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# Re-analysis of the (100), (001), and (020) rotational structure of $\mathrm{SO}_{2}$ on the basis of high resolution FTIR spectra 

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#### Abstract

Three infrared spectra, weak (W), medium (M), and strong (S), of the ${ }^{32} \mathrm{SO}_{2}$ molecule were recorded with high resolution in the $1000-1500 \mathrm{~cm}^{-1}$ region. Spectra were recorded with the Fourier Transform interferometer Bruker IFS-120 HR in Oulu (Finland) with different pressures, absorption path lengths, and recording time. That allowed us to record not only the $\nu_{1}$ and $\nu_{3}$ bands with higher values of quantum numbers $J$ and $K_{a}$ than it was made earlier, but to record for the first time very weak $2 \nu_{2}$ band. In this case, transitions with the values $J^{\max .} / K_{a}^{\max }$. equal to $89 / 37,109 / 28$, and $54 / 9$ were assigned in the experimental spectra for the bands $\nu_{1}, \nu_{3}$, and $2 \nu_{2}$, respectively. As it became clear in the course of the analysis, the rotational parameters of the ground vibrational state, known in the literature, do not describe suitably the ground state combination differences (GSCD) for the states with the value $K_{a}>26-27$. As a consequence, the ground state rotational parameters were improved on the basis of our experimental data. The 12131 transitions assigned in the experimental spectrum ( 7618,3952 , and 561 transitions of the bands $\nu_{1}, \nu_{3}$, and $2 \nu_{2}$, respectively) were used for determination of ro-vibrational energy values of the vibrational states (100), (001), and (020). The lasts were used then in the fit procedure together with known in the literature high accurate sub-millimeter wave data. Resonance interactions between all three vibrational states have been taken into account in the Hamiltonian used for the fit. As a result, the 51 varied parameters, obtained from the fit, reproduce 4063 ro-vibrational energies of the states (100), (001), and (020) (12 131 initial experimental transitions) with accuracies close to experimental uncertainties: the rms deviation is $6.1 \times 10^{-5} \mathrm{~cm}^{-1}, 9.7 \times 10^{-5} \mathrm{~cm}^{-1}$, and $13.9 \times 10^{-5} \mathrm{~cm}^{-1}$ for our FTIR data (for the (100), (001), and (020) states, respectively), and comparable with experimental uncertainties for heterodyne data.


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## 1. Introduction

Sulfur dioxide is an important chemical species in many fields such as chemistry, astrophysics, laser techniques. It

[^0]plays an important role as a pollutant in the terrestrial atmosphere. In particular, sulfur dioxide is one of the major air pollutants released in the atmosphere as a result of volcanic eruptions and of fuel combustion in human activities; it contributes to the generation of smog and constitutes a serious health hazard for the respiratory system. To solve the problems of propagation of monochromatic radiation in the atmosphere, laser sounding, information transfer,
the remote detection and monitoring of $\mathrm{SO}_{2}$ in situ, etc., one also should have a good knowledge of the fine structure of the $\mathrm{SO}_{2}$ absorption spectra in different parts of the electromagnetic spectrum, in particular, in the infrared. Therefore, spectroscopic studies of the sulfur dioxide molecule have been made during many years both in the microwave, and sub-millimeter wave and infrared regions (see, Refs. [1-35].

In this paper we follow up on our recent studies of the high resolution spectra of $\mathrm{SO}_{2}$, Refs. [36-40]. The subject of the present study is the region of $1000-1500 \mathrm{~cm}^{-1}$. This region, where the strongest $\mathrm{SO}_{2} \nu_{3}$ and $\nu_{1}$ bands, and very weak $2 \nu_{2}$ band are located, is ideal for infrared atmospheric measurements of that molecule. Earlier the $\nu_{1}$ and $\nu_{3}$ bands were discussed in Refs. [5,12,16,21,23]. All information containing in these references were then generalized in Ref. [26]. But the information about the band $2 \nu_{2}$ was absent, and ro-vibrational energies of the (020) state were determined from the hot $2 \nu_{2}-\nu_{2}$ band [21]. In our present analysis, we were able to assign transitions of the $\nu_{1}$ and $\nu_{3}$ bands with values of quantum numbers $J$ and $K_{a}$ considerably higher than in Ref. [26]. Moreover, transitions of the very weak $2 \nu_{2}$ band also were recorded and assigned for the first time. Section 2 describes the experimental conditions for the recording spectra. In Section 3 we briefly present the Hamiltonian model used to fit the experimental line positions. Description of the spectra, assignment of transitions, and the problem of improvement of the ground state rotational parameters are discussed in Section 4. The results of analysis of the high resolution spectra of the bands $\nu_{1}, \nu_{3}$ and $2 \nu_{2}$ and determination of spectroscopic
parameters of the states (100), (001), and (020) are presented in Section 5.

## 2. Experimental details

For this molecular research three infrared spectra (weak, medium, and strong) of the ${ }^{32} \mathrm{SO}_{2}$ molecule in the region from 1000 to $1500 \mathrm{~cm}^{-1}$ were recorded with a high resolution Fourier Transform spectrometer Bruker IFS-120 HR in the infrared laboratory at the University of Oulu (Finland). In these measurements two absorption cells were used. In the W and M registrations the $\mathrm{SO}_{2}$ sample, made by SigmaAldrich Inc., was introduced in a White type cell, Ref. [41], at the pressure of 11.7 and 110.7 Pa , respectively. The absorption path length (APL) was 3.2 m . In the S measurement the sample was at pressure of 110.7 Pa in another cell, Ref. [42], with APL= 163.2 m . The both cells have been optimized for the used spectrometer, and they were provided with potassium bromide ( KBr ) windows in these measurements. The instrumental resolution at $1100 \mathrm{~cm}^{-1}$ was better than $0.0019 \mathrm{~cm}^{-1}$ in all three registrations. A globar source, a germanium film between two KBr plates, as the beamsplitter, and mercury cadmium telluride (MCT) detector were used. The registration times were 42.4, 79.9 and 99.9 h in the $\mathrm{W}, \mathrm{M}$ and S measurements, respectively. The spectra were calibrated with peaks of the OCS $2 \nu_{2}$ band [43] measured in fourth registration together with the sample spectrum. The peak positions were calculated with an optimized center of gravity method [44].


Fig. 1. Survey weak $(W)$ spectrum of $\mathrm{SO}_{2}$ in the region of $1050-1450 \mathrm{~cm}^{-1}$. Experimentally recorded strong bands $\nu_{1}$ and $\nu_{3}$ can be seen in the upper part of the figure. Experimental conditions: absorption path length is 3.2 m ; sample pressure is 11.7 Pa ; room temperature; registration times is 42.4 h . Synthetic spectrum is shown on the lower part of the figure. In the $1400-1450 \mathrm{~cm}^{-1}$ spectral range, some lines that are appeared in the experimental spectrum but not in the calculated one, belong to $\mathrm{H}_{2} \mathrm{O}$.


Fig. 2. Survey strong $(\mathrm{S})$ spectrum of $\mathrm{SO}_{2}$ in the region of the $\nu_{1}$ and $2 \nu_{2}$ bands. The P-branch of the weak $2 \nu_{2}$ band is seen in the left part of the figure. Experimental conditions: absorption path length is 163.2 m ; sample pressure is 110.7 Pa ; room temperature; registration times is 99.9 h .


Fig. 3. A small part of the high resolution spectrum of $\mathrm{SO}_{2}$ in the region of $R$-branch of the $\nu_{1}$ band (upper part). Experimental conditions correspond to the weak spectrum W . Lines of ${ }^{32} \mathrm{SO}_{2}$ are marked by dark circle. Some weak lines which are absent in the synthetic spectrum (lower part), belong to the hot band $\nu_{1}+\nu_{2}$ (they are marked by dark square), or to the ${ }^{34} \mathrm{SO}_{2}$ isotopic species (they are marked by dark triangle).

