CHAPTER 2. PROJECTION METHOD

Images of three-dimensional space objects on a plane are made by the projection method.

The projector includes a projectable object, projecting beams and the plane on which the picture of the object appears.

2.1 Central Projection

Central projection is the general case when geometric images are projected on a given plane from a selected outside centre. Figure 2.1 – Point *S* is the Projection Centre, Plane *P* is the Projection Plane. To make a central projection of a point, draw a projecting line through the point and the projection centre. The intersection point of this line and the projection plane turns out to be the central projection of the given point on the selected plane.

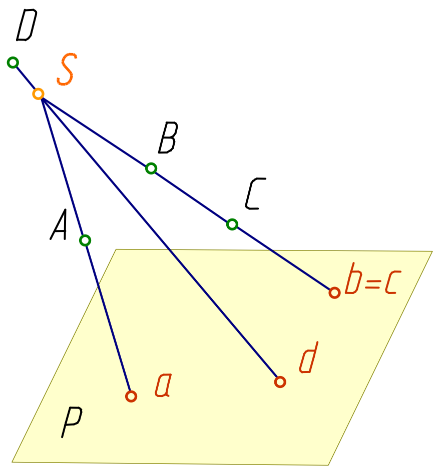

Fig.2.1

The *a, b, c, d* points are the central projections of the *A, B, C, D* points on the *P* plane.

B and *C* are two different points which are situated on one projecting line. That is why the central projections *b* and *c* coincide. The whole set of points of one projecting line has only one central projection if there is only one projection centre.

So, on the given plane and given projection centre one point in space has one central projection. But one central projection of a point does not allow us to determine the location of the point in space unambiguously, i.e. in central projecting there is no reversibility of drawing and additional conditions are required to provide drawing reversibility.

Central projection provides imaging of any line or surface as a set of projections of all its points. With this, the projecting lines, collectively, drawn through all points of a curved line, either make a projecting conical surface (Fig. 2.2) or can appear on one plane (Fig.2.3).

The projection of a curved line is a line of intersection of the projecting conical surface and projection plane. Fig.2.2 - the projecting conical surface Φ intersects the projection plane *P* along an *ab* curve which is an *AB* line projection. However, one projection of a line does not determine the line projected as there can be an infinite quantity of lines projecting to one and the same line in the projection plane.

