9. Solving optimization problems

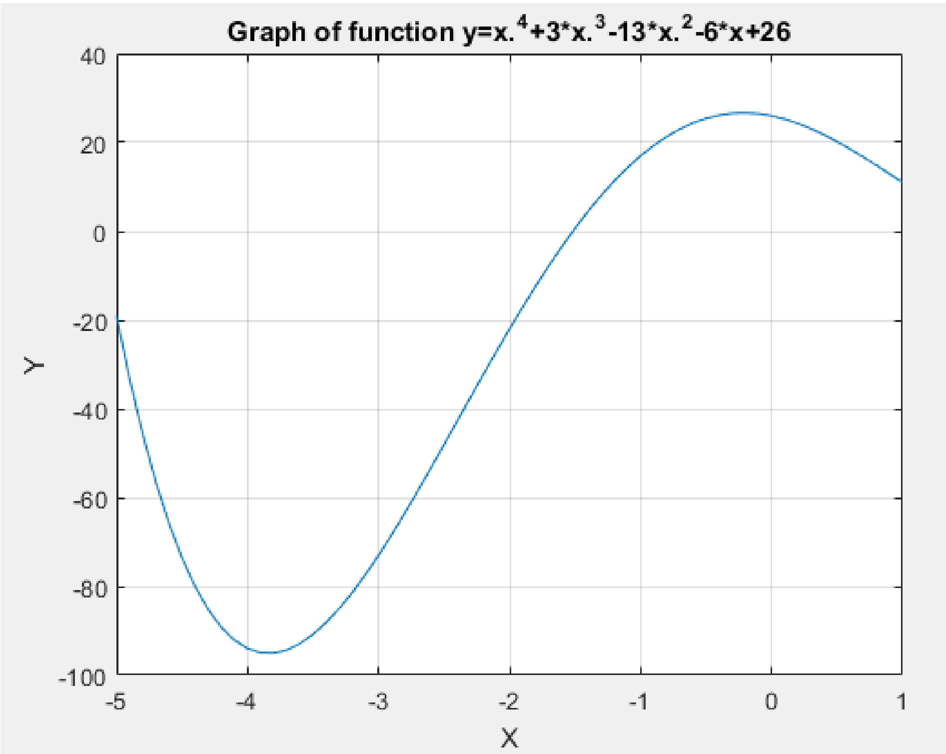
9.1. Search minimum of one variable

As a simple optimization problem, consider searching for a local minimum of a function of one variable.

Problem 9.1.

Find the minimum of $f(x) = x^4 + 3x^3 - 13x^2 - 6x + 26$.

```
x=-5:0.1:1;
y=x.^4+3*x.^3-13*x.^2-6*x+26;
plot(x,y);
grid on;
title('Graph of function y=x.^4+3*x.^3-13*x.^2-6*x+26')
xlabel('X');
ylabel('Y');
```



The graph shows that the function has a minimum near the point of -4. To find a more accurate value of the minimum function in Matlab use the command:

x = **fminbnd**(fun,x1,x2)

returns a value x that is a local minimizer of the function that is described in fun in the interval x1 < x < x2. fun is a function handle.

$$f(x) = x^4 + 3x^3 - 13x^2 - 6x + 26$$

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Pr9_1a.m

```
f=@(x)x^4+3*x^3-13*x^2-6*x+26;
xmin=fminbnd(f,-4,-3.5)
fmin=f(xmin)
```

Command Window

```
>> Pr9_1a

xmin =

   -3.8407


fmin =

  -95.0894
```

Similarly, we can find a minimum of every other function of one variable, the main objective is the problem of the correct choice of the point of the initial approximation. But this problem is not the Matlab, but a mathematical problem.

9.2. Search minimum function of many variables

Problem 9.2.

