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High resolution study of the $3v_1$ band of SO₂

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1. Introduction

Sulfur dioxide is an important chemical species in many fields such as chemistry, astrophysics, atmospheric optics, laser techniques, etc. (see, e.g., reviews in [1,2]). Therefore, spectroscopic studies of the sulfur dioxide molecule have been made during many years in microwave (see, e.g., review in Ref. [3]), submillimeter wave, and infrared (see, Refs. [4-33]) regions. We report in this communication the first high resolution analysis of the 3340-3520 cm⁻¹ spectral region where the second stretching overtone $3v_1$ is located. Bands corresponding both to single and binary excitation of stretching quanta have been studied earlier in details in number of publications, see, e.g., [2], and references cited therein. At the same time, as to our knowledge of high resolution studies of bands corresponding to triple excitation of stretching quanta, only two of them have been analyzed before with high resolution: the $3v_3$ band has been discussed in Refs. [1,26], and $2v_1 + v_3$ band has been considered in Ref. [31].

2. Experimental details

The experimental measurements over the $3v_1$ region were performed in the infrared laboratory of Oulu using a Bruker IFS-120 HR Fourier spectrometer. The SO₂ sample, obtained from Sigma– Aldrich Inc. with purity of 99.9%, was used in the White type cell [34] at a pressure of 630 Pa. The absorption path length was 154 m and the cell was provided with two 6 mm thick potassium bromide windows. A tungsten source, a quartz beam splitter, and an indium antimonide semiconductor detector were used. Optical band-pass filters were used to limit the wavenumber region be-

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ABSTRACT

The second overtone band $3v_1$ of sulfur dioxide has been studied for the first time with high resolution rotation-vibration spectroscopy. About 3000 transitions involving about 900 upper state energy levels with $J^{\text{max}} = 66$ and $K_a^{\text{max}} = 24$ have been assigned to the $3v_1$ band. In the analysis, an effective Hamiltonian taking into account accidental interactions between the vibrational states (300), (220), and (041) was used. The Watson operator in *A*-reduction and I^r representation was used in the diagonal blocks of the Hamiltonian. As the result of analysis a set of parameters reproducing the initial experimental data with the rms = 0.00028 cm⁻¹ was obtained.

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tween 3000 and 4000 cm⁻¹. The obtained spectral resolution in the $3v_1$ region was 0.0075 cm⁻¹ determined by aperture broadening (0.0040 cm⁻¹) and maximum optical path difference (0.0034 cm⁻¹) together with Doppler and pressure broadenings. The total registration time was more than 85 h leading to signal to noise ratio better than 1000 and even 3000 for optimal region. The spectrum was calibrated with 43 water lines [35] and the peak positions were calculated using the method of optimized center of gravity [36].

3. Analysis and results

Survey spectrum of the $3v_1$ band of SO_2 molecule is shown on Fig. 1. One can see all three clearly pronounced branches, *P*, *Q*, and *R*, as well as the structure of sets of transitions in the *P* and *R* branches. The upper state (300) of the $3v_1$ band is of A_1 symmetry and therefore the observed transitions are of *b*-type. Moreover, due to nuclear spin statistics, only transitions with even values of $K_a'' + K_c''$ of the lower quantum numbers are available. The transitions were assigned using the method of ground state combination differences (GSCD). For this purpose, the ground state rotational energies were calculated using the parameters from Ref. [33]. As the result of assignment, about 3000 transitions with upper quantum numbers $J^{max} = 66$ and $K_a^{max} = 24$ were found for the $3v_1$ band. On that basis about 900 upper state rovibrational energies were obtained. List of the obtained energies is presented in the column 2 of Table 1.

To give an impression of the relative strength of the $3v_1$ band in comparison with the earlier studied $2v_1 + v_3$ band, Fig. 2 presents the spectrum of SO₂ in more wide region. Fig. 3 shows, also for illustration, a small portion of the high resolution spectrum in the region of the *P*-branch of the $3v_1$ band.

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Fig. 1. Survey spectrum of SO₂ in the region of 3340–3520 cm⁻¹: I is the experimentally recorded spectrum (lines of CO₂ are seen also); II is simulated spectrum (in this case, only one dipole moment parameter was used in the simulation).

The SO₂ molecule is an asymmetric top with the value of asymmetry parameter, $\kappa = (2B - A - C)/(A - C) \approx -0.94$, close to the prolate symmetric top limit. However, the SO₂ molecule is relatively heavy, and, in spite of the small absolute value of the difference between *B* and *C* rotational constants, the centrifugal distortion parameters are considerably smaller. On that reason the *A* reduction and *I*^{*r*} representation looks suitable enough and can be applied in the analysis (see, e.g., cited references).