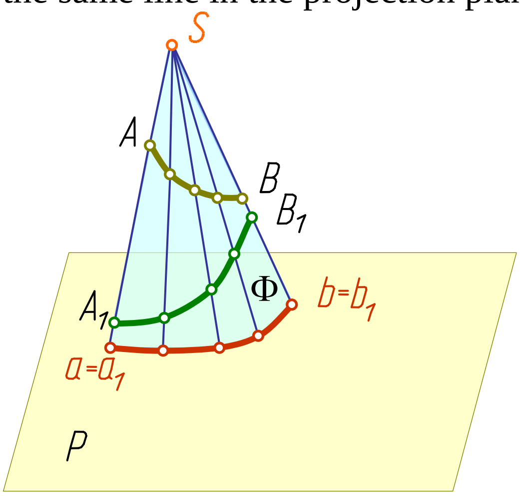

Fig.2.2

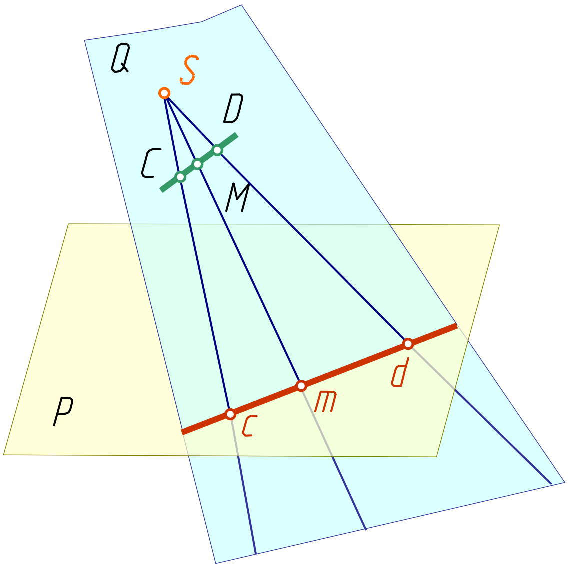

Fig.2.3

The plane serves as a projection surface if a line which does not pass through the projection center is being projected. Figure 2.3 - the projecting plane *Q* formed by the *SC* and *SD* projection lines which pass through the *C* and *D* points of a straight line, intersects the projection plane *P* along the *cd* line which is the projection of the above line. As the *M* point belongs to the *CD* line then its projection - *m* point - belongs to the *cd* projection.

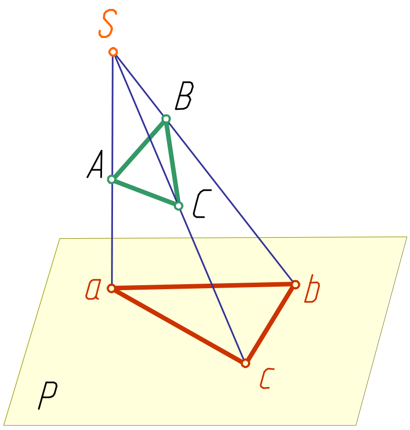

Fig.2.4

The construction of projections of lines, surfaces or bodies very often requires the construction of projections of some characteristic points only. For example, to construct a projection of the *ABC* triangle (Fig.2.4) it is enough to build projections of its three points - *A*, *B*, *C* vertices.

To summarize the material above let us note the following properties of central projection:

I. Under central projection

a) a point projects to a point;

b) a line not passing through the projection center projects to a line (projecting line - to a point);

c) a plane (two-dimensional) figure not belonging to the projecting plane projects to a two-dimensional figure (figures belonging to the projecting plane project to straight lines);

d) a three-dimensional figure projects to a two-dimensional one;

e) a central projection of figures keep mutual belonging, continuity and some other geometric properties

2. Under the given projection center figures of parallel planes are similar.

3. Central projection provides unambiguous similarity of the figure and its projection, for example, projection on a screen or a film.

In spite of visualization, a great positive property, central projections still have some disadvantages such as the complexity of a subject image construction and its real dimension determination. That is why this method is limited in usage. Central projection is used in constructing building perspectives, in painting, etc.

2.2 Parallel Projection

Parallel projection is one of the cases of central projection, where the center of projection is infinitely distant ($S\infty$). In this case parallel projecting lines are used, drawn in the given direction relative to the plane of projections. If the direction of projecting is perpendicular to the plane of projections, the last are called rectangular or orthogonal. In other cases they are called oblique projections. In Fig.2.5 the direction of projection is shown by the arrow at an angle $\alpha \neq 90°$ with the projection plane *P*.

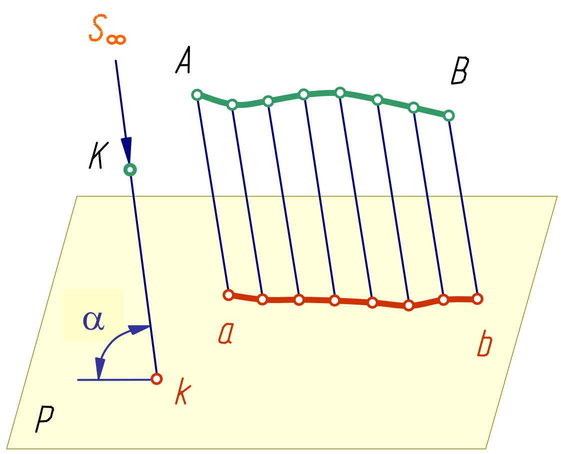

Fig.2.5

Parallel projection possesses all the properties of central projection and the following new ones:

1. Parallel projections of mutually parallel lines are parallel, and the ratio of those lines segments is equal to the ratio of their projections lengths.
2. A plane figure parallel to a projection plane projects to the same figure on the above plane.
3. A parallel transfer of a figure in space or in projection plane does not change the shape or dimensions of the figure projection.