## 3. Hamiltonian model

The $\mathrm{SO}_{2}$ molecule is an asymmetric top of the $C_{2 v}$ symmetry. On this reason its three vibrational modes possess the following symmetry: $q_{\lambda} \in A_{1}$ for $\lambda=1,2$, and
$q_{\lambda} \in B_{1}$ for $\lambda=3$. As a consequence, two type bands are allowed in absorption: parallel (or $A_{1}$ ) type bands that correspond to the vibrational transitions $\left(v_{A_{1}}\right) \leftarrow\left(v_{g r}\right.$. ) and perpendicular (or $B_{1}$ ) type bands that correspond to the vibrational transitions ( $\left.v_{B_{1}}\right) \leftarrow\left(v_{g r}\right)$. The selection rules for

Table 1
High accurate sub-millimeter wave transitions of the $\mathrm{SO}_{2}$ molecule.

| Transition <br> 1 |  |  |  |  |  |  | Frequency, exp. ${ }^{\text {a }}$ in MHz 2 | $\begin{aligned} & \Delta(\text { calc })^{\mathrm{b}} \text { in } \mathrm{kHz} \\ & 3 \end{aligned}$ | $\begin{aligned} & \Delta(\text { calc })^{c} \text { in } \mathrm{kHz} \\ & 4 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 70 | 8 | 62 | $\leftarrow$ | 70 | 7 | 63 | 492056.427 (10) | 0.3 | 2.2 |
| 68 | 8 | 60 | $\leftarrow$ | 68 | 7 | 61 | $496010.525(15)$ | -1.0 | 4.4 |
| 69 | 8 | 62 | $\leftarrow$ | 68 | 9 | 59 | 498109.421 (20) | -16.2 | -5.1 |
| 49 | 15 | 35 | $\leftarrow$ | 50 | 14 | 36 | 498 657.914(10) | -7.0 | 2.5 |
| 70 | 9 | 61 | $\leftarrow$ | 69 | 10 | 60 | 498 920.109(20) | 8.0 | 5.3 |
| 72 | 8 | 64 | $\leftarrow$ | 72 | 7 | 65 | 499 505.309(10) | 0.9 | -0.6 |
| 54 | 16 | 38 | $\leftarrow$ | 55 | 15 | 41 | $500111.014(10)$ | 1.5 | 1.9 |
| 59 | 17 | 43 | $\leftarrow$ | 60 | 16 | 44 | 501179.560 (15) | 10.0 | 4.6 |
| 64 | 18 | 46 | $\leftarrow$ | 65 | 17 | 49 | $501871.632(20)$ | 16.1 | 4.2 |
| 53 | 16 | 38 | $\leftarrow$ | 54 | 15 | 39 | 519 365.544(10) | -0.5 | 2.0 |
| 58 | 17 | 41 | $\leftarrow$ | 59 | 16 | 44 | 520 382.480(15) | 3.8 | 1.6 |
| 63 | 18 | 46 | $\leftarrow$ | 64 | 17 | 47 | 521 011.924(20) | 15.4 | 8.6 |
| 73 | 20 | 54 | $\leftarrow$ | 74 | 19 | 55 | 521 137.011(100) | 42.9 | -10.3 |
| 68 | 19 | 49 | $\leftarrow$ | 69 | 18 | 52 | 521 261.295(40) | 15.4 | -4.0 |
| 47 | 15 | 33 | $\leftarrow$ | 48 | 14 | 34 | 537 227.581(10) | -11.9 | 1.2 |
| 52 | 16 | 36 | $\leftarrow$ | 53 | 15 | 39 | 538 602.765(10) | -3.4 | 1.3 |
| 77 | 7 | 71 | $\leftarrow$ | 78 | 4 | 74 | 539 061.368(60) | -13.6 | 21.8 |
| 57 | 17 | 41 | $\leftarrow$ | 58 | 16 | 42 | 539 570.174(10) | 2.4 | 3.0 |
| 78 | 6 | 72 | $\leftarrow$ | 79 | 5 | 75 | 539 807.786(100) | -9.2 | 22.4 |
| 72 | 20 | 52 | $\leftarrow$ | 73 | 19 | 55 | $540112.736(40)$ | 34.9 | -3.8 |
| 62 | 18 | 44 | $\leftarrow$ | 63 | 17 | 47 | 540 139.119(20) | 9.6 | 7.0 |
| 75 | 9 | 67 | $\leftarrow$ | 74 | 10 | 64 | 543 973.462(10) | -11.9 | -6.3 |
| 80 | 9 | 71 | $\leftarrow$ | 80 | 8 | 72 | $548752.732(20)$ | -15.6 | -8.7 |
| 76 | 8 | 68 | $\leftarrow$ | 76 | 7 | 69 | 549 848.964(10) | -1.4 | -9.7 |
| 78 | 9 | 69 | $\leftarrow$ | 78 | 8 | 70 | 551 403.734(40) | 13.3 | 22.8 |
| 41 | 7 | 35 | $\leftarrow$ | 42 | 4 | 38 | 564 029.767(15) | -28.2 | 2.5 |
| 51 | 8 | 44 | $\leftarrow$ | 52 | 5 | 47 | 564 469.319(10) | -20.6 | 6.4 |
| 72 | 9 | 63 | $\leftarrow$ | 71 | 10 | 62 | 564 665.735(10) | 10.0 | 2.0 |
| 76 | 9 | 67 | $\leftarrow$ | 76 | 8 | 68 | 564 896.451(20) | -9.0 | 2.2 |
| 60 | 4 | 56 | $\leftarrow$ | 61 | 3 | 59 | 567 304.250(20) | 8.9 | -1.8 |
| 70 | 7 | 63 | $\leftarrow$ | 70 | 6 | 64 | 569 387.048(10) | 6.3 | -6.9 |
| 69 | 6 | 64 | $\leftarrow$ | 70 | 3 | 67 | 571 153.962(20) | -7.3 | -6.8 |
| 51 | 4 | 48 | $\leftarrow$ | 52 | 1 | 51 | 574 052.463(10) | 6.1 | 1.2 |
| 50 | 16 | 34 | $\leftarrow$ | 51 | 15 | 37 | 577 028.807(10) | -9.6 | -1.4 |
| 79 | 7 | 73 | $\leftarrow$ | 80 | 4 | 76 | 577 256.699(60) | 14.7 | 56.4 |
| 55 | 17 | 39 | $\leftarrow$ | 56 | 16 | 40 | 577 902.912(10) | -2.5 | 2.6 |
| 70 | 20 | 50 | $\leftarrow$ | 71 | 19 | 53 | 578 039.819(30) | 1.9 | -13.8 |
| 60 | 18 | 42 | $\leftarrow$ | 61 | 17 | 45 | 578 356.855(15) | -4.6 | -0.1 |
| 65 | 19 | 47 | $\leftarrow$ | 66 | 18 | 48 | 578 399.767(20) | -22.9 | -22.1 |
| 70 | 5 | 65 | $\leftarrow$ | 71 | 4 | 68 | 579 237.260(20) | -1.9 | -0.8 |
| 84 | 9 | 75 | $\leftarrow$ | 84 | 8 | 76 | 579739.581 (30) | 8.6 | 9.1 |



| 766 833.091(50) | 9.0 | -30.2 |
| :---: | :---: | :---: |
| 767466.649 (120) | -62.7 | -75.7 |
| 774 876.880(40) | 29.2 | 28.4 |
| 780 862.523(30) | -9.6 | -18.3 |
| 784 626.891(10) | 8.4 | 18.7 |
| 793524.340 (20) | 5.5 | 4.2 |
| 798 621.881(40) | -18.2 | -29.9 |
| 798 697.330(150) | 56.8 | 106.5 |
| 801319.331 (30) | 7.8 | 14.8 |
| 803 461.207(15) | -15.5 | -7.6 |
| 1927 842.304(30) | -4.5 | 4.6 |
| 1927 998.798(40) | -42.5 | -35.9 |
| 1928 161.366(30) | 2.5 | 6.6 |
| 1928 329.684(30) | 7.9 | 9.9 |
| 1928 503.566(40) | -1.1 | -1.4 |
| 1928 682.827(20) | 9.4 | 6.9 |
| 1928 867.227(20) | 27.6 | 22.8 |
| 1929 056.510(30) | 33.2 | 26.5 |
| 1929 448.742(20) | 11.9 | 1.8 |
| 1929 651.196(40) | 4.8 | -6.9 |
| 1929 857.538(25) | 19.5 | 6.3 |
| 1930 067.441(30) | 6.9 | -7.5 |
| 1930 280.606(40) | -45.8 | -61.2 |
| 1930 496.853(40) | -24.7 | -40.9 |
| 1930 715.784(30) | -25.6 | -42.3 |
| 1930 937.131(50) | -5.9 | -23.3 |
| 1931 160.518(30) | -24.8 | -42.2 |
| 1931 385.703(40) | 2.6 | -15.0 |
| 1931 571.663(30) | -29.0 | -40.3 |
| 1931 612.305(30) | 28.1 | 10.5 |
| 1931 839.941(24) | 9.9 | -7.3 |
| 1932 068.330(30) | 15.6 | -0.9 |
| 1932 297.098(20) | 28.0 | 12.2 |
| 1932 525.788(50) | -46.2 | -61.2 |
| 1932 754.238(30) | 2.0 | -12.0 |
| 1932 958.812(30) | -53.7 | -44.0 |
| 1932 981.907(20) | 10.7 | -2.5 |
| 1933 208.376(60) | -53.7 | -65.7 |
| 1933 324.398(30) | -14.7 | -36.8 |
| 1933 433.460(20) | 17.3 | 6.6 |
| 1933 656.548(20) | 13.6 | 4.0 |
| 1933 877.304(40) | 6.5 | -2.1 |
| 1934 095.318(50) | 0.7 | 6.7 |
| 1934 310.166(60) | -5.5 | -11.9 |
| 1934 550.591(40) | -22.3 | 7.7 |
| $1934705.394(40)$ | -18.9 | 20.9 |
| 1934 728.672(30) | 11.9 | 6.6 |
| 1934 931.429(40) | 13.4 | 8.4 |
| 1935 046.840(20) | -7.9 | -8.3 |
| 1935117.660 (70) | 37.2 | 79.9 |
| 1935 129.290(40) | 43.0 | 37.6 |
| 1935 673.179(30) | 10.9 | 5.2 |
| 1935 870.169(20) | 27.4 | 20.7 |
| $1936732.689(100)$ | -83.7 | -135.4 |