As has been discussed earlier (e.g., in [24,26,31,32]), weak Fermi- and/or Coriolis-type interactions should be taken into account when analyzing high resolution spectra of sulfur dioxide. On that reason we used in our analysis the Hamiltonian model which takes into account both types of these interactions:

$$H^{\text{vib.-rot.}} = \sum_{\nu,\tilde{\nu}} |\nu\rangle \langle \tilde{\nu} | H^{\nu\tilde{\nu}}, \tag{1}$$

where v and \tilde{v} denote interacting vibrational states. In this model the diagonal block operators are the traditional Watson type operators in *A* reduction and I^r representation, Ref. [37]

$$\begin{split} H^{\nu\nu} &= E^{\nu} + \left[A^{\nu} - \frac{1}{2} (B^{\nu} + C^{\nu}) \right] J_{z}^{2} + \frac{1}{2} (B^{\nu} + C^{\nu}) J^{2} + \frac{1}{2} (B^{\nu} \\ &- C^{\nu}) J_{xy}^{2} - \Delta_{k}^{\nu} J_{z}^{4} - \Delta_{Jk}^{\nu} J_{z}^{2} J^{2} - \Delta_{J}^{\nu} J^{4} - \delta_{K}^{\nu} \left[J_{z}^{2} , J_{xy}^{2} \right]_{+} - 2 \delta_{J}^{\nu} J^{2} J_{xy}^{2} \\ &+ H_{K}^{\nu} J_{z}^{6} + H_{KJ}^{\nu} J_{z}^{4} J^{2} + H_{JK}^{\nu} J_{z}^{2} J^{4} + H_{J}^{\nu} J^{6} \\ &+ \left[h_{K}^{\nu} J_{z}^{4} + h_{JK}^{\nu} J_{z}^{2} J^{2} + h_{J}^{\nu} J^{4} , J_{xy}^{2} \right]_{+} + L_{KJ}^{\nu} J_{z}^{8} + L_{KKJ}^{\nu} J_{z}^{6} J^{2} + L_{JK}^{\nu} J_{z}^{4} J^{4} \\ &+ L_{JJK}^{\nu} J_{z}^{2} J^{6} + L_{J}^{\nu} J^{8} + \left[l_{K}^{\nu} J_{z}^{6} + l_{KJ}^{\nu} J_{z}^{4} J^{2} + l_{JK}^{\nu} J_{z}^{2} J^{4} + l_{J}^{\nu} J_{z}^{6} , J_{xy}^{2} \right]_{+} \\ &+ P_{KJ}^{\nu} J_{z}^{10} + P_{KKJ}^{\nu} J_{z}^{8} J^{2} + P_{KJ}^{\nu} J_{z}^{6} J^{4} + P_{JK}^{\nu} J_{z}^{4} J^{6} + \dots + S_{KJ}^{\nu} J_{z}^{12} \\ &+ S_{KKJ}^{\nu} J_{z}^{10} J^{2} + \dots, \end{split}$$

where $J_{\alpha}(\alpha = x, y, z)$ are the components of the angular momentum operator defined in the molecule-fixed coordinate system, $J_{xy}^2 = J_x^2 - J_y^2$, and $[\dots, \dots]_+$ denotes anticommutator. Because of the symmetry arguments of the SO₂ molecule, the nondiagonal block operators can only be of two types. If the states v and \tilde{v} in Eq. (1)

are of same symmetry, the corresponding operator $H^{v\bar{v}}$ describes the Fermi-type interaction and has the form:

$$\begin{aligned} H_{F}^{\nu\tilde{\nu}} &= F^{\nu\tilde{\nu}} + F_{K}^{\nu\tilde{\nu}} J_{z}^{2} + F_{J}^{\nu\tilde{\nu}} J^{2} + \dots + F_{xy}^{\nu\tilde{\nu}} J_{xy}^{2} + F_{xyk}^{\nu\tilde{\nu}} \left[J_{xy}^{2}, J_{z}^{2} \right]_{+} \\ &+ F_{xyJ}^{\nu\tilde{\nu}} J_{xy}^{2} J^{2} + \dots. \end{aligned}$$

$$(3)$$

If the symmetries of the states v and \tilde{v} are different (they can be of type A_1 and B_1 only), then the operator $H^{v\tilde{v}}$ describes the Coriolis-type interaction and has the following form:

$$\begin{aligned} H_{\mathcal{C}_{y}}^{\nu\bar{\nu}} &= (2B\zeta^{y})^{\nu\bar{\nu}}iJ_{y} + C_{yK}^{\nu\bar{\nu}}\left[iJ_{y},J_{z}^{2}\right]_{+} + C_{yJ}^{\nu\bar{\nu}}iJ_{y}J^{2} + \dots + C_{xz}^{\nu\bar{\nu}}[J_{x},J_{z}]_{+} \\ &+ C_{xzK}^{\nu\bar{\nu}}[[J_{x},J_{z}]_{+},J_{z}^{2}]_{+} + C_{xzJ}^{\nu\bar{\nu}}[J_{x},J_{z}]_{+} \dots . \end{aligned}$$

