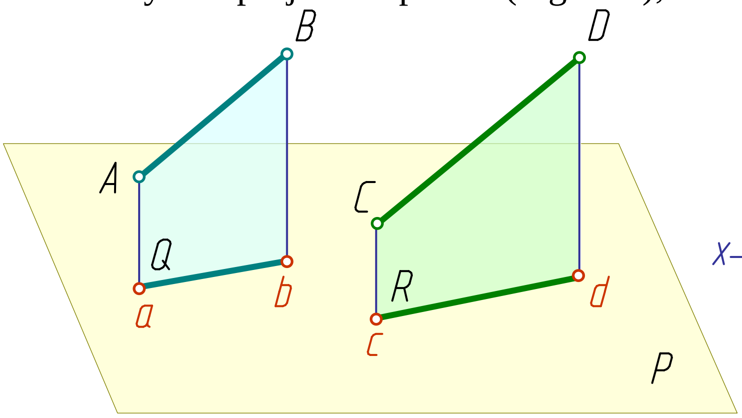


Fig.3.21

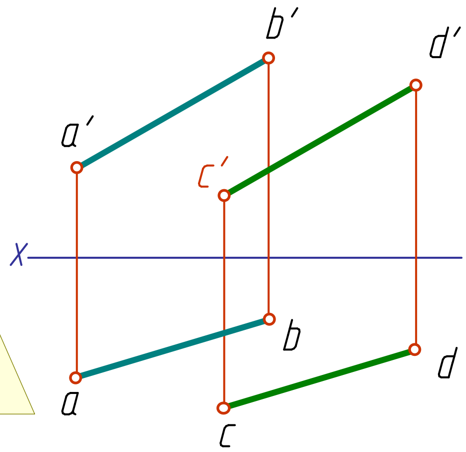
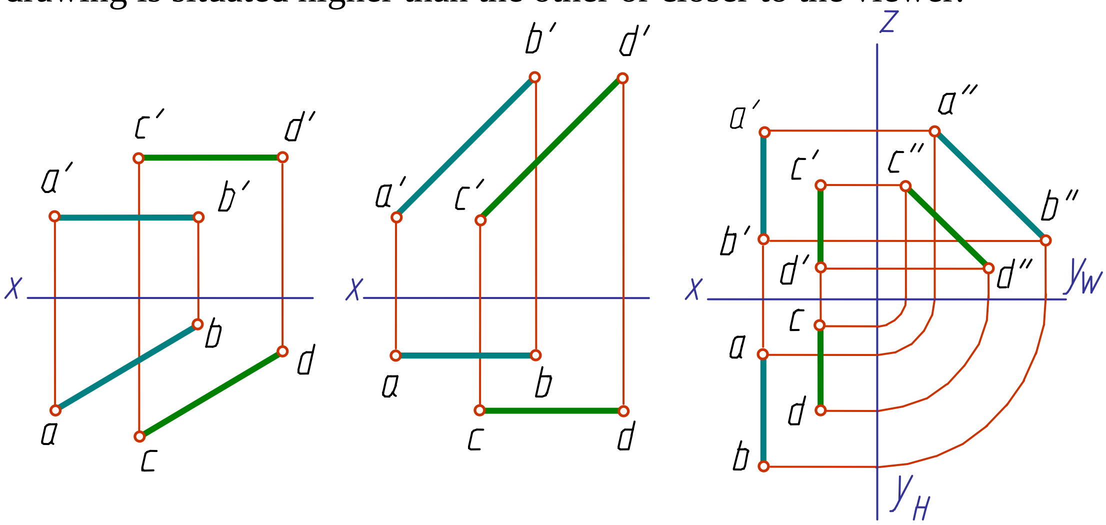


Fig.3.22

for the lines of particular position - their like projections are parallel to one of the projection axis and their like projections are parallel to each other on that projection plane, to which the above lines are parallel (Fig.3.23).

Skew lines. If the lines in space do not intersect but cross (Fig.3.24), their like projections in a drawing may intersect , but the intersection points of projections do not lie on one connecting line. These points are not common to the above lines.

Comparing the loci of such points, determine which of the lines in a drawing is situated higher than the other or closer to the viewer.