Table 1 (continued)

| Transition |
| :--- | ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 |

"allowed" transitions in the parallel bands are
$\Delta J=0, \pm 1, \quad \Delta K_{a}=0, \quad \Delta K_{c}= \pm 1$.
The selection rules for the "allowed" transitions in the perpendicular bands are
$\Delta J=0, \pm 1, \quad \Delta K_{a}= \pm 1, \quad \Delta K_{c}= \pm 1$.
Besides "allowed" transitions, weak "forbidden" ones are allowed both in the parallel and perpendicular bands. The selection rules have the following form:
$\Delta J=0, \pm 1, \quad \Delta K_{a}=$ even,$\quad \Delta K_{c}=$ odd,
and
$\Delta J=0, \pm 1, \quad \Delta K_{a}=$ odd,$\quad \Delta K_{c}=o d d$.
In accordance with the symmetry properties, the effective Hamiltonian of the $\mathrm{XY}_{2}$-type molecule of the $C_{2 v}$ symmetry has been discussed in the spectroscopic literature many times (see, e.g., Refs. [45-47]). For consistency, we will briefly present it here without detailed

Table 2
Spectroscopic parameters of the ground vibrational state of the $\mathrm{SO}_{2}$ molecule (in $\mathrm{cm}^{-1}$ ). ${ }^{\text {a }}$

|  |  |  |
| :--- | :--- | :---: |
| Parameter | PS $^{\mathrm{b}}$ | Ref. $[35]^{\mathrm{c}}$ |
| 1 | 2 | 3 |
| $E$ |  |  |
| $A$ | $2.02735420407(163)$ | 2.02735433 |
| $B$ | $0.344173882136(410)$ | 0.3441739084 |
| $C$ | $0.293526503766(428)$ | 0.293526529 |
| $\Delta_{K} \times 10^{4}$ | $0.864015421(289)$ | 0.8640369 |
| $\Delta_{J K} \times 10^{5}$ | $-0.390123832(126)$ | -0.3901187 |
| $\Delta_{J} \times 10^{6}$ | $0.220539487(159)$ | 0.220549 |
| $\delta_{K} \times 10^{6}$ | $0.846291509(854)$ | 0.846284 |
| $\delta_{J} \times 10^{7}$ | $0.567423257(229)$ | 0.5674232 |
| $H_{K} \times 10^{7}$ | $0.12360428(155)$ | 0.12375 |
| $H_{K J} \times 10^{9}$ | $-0.64960685(842)$ | -0.64936 |
| $H_{J K} \times 10^{11}$ | $0.1160311(386)$ | 0.116 |
| $H_{J} \times 10^{12}$ | $0.3746170(246)$ | 0.37589 |
| $h_{K} \times 10^{9}$ | $0.5679872(101)$ | 0.5670 |
| $h_{J K} \times 10^{12}$ | $-0.243030(283)$ | -0.23 |
| $h_{J} \times 10^{12}$ | $0.18300674(481)$ | 0.1829 |
| $L_{K} \times 10^{11}$ | $-0.2608919(349)$ | -0.265 |
| $L_{K K J} \times 10^{12}$ | $0.1807804(411)$ | 0.180 |
| $L_{J K} \times 10^{13}$ | $-0.1097261(451)$ | -0.109 |
| $L_{J K} \times 10^{17}$ | $-0.99508(476)$ | -0.88 |
| $L_{J} \times 10^{17}$ | $-0.110360(128)$ | -0.116 |
| $l_{K} \times 10^{12}$ | $-0.319192(117)$ | -0.32 |
| $l_{K J} \times 10^{14}$ | $0.254416(212)$ | 0.27 |
| $l_{J K} \times 10^{17}$ |  | -0.2 |
| $l_{J} \times 10^{18}$ | $-0.607568(294)$ | -0.597 |
| $P_{K} \times 10^{15}$ | $0.584163(354)$ | 0.649 |
| $P_{K K J} \times 10^{16}$ | $-0.408713(675)$ | -0.394 |
| $P_{K J} \times 10^{18}$ | $-0.623763(944)$ | -0.703 |
| $P_{J K} \times 10^{19}$ | $0.649369(650)$ | 0.778 |
| $S_{K} \times 10^{18}$ | $-0.088587(130)$ | -0.12 |
| $S_{K K J} \times 10^{20}$ | $0.81536(427)$ | 0.70 |
|  |  |  |
|  |  |  |

[^1]explanations:
$H^{v .-r .}=\sum_{v, \tilde{v}}|v\rangle\langle\tilde{v}| H_{v \tilde{v}}$,
where the summation extends over all (in our case, three) interacting vibrational states: $|1\rangle \equiv\left(100, A_{1}\right),|2\rangle \equiv\left(020, A_{1}\right)$, and $|3\rangle \equiv\left(001, B_{1}\right)$. The diagonal operators $H_{v v}$ describe unperturbed rotational structures of the corresponding vibrational states. The nondiagonal operators $H_{v \tilde{v}},(v \neq \tilde{v})$ describe resonance interactions (Fermi, or Coriolis) between the states $|v\rangle$ and $|\tilde{v}\rangle$. The diagonal block operators have the same form for all the three vibrational states involved (Watson's Hamiltonian in the $A$-reduction and $I^{r}$ representation):
\[

$$
\begin{align*}
H_{v v}= & E^{v}+\left[A^{v}-\frac{1}{2}\left(B^{v}+C^{v}\right) J_{z}^{2}+\frac{1}{2}\left(B^{v}+C^{v}\right) J^{2}+\frac{1}{2}\left(B^{v}-C^{v}\right) J_{x y}^{2}\right. \\
& -\Delta_{K}^{v} J_{z}^{4}-\Delta_{J K}^{v} J_{z}^{2} J^{2}-\Delta_{J}^{v} J^{4}-\delta_{K}^{v}\left[J_{z}^{2}, J_{x y}^{2}\right]-2 \delta_{J}^{v} J^{2} J_{x y}^{2} \\
& +H_{K}^{v} J_{z}^{6}+H_{K J}^{v} J_{z}^{4} J^{2}+H_{J K}^{v} J_{z}^{2} J^{4}+H_{J}^{v} J^{6} \\
& +\left[J_{x y}^{2}, h_{K}^{v} J_{z}^{4}+h_{J J}^{v} J^{2} J_{z}^{2}+h_{J}^{v} J^{4}\right] \\
& +L_{K}^{v} J_{z}^{8}+L_{K K J}^{v} J_{z}^{6} J^{2}+L_{J K}^{v} J_{z}^{4} J^{4}+L_{K J J}^{v} J_{z}^{2} J^{6}+L_{J}^{v} J^{8} \\
& +\left[J_{x y}^{2}, l_{K}^{v} J_{z}^{6}+l_{K J}^{v} J^{2} J_{z}^{4}+l_{J K}^{v} J^{4} J_{z}^{2}+l_{J}^{v} J^{6}\right] \\
& +\cdots+P_{K}^{v} J_{z}^{10}+P_{K K K J}^{v} J_{Z}^{8} J^{2}+P_{K K J}^{v} J_{z}^{6} J^{4}+P_{J J K}^{v} J_{Z}^{4} J^{6} \\
& +S_{K}^{v} J_{z}^{12}+S_{K K K K J}^{v} J_{z}^{10} J^{2}+\cdots \tag{2}
\end{align*}
$$
\]

where $J_{\alpha}(\alpha=x, y, z)$ are the components of the angular momentum operator defined in the molecule-fixed coordinate system; $J_{x y}^{2}=J_{x}^{2}-J_{y}^{2} ; A^{v}, B^{v}$, and $C^{v}$ are the effective rotational constants connected with the vibrational states $v$, and the other parameters are the different order centrifugal distortion coefficients.

We may distinguish between two types of coupling operators $H^{v \tilde{\nu}},(v \neq \tilde{v})$, corresponding to the two different types of resonance interactions which can occur in $\mathrm{XY}_{2}$ $\left(C_{2 v}\right)$ asymmetric top molecules. If the product $\Gamma=\Gamma^{v} \otimes \Gamma^{\tilde{v}}$ of the symmetry species of the states $v$ and $\tilde{v}$ is equal to $A_{1}$ (i.e., $\Gamma^{v}=\Gamma^{\tilde{v}}$ ), then the states $v$ and $\tilde{v}$ are connected by an anharmonic Fermi resonance interaction, and the corresponding interaction operator has the form

$$
\begin{align*}
H_{v \tilde{v}}= & { }^{v \tilde{v}} F_{0}+{ }^{v \tilde{v}} F_{K} J_{z}^{2}+{ }^{\mathrm{v}} F_{J} J^{2}+{ }^{\mathrm{v}} F_{K K} J_{z}^{4}+{ }^{\mathrm{v} \tilde{{ }^{v}}} F_{K} J_{z}^{2} J^{2} \\
& +{ }^{v} F_{J J} J^{4}+\cdots+{ }^{v \tilde{v}} F_{x y}\left(J_{x}^{2}-J_{y}^{2}\right)+{ }^{\mathrm{v} \tilde{v}} F_{K x y}\left\{J_{z}^{2},\left(J_{x}^{2}-J_{y}^{2}\right)\right\}_{+} \\
& +2^{v \tilde{v}} F_{J x y} J^{2}\left(J_{x}^{2}-J_{y}^{2}\right)+\cdots \tag{3}
\end{align*}
$$