$$(4)$$

During the analysis it became evident that local resonances between vibrational state (300) and the states (220) as well as (041) are essential in the study of the $3v_1$ band. As follows from estimation on the basis of parameters from Ref.[27], the separation of band centers of the bands $3v_1$ and $2v_1 + 2v_2$ is about 100 cm^{-1} and that of the bands $3v_1$ and $4v_2 + v_3$ is 19 cm⁻¹ only.At the same time, we were not able to undoubtedly assign transitions to the $2v_1 + 2v_2$ and/or $4v_2 + v_3$ bands and, therefore, it was important to preliminary estimate the values of their band centers and main rotational and centrifugal parameters.In our case, the values of the band centers and effective rotational parameters were predicted using the values of harmonic frequencies, ω_{λ} , and anharmonic coefficients, $x_{\lambda\mu}$, from Ref.[27], and of equilibrium rotational parameters, X_{β} , and the vibration–rotation constants, α_{λ}^{β} and $\gamma_{\lambda\mu}^{\beta}$, from [26].To estimate the values of some of centrifugal distortion coefficients, Δ_K , Δ_{JK} , δ_K , H_K , and H_{KJ} , we made a linear extrapolation of the values of corresponding parameters of states (010), (020), (100), (200), (001), (002), (003), (101), (201), (110), (210), (011), and (111) (see, Refs.[21-26,31-33]).All other centrifugal distortion coefficients were constrained to the corresponding ground state values from Ref.[33].All the estimated values are collected in columns 2, 4 and 6 of Table 2, and the ground state parameters are presented in column 5 of Table 3. The estimated values were used then as an initial approximation in Hamiltonian(1)-(4).

Table 1

able 1	
xperimental rovibrational term values for the (300) vibrational state of the SO_2 molecule (in cm ⁻¹	'). ^a

J	Ka	Kc	Ε	Δ	δ	J	Ka	Kc	Е	Δ	δ	J	Ka	Kc	Ε	Δ	δ
1			2	3	4	1			2	3	4	1			2	3	4
1	1	1	3434.60712	17	-28	11	1	11	3473.67250	15	-14	15	9	7	3646.23618	9	7
2	0	2	3434.17186	16	-25	11	2	10	3480.51733	27	2	15	10	6	3678.54704	3	-9
2	1	1	3435,96397	13	-22	11	3	9	3489.29655	8	-11	15	11	5	3714,18903	7	-7
2	2	0	3441.03765		-22	11	4	8	3501.27139	4	-1	15	12	4	3753,14054	6	2
3	1	3	3437.62478	26	21	11	5	7	3516.66809	9	9	15	13	3	3795,37833	9	-11
3	2	2	3442,92220	20	-17	11	6	6	3535,47974	5	10	15	14	2	3840.87847	15	-4
3	3	1	3451,49839	37	-18	11	7	5	3557.69014	11	13	15	15	1	3889.61481	10	-17
4	0	4	3438 55645	11	3	11	8	4	3583 28355	18	9	16	0	16	3515 22593	3	15
4	1	3	3440 53532	5	_9	11	9	3	3612 24345	19	-3	16	1	15	3521 84926	4	-16
4	2	2	3445.45181	29	-8	11	10	2	3644.55243	21	26	16	2	14	3526.60669	10	-0
4	3	1	3454 01485	10	-18	11	11	1	3680 19024		21	16	3	13	3533 83037	9	-4
4	4	0	3466.01135	14	-3	12	0	12	3480 33896	8	3	16	4	12	3545 45772	28	-6
5	1	5	3443 04988	20	1	12	1	11	3484 66828	3	-6	16	5	11	3560 78321	29	13
5	2	4	3448 57576	5	_2	12	2	10	3488 97495	8	_0	16	6	10	3579 56614	12	20
5	2	3	3457 16099	28	39	12	2	0	3406.03780	13	4	16	7	0	3601 76281	11	20
5	4	2	3469 15671	20	-7	12	4	8	3508 83800	8	6	16	8	8	3627 35026	19	25
5		1	3484 56919		- /	12	5	7	3524 22609	31	20	16	9	7	3656 30960	13	5
6	0	6	3445 41057	2	1	12	6	6	3543 03446	19	16	16	10	6	3688 62074	10	1
6	1	5	3445,41057	10	-1	12	7	5	3565 24228	20	10	16	11	5	3724 26370	30	-4
c	1	3	2452 41720	10	5	12	,	3	3500.03051	50	12	10	12	1	3724.20370	20	5
c	2	4	3452.41720	14	-5	12	8	4	3090.83001	10	12	10	12	4	3703.21033	30	-5
c	3	2	3400.93934	14	2	12	10	2	3019./90/8	10	8	10	13	2	3803.43620	30	15
c	4	2	34/2.93204	24	12	12	10	2	3032.10003	30	-/	10	14	2	3830.93800	17	-13
0	5	1	3488.34323	15	15	12	11	1	3087.74507	2	1	16	15	1	3899.09093	15	-/
0	0	0	3507.16076	12	2	12	12	0	3720.09230	15	-2	10	16	0	3951.04512	2	-25
/	1		3450.87430	5	-3	13	1	13	3488.62013	2	-6	17	1	1/	3525.55174	5	-2
_	2	0	3430.73295	15	0	15	2	12	5496.11688	20	-23	17	2	16	3534.64704	13	
7	3	5	3465.34581	91	-9	13	3	11	3505.06693	9	-22	17	3	15	3544.18681	9	-4
7	4	4	34/7.33766	30	22	13	4	10	3517.03426	19	18	17	4	14	3556.17680	20	16
7	5	3	3492.74744	39	2	13	5	9	3532.41553	18	-18	17	5	13	35/1.505//	6	1
7	6	2	3511.56498	26	27	13	6	8	3551.22015	14	24	17	6	12	3590.27903	3	8
7	7	1	3533.77685	11	7	13	7	7	35/3.42696	10		17	7	11	3612.47012	23	-7
8	0	8	3454.69105	22	-8	13	8	6	3599.01956	7	18	17	8	10	3638.05532	24	11
8	1	7	3457.47109	9	-11	13	9	5	3627.97992	4	12	17	9	9	3667.01350	10	15
8	2	6	3461.96906	39	-17	13	10	4	3660.29015	12	26	17	10	8	3699.32464	9	8
8	3	5	3470.39215	29	9	13	11	3	3695.92988	15		17	11	7	3734.96810	27	-3
8	4	4	3482.37373	6	9	13	12	2	3734.87841	3	-11	17	12	6	3773.92229	21	9
8	5	3	3497.78189	18	16	13	13	1	3777.11307	17	3	17	13	5	3816.16349	27	-13
8	6	2	3516.59866	27	26	14	0	14	3496.63521	5	-2	17	14	4	3861.66799	28	11
8	7	1	3538.81063	22	18	14	1	13	3502.03410	5	-14	17	15	3	3910.40905	20	-6
8	8	0	3564.40381	6	15	14	2	12	3506.46592	12	1	17	16	2	3962.35996	8	-21
9	1	9	3461.08623	5	-5	14	3	11	3514.07525	13	19	17	17	1	4017.49215		-54
9	2	8	3467.38421	31	-5	14	4	10	3525.87380	14	-1	18	0	18	3536.11006	11	22
9	3	7	3476.05666	11	5	14	5	9	3541.23827	11	6	18	1	17	3544.05887	4	-13
9	4	6	3488.04081	21	-5	14	6	8	3560.03679	15	-1	18	2	16	3549.36924	8	-1
9	5	5	3503.44642		7	14	7	7	3582.24117	20	-6	18	3	15	3556.23935		-89
9	6	4	3522.26195	20	-1	14	8	6	3607.83268	16	10	18	4	14	3567.60353	14	1
9	7	3	3544.47377	11	0	14	9	5	3636.79293	13	3	18	5	13	3582.86580	11	-3
9	8	2	3570.06736	5	13	14	10	4	3669.10355	24	2	18	6	12	3601.62497	10	7
9	9	1	3599.02655	12	24	14	11	3	3704,74477	21	23	18	8	10	3649,39100	10	-6
10	0	10	3466.34825	11	-6	14	12	2	3743.69460	11	2	18	9	9	3678.34769	7	3
10	1	9	3469.80082	13	-0	14	13	1	3785,93066	13	-11	18	10	8	3710.65858	10	0
	1	5	5405.00002	15	0	14	15		5765.55000	15	- 11	10	10	0	(con	tinuad on n	avt naga)