AB//H

a

AB//V

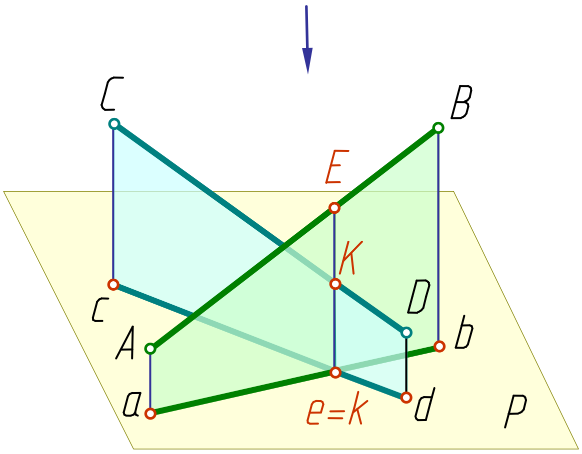
b

AB//W

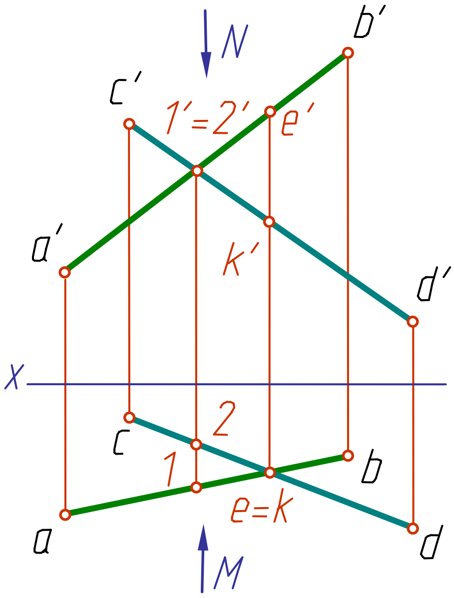
c

Fig.3.23

Fig.3.24, a - the point *E* (belonging to the line *AB*) is situated above the point *K* (belonging to the line *CD*) and if viewed from above, in the direction of the arrow, the point *E* covers the point *K*. A similar situation is in Fig.3.24, b - the frontal projection *e′* is located higher than the frontal projection *k′*, and if viewed from above, in the direction of the arrow *N*, the point *e*, being projected on the plane *H*, covers the point *k*. The straight line *AB* runs above the line *CD*.



a



b

Fig.3.24

The frontal projections *1′* and *2′* of the lines *AB* and *CD* coincide on the plane *V*. If viewed in the direction of the arrow *M*, the point *1* of the *AB* line is situated closer to the observer, so, being projected on the plane *V* this point covers point *2* of the line *CD*. The line *AB* lies closer to the observer.

Considered the above points of skew lines, projections of which coincide on one of the planes, are referred to as competitive points.

3.7. Projecting of Plane Angles

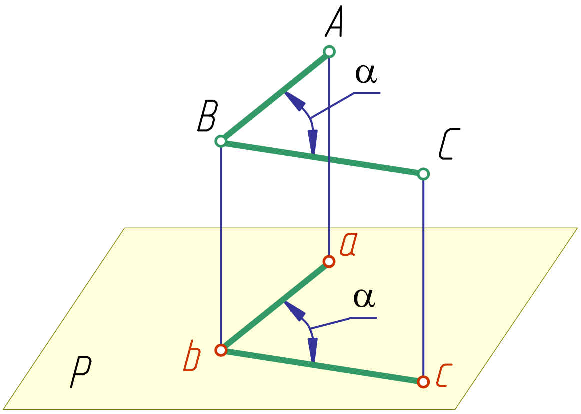


Fig.3.25

Any linear angle is formed by two intersecting lines. It is usually projected onto the projection planes in distortion. However, if both arms of the angle are parallel to one of the projection planes, the angle is projected on this plane without changing it, i.e. in true size. For example, the arms of *ABC* angle (Fig.3.25) are parallel to the horizontal plane *P*, hence, the angle α is projected on it in true size. *Note.* A right angle with at least one side parallel to one of the projection planes is projected on this plane also as a right angle. Consider a theorem on a right angle projecting.

Theorem. A right angle is projected as a right angle, when one of its arms is parallel to a projection plane and the second arm is not perpendicular to it.

Given: the arm *ED* of the right angle *KED* is parallel to the plane *P*, the arm *KE* is not perpendicular to it (Fig.3.26). Prove that the angle projection *ked* = 90°

$$(ED \perp EK) \wedge (ED \,//\, P)\ (EK \not\perp P) \Rightarrow (ed) \perp (ek).$$