If the product is $\Gamma=B_{1}$, then the states $v$ and $\tilde{v}$ are connected by a Coriolis resonance interaction of the form

$$
\begin{align*}
H_{v \tilde{v}}= & \left.i J_{y} H_{v \tilde{v}}^{(1)}+H_{v \tilde{v}}^{(1)} i J_{y}+\left\{J_{x}, J_{z}\right\}_{+} H_{v \tilde{v}}^{(2)}+H_{v \tilde{v}}^{(2)} J_{x}, J_{z}\right\}_{+} \\
& +\left\{i J_{y},\left(J_{x}^{2}-J_{y}^{2}\right)\right\}_{+} H_{v \tilde{v}}^{(3)}+H_{v \tilde{v}}^{(3)}\left\{i J_{y},\left(J_{x}^{2}-J_{y}^{2}\right)\right\}_{+}+\cdots \tag{4}
\end{align*}
$$

The operators $H_{v \dot{v}}^{(i)}, i=1,2,3, \ldots$ in Eq. (4) have the form

$$
\begin{align*}
H_{v \tilde{v}}^{(i)}= & \frac{1}{2}{ }^{v \tilde{v}} C^{i}+{ }^{v \tilde{v}} C_{K}^{i} J_{z}^{2}+\frac{1}{2}{ }^{v \tilde{v}} C_{J}^{i} J^{2}+{ }^{v \tilde{v}} C_{K K}^{i} J_{z}^{4}+{ }^{v \tilde{v}} C_{K J}^{i} J_{z}^{2} J^{2} \\
& +\frac{1}{2}{ }^{2} C_{J J}^{i} J^{4}+{ }^{v \tilde{v}} C_{K K K}^{i} J_{Z}^{6}+{ }^{v \tilde{v}} C_{K K J}^{i} J_{Z}^{4} J^{2} \\
& +{ }^{\mathrm{v}} C_{K J J}^{i} J_{Z}^{2} J^{4}+\frac{1}{2}{ }^{v} C_{J J J}^{i} J^{6}+\cdots \tag{5}
\end{align*}
$$

To prevent confusion in the label notations used, we should mention that the axis of symmetry of the $\mathrm{XY}_{2}$ molecule in our case is not the $z$-axis, but the $x$-axis. This fact is the consequence of using the diagonal blocks (2) of the Hamiltonian (1) in the form of Watson's operator in $A$-reduction and $I^{r}$-representation. However, for the point group symmetry assignment of normal modes we use the

Table 3
Statistical information for the $\nu_{1}, \nu_{3}$, and $2 \nu_{2}$ bands of $\mathrm{SO}_{2}$ molecule.

| Band | Center in $\mathrm{cm}^{-1}$ | $J^{\text {max }}$ | $K_{a}^{\max }$ | $N_{l}{ }^{\text {a }}$ | $r m s_{l}$ <br> in $10^{-5} \mathrm{~cm}^{-1}$ | $\begin{aligned} & m_{l 1}{ }^{\mathrm{b}} \\ & \text { in \% } \end{aligned}$ | $\begin{aligned} & m_{12}{ }^{\mathrm{b}} \\ & \text { in \% } \end{aligned}$ | $\begin{aligned} & m_{13}{ }^{\mathrm{b}} \\ & \text { in \% } \end{aligned}$ | $N_{t}{ }^{\text {c }}$ | $r m s_{t}$ in $10^{-5} \mathrm{~cm}^{-1}$ | $\begin{aligned} & m_{t 1}{ }^{\mathrm{b}} \\ & \text { in \% } \end{aligned}$ | $\begin{aligned} & m_{\mathrm{t} 2}{ }^{\mathrm{b}} \text { in } \\ & \% \end{aligned}$ | $\begin{aligned} & m_{t 3}{ }^{\text {b }} \\ & \text { in \% } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Ground |  |  |  |  |  |  |  |  |  |  |  |  |  |
| With param. from |  | 92 | 23 |  |  |  |  |  | 149 | $28.2{ }^{\text {d }}$ |  |  |  |
| Ref. [35] ${ }^{\text {d }}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| With our param. ${ }^{\text {d }}$ |  | 92 | 23 |  |  |  |  |  | 149 | $22.5{ }^{\text {d }}$ |  |  |  |
| With param. from |  | 53 | 34 |  |  |  |  |  | 78 | 269.1 | 3.8 | 11.5 | 84.7 |
| Ref. [35] ${ }^{\text {e }}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| With our param. ${ }^{\text {e }}$ |  | 53 | 34 |  |  |  |  |  | 78 | 10.8 | 73.1 | 20.5 | 6.4 |
| $\nu_{1}$ | 1151.71295 |  |  |  |  |  |  |  |  |  |  |  |  |
| Ref. [26] |  | 69 | 28 | 1216 | 21.0 |  | 70.7 | 29.3 |  |  |  |  |  |
| ( $\left.J \leq 69 / K_{a} \leq 28\right)$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Our ( $J \leq 69 / K_{a} \leq 28$ ) |  | 69 | 28 | 1562 | 3.9 | 97.3 | 2.4 | 0.3 | 6925 | 12.2 | 80.4 | 9.4 | 10.2 |
| Our (total) |  | 89 | 37 | 1913 | 6.1 | 93.3 | 5.1 | 1.6 | 7618 | 12.7 | 78.4 | 10.6 | 11.0 |
| $\nu_{3}$ | 1362.06030 |  |  |  |  |  |  |  |  |  |  |  |  |
| Ref. [26] |  | 89 | 24 | 884 | 21.0 |  | 70.7 | 29.3 |  |  |  |  |  |
| ( $J \leq 89 / K_{a} \leq 24$ ) |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Our ( $J \leq 89 / K_{a} \leq 24$ ) |  | 89 | 24 | 1653 | 8.2 | 87.7 | 9.2 | 3.1 | 3695 | 15.0 | 69.9 | 14.3 | 15.8 |
| Our (total) |  | 109 | 28 | 1838 | 9.7 | 84.1 | 10.6 | 5.3 | 3952 | 15.6 | 68.3 | 14.7 | 17.0 |
| $2 \nu_{2}$ | 1035.12639 |  |  |  |  |  |  |  |  |  |  |  |  |
| Our |  | 54 | 9 | 312 | 13.9 | 68.0 | 20.5 | 11.5 | 561 | 21.1 | 43.6 | 25.1 | 31.3 |

[^2]standard convention with $z=\mathrm{C}_{2}$. For further, more detailed discussion of this type of effective Hamiltonian model we refer to [45].

## 4. Analysis of spectra and re-determination of ground state spectroscopic parameters

Survey weak spectrum of the $\nu_{1}$ and $\nu_{3}$ bands of the $\mathrm{SO}_{2}$ molecule is shown on the upper part of Fig. 1. Fig. 2 presents the S-spectrum of the $\nu_{1}$ band. In this case, the P-branch of very weak $2 \nu_{2}$ band is seen in the lower wavenumber region. For illustration of quality of the experimentally recorded spectra, upper part of Fig. 3 presents small portion of the high resolution spectrum in the R-branch of the $\nu_{1}$ band. As the analysis showed, in both the $\nu_{1}$ and $\nu_{3}$, and $2 \nu_{2}$ bands, at least, two of three (P-, Q-, and/or R-) branches are clearly pronounced. As a consequence, one can expect a good fulfillment of the "ground state combination differences" principle.

In the analysis of the recorded spectra we were able to assign transitions with the maximum values of quantum numbers $J^{\text {max. }}=109$ and $K_{a}^{\text {max. }}=37$. Correctness of assignments was controlled by the construction of corresponding "experimental" Ground State Combination Differences. It was found that, starting with the value of quantum number $K_{a}=26-27$, difference between GSCD values obtained from experimental transitions and calculated ones obtained by the best set of the ground state rotational parameters, Ref. [35], starts increasing quickly with increase of quantum number $K_{a}$. In this case, differences between "experimental" and "calculated" GSCD values were increased from about $1-1.5 \times 10^{-4} \mathrm{~cm}^{-1}$ for $K_{a} \leq 25$ up to $60.5 \times 10^{-4} \mathrm{~cm}^{-1}$ for $K_{a}=34$. This circumstance becomes clear if one takes into account that in [35] the ground state rotational parameters were obtained on the basis of high-accurate experimental sub-millimeter wave transitions with the maximum value of quantum number $K_{a}^{\max }=23$. For this reason, in the present paper we re-analyze the rotational structure of the $\mathrm{SO}_{2}$ ground vibrational state on the basis of the ground state