Using parallel projection of a point and a line, one can draw parallel projections of a surface and a body. Neither parallel nor central projections provide drawing reversibility.

2.3. Methods of Projection Drawings Supplement

In projecting on one projection plane there is no one-to-one correspondence between the figure and its projection. At the given position and set direction, only one projection corresponds to the subject projected.

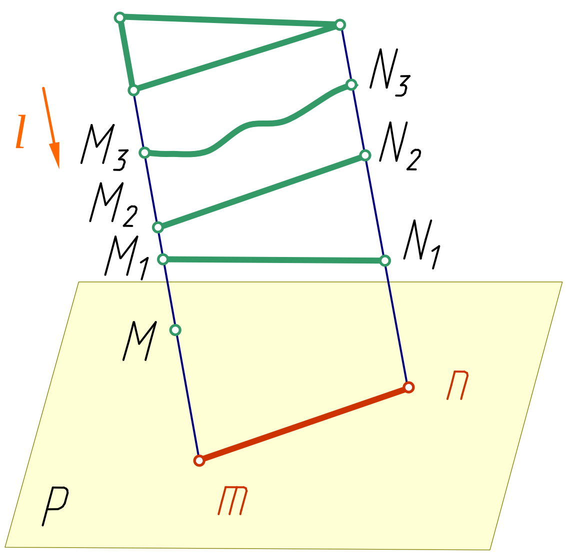

Fig.2.6

However, the figure obtained can be a projection of an infinite set of other figures, different in size and form. Figure 2.6 shows that m is the only projection of the spatial point M on the plane P. At the same time the point m is the projection of a set of points situated on the projecting line (M, M_1, M_2, M_3). The segment mn can be a projection not only of the segment M_1N_1 or M_2N_2, but a projection of the curve M_3N_3 and of any plane figure situated in the projecting plane.

Therefore, representation of the figure is not complete. The drawing can be understood clearly only if there are some additional clarifications.

Let us consider a few supplementary methods of the projection representation which provide the opportunity to make it “reversible”, i.e. unambiguously similar to the object projected.

Method of Projection with Digital Marks

This method is primarily used in drawings of landscape plans and some engineering facilities (such as dams, roads, dikes, etc.). The essence of the method is that the position of any point in space is specified by its rectangular projection on a horizontal plane taken for a zero plane (Fig.2.7).

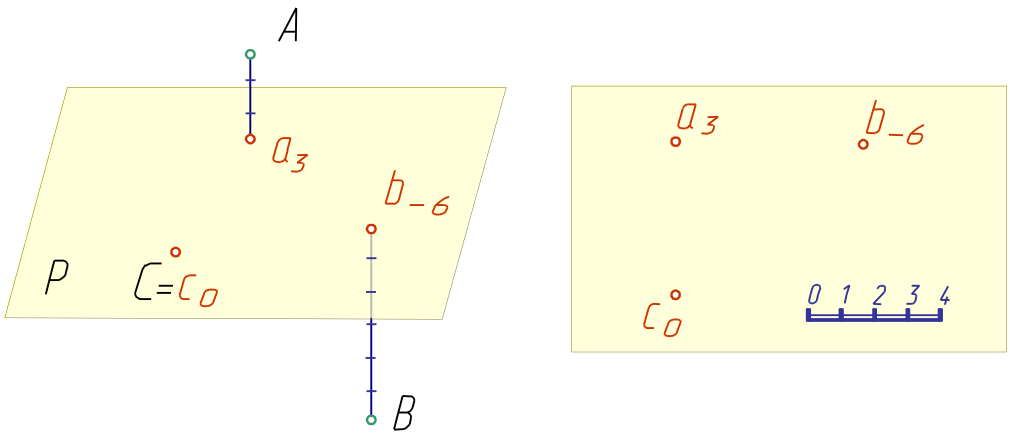

Fig.2.7

Near the projections of the points (*a*, *b*, *c*) their marks are shown, i.e. a number of length units, determining the distance from a point to the projection plane.

