## Topic \#1: Charging of interest. Discounting. Profitability

Mathematical Elements of Financial Management

## Content

- Using simple interest in practice;
- Fixed and variable interest-rate values;
- The essence of charging compound interest;
- The difference between simple and compound interest rates;
- Calculation on variable rates of compound interest;
- Determination of the term of a loan and the level of interest rates;
- The concept of discount. the notion of discounting;
- The specified sum and coefficient of reduction;
- Types of discounting: Mathematical discounting and bank accounts;
- Formula for determining the amount of money obtained by taking into account financial obligations;
- The profitability of financial operations;
- Types and methods of calculating the returns.
- Time value of money - refers to the fact that $\$ 1$ received today is worth more than $\$ 1$ received tomorrow. This is due, for example investing capital today can generate revenue in the future. One can eventually argue that inflation devalues the money supply and therefore " $\$ 1$ received today" is more valuable than " $\$ 1$ received tomorrow".
- Compounding and discounting form the basis for the valuation process used in finance.


## Definitions

## - Compounding

- You put money in an account today (its present value - PV) for a promised rate of return (interest - INT) for a number of periods (NPER - usually months or years). The interest received in reinvested at the end of each period - it compounds. The future value (FV) is the value of the investment compounded at the end of a given number of periods. We know the value of our initial investment and the interest rate, and can calculate the $\mathbf{F V}$ at the end of any period.


## - Discounting

- It is the reverse of compounding. We know the how much we need on a specific date in the future (FV) and calculate how much we need to invest today, PV, at an interest rate. Work from the future back to the present.


## Illustration Compounding and Discounting



## Basic Compounding

The table shows the ending wealth that an investor could have accumulated by the end of 1998 had he invested $\$ 1000$ in 1938

|  | 1938 | 19938 | 1958 | 1968 | 1978 | 1988 | 1998 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stocks | 1，091 | 2，103 | 10，128 | 27，539 | 51，038 | 193，038 | 485，058 |
| Bonds | 1，056 | 1，434 | 1，623 | 2，084 | 3，691 | 10，480 | 37，720 |
| 丁－BiJ］s | 1，006 | 1，058 | 1，2年尓 | 1，893 | 3，678 | 11，489 | 23，253 |
| U．S． stocks | 1，3我年 | 2，651 | 15，602 | 白4，804 | 65，815 | 303，322 | 2，260，431 |

## Compound and Discounting Variables

$\mathrm{P}=$ current cash flow
F = future cash flow
$\mathrm{A}=$ the amount of annuity
$\mathrm{i}=$ the stated (or nominal) interest rate
I = dollar amount of interest
$r=$ the effective period rate of return
$\mathrm{n}=\#$ of periods under consideration
$\mathrm{m}=$ \# of compounding periods per year
$\mathrm{PV}=$ present value of a future cash flow(s)
$\mathrm{FV}=$ future value of a cash flow(s)

## Accruing compound interest

$$
\begin{gathered}
\mathbf{F}_{\mathbf{n}}=\mathbf{P}(\mathbf{1}+\mathbf{r})^{\mathbf{n}} \\
\mathbf{o r} \\
\mathbf{F V}=\mathbf{P V}(\mathbf{1}+\mathbf{r})^{\mathbf{n}}
\end{gathered}
$$

- The equations represent the compounding relationship that is the basis for determining equivalent future and present values of cash flows


## Discounting

$$
P V=\frac{F V}{\left(1+i_{n} / m\right)^{m n}}
$$

- Discounting - the process of converting future values of cash flows (FV) into their present value equivalents (PV)


## Example

- How much money you need to put in the bank at $20 \%$ per annum (per year) (assuming an annual capitalization) to get 250 thousand rubles in 2 years?

$$
P V=\frac{F V \times 1}{(1+r)^{n}} \quad P V=\frac{250}{(1+0.20)^{2}}=\frac{250}{1.44}=173.61
$$

Therefore, we need 173.61 thousand rubles in today's money at $20 \%$ to get 250 thousand in two year

## Account of the promissory notes

Banks accounts can be use to buy obligations such as promissory notes (notes), at a price less than the nominal specified amount. In this case the promissory notes is taken into account, and the client will receive the sum:

$$
P=S-D
$$

where S - the nominal value of the obligation;
P - purchase price of bank promissory notes;
D - the discount, the amount of money in interest.