Table 4
Spectroscopic parameters of the (100), (001), and (020) vibrational states of the $\mathrm{SO}_{2}$ molecule (in $\mathrm{cm}^{-1}$ ). ${ }^{\text {a }}$

| Parameter <br> 1 | (100), our $2$ | $\begin{aligned} & (100) \text { from [26] } \\ & 3 \end{aligned}$ | $\begin{aligned} & \text { (001), our } \\ & 4 \end{aligned}$ | $\begin{aligned} & (001) \text { from [26] } \\ & 5 \end{aligned}$ | (020), our $6$ | $\begin{aligned} & \text { (020) from [26] } \\ & 7 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E | 1151.68575415 (466) | 1151.712950 | 1362.06030242(716) | 1362.060336 | 1035.1535902 (118) | 1035.126371 |
| A | 2.0284219524 (969) | 2.028436263 | 2.006648486 (197) | 2.006644631 | 2.107666447 (651) | 2.107649092 |
| B | 0.3425072831 (280) | 0.3425122666 | 0.3430035682 (288) | 0.3430079384 | 0.3443180685 (580) | 0.3443172408 |
| C | 0.2922516577 (118) | 0.2921140821 | 0.2922918154 (122) | 0.2924294870 | 0.2924639658 (337) | 0.2924635974 |
| $\Delta_{K} \times 10^{4}$ | 0.87785264 (873) | 0.87644839 | 0.8487019 (198) | 0.8503404 | 1.0631021 (904) | 1.0638447 |
| $\Delta_{J K} \times 10^{5}$ | -0.3905198 (726) | -0.37655179 | -0.3921223 (802) | -0.4069928 | -0.4255028 (501) | -0.4257225 |
| $\Delta_{J} \times 10^{6}$ | 0.2193355 (109) | 0.21977784 | 0.2234330 (110) | 0.22308590 | 0.2216133 (260) | 0.2212268 |
| $\delta_{K} \times 10^{6}$ | 0.846291509 | 0.922469 | 0.8542019 (820) | 0.7891669 | 1.23853 (158) | 1.236181 |
| $\delta_{J} \times 10^{7}$ | 0.5676407 (604) | 0.5653892 | 0.5719439 (545) | 0.5743462 | 0.574481 (140) | 0.5742303 |
| $H_{K} \times 10^{7}$ | 0.1272614 (117) | 0.1283702 | 0.1202984 (677) | 0.120473 | 0.182918 | 0.188955 |
| $H_{K J} \times 10^{9}$ | -0.656738 (253) | -0.715856 | -0.651555 (295) | -0.608500 | -0.8644971 | -0.86163 |
| $H_{J K} \times 10^{11}$ | 0.1160311 | 0.1325 | 0.1160311 | 0.94826 | 0.79840 | 0.554 |
| $H_{J} \times 10^{12}$ | 0.371073 (328) | 0.38265 | 0.383059 (564) | 0.37734 | 0.373265 | 0.26162 |
| $h_{K} \times 10^{9}$ | 0.5679872 | 0.553916 | 0.577289 (542) | 0.90549 | 0.954963 | 0.927 |
| $h_{J K} \times 10^{12}$ | -0.243030 | -0.37865 | -0.243030 | -0.37865 | -1.3097 | -2.78 |
| $h_{J} \times 10^{12}$ | 0.184502 (319) | 0.178527 | 0.184898 (178) | 0.190584 | 0.183500 | 0.194 |
| $L_{K} \times 10^{11}$ | -0.271360 (112) | -0.280590 | -0.24940 (108) | -0.26921 | -0.478860 | -0.437 |
| $L_{K K J} \times 10^{12}$ | 0.1868310 (860) | 0.125454 | 0.175030 (216) | 0.16636 | 0.29657 | 0.158 |
| $L_{J K} \times 10^{13}$ | -0.110391 (183) | 0.007728 | -0.1097261 | 0.007728 | -0.1993 |  |
| $L_{J J K} \times 10^{16}$ | -0.099508 | -1.2014 | -0.099508 | -1.8914 | -0.7190 |  |
| $L_{J} \times 10^{17}$ | -0.110360 | -0.3099 | -0.10166 (312) | -0.2910 | -0.0957 |  |
| $l_{K} \times 10^{12}$ | -0.324991 (886) |  | -0.315084(552) |  | -0.5895 |  |
| $l_{K J} \times 10^{14}$ | 0.284853 (540) |  | 0.254416 |  | 0.273 |  |
| $l_{J K} \times 10^{17}$ |  |  |  |  | -0.24 |  |
| $l_{J} \times 10^{18}$ | -0.607568 |  | -0.607568 |  | -0.69185 |  |
| $P_{K} \times 10^{15}$ | 0.599887 (390) | 0.68478 | 0.55386 (604) | 0.7101 | 1.443 |  |
| $P_{\text {KKJ }} \times 10^{16}$ | -0.408713 | -0.26122 | -0.408713 | -0.26122 | -0.78360 |  |
| $P_{K J} \times 10^{18}$ | -0.623763 |  | -0.623763 |  | -1.286 |  |
| $P_{J K} \times 10^{19}$ | 0.649369 |  | 0.649369 |  | 1.666 |  |
| $S_{K} \times 10^{19}$ | -0.88587 |  | -0.88587 |  | -3.30 |  |
| $S_{K K J} \times 10^{20}$ | 0.81536 |  | 0.81536 |  | 1.615 |  |

[^3]combination differences determined in the frame of analysis of our FTIR experimental data (in this case, 78 FTIR combination differences with the values $J^{\text {max. }}=53$ and $K_{a}^{\text {max. }}=34$ were used). The 149 sub-millimeter wave transitions from Ref. [35] (they are reproduced in column 2 of Table 1) were also added to the set of initial data used in the fit of the ground state spectroscopic parameters. The values of parameters obtained from the weighted fit are presented in column 2 of Table 2. Values in parenthesis are $1 \sigma$ statistical confidence intervals. For comparison, column 3 of Table 2 presents the values of the ground state spectroscopic parameters that are reproduced from

Table 5
Resonance interaction parameters for the (100), (020), and (001) vibrational states of the $\mathrm{SO}_{2}$ molecule (in $\mathrm{cm}^{-1}$ ). ${ }^{\text {a }}$

| Parameter | Value |
| :--- | :---: |
| ${ }_{(100),(020)} F_{0}$ | 1.78 |
| ${ }^{(100),(001)} C^{1}$ | -0.17 |
| ${ }^{(100),(001)} C^{3} \times 10^{6}$ | $0.31773(644)$ |
| ${ }^{(100),(020)} F_{K x y} \times 10^{7}$ | $0.714042(816)$ |
| ${ }^{(100),(001)} C_{K}^{1} \times 10^{4}$ | $-0.16849(214)$ |
| ${ }^{(100),(001)} C_{J}^{3} \times 10^{11}$ | $-0.2335(125)$ |
| ${ }_{(100),(001)} C^{2} \times 10^{2}$ | $0.20827(167)$ |

${ }^{\text {a }}$ Values in parentheses are $1 \sigma$ standard errors. Parameters presented without standard errors have been estimated on the basis of the potential energy surface parameters of $\mathrm{SO}_{2}$ from Ref. [48] and were constrained in the fit.

Ref. [35]. One can see good correlation between both sets of parameters. At the same time our new set of parameters not only reproduce better the FTIR Ground State Combination Differences (the rms deviation of 78 GSCD, $29 \geq K_{a} \geq 34$, obtained from our experimental data, being calculated with the parameters from Ref. [35] is $269.1 \times 10^{-5} \mathrm{~cm}^{-1}$; being calculated with our set of parameters, that value is decreased up to $10.8 \times 10^{-5} \mathrm{~cm}^{-1}$, see statistical Table 3 for more details), but also reproduces with the better rmsdeviation values the sub-millimeter wave line positions from Ref. [35] (compare columns 3 and 4 of Table 1; the rms deviation is 28.2 kHz and 22.5 kHz at calculation with parameters from [35] and our parameters from column 2 of Table 2, respectively). The number of fitted parameters in our case is even less than in [35].

## 5. Re-analysis of the rotational structures of the vibrational states (100), (001), and (020)

The new ground state parameters obtained in Section 4 were used then for calculation of ground state rotational energies which, in turn, were used in the re-assignments of transitions in the recorded FTIR spectra. The list of more than 12130 finally assigned transitions is presented in the Supplementary Materials (see also the statistical information in Table 3). From these transitions we obtained 4063 upper ro-vibrational energies (1913, 1838, and 312 upper energies for the states (100), (001), and (020), respectively) which were used then as input data in a weighted least square fit with the aim to determine rotational, centrifugal distortion, and resonance interaction parameters of the

Table 6
Heterodyne frequencies of $\mathrm{SO}_{2}$ in the $\nu_{3}$ band.