Find the minimum of Rosenbrock's function:

$$f(x, y) = 100(y - x^2)^2 + (1 - x)^2$$

Graph of Rosenbrock's function is shown in Figure

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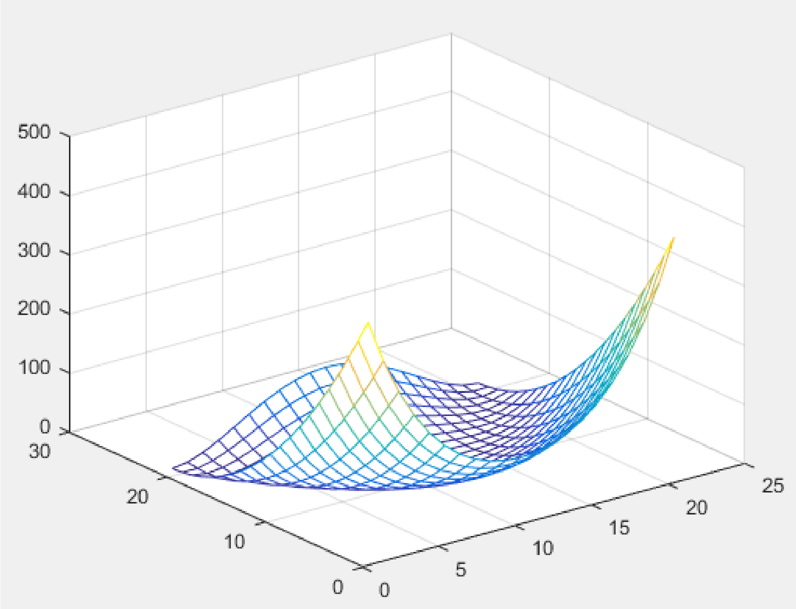
Pr9_2.m

```
[x,y]=meshgrid(-1:0.1:1, -1:0.1:1);
z=100*(y-x.^2).^2+(1+x).^2;
figure
mesh(z);
```

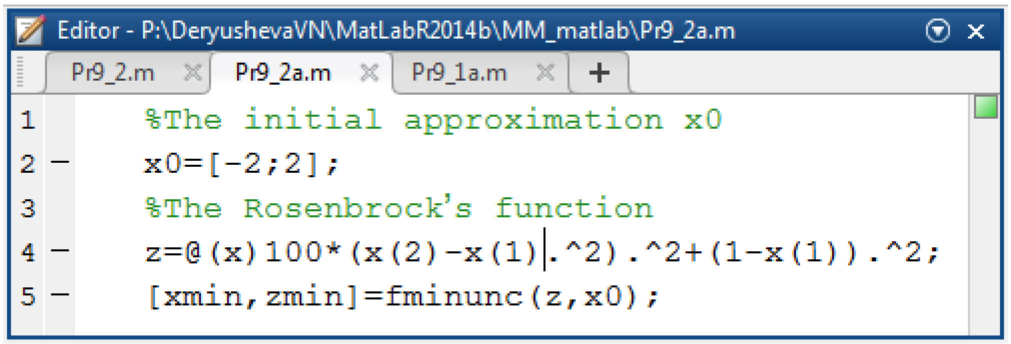


As is known, the Rosenbrock's function has a minimum at the point (1, 1) equal to 0. Rosenbrock function is a test for the minimization algorithms. Let us find the minimum of this function with the function **fminunc:**

x = **fminunc**(fun,x0)

starts at the point x0 and attempts to find a local minimum x of the function described in fun. The point x0 can be a scalar, vector, or matrix.

[x,fval] = **fminunc**(___), for any syntax, returns the value of the objective function fun at the solution x.



```
Command Window
>> xmin,zmin

xmin =

    1.0000
    1.0000


zmin =

   5.6305e-11

fx >>
```

9.3. Search a maximum of function one variable

There is no special command found the maximum of the function in Matlab. It is not needed: enough is already known **fminbnd**. Indeed, the maximum of the function f (t) is the minimum function -f (t). Therefore, finding the maximum order is the same as above, but we will work with the function -f (t) and change in the initial approximation x0 = -1, as the schedule (the problem 9.1.).

Problem 9.3.