Notes - a written promissory note strictly statutory form, which is issued by the borrower (the maker or issuer) creditor (note holder or payee) and gives the right to the holder to require the borrower to pay by a certain date of the loan and interest amounts.
Interest income is determined by the buyer's promissory notes, for example, a simple discount rate:

$$
d \%=\frac{D}{S} 100 \%
$$

If the term of the account ( n ) until the maturity date will be during the year, then the discount is determined by the following formula:

$$
D=n \cdot d \cdot S=\frac{t}{T} \cdot d \cdot S
$$

where d - the relative size of the discount rate;
$t$ - accrual period in days;
T - number of days in a year.
The amount of money the bearer of the note can be describe as:

$$
P=S-D=S(1-n d)=S\left(1-\frac{t}{T} d\right)
$$

## Example

A Bank considers a promissory note at $25 \%$ per year. until maturity there are 90 days, the nominal value of the note is 100 thousand Rubles. How much money will the bearer of the note get?

$$
P=100 \cdot\left(1-\frac{90}{365} \cdot 0,25\right) \approx 100 \cdot 0,93836 \approx 93,836
$$

The bearer will receive 93.836 thousand rubles.

It should be noted that discounting may be associated with the conduct of credit operations. In this case, interest is charged at the beginning of the accrual period and the borrower gets the amount of $\boldsymbol{P}$ minus the percentage of money $\boldsymbol{D}$ of the loan amount $\boldsymbol{S}$, to be returned. In this case, during the operation to a simple discount rate d should use this formula

$$
S=\frac{P}{1-n d}
$$

During the operation difficult discount rate dc\% should use the formula:

$$
S=\frac{P}{\left(1-d_{c}\right)^{n}}
$$

In the development of the terms of contracts or their analysis sometimes necessary in solving inverse problems - determining the term of the loan or the interest rate level.
Formulas for calculating the duration of the loan and the value of the discount rate we obtain by solving the equation with respect to n and d .

$$
n=\frac{\ln P / S}{\ln \left(1-d_{c}\right)} ; \quad d_{c}=1-\sqrt[n]{\frac{P}{S}}
$$

Profitability of the accrual of interest on a discount rate to the lender or the borrower depends on the interest rate and loan term.

From these models through simple transformations we can obtain the formula to calculate various indicators of financial transactions.

## Practical exercises

1. We need to calculate the present value of receiving a single amount of $\$ 1,000$ in 20 years. The interest rate for discounting the future amount is estimated at $10 \%$ per year compounded annually.
2. How much money do you need to put in the bank at $18 \%$ per year to receive 25 thousand Rubles in 3 years provided compounded quarterly?
3. The promissory note in the amount of 20000 Rubles with a maturity date in the 27 of November 2017. was taken into the account by the bank the $11^{\text {th }}$ of August of 2017 on a simple discount rate of $12 \%$ per year. Determine how much money has been paid into account?.
4. The nominal value of the promissory notes is 11,200 USD The bank buys a company bill for 93 days before its maturity at an interest rate of $12 \%$ per annum.
What is the amount of discount on the note? and what's the amount the company will pay the bank?

## Topic \#2: Equivalence of payments and interest rates. Change of a contract conditions.

Mathematical Elements of Financial Management

## Content

- Nominal and effective interest rates.;
- The principle of equivalence payments and its application in contracts when conditions change;
- Merging ( consolidating ) payments;
- The concept of equivalence in interest rates and their use in quantitative financial analysis;
- Using equivalence equations;
- The formula for determining equivalent values of simple interest rates and simple and compound interest rates, effective and nominal rates of compound interest;
- The essence of the method of calculating the limit values of the parameters of agreements (contracts).


## Equivalence of interest rates

At the conclusion of financial contracts, each participant of the transaction is committed to sign a contract on the most favorable terms. Contract terms may be different, and we should be able to compare contracts. Moreover, different contracts may provide for different types of interest calculation and comparison of such contracts should be to develop a method of bringing different interest rates to the same species.