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J	Ka	Kc	Ε	Δ	δ	J	Ka	Kc	Ε	Δ	δ	J	Ka	Kc	Е	Δ	δ
1			2	3	4	1			2	3	4	1			2	3	4
10	2	8	3474.14552	14	-18	14	14	0	3831.42884	12	0	18	11	7	3746.30269	3	-3
10	3	7	3482.38537	19	-3	15	1	15	3505.91635	11	-36	18	12	6	3785.25790	38	-12
10	4	6	3494.34026	18	-18	15	2	14	3514.16653	4	-2	18	13	5	3827.50138	2	20
10	5	5	3509.74174	17	10	15	3	13	3523.36592	31	-0	18	14	4	3873.00741	4	-17
10	6	4	3528.55574	12	12	15	4	12	3535.33423	25	6	18	15	3	3921.75116	8	-14
10	7	3	3550.76698	13	8	15	5	11	3550.69347	10	2	18	16	2	3973.70517	21	2
10	8	2	3576.36062	5	18	15	6	10	3569.48543	12	8	18	17	1	4028.84038	16	-33
10	9	1	3605.32011	16	6	15	7	9	3591.68650	25	15	18	18	0	4087.12869		23
10	10	0	3637.62807	7	12	15	8	8	3617.27636		19	19	1	19	3547.51695	4	0
19	2	18	3557.53925	23	26	22	3	19	3609.13374	6	-12	24	19	5	4229.90643	35	27
19	3	17	3567.51966	31	3	22	4	18	3619,66068	17	13	24	20	4	4294,43080	44	5
19	4	16	3579.56549	4	22	22	5	17	3634.67235	30	-1	24	21	3	4362.01446	45	5
19	5	15	3594.85772	9	10	22	6	16	3653,34754	5	6	24	22	2	4432.62454		55
19	6	14	3613.60415		3	22	7	15	3675.48998	2	5	24	23	1	4506.22607		34
19	7	13	3635 78072	11	_4	22	8	14	3701 04830	28	-13	24	24	0	4582 78576		34
19	8	12	3661 35817	22	10	22	9	13	3729 99304	16	1	25	1	25	3627 33804	2	-4
19	g	11	3690 31274	10	11	22	10	12	3762 29894	17	_11	25	2	24	3640 49973	16	25
19	10	10	3722 62292	6	0	22	11	11	3797 94320	16	-11	25	3	23	3652 43035	14	-38
19	10	9	3758 26740	17	-5	22	12	10	3836 90215	14	2	25	4	22	3665.01046	36	17
19	12	8	3797 22390	16	2	22	13	9	3879 15164	8	_4	25	5	21	3680 20285	15	-11
10	12	7	3830 46862	5	12	22	14	6	3074 66686	6	25	25	6	20	3608 80000	0	- 11
19	15	6	2004 07740	20	-12	22	14	07	2072 42054	0	17	25	7	20	2720.00001	16	-1
19	14	5	2022 72220	10	10	22	15	6	4025 29621		-17	25	, ,	19	3720.90001	10	10
19	15	3	2005 60007	15	-28	22	10	5	4023.36031	27	-11	25	0	10	3740.42814	10	5
19	10	4	4040 91965	12	-21	22	10	1	4000.35359	27	22	25	10	16	2007 65225	15	-0
19	17	1	4040.01003	12	-35	22	10	4	4130.03723	17	1	25	10	10	2842 20205	2	-0
19	19	20	4100.32304	22	-10	22	19	2	4200.20160	20	-1	25	11	14	2002 25221	2	-2
20	0	20	3559.29005	33	2	22	20	2	4204.77700	30	12	25	12	14	3882.23231	3	-9
20	1	19	3508.01000	12	4	22	21	1	4332.33207		30	25	13	13	3924.50554	40	5
20	2	18	35/4./1421	20	-0	22	22	22	4402.95196	0	66	25	14	12	3970.02617	0	-9
20	5	17	2502 22022	18	22	23	1	23	3598.41414	8 10	20	25	15	10	4018.78796	10	-18
20	4	16	3392.32933	35	-22	23	2	22	3610.48210	10	-20	25	10	10	4070.76311	10	-9
20	5	15	3607.49305	10	12	23	د	21	3621.66040	/	0	25	1/	9	4125.92241	13	-17
20	6	14	3626.21/3/	13	1	23	4	20	3633.98529	2	11	25	18	8	4184.23639	8	2
20	7	13	3648.38435	8	-1	23	5	19	3649.20103	20	-15	25	19	7	4245.67394		19
20	8	12	36/3.95654	6	6	23	6	18	3667.86485	14	6	25	20	6	4310.20360		61
20	9	11	3702.90837	11	-3	23	7	17	3689.99263	7	0	25	21	5	4377.79190		34