Find the maximum of $f(x) = x^4 + 3x^3 - 13x^2 - 6x + 26$.

```
Editor - P:\DeryushevaVN\MatLabR2014b\MM_matlab\Pr9_3a.m
Pr9_2.m | Pr9_2a.m | Pr9_3a.m | +
f=@(x)-x^4-3*x^3+13*x^2+6*x-26;
xmax=fminbnd(f,-1,0)
fmax=-f(xmax)
```

```
Command Window
>> Pr9_3a

xmax =

   -0.2162


fmax =

   26.6614

fx
```

9.4. The solution of linear programming problems

Another frequently encountered in the practice of optimization problem is a linear programming problem. Introduction to linear programming problems begin by the example of an optimal diet.

The problem of the optimal diet. There are four kinds of food: $П_1$, $П_2$, $П_3$, $П_4$. The Known cost per unit of each product is c_1, c_2, c_3, c_4. It is Necessary to make the diet of these products, which must be at least b1 units of proteins, at least b_2 units of carbohydrate, at least b_3 units of fats. And we know that the product $П_1$ contains a_{11} units of proteins, a_{12} units of carbohydrates and a_{13} units of fats, etc. (See. Table 9.1).

It is Required to make a diet to provide specified conditions at minimal cost.

Table 9.1. The content of proteins, carbohydrates and fats in foods

Product	Proteins	Carbohydrates	Fats
$П_1$	a_{11}	a_{12}	a_{13}
$П_2$	a_{21}	a_{22}	a_{23}
$П_3$	a_{31}	a_{32}	a_{33}
$П_4$	a_{41}	a_{42}	a_{43}

Let x_1, x_2, x_3, x_4 – number of products $П_1$, $П_2$, $П_3$, $П_4$. The total value of the diet is

$$L = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 = \sum_{i=1}^{4} c_ix_i. \qquad (9.1)$$

Formulate a limit on the amount of proteins, carbohydrates and fats in the form of inequalities.

The product $П_1$ contains a_{11} units of proteins, in x_1 units contain $a_{11}x_1$, in x_2 units of product $П_2$ contain $a_{21}x_2$ units of proteins and etc.

Consequently, the total amount of proteins in the four types of product equal

$$\sum_{j=1}^{4} a_{j1}x_j$$

and should be less than b_1. We get the first limitation:

$$a_{11}x_1 + a_{21}x_2 + a_{31}x_3 + a_{41}x_4 \geq b_1. \qquad (9.2)$$

Similar limitations for the fats and carbohydrates have the form:

$$a_{12}x_1 + a_{22}x_2 + a_{32}x_3 + a_{42}x_4 \geq b_2. \qquad (9.3)$$

$$a_{13}x_1 + a_{23}x_2 + a_{33}x_3 + a_{43}x_4 \geq b_3. \qquad (9.4)$$

Taking into account that the x_1, x_2, x_3, x_4 are positive values, we obtain four constraints:

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0 \qquad (9.5)$$

Thus, the problem of a rational diet can be formulated as follows: find the values of the variables x_1, x_2, x_3, x_4, satisfying the system constraints (9.2) - (9.5), in which the linear function (9.1) would take the minimum.

The problem of the optimal diet is a linear programming problem, the function (9.1) is called the objective function and constraints (9.2) - (9.5) -system limitations of linear programming problem.

In problems of linear programming the objective function L and system constraints are linear.

In general, the linear programming problem can be formulated as follows. Find those values x_1; x_2; …; x_n, satisfy the system constraints (9.6), in which the objective function L (9.7) reaches its minimum (maximum) values:

$$\sum_{j=1}^{n} a_{ji}x_j \le b_i, \qquad i = 1, \dots, m, \qquad x_i \ge 0 \qquad (9.6)$$

$$L = c_1x_1 + c_2x_2 + \cdots + c_nx_n = \sum_{i=1}^{n} c_ix_i. \qquad (9.7)$$

In order to solve linear programming problems in Matlab is a function **linprog** the following structure:

x = **linprog**(f,A,b,Aeq,beq,lb,ub)

solves min f'*x such that A*x ≤ b

defines a set of lower and upper bounds on the design variables, x, so that the solution is always in the range lb ≤ x ≤ ub. Set Aeq = [] and beq = [] if no equalities exist.