Formula for calculating the sum $\boldsymbol{S}$ Accreted for all types of interest rates:

$$
\begin{array}{lll}
S=P(1+n i) & - \text { simple interest } & S=\frac{P}{\left(1-d_{c}\right)^{n}}-\text { accounting for complex discou } \\
S=P\left(1+i_{c}\right)^{n} & - \text { compound interest } & \\
S=P\left(1+\frac{i_{n}}{m}\right)^{m \cdot n} & - \text { accrual of interest (m) once a year } & \\
S=\frac{P}{1-n d} & - \text { simple discount } & S=\frac{P}{\left(1-\frac{d_{n}}{m}\right)^{m \cdot n}}
\end{array}
$$

## Example

Determine the value of the bank interest rate equivalent to the rate of interest of $40 \%$ per annum.

To calculate the efficiency of financial operations using the comparative yield, which is based on assumptions about equality of financial results of different variants of investment leads to the concept of equivalent rates of simple or compound interest. This allows you to get the correct comparison tool.
Effective interest rate measures the relative income, which is produced in total for the year from the accrual of interest several tirnes a year:

$$
j=\left(1+\frac{i_{n}}{m}\right)^{m}-1
$$

In other words, the effective rate shows what the annual rate of compound interest has the same financial result ( $m$ ) -one-time per year at a rate of ( $i / m$ ).

Under the effective discount rate to understand complex annual interest rate equivalent to the nominal value given ( m ) and calculate is used the following formula:

$$
f=\left(1-\frac{d_{c}}{m}\right)^{m}-1
$$

## Example

The Bank shall calculate the interest on the contributions on the basis of nominal rates $12 \%$ per year. Determine the effective (annual) rate with daily interest capitalization.

$$
j=\left(1+\frac{i_{n}}{m}\right)^{m}-1=\left(1+\frac{0,12}{365}\right)^{365}-1=0,12747 \quad \text { or } 12,75 \%
$$

## Practical exercises

1)The deadline for payment of a debt obligation is six months. The discount rate is $18 \%$. What is the profitability of the operation, measured as a simple loan interest rates? What is the profitability of the operation if measured in the form of a complex loan interest rate?
2) Determine at what interest rate is more favorable to place the capital in 10 million monetary units for 5 years:
a) a simple $30 \%$ per annum;
b) under the difficult interest rate of $25 \%$ per annum with quarterly compounding.

## Topic \#3: Cash flow models. Model operations with bonds.

Mathematical Elements of Financial Management

## Content

- The essence of the flow of payments and financial rent;
- Types of financial rents;
- General characteristic of financial flows: the sum of accreted and current cash flow value;
- Determining usual rent;
- Rate of return coefficient and how it is defined;
- Current value of an ordinary annuity;
- Coefficient of reduction of a rent and ways of his definition;
- Defining parameters of financial rents: annuities and the period of annuities.;
- Credit calculations;
- Methods of repayment of a loan;
- Debt repayment in a single one-time payment;
- Debt repayment in installments.;
- Repayment of debt in differentiated payments


## Cash flows

- Cash flows are an integral part of virtually any field of activity. In Commerce, they form a nutritious commodity circulation. In the economic, financial, production and other fields, these flows also generate interest.
- Examples of such flows are: cash receipts from sales of goods and services; cash receipts from rents, royalties or fees, commissions and other sources of revenue; cash payments to suppliers of goods and services; cash payments to employees; cash payments as advances; cash receipts and payments on contracts for commercial or trade purposes; interest on loans.
- This sort of things may occur several consecutives payments (R) that make up the cash flow.


## Annuity and financial rent

- A series of consecutive payments made at regular intervals ( T ) is called financial rent or an annuity

An example of an annuity can be regular contributions to a pension or other funds, the payment of interest on securities, such as stocks and so on. Annuities or Financial rent are determined by the following main characteristics:

- Rent payments (Ri) - the value of each individual payment;
- Rent interval (Ti) - time interval between two payments;
- Term of rent ( t or n ) - the time from the beginning of the realization of the rent until the last payment (there are perpetual rents);
- Interest rate for calculation of accrual or discounting of payments;
- The increased future rent (S), which includes all members of the payment stream with interest on the date of the last payment;
- Current (reduced) value of rent (A )- the sum of all members of the flow of payments, discounted (reduced) by the amount of the interest rate at the initial moment of time of rent.