20	10	10	3/35.21//6	8	5	23	8	16	3/15.5423/	37	-11	25	22	4	4448.40683		56
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27	19	9	4279 09918		-27	30	8	22	3834 72498	32	-24	33	4	30	3814 28701	4	-7	ŝ
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27	20	7	4411 22786		70	30	10	20	3805 86604	12	6	22	6	20	3848 12613	7	10	Cu
27	21	6	4411.25780		33	30	11	10	2021 /0125	5	6	33	7	20	3860.05466	22	7	lar
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20	19	9	4290.75762		17	31	10	22	5915.40515	48	12	34	6	28	2009./2098	32 invad or in	-19	
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J	Ka	Kc	Ε	Δ	δ	J	Ka	Kc	Ε	Δ	δ	J	Ka	Kc	Ε	Δ	δ
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37	22	16	4686.95729		97	41	2	40	3964.38516	21	-9	44	6	38	4121.69735		25	R
38	0	38	3871 67825		21	41	3	39	3984 70297	71	0	44	7	37	4141 86170	11	6	ole
44	8	36	4166 54926	18	4	47	13	35	4430 45860	70	9	51	7	45	4355 13987		Ő	cul
44	9	35	4194 99744	32	-8	47	14	34	4475 93434	70	-14	51	10	42	4439 18268		-32	ą
44	10	34	4226 99478	52	81	47	15	33	4524 70009		-33	51	11	41	4474 42785		-43	Spe
44	11	33	4262 44556	36	-2	47	16	32	4576 71198		2	51	12	40	4513 13566		-22	Ct
44	12	32	4301 29192	50	-13	47	17	31	4631 93361		-40	51	15	37	4649 42643			SO.
44	14	30	4388 98941	12	7	47	18	30	4690 33203		5	52	0	52	4243 79749		-26	do
44	15	29	4437 76534	12	2	48	0	48	4125 96219	31	2	52	° 2	50	4299 55295	2	_23	V N
44	17	23	4544 99822	15	36	40	1	40	4152 55439	51	44	52	3	49	4359 62570	2	37	5 5
44	18	26	4603 38788	15	43	40	2	46	4176 97693	20	-15	52	6	46	4344 47519		4	2
44	19	25	4664 91464		15	40	2	45	4198 67297	20	28	52	7	45	4389 34575		60	200
45	15	45	4043 62839	11	_13	40	4	43	4216.03832	22	20	52	8	43	4412 60512		75	3
45	2	43	4068 45679	12	10	40	5	43	4278 49194		24	52	11	41	4507 24641		_74	11
45	2	43	4001 13450	64	10	40	6	42	4220.45154		20	52	12	40	4545 02764		54	-
45	1	43	4051.15450	04	21	40	7	42	4241.45522		20	52	14	20	4545.52704		- 34	21
45		42	4111.51050		-51	40	, e	40	4200.23780	50	-23	52	15	37	4692 19099	0	-45	
45	5	20	4150.10104		26	40	0	20	4204.40229	25	-1	52	15	57	4062.16066	9	- 50	
45	0	29	41/0.2/393		-30	40	10	29	4312.39193	12	0	53	2	55	4274.09402		-35	
45	0	26	4193.03003	45	-21	40	10	20	4344.42364	40	-4	53	2	52	4304.22437		10	
45	10	20	4233.33363	45	-9	40	11	20	43/9.//004	42	4	55	2	50	4351.04073		10	
45	11	33	4290.82850	22	15	48	12	30	4418.0000	45	23	53	4	30	4330.70334	C	27	
40	12	34	4329.03912		-35	48	15	22	4554.95758		-21	53	0	48	4400.35200	0	-15	
40	13	22	43/1.84/30		15	48	10	32	4600.94720		-42	53	0	47	4421.88517	27	20	
40	14	32	4417.54001	4	-35	48	10	20	4002.17018		-1	53	ð 10	40	4440.10354	27	-28	
40	10	30	4518.12798	4	22	48	18	40	4/20.5/050	20	14	53	10	44	4505.53597		35	
40	17	29	43/3.34/96		20	49	1	49	4154.55799	20	-14	22	11	45	4540.70029		-11	
															(con	maea on ne	ext page)	