| Transition 1 | Frequency, exp. ${ }^{\text {a }}$ in MHz 2 | $\begin{aligned} & \Delta I,^{\mathrm{b}} \text { in } \mathrm{MHz} \\ & 3 \end{aligned}$ | $\begin{aligned} & \Delta I I,{ }^{\mathrm{c}} \text { in } \mathrm{MHz} \\ & 4 \end{aligned}$ | Transition 1 | Frequency, exp. ${ }^{\text {a }}$ in MHz 2 | $\begin{aligned} & \Delta I,,^{\mathrm{b}} \text { in } \mathrm{MHz} \\ & 3 \end{aligned}$ | $\begin{aligned} & \Delta I I,{ }^{\mathrm{C}} \text { in } \mathrm{MHz} \\ & 4 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{q} \mathrm{P}_{6}$ (18) | 40457 019.8(5.0) | 1.5 | -0.3 | ${ }^{q} \mathrm{R}_{1}(13)$ | 41079 955.2(5.0) | 1.5 | 0.3 |
| ${ }^{q} \mathrm{P}_{0}(20)$ | 40458 986.4(5.0) | 1.4 | 0.0 | ${ }^{q} \mathrm{R}_{5}(13)$ | 41080 193.8(3.0) | -3.1 | -4.6 |
| ${ }^{q} \mathrm{P}_{10}(16)$ | 40460 775.5(9.0) | 3.5 | 2.0 | ${ }^{q} \mathrm{R}_{10}$ (26) | 41267 578.4(5.0) | -1.0 | -1.9 |
| ${ }^{q} \mathrm{P}_{8}(17)$ | 40461 362.9(5.0) | 0.3 | -1.2 | ${ }^{q} \mathrm{R}_{6}(24)$ | 41271 076.8(4.0) | 0.6 | -0.9 |
| ${ }^{q} \mathrm{P}_{2}(19)$ | 40462 193.6(5.0) | 2.1 | 0.6 | ${ }^{9} \mathrm{R}_{8}(25)$ | 41271 318.4(4.0) | -0.7 | -1.9 |
| ${ }^{q} \mathrm{P}_{5}(18)$ | 40463 046.5(5.0) | 2.1 | 0.6 | ${ }^{q} \mathrm{R}_{11}(27)$ | 41272 779.1(4.0) | -1.4 | -1.7 |
| ${ }^{q} \mathrm{P}_{2}(18)$ | 40464 934.6(5.0) | 1.5 | 0.3 | ${ }^{q} \mathrm{R}_{1}(25)$ | 41275 291.4(4.0) | 1.3 | -0.2 |
| ${ }^{q} \mathrm{Q}_{12}(21)$ | 40734 536.6(4.0) | 0.4 | -0.5 | ${ }^{q} \mathrm{R}_{8}(38)$ | 41494 155.7(6.0) | 1.2 | 0.6 |
| ${ }^{q} \mathrm{P}_{2}$ (5) | 40734 999.1(4.0) | -0.8 | -1.7 | ${ }^{q} \mathrm{R}_{12}(41)$ | 41495 464.1(8.0) | -0.8 | -0.5 |
| ${ }^{q} \mathrm{Q}_{12}$ (20) | 40735 926.4(4.0) | -0.5 | -1.7 | ${ }^{9} \mathrm{R}_{6}(37)$ | 41496 666.6(4.0) | 1.0 | -0.2 |
| ${ }^{q} \mathrm{Q}_{12}(19)$ | 40737 249.7(4.0) | -1.7 | -2.9 | ${ }^{q} \mathrm{R}_{4}(37)$ | 41499 389.5(4.0) | 0.0 | -1.8 |
| ${ }^{q} \mathrm{Q}_{12}(18)$ | 40738 508.3(4.0) | -1.3 | -2.5 | ${ }^{9} \mathrm{R}_{3}(36)$ | 41500 253.4(4.0) | 1.1 | -0.7 |
| ${ }^{q} \mathrm{P}_{1}$ (5) | 40740 637.5(4.0) | 1.8 | 0.9 | ${ }^{q} \mathrm{R}_{5}(37)$ | 41502 232.2(4.0) | 1.0 | -0.5 |
| ${ }^{q} \mathrm{Q}_{12}(16)$ | 40740 829.0(4.0) | 1.4 | 0.2 | ${ }^{q} \mathrm{R}_{0}(40)$ | 41504 180.9(6.0) | 2.2 | 1.6 |
| ${ }^{q} \mathrm{Q}_{11}(25)$ | 40741 496.8(4.0) | 1.3 | 0.7 | ${ }^{q} \mathrm{R}_{7}(38)$ | 41504 742.1(5.0) | 0.1 | -0.8 |
| ${ }^{q} \mathrm{Q}_{12}(15)$ | 40741 887.7(4.0) | 0.6 | -0.9 | ${ }^{q} \mathrm{R}_{10}$ (40) | 41504 915.0(6.0) | -1.1 | -1.1 |
| ${ }^{q} \mathrm{Q}_{12}(13)$ | 40743 809.8(6.0) | 2.3 | 0.8 | ${ }^{9} \mathrm{R}_{2}(38)$ | 41505 369.2(4.0) | 1.3 | -0.8 |
| ${ }^{q} \mathrm{R}_{0}(2)$ | 40890 362.9(6.0) | 13.1 | 12.0 | ${ }^{q} \mathrm{R}_{8}(56)$ | 41792 579.5(4.0) | -1.0 | -0.1 |
| ${ }^{q} \mathrm{R}_{3}(7)$ | 40979 127.8(20.0) | 5.7 | 4.5 | ${ }^{q} \mathrm{R}_{10}(58)$ | 41793 377.8(5.0) | -1.9 | -5.8 |
| ${ }^{q} \mathrm{R}_{6}(8)$ | 40981712.6 (10.0) | -1.0 | -2.4 | ${ }^{q} \mathrm{R}_{11}(59)$ | 41794 759.0(4.0) | -2.5 | -1.6 |
| ${ }^{q} \mathrm{R}_{8}(9)$ | 40983 895.9(9.0) | 2.6 | 1.1 | ${ }^{q} \mathrm{R}_{3}(58)$ | 41796023.8 (6.0) | -3.7 | -3.7 |
| ${ }^{q} \mathrm{R}_{5}(8)$ | 40988 165.8(15.0) | -4.4 | -5.9 | ${ }^{q} \mathrm{R}_{1}(60)$ | 41796 222.6(5.0) | 0.5 | 2.9 |
| ${ }^{q} \mathrm{R}_{8}(14)$ | 41075 376.3(3.0) | 1.3 | -0.2 | ${ }^{q} \mathrm{R}_{5}(57)$ | 41796 860.0(5.0) | 0.0 | -0.6 |
| ${ }^{q} \mathrm{R}_{11}(16)$ | 41078 597.4(6.0) | 2.6 | 1.1 |  |  |  |  |

[^4]Table 7
Heterodyne frequencies of $\mathrm{SO}_{2}$ in the $\nu_{1}$ band.