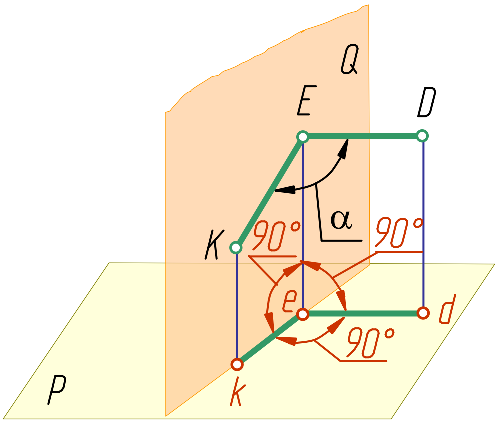


Fig.3.26

Proof. Pass the auxiliary plane *Q* through the lines *KE* and *Ee*. The plane *Q* (being passed through the line *Ee*, perpendicular to the plane *P*) is perpendicular to the plane *P*. The line *ED* is perpendicular to the plane *Q*, as it is perpendicular to two lines (*EK* and *Ee*) of this plane. The line *ed* is also perpendicular to the plane *Q*, as the line *ED* and its projection *ed* are parallel to each other (see given conditions). Hence, the line *ed* is perpendicular to any line, lying in this plane, the line *ek* among them, which means, the angle *ked* is a right angle. As was to be proved.

3.8 Determining the True Size of a Line-Segment

No segments of the lines of general position are projected on the planes of projection in true size. But the solution of some problems requires determining by a drawing the length of a line-segment and the angle of the inclination of a line to the projection planes.

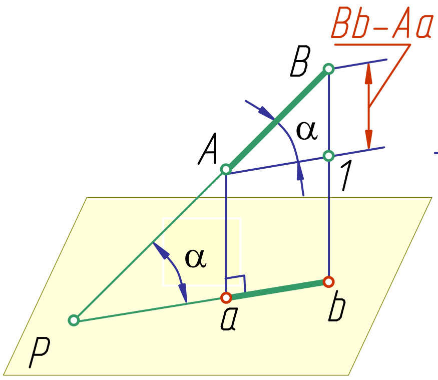
In this case the method of a right triangle construction is used.

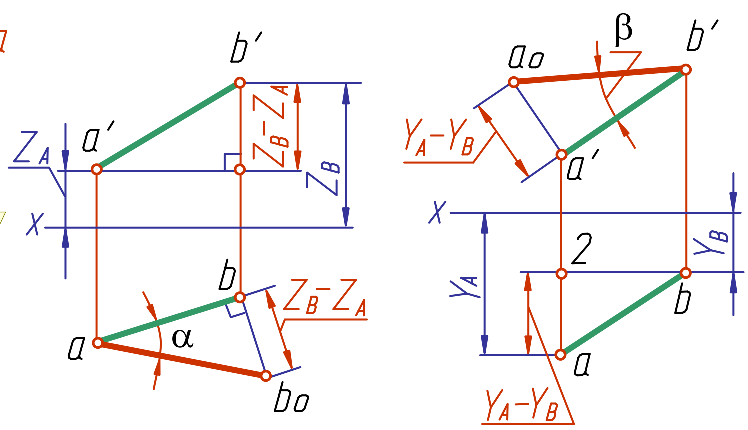
Theorem. The true length of a segment of a line of general position is equal to the hypotenuse of a right triangle one leg of which is a projection of the given line-segment on one of the projection planes, the second leg being equal to the absolute value of the algebraic difference of the distances from the ends of the line-segment to the same plane.

Proof. Fig.3.27 shows that the true length of the line-segment *AB* is the hypotenuse of the right triangle *AB1* one leg of which is equal to the projection of the given segment, the second leg - to the absolute value of the algebraic difference of the distances from the ends of the line-segment to the projection plane.

Determine the true length of the line segment *AB* and the angle of its inclination to the plane *H* (angle α) given two projections of the line-segment (Fig.3.28, a). Construct a right triangle given its two legs: the horizontal projection of the line-segment and a line-segment of length equal to the value of the algebraic difference of the distances from the line-segment ends to the plane *H* (difference of *z* coordinates of the points *A* and *B*). The hypotenuse ab_0 of this triangle will yield the true length of *AB*, and the angle bab_0 - its inclination angle to the plane *H*.

Fig.3.28, b shows determination of the true size of the line-segment *AB* and its inclination angle to the plane *V* (angle β).





a b

Fig.3.27 Fig.3.28

Questions to Chapter 3

1. What location relative to the projection planes causes a line to be called “a line of general position”?
2. What is the locus of a line in the system of the planes *H*, *V*, *W* given all three projections of the line are equal in length?
3. How do we construct a profile projection of a line of general position given its frontal and horizontal projections?
4. What positions of a straight line in the system of *H*, *V*, *W* planes are considered to be the particular ones?
5. What is the position of a frontal projection of a line-segment given its horizontal projection is equal to the line-segment proper?
6. What is the position of a horizontal projection of a line-segment given its frontal projection is equal to the line-segment proper?
7. What is referred to as “the trace of a straight line on a projection plane”?
8. Which coordinate is equal to zero:
   a) for a frontal trace of a line;
   b) for a horizontal trace of a line?