Table 1 (continued)

I	Ka	Kc	E	Δ	δ	I	Ka	Ke	E	Δ	δ	I	Ka	Ke	E	Δ	δ
1			2	3	4	1	u		2	3	4	1	u		2	3	4
45	10	20	4621 74065	-	20	40	2	49	4191 72020	10	24	52	14	40	-		- 44
45	10	20	4031.74003		29	49	2	40	4101.75950	10	4	55	14	40	4000.85000		-44
45	19	27	4095.27070		-91	49	- 4	40	4229.46097		45	54	2	51	4300.10047		55
40	1	40	4070.43710	20	-14	45	7	43	4249.95107		-7	54	4	50	4383.82334	25	14
46	2	45	4035.51157	23	_3	45	8	43	4230.33100	16	12	54	7	47	4411.55174	22	-10
46	2	43	4139.46400	25	31	40	9	41	4343 58394	25	12	54	8	46	4480 61948		-10
46	4	42	4155 15030	18	34	49	11	39	4410 69315	25	-6	54	9	45	4508 18078	35	-10
46	5	41	4166 68257	25	20	49	14	36	4537 04817		_20	54	10	44	4539 66973	55	27
46	6	40	4180 19255	14	-15	49	16	34	4637 81194	25	-29	54	12	42	4613 41012		-35
46	7	39	4199 73866	••	15	50	0	50	4183 72937	20	-3	55	1	55	4338 21250	89	-20
46	8	38	4224.18733		6	50	1	49	4211.49653		-54	55	3	53	4397.51305	15	-17
46	10	36	4284.44100	35	-8	50	3	47	4260.12970		39	55	4	52	4423.76894	15	31
46	11	35	4319.84410		-24	50	4	46	4237.11897		78	55	10	46	4574,43822		0
46	12	34	4358.65850	8	-22	50	5	45	4292.83246	30	29	55	11	45	4609.51047		-67
46	13	33	4400.83723	-	22	50	6	44	4279.15557		47	56	0	56	4370.83388	65	9
46	14	32	4446.32279		12	50	7	43	4323,44931	12	15	56	2	54	4431.30549		-49
46	15	31	4495.09245		-8	50	8	42	4347.20400	54	8	56	3	53	4458.08710		-29
46	17	29	4602.32643		-7	50	9	41	4375,22405	45	20	56	4	52	4481.71168		54
46	18	28	4660.72156		-34	50	10	40	4406.96270		26	57	1	57	4404.02974		38
47	1	47	4097.94213	21	14	50	12	38	4480.97590	64	-11	57	2	56	4435.90365	30	-9
47	2	46	4123.94668		-35	50	14	36	4568.55011		-33	57	3	55	4465.68098		-39
47	4	44	4169.33655	38	-6	50	15	35	4617.30081		0	57	11	47	4680.86283		67
47	6	42	4208.10875		40	50	16	34	4669.30534		-39	58	1	57	4470.25863		2
47	7	41	4229.32824	9	14	50	18	32	4782.93252		9	59	2	58	4505.18783		-15
47	8	40	4253.94159		-5	51	1	51	4213.47571		-29	60	0	60	4507.06087		-63
47	9	39	4282.23356		23	51	2	50	4241.83134		-41	62	2	60	4646.13565	20	50
47	10	38	4314.11750	22	-1	51	3	49	4268.06513	6	21	64	0	64	4652.47163		29
47	12	36	4388.28982		-9	51	6	46	4333.79322		27	66	0	66	4728.61653	15	13