| Transition <br> 1 | $\begin{aligned} & \text { Wavenumber, exp. }{ }^{\mathrm{a}} \text { in } \mathrm{cm}^{-1} \\ & 2 \end{aligned}$ | $\Delta I^{\text {b }}$ 3 | $\begin{aligned} & \Delta I I^{\mathrm{C}} \\ & 4 \end{aligned}$ | Transition <br> 1 | $\begin{aligned} & \text { Wavenumber, exp. }{ }^{\mathrm{a}} \text { in } \mathrm{cm}^{-1} \\ & \end{aligned}$ | $\Delta I^{\text {b }}$ 3 | $\begin{aligned} & \Delta I I^{\mathrm{c}} \\ & 4 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{p} \mathrm{P}_{10}$ (48) | 1085.8791(2) | 3.0 | 2.5 | ${ }^{p} \mathrm{P}_{13}(43)$ | 1079.6768(2) | 0.7 | 0.3 |
| ${ }^{p} \mathrm{P}_{13}(35)$ | 1085.7302(2) | -0.6 | -0.8 | ${ }^{p} \mathrm{P}_{17}(26)$ | 1079.6722(2) | 0.9 | 0.0 |
| ${ }^{p} \mathrm{P}_{16}$ (22) | 1085.6640(2) | -0.4 | -1.2 | ${ }^{p} \mathrm{P}_{18}(23)$ | 1078.7424(2) | -1.6 | -2.7 |
| ${ }^{p} \mathrm{P}_{11}(45)$ | 1084.7518(2) | 1.5 | 1.2 | ${ }^{p} \mathrm{P}_{14}(40)$ | 1078.7327(2) | -3.4 | -3.8 |
| ${ }^{p} \mathrm{P}_{14}(32)$ | 1084.7412(2) | 1.0 | 0.7 | ${ }^{p} \mathrm{P}_{15}(36)$ | 1078.5654(2) | -0.9 | -1.4 |
| ${ }^{p} \mathrm{P}_{17}(19)$ | 1084.6661(2) | 0.2 | -0.9 | ${ }^{p} \mathrm{P}_{19}$ (19) | 1078.5249(2) | 1.5 | 0.2 |
| ${ }^{p} \mathrm{P}_{15}(28)$ | 1084.4941(2) | 0.6 | 0.1 | ${ }^{p} \mathrm{P}_{16}(32)$ | 1078.3954(2) | 0.3 | -0.3 |
| ${ }^{p} \mathrm{P}_{10}$ (51) | 1083.6276(2) | 0.9 | 0.3 | ${ }^{p} \mathrm{P}_{17}(29)$ | 1077.4839(2) | 2.4 | 1.6 |
| ${ }^{p} \mathrm{Q}_{21}(31)$ | 1083.5631(2) | 0.3 | 1.5 | ${ }^{p} \mathrm{P}_{13}(46)$ | 1077.3667(2) | 0.7 | 0.1 |
| ${ }^{p} \mathrm{P}_{16}(25)$ | 1083.5164(2) | -0.8 | -1.5 | ${ }^{p} \mathrm{P}_{25}(18)$ | 1077.3043(2) | 0.2 | -0.8 |
| ${ }^{p} \mathrm{P}_{13}(38)$ | 1083.4793(2) | 0.5 | 0.2 | ${ }^{p} \mathrm{P}_{12}(52)$ | 1076.0146(2) | -1.7 | -2.6 |
| ${ }^{p} \mathrm{Q}_{21}(32)$ | 1083.4529(2) | 1.5 | 2.8 | ${ }^{p} \mathrm{P}_{17}(31)$ | 1076.0089(2) | -0.1 | -0.9 |
| ${ }^{p} \mathrm{P}_{9}(56)$ | 1083.4368(2) | 0.1 | -0.4 | ${ }^{p} \mathrm{P}_{18}(27)$ | 1075.8530(2) | -0.2 | -1.1 |
| ${ }^{p} \mathrm{Q}_{21}(33)$ | 1083.3391(2) | 0.2 | 1.6 | ${ }^{p} \mathrm{P}_{20}$ (20) | 1074.7958(2) | 1.1 | 0.2 |
| ${ }^{p} \mathrm{P}_{14}(34)$ | 1083.2556(2) | -3.0 | -3.3 | ${ }^{p} \mathrm{P}_{15}(41)$ | 1074.7655(2) | -1.1 | -1.8 |
| ${ }^{p} \mathrm{P}_{17}(22)$ | 1082.5452(2) | -0.6 | -1.6 | ${ }^{p} \mathrm{P}_{16}(37)$ | 1074.6457(2) | 0.4 | -0.3 |
| ${ }^{p} \mathrm{P}_{14}(35)$ | 1082.5091(2) | 0.9 | 0.5 | ${ }^{p} \mathrm{P}_{17}(33)$ | 1074.5217(2) | 0.1 | -0.6 |
| ${ }^{p} \mathrm{P}_{11}(48)$ | 1082.4653(2) | -1.0 | -1.6 | ${ }^{p} \mathrm{P}_{19}(28)$ | 1072.0598(2) | -0.3 | -1.2 |
| ${ }^{p} \mathrm{P}_{15}(31)$ | 1082.2932(2) | -0.5 | -1.0 | ${ }^{p} \mathrm{P}_{20}(24)$ | 1071.9389(2) | 1.0 | 0.3 |
| ${ }^{p} \mathrm{P}_{18}$ (18) | 1082.2808(2) | -1.0 | -2.3 | ${ }^{p} \mathrm{P}_{14}(49)$ | 1071.7722(2) | -2.0 | -2.8 |
| ${ }^{p} \mathrm{P}_{12}(44)$ | 1082.1895(2) | 0.1 | -0.3 | ${ }^{p} \mathrm{P}_{18}(34)$ | 1070.6730(2) | 1.0 | 0.2 |
| ${ }^{p} \mathrm{P}_{16}$ (27) | 1082.0690(2) | 0.9 | 0.3 | ${ }^{p} \mathrm{P}_{20}$ (26) | 1070.4904(2) | -0.1 | -0.6 |
| ${ }^{p} \mathrm{P}_{16}(28)$ | 1081.3403(2) | -1.0 | -1.7 | ${ }^{p} \mathrm{P}_{22}(21)$ | 1070.3808(2) | -0.1 | 0.3 |
| ${ }^{p} \mathrm{P}_{17}(24)$ | 1081.1152(2) | 0.7 | -0.2 | ${ }^{p} \mathrm{P}_{18}(36)$ | 1069.1647(2) | 1.3 | 0.5 |
| ${ }^{p} \mathrm{P}_{14}(37)$ | 1081.0069(2) | 0.5 | 0.2 | ${ }^{p} \mathrm{P}_{19}(32)$ | 1069.1017(2) | -0.3 | -1.0 |
| ${ }^{p} \mathrm{P}_{18}(20)$ | 1080.8756(2) | 1.5 | 0.2 | ${ }^{p} \mathrm{P}_{20}(28)$ | 1069.0289(2) | 0.9 | 0.5 |
| ${ }^{p} \mathrm{P}_{12}(47)$ | 1079.8872(2) | -0.2 | -0.4 | ${ }^{p} \mathrm{P}_{21}(24)$ | 1068.9440(2) | 0.6 | 1.1 |
| ${ }^{p} \mathrm{P}_{16}$ (30) | 1079.8739(2) | -1.6 | -2.2 |  |  |  |  |

${ }^{\text {a }}$ Values in parentheses in column 2 are experimental uncertainties (in $10^{-4} \mathrm{~cm}^{-1}$ ).
${ }^{\mathrm{b}}$ The value $\Delta I=\nu^{\text {exp. }}-\nu^{\text {calc. ( (in }} 10^{-4} \mathrm{~cm}^{-1}$ ), where $\nu^{\text {exp. }}$ is taken from Refs. [12,26], and $\nu^{\text {calc. }}$ is calculated with the parameters from column 2 of Table 2 , columns 2, 4, 6 of Tables 4 and 5.
${ }^{\text {c }}$ The values $\Delta I I=\nu^{\text {exp. }}-\nu^{\text {calc. ( }}$ (in $10^{-4} \mathrm{~cm}^{-1}$ ) are reproduced from the before last column of Table VI, Ref. [26].
states (100), (001), and (020). Numerous resonance interactions between all three vibrational states were taken into account.

The fit was made with the Hamiltonian model discussed in Section 3. The initial values of rotational and centrifugal distortion parameters of the (100) and (001) states were taken to be equal to the values of corresponding parameters of the ground vibrational state. The initial values of spectroscopic parameters of the (020) state were estimated in accordance with the formula
$P^{(020)}=P^{(000)}+2\left(P^{(010)}-P^{(000)}\right)$,
where $P^{(000)}$ or $P^{(010)}$ is any of rotational or centrifugal distortion parameter of the ground or (010) vibrational state from Ref. [35], respectively; $P^{(020)}$ is a corresponding estimated parameter of the (020) vibrational state. The values of two main resonance interaction parameters, ${ }^{(100),(020)} F_{0}$ and ${ }^{(100),(001)} \mathrm{C}^{1}$, have been estimated on the basis of the potential energy surface parameters of $\mathrm{SO}_{2}$ from Ref. [48] and were constrained in the fit. The reason for the last was in the following. It is known (see, e.g., Refs. [49,50]) that the main both Fermi- and Coriolis-interaction parameters are strongly correlated with pure vibrational energies, $E^{v}$, or effective rotational parameters, $A^{v}, B^{v}, C^{v}$, respectively. In our case, when we tried to vary the ${ }^{(100),(020)} F_{0}$ and/or ${ }^{(100),(001)} C^{1}$ parameters, their absolute values change more than twice that is absolutely unsuitable from the physical point of view.

For this reason we preferred to fix values of these parameters to theoretically estimated ones.

Results of the fit with the Hamiltonian, Eqs. (1)-(5), are presented in columns 2, 4, and 6 of Table 4 and in Table 5 (values in parentheses are $1 \sigma$ statistical confidence intervals). Parameters presented without confidence intervals have been constrained to their initial values (see above). To take into account numerous accidental resonance interactions between all three studied vibrational states, (100), (001), and (020) (especially for high values of quantum numbers $J$ and $K_{a}$ ), we introduced into consideration some resonance interaction parameters. They are presented in Table 5 together with their $1 \sigma$ confidence intervals (in parentheses). In this case, values of the main resonance interaction parameters, ${ }^{(100),(020)} F_{0}$ and ${ }^{(100),(001)} C^{1}$, were constrained to their predicted ones (see above). To compare results of the present study with the previous ones, columns 3, 5, and 7 of Table 4 show the values of spectroscopic parameters from earlier study, Ref. [26]. One can see good correlation between both sets of parameters. At the same time our parameters reproduce better our FTIR experimental data. In particular, the 4063 ro-vibrational energies obtained from our experimental data are reproduced by our set of parameters with the $d_{\text {rms }}=8.7 \times 10^{-5} \mathrm{~cm}^{-1}$. For comparison, the same 4063 rovibrational energies are reproduced by the parameters from Ref. [26] with the $d_{r m s}=3329.5 \times 10^{-5} \mathrm{~cm}^{-1}$ (see

Table 8
MW frequencies of $\mathrm{SO}_{2}$ in the $\nu_{1}$ band.