Problem 9.4.

Find the values of the variables x_1; x_2; x_3; x_4, in which the objective function L

$$L = -x_2 - 2x_3 + x_4$$

It reaches its minimum value, and cateres restrictions:

$$\begin{cases} 3x_1 - x_2 \le 2 \\ x_2 - 2x_3 \le -1 \\ 4x_3 - x_4 \le 3 \\ 5x_1 + x_4 \ge 6 \\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0 \end{cases}$$

Note that in the fourth limited presence of the sign ">". To bring the system of constraints mean (13.1) must be the fourth equation is multiplied by -1.

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Pr9_4.m | Pr9_exam.m

```
%The coefficients of leside of inequality
A=[3 -1 0 0; 0 1 -2 0; 0 0 4 -1; -5 0 0 -1];
%The coefficients of right side of inequality
b=[2;-1;3;-6];
% The coefficients of the objective function
f=[0;-1;-2;1];
lb=[0;0;0;0];
x=linprog(f,A,b,[],[],lb)
```

```
>> Pr9_4
Optimization terminated.

x =

    0.8795
   72.3113
   36.6556
  143.6226
```

Problem 9.5.

Travel agency signed a contract with two tourist bases at one of the seaside resorts, with capacity respectively for 200 and 150 people. Tourists are offered to explore the Aquarium in the city, the Botanical Gardens and hiking in the mountains. Create the route ~~of~~ for tourists so that it may cost less if the Aquarium receives daily 70 organized tourists, the Botanical Gardens - 180, and the mountain hike can take up to 110 people a day.

The cost of a single visit to the table is expressed in Table 9.2

Table 9.2.

Camp site	Aquarium	Botanical Garden	Hiking in the mountains
1	5	6	20
2	10	12	5

To solve the problem, we introduce the following notation:

x_1 – the number of tourists first camp site, visiting of the Aquarium;

x_2 – the number of tourists first camp site, visiting of the Botanical Gardens;

x_3 – the number of tourists first camp site, go hiking in the mountains;

x_4 – the number of tourists second camp site, visiting the Aquarium;

x_5 – the number of tourists second camp site, visiting of the Botanical Gardens;

x_6 – the number of tourists second camp site, go hiking in the mountains;

Construct the objective function is to minimize the cost of activities of the company:

$$Z = 5x_1 + 6x_2 + 20x_3 + 10x_4 + 12x_5 + 5x_6$$

Guided by the conditions of the problem, define the limits:

$$\begin{cases} x_1 + x_4 \leq 70 \\ x_2 + x_5 \leq 180 \\ x_3 + x_6 \leq 110 \\ x_1 + x_2 + x_3 = 200 \\ x_4 + x_5 + x_6 = 150 \end{cases}$$

Furthermore, the number of tourists participating in events can not be negative: $x_1 > 0$, $x_2 > 0$, $x_3 > 0$, $x_4 > 0$, $x_5 > 0$, $x_6 > 0$.

The values of x will store the values x_1, x_2, x_3, x_4, x_5, x_6. The coefficient matrix A of the first two lines reflect the equality of restrictions, therefore enabled k = 2.

Vector lb represents the lower limit of the unknown, they they must be more than zero.

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Pr9_5.m

```
f=[5;6;20;10;12;5];
A=[
1 0 0 1 0 0
0 1 0 0 1 0
0 0 1 0 0 1
];
b=[70;180;110];
Aeq=[
1 1 1 0 0 0
0 0 0 1 1 1
];
beq=[200;150];
lb=[0;0;0;0;0;0];
x=linprog(f,A,b,Aeq,beq,lb)
```

Command Window

```
>> Pr9_5
Optimization terminated.

x =

   30.0000
  170.0000
    0.0000
   40.0000
    0.0000
  110.0000
```