 $\frac{1}{2}$ $\frac{1}$

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Fig. 2. Overview spectrum of SO_2 in the region of 3340–3800 cm⁻¹.

In the final fit 20 free Hamiltonian parameters (18 diagonal parameters and 2 interaction parameters) and about 900 ($J^{\text{max.}} = 66$ and $K_a^{\text{max.}} = 24$) experimental rovibrational energies were used. Results are presented in Table 3 together with 1 σ statistical confidence intervals. Parameters without confidence intervals in Table 3 have been constrained to the values estimated by above discussed procedure. The rms value of the fit is 0.00028 cm⁻¹ corresponding to the experimental uncertainties. To illustrate the quality of experimental data and results of the fit, the list of energy values, $E^{\text{exp.}}$, in cm⁻¹, is presented in column 2 of Table 1 together with their experimental uncertainties Δ in column 3 and $\delta = (E^{\text{exp.}} - E^{\text{calc.}})$ in column 4. The values of $E^{\text{calc.}}$ have been calculated using the parameters of Table 3. One can see good correlation between experimental and calculated values and no systematic deviations exist.

When using constrained Hamiltonian parameters in the fit, the reliability of the approximated fixed values should be discussed. Therefore in Table 2 together with the calculated band centers and extrapolated rotational and centrifugal parameters, the corresponding values, when available from the final fit in Table 3, have been collected for comparison. In case of the (300) state, all experimental and predicted values are equal within 1σ confidence interval (also ν close enough). This confirms that the fixed values for states (220) and (041) can be considered reliable.

As was mentioned above, local resonance interactions are essential between the states (300), on the one hand, and the states (220) and (041), on the other hand. In this case, because (as follows from Table 3) the distance between the centers of the bands $3v_1$ and $4v_2 + v_1$ is 18 cm⁻¹ only, interactions between sets of rovibrational states with different values of quantum number K_a



Fig. 3. Small portion of the recorded high resolution spectrum of the SO₂ molecule in the P-branch of the 3v₁ absorption band: I is the experimentally recorded spectrum; II is simulated spectrum (one dipole moment parameter was used in the simulation).

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Table 2 Some spectroscopic parameters of the (300), (220), and (041) states of the SO₂ molecule (in cm⁻¹).^a

-									
Parameter	(300)		(220)		(041)				
	Predicted	Fitted	Predicted	Fitted	Predicted	Fitted			
1	2	3	4	5	6	7			
ν	3432.31	3432.28768(26)	3317.91	3317.17(69)	3413.18	3414.59(70)			
Α	2.030	2.0304887(644)	2.110	2.1071(513)	2.167	2.167			
В	0.339	0.3392004(131)	0.341	0.3409(170)	0.343	0.3259(794)			
С	0.289	0.2893093(110)	0.290	0.2914(169)	0.290	0.2983(791)			
$\Delta_K imes 10^4$	0.907	0.9005(864)	1.088	1.088	1.286	1.286			
$\Delta_{lK} imes 10^5$	-0.347	-0.3445(678)	-0.397	-0.397	-0.478	-0.478			
$\delta_K \times 10^5$	0.110	0.1116(105)	0.140	0.140	0.157	0.157			
$H_K imes 10^7$	0.135	0.1348(135)	0.197	0.197	0.250	0.250			
$H_{KJ} imes 10^9$	-0.909	-0.9466(149)	-1.035	-1.035	-1.033	-1.033			

^a Values presented in columns 2, 4, and 6 have been theoretically estimated (see text for details). Values in columns 3, 5, and 7 were obtained from the fit (see, for more details, Table 3). In this case, corresponding 1 σ statistical confidence interval for fitted parameter is given also. Parameters presented in columns 3, 5, 7 without confidence intervals have been constrained to the predicted ones.