| Transition 1 |  |  |  |  |  |  | Frequency, exp. in MHZ 2 | $\begin{aligned} & \Delta,{ }^{\mathrm{a}} \text { in MHZ } \\ & 3 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | $\leftarrow$ | 0 | 0 | 0 | 69566.06 | 0.09 |
| 1 | 1 | 1 | $\leftarrow$ | 2 | 0 | 2 | 12522.89 | 0.17 |
| 2 | 1 | 1 | $\leftarrow$ | 2 | 0 | 2 | 53595.80 | 0.10 |
| 3 | 2 | 2 | $\leftarrow$ | 4 | 1 | 3 | 70293.43 | 0.04 |
| 4 | 1 | 3 | $\leftarrow$ | 4 | 0 | 4 | 59260.96 | -0.21 |
| 4 | 2 | 2 | $\leftarrow$ | 5 | 1 | 5 | 70765.92 | 0.09 |
| 5 | 2 | 4 | $\leftarrow$ | 6 | 1 | 5 | 24275.46 | 0.10 |
| 6 | 1 | 5 | $\leftarrow$ | 6 | 0 | 6 | 68951.16 | -0.69 |
| 6 | 2 | 4 | $\leftarrow$ | 7 | 1 | 7 | 44794.50 | 0.02 |
| 8 | 1 | 7 | $\leftarrow$ | 7 | 2 | 6 | 24301.59 | 0.35 |
| 8 | 2 | 6 | $\leftarrow$ | 9 | 1 | 9 | 24888.00 | 0.04 |
| 10 | 2 | 8 | $\leftarrow$ | 11 | 1 | 11 | 12597.69 | 0.14 |
| 10 | 3 | 7 | $\leftarrow$ | 11 | 2 | 10 | 53378.90 | -0.29 |
| 12 | 2 | 10 | $\leftarrow$ | 13 | 1 | 13 | 9172.50 | 0.26 |
| 12 | 3 | 9 | $\leftarrow$ | 13 | 2 | 12 | 21768.64 | 0.20 |
| 14 | 2 | 12 | $\leftarrow$ | 13 | 3 | 11 | 45911.10 | -0.11 |
| 14 | 2 | 12 | $\leftarrow$ | 15 | 1 | 15 | 15230.50 | 0.14 |
| 15 | 4 | 12 | $\leftarrow$ | 16 | 3 | 13 | 40720.80 | -0.07 |
| 16 | 2 | 14 | $\leftarrow$ | 17 | 1 | 17 | 30700.50 | -0.05 |
| 17 | 2 | 16 | $\leftarrow$ | 16 | 3 | 13 | 27386.15 | -0.07 |
| 16 | 4 | 12 | $\leftarrow$ | 17 | 3 | 15 | 33996.10 | 0.05 |
| 19 | 2 | 18 | $\leftarrow$ | 18 | 3 | 15 | 41642.40 | 0.03 |
| 21 | 2 | 20 | $\leftarrow$ | 20 | 3 | 17 | 46929.50 | 0.38 |
| 21 | 3 | 19 | $\leftarrow$ | 20 | 4 | 16 | 35084.30 | -0.02 |
| 21 | 5 | 17 | $\leftarrow$ | 22 | 4 | 18 | 26977.80 | -0.04 |
| 22 | 5 | 17 | $\leftarrow$ | 23 | 4 | 20 | 14974.30 | 0.03 |
| 24 | 4 | 20 | $\leftarrow$ | 23 | 5 | 19 | 19226.75 | -0.05 |
| 25 | 4 | 22 | $\leftarrow$ | 24 | 5 | 19 | 23788.00 | 0.04 |
| 26 | 6 | 20 | $\leftarrow$ | 27 | 5 | 23 | 37515.60 | -0.07 |
| 27 | 6 | 22 | $\leftarrow$ | 28 | 5 | 23 | 12563.00 | -0.04 |
| 30 | 5 | 25 | $\leftarrow$ | 29 | 6 | 24 | 32381.80 | -0.13 |
| 31 | 5 | 27 | $\leftarrow$ | 30 | 6 | 24 | 43878.40 | -0.18 |
| 31 | 7 | 25 | $\leftarrow$ | 32 | 6 | 26 | 40163.00 | 0.01 |
| 32 | 7 | 25 | $\leftarrow$ | 33 | 6 | 28 | 20835.10 | -0.03 |
| 35 | 6 | 30 | $\leftarrow$ | 34 | 7 | 27 | 20689.80 | -0.02 |
| 36 | 8 | 28 | $\leftarrow$ | 37 | 7 | 31 | 45834.00 | -0.16 |
| 37 | 8 | 30 | $\leftarrow$ | 38 | 7 | 31 | 24431.80 | -0.15 |
| 40 | 7 | 33 | $\leftarrow$ | 39 | 8 | 32 | 17994.70 | 0.23 |
| 41 | 7 | 35 | $\leftarrow$ | 40 | 8 | 32 | 37593.40 | 0.27 |
| 42 | 9 | 33 | $\leftarrow$ | 43 | 8 | 36 | 29668.00 | -0.12 |
| 46 | 8 | 38 | $\leftarrow$ | 45 | 9 | 37 | 33733.40 | 0.09 |

${ }^{\text {a }}$ The value $\Delta=\nu^{\text {exp. }} \nu_{\nu}^{\text {calc. }}$, where $\nu^{\text {exp. }}$ is taken from Ref. [26], and $\nu^{\text {calc. }}$ is calculated with the parameters from column 2 of Table 2, columns 2, 4, 6 of Tables 4 and 5 .
also statistical information in Table 3). In this case, differences between experimental energy values and ones calculated with the parameters from Ref. [26] are increased quickly with the increase of $K_{a}$ quantum number, and for $K_{a}=37$ the difference reaches the value of $0.62 \mathrm{~cm}^{-1}$.

As one more illustration of the correctness of our results, we send the reader to Tables $6-8$. Columns 2 of these tables reproduce high accurate heterodyne experimental line positions of the $\nu_{3}$ band from Ref. [23] (Table 6), $\nu_{1}$ band from Ref. [12] (Table 7), and microwave rotational transitions in the excited (100) vibrational state (Table 8) from Ref. [26], respectively. Columns 3 of these three tables present the values of differences $\Delta$ between experimental line positions and corresponding values calculated with our parameters. One can see that values of $\Delta$ are not worse than the experimental uncertainties (the lasts are given in
parentheses in column 2 of Tables 6 and 7). Columns 4 of Tables 6 and 7 present, for comparison, corresponding $\Delta$-values from Ref. [26]. It is important to note that the experimental data from columns 2 of Tables 6-8 were not used in our fit as an input data. It means that one can consider results shown in the columns 3 of Tables 6-8, as a prediction. At the same time, our predicted $\Delta$-values are, at least, not worse than ones from Ref. [26] (compare columns 3 and 4 of Tables 6 and 7) in spite of the fact that both the heterodyne and MW data reproduced in column 2 of Tables 6-8 were used in the fit of Ref. [26].

## 6. Conclusion

We re-analyzed the high resolution ro-vibrational structures of the $\nu_{1}$ and $\nu_{3}$ vibrational bands and, as a result, assigned about two times more transitions than was made before. Moreover, the weak $2 \nu_{2}$ band has been observed and analyzed for the first time. We determined parameters of the Hamiltonian that reproduce the initial "experimental" ro-vibrational energies of the states (100), (001), and (020) with the rms deviation of $6.1 \times 10^{-5} \mathrm{~cm}^{-1}$, $9.7 \times 10^{-5} \mathrm{~cm}^{-1}$, and $13.9 \times 10^{-5} \mathrm{~cm}^{-1}$, respectively, that is close to uncertainties in experimental line positions.

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## Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org.10.1016/j. jqsrt.2013.04.011.

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[^1]:    ${ }^{\text {a }}$ Values in parentheses are $1 \sigma$ confidence intervals (in last digits).
    ${ }^{\mathrm{b}}$ Obtained from the fit in the present study.
    ${ }^{\text {c }}$ Recalculated from Table 3 of Ref. [35]. Number of kept digits in the values of parameters corresponds to the number of digits in the initial values from Ref. [35].

[^2]:    ${ }^{\text {a }} N_{l}$ is the numbers of upper levels.
    ${ }^{\mathrm{b}}$ Here $m_{i}=n_{i} / N \times 100 \%(i=1,2,3) ; n_{1}, n_{2}$, and $n_{3}$ are the numbers of levels (transitions) for which the differences $\delta=\left|E^{\text {exp. }}-E^{\text {calc. }}\right|$ (for columns $7-9$ ), or $\delta=\left|\nu^{\text {exp }}-\nu^{\text {calc }}\right|$ (for columns 12-14) satisfy the conditions $\delta \leq 10 \times 10^{-5} \mathrm{~cm}^{-1}, 10 \times 10^{-5} \mathrm{~cm}^{-1}<\delta \leq 20 \times 10^{-5} \mathrm{~cm}^{-1}$, and $\delta>20 \times 10^{-5} \mathrm{~cm}^{-1}$.
    ${ }^{\text {c }} N_{t}$ is the numbers of transitions.
    ${ }^{\text {d }}$ Reproduction of sub-millimeter wave transitions from Ref. [35]: $K_{a} \leq 23$. The rms in column 11 is in kHz .
    ${ }^{\mathrm{e}}$ Reproduction of Ground State Combination Differences obtained from analysis of our FTIR experimental data: $29 \leq K_{a} \leq 34$.

[^3]:    ${ }^{\text {a }}$ Values in parentheses are $1 \sigma$ standard errors. Parameters presented without standard errors have been fixed to the values of the ground vibrational state ( $(100)$ and (001) states) from column 2 of Table 2, or to the values ( $(020)$ state) estimated in accordance with the formula (6).

[^4]:    ${ }^{\text {a }}$ Values in parentheses are experimental uncertainties (in MHz).
    ${ }^{\mathrm{b}}$ The value $\Delta I=\nu^{\text {exp. }}-\nu^{\text {calc. }}$, where $\nu^{\text {exp. }}$ is taken from Ref. [23], and $\nu^{\text {calc. }}$ is calculated with the parameters from column 2 of Table 2 , columns $2,4,6$ of Table 4 and 5.
    ${ }^{\mathrm{c}}$ The values $\Delta I I=\nu^{\text {exp. }}-\nu^{\text {calc. }}$. are reproduced from the last column of Table V, Ref. [26].