Method of Vector Projections

Academician E.S. Fyodorov suggested the representation of the points altitudes (heights) by means of parallel segments in the projection plane, the beginnings of which are situated on the projections of the respective points. The directions of all altitude segments are arbitrary. If the points are located above the horizontal plane, the altitude segments, as well as digital marks are considered to be positive. In the opposite case (if the points are below the plane) - negative. Positive and negative altitude segments in “Fyodorov’s projections” differ in direction. Drawings produced by method of “Fyodorov’s projections” are used in geology, mining, topography (Fig.2.8).

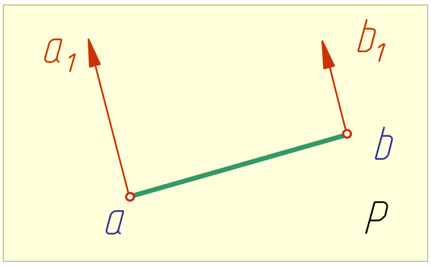

Fig.2.8

The Method of Orthogonal Projections

Drawing in a system of rectangular projections is constructed by projecting a component on two or three mutually perpendicular planes of projection. This method is a special case of parallel projection, when the direction of projecting is perpendicular to the projection plane. The foot of the perpendicular drawn from a point to the projection plane is called the rectangular (*orthogonal*) projection of the point.

Jasper Monge was the first to make the projection on two mutually perpendicular planes. Such a kind of projection ensures drawing reversibility, i.e. an unambiguous determination of a point position in space by its projections. One of the planes is usually arranged horizontally, and it is called the horizontal projection plane *H*, the other is perpendicular to it. Such a vertical plane is called the frontal projection plane *V*. The line of intersection of the above planes is called the projection (or coordinate) axis (Fig.2.9).

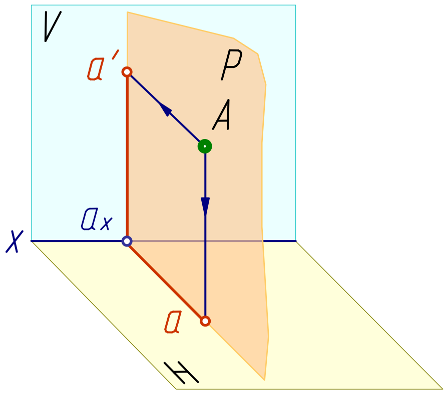

Fig.2.9

To obtain a point projection on a plane drop perpendiculars (projecting beams) from the spatial point *A* to the planes *H* and *V*. The projecting beams produce the plane *P* which is perpendicular to the planes *H* and *V* and cuts these planes along the lines perpendicular to the projection axis, and the axis proper - in the point a_x. Consequently, the lines aa_x, $a'a_x$ and the axis *x* are mutually perpendicular.

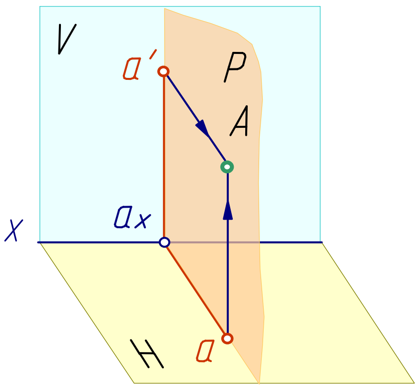

Fig.2.10

Construction of a certain spatial point *A* from given its two projections (the horizontal one *a* and the frontal one *a′*) is shown in Fig.2.10. Find the point *A* in the intersection of the perpendiculars drawn from the projection *a* onto the plane *H* and from the projection *a'* onto the plane *V*. The above perpendiculars belong to one plane P perpendicular to *H* and *V* planes, and intersect in the only spatial point *A*, which is the desired one.

Thus, two orthogonal projections of a point fully determine its position in space relatively to a given system of mutually perpendicular projection planes.

Questions to Chapter 2

1. What is the method of construction of the central projection of a point?
2. In what case is the central projection of a straight line represented by a point?
3. What is the essence of the parallel projection method?
4. How is the parallel projection of a line constructed?
5. Can the parallel projection of a line be represented by a point?
6. What are the positions of a point and a line projections if the point lies on the line?