Table 3Spectroscopic parameters of the (300), (220), (041), and ground vibrational states of the SO2 molecule (in cm^{-1}).^a

Parameter	(300)	(220) ^b	$(041)^{b}$	(000) ^c
1	2	3	4	5
Е	3432.28768(26)	3317.17(69)	3414.59(70)	
Α	2.0304887(644)	2.1071(513)	2.167†	2.02735433
В	0.3392004(131)	0.3409(170)	0.3259(794)	0.3441739084
С	0.2893093(110)	0.2914(169)	0.2983(791)	0.293526529
$\Delta_K imes 10^4$	0.9005(864)	1.088†	1.286†	0.8640369
$\Delta_{I\!K} imes 10^5$	-0.3445(678)	-0.397†	-0.478†	-0.3901187
$\Delta_I imes 10^6$	0.2186(206)	0.220549	0.220549	0.220549
$\delta_K \times 10^5$	0.1116(105)	0.140†	0.157†	0.0846284
$\delta_I \times 10^7$	0.5627(371)	0.5674232	0.5674232	0.5674232
$\dot{H}_K \times 10^7$	0.1348(135)	0.197†	0.250†	0.12375
$H_{Kl} imes 10^9$	-0.9466(149)	-1.035†	-1.033†	-0.64936
$H_{lK} \times 10^{11}$	0.116‡	0.116‡	0.116‡	0.116
$H_{I} \times 10^{12}$	0.37589‡	0.37589‡	0.37589‡	0.37589
$h_K \times 10^9$	0.5670‡	0.5670‡	0.5670‡	0.5670
$h_{IK} \times 10^{12}$	-0.23‡	-0.23‡	-0.23‡	-0.23
$\dot{h_{l}} \times 10^{12}$	0.1829‡	0.1829‡	0.1829‡	0.1829
$L_{K} \times 10^{11}$	-0.265‡	-0.265‡	-0.265‡	-0.265
$L_{KKI} \times 10^{12}$	0.180‡	0.180‡	0.180‡	0.180
$L_{lK} \times 10^{13}$	-0.109‡	-0.109‡	-0.109‡	-0.109
$L_{IJK} \times 10^{19}$	-0.88‡	-0.88‡	-0.88‡	-0.88
$L_{I} \times 10^{19}$	-0.116‡	-0.116‡	-0.116‡	-0.116
$l_{K}^{\prime} \times 10^{12}$	-0.32‡	-0.32‡	-0.32‡	-0.32
$l_{KI} imes 10^{14}$	0.27‡	0.27‡	0.27‡	0.27
$l_{JK} \times 10^{17}$	-0.2‡	-0.2‡	-0.2‡	-0.2
$l_{J} \times 10^{18}$	-0.597‡	-0.597‡	-0.597‡	-0.597
$P_K \times 10^{15}$	0.649‡	0.649‡	0.649‡	0.649
$P_{KKI} \times 10^{16}$	-0.394‡	-0.394‡	-0.394‡	-0.394
$P_{KI} \times 10^{18}$	-0.703‡	-0.703‡	-0.703‡	-0.703
$P_{IK} \times 10^{19}$	0.778‡	0.778‡	0.778‡	0.778
$S_{K} \times 10^{18}$	-0.12‡	-0.12‡	-0.12‡	-0.12
$S_{KKJ} imes 10^{20}$	0.70‡	0.70‡	0.70‡	0.70
$F_{xy}^{300-220} \times 10^4$	0.57(23)			
$(2B\zeta^y)^{300-041} \times 10^4$	0.61(21)			

^a Values in parentheses are 1*σ* confidence intervals (in last digits). Parameters presented without confidence intervals were constrained to the predicted values (see text for details). Values marked by "‡" and "†" have been fixed to the values of corresponding parameters of the ground vibrational state and values quated in Table 2, respectively. ^b There are no experimental transitions assigned to this state in our analysis.

^c Recalculated from Table 3 of Ref. [33]. Number of kept digits in the values of parameters corresponds the number of digits in the initial values from Ref. [33].

should be taken into account. At the same time, if to speak about interaction between the states (300) and (220), we found for our experimental data only one essential set of strongly interacting states. They are states $[JK_a = 13K_c](300)$, on the one hand, and states $[JK_a = 15K_c](220)$, on the other hand. Fig. 4 illustrates this kind of resonance interaction. The curves I and II give the dependences of the value $\Delta_J = E^{\text{exp.}}_{[JK_a = 13K_c](300)} - E^{\text{calc.}}_{[JK_a = 13K_c](300)}$ as functions of the quantum number J for the situations without and with resonance operator, Eq. (3), in the Hamiltonian of the molecule. One can see that the presence of resonance operator improves the value

of difference Δ . To illustrate a strength of the discussed local interaction, the top panel of Fig. 4 shows the dependence of the value $\Delta E_J = E_{JK_a=13K_c](300)} - E_{JK_a=15K_c](220)}$ on the value of quantum number *J*.

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Fig. 4. Illustration of a local resonance interaction in the (300) and (220) vibrational states of SO₂. The dependences of the value $\Delta_J = E_{IK_a=13K_c|(300)}^{exp} - E_{IJK_a=13K_c|(300)}^{calc}$, as functions of the quantum number *J*, are given by the curves I and II. The curves I and II have been constructed with the energies, E^{calc} , obtained without and with resonance operator, Eq. (3), in the Hamiltonian, Eq. (1), respectively. The top panel illustrates the dependence of the difference $\Delta E_J = E_{JK_a=13K_c|(3\,0\,0)} - E_{JK_a=15K_c|(2\,2\,0)}$ on the value of quantum number J.

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