Rents are divided into permanents (perpetuity), when payments of the rent are: $R_{1}=R_{2}=R_{3}=\ldots=R_{n}$, and variables, when the magnitude of the payments differ.

Considering models of annual payment flows, which are accrued an the end of each year by the compound interest method.

The formula for the accumulated annuity sum is:

$$
S=R \frac{\left(1+i_{c}\right)^{n}-1}{i_{c}}
$$

The received models allow to determine, for example, the amount of payment:

$$
R=\frac{S \cdot i_{c}}{\left(1+i_{c}\right)^{n}-1},
$$

To determine the term (years, months, quarters, half a year) of rent, the following formulas can be used:

$$
n=\frac{\ln \left[(S / R) i_{c}+1\right]}{\ln \left(1+i_{c}\right)}
$$

The present value of the rent will be calculated by the formula:

$$
A=R \frac{1-\left(1+i_{c}\right)^{-n}}{i_{c}}
$$

This formula can be used when a borrower takes credit for his maturity in the future equal payments annually:

$$
R=\frac{A \cdot i_{c}}{1-\left(1+i_{c}\right)^{-n}}
$$

## Example

A borrower takes a loan in the amount of 100 thousand rubles for five years at $25 \%$ per annum and he is going to repay each year in equal amounts. What is the value of these payments?

$$
\begin{gathered}
R=\frac{A \cdot i_{c}}{1-\left(1+i_{c}\right)^{-n}} . \\
R=\frac{100 \cdot 0,25}{1-\frac{1}{(1+0,25)^{5}}}=\frac{25}{0,67232} \approx 37,18467
\end{gathered}
$$

The borrower at the end of each year will pay the amount of 37.18467 thou. rubles within five years.
Depending on the source data for each task generates a corresponding set of models to determine the quantitative values contract.

## Practical exercises

- 1. 5000 roubles payments are paid monthly for 6 years with interest at a rate of $11.5 \%$ per annum compound interest. calculate the amount of the annuity and the build-up ratio( $\mathrm{K}_{\text {на }}$ ) .
- 2. A loan for 40 thousand rubles is issued for 4 years under $24 \%$ per year, the debtor is obliged to pay the contract in equal instalments together with interest. Calculate the amount of monthly payments.
- 3. A loan amounting to 180 thousand rubles for 3 year under $12 \%$ per year. 5 January. The first payment made February 10 in the amount of 30 thousand Rub. Calculate the minimum size for the next payment if it is planned to pay March 15


## Bond

## What is a 'Bond'

A bond is a debt investment in which an investor loans money to an entity (typically corporate or governmental) which borrows the funds for a defined period of time at a variable or fixed interest rate. Bonds are used by companies, municipalities, states and sovereign governments to raise money and finance a variety of projects and activities. Owners of bonds are debtholders, or creditors, of the issuer.

## BREAKING DOWN 'Bond'

Bonds are commonly referred to as fixed-income securities and are one of the three main generic asset classes, along with stocks (equities) and cash equivalents. Many corporate and government bonds are publicly traded on exchanges, while others are traded only over-the-counter (OTC).

## How Bonds Work

When companies or other entities need to raise money to finance new projects, maintain ongoing operations, or refinance existing other debts, they may issue bonds directly to investors instead of obtaining loans from a bank. The indebted entity (issuer) issues a bond that contractually states the interest rate (coupon) that will be paid and the time at which the loaned funds (bond principal) must be returned (maturity date).
The issuance price of a bond is typically set at par, usually $\$ 100$ or $\$ 1,000$ face value per individual bond. The actual market price of a bond depends on a number of factors including the credit quality of the issuer, the length of time until expiration, and the coupon rate compared to the general interest rate environment at the time.

## Blond yield

When calculating the bond yield, the concept of the exchange is used:

$$
P_{k}=\frac{P}{N} \cdot 100 \%,
$$

where $P=$ the price of the bond;
$N=$ the nominal value of the bond;
$P k=$ rate of the bond.
The price of bonds in a given course is determined by the formula:

$$
P=\frac{P_{k} \cdot N}{100 \%}
$$

## Practical exercise

1. The bond was purchased at a rate of $85 \%$ and will be repaid after 5 years after the purchase. Annual coupon payments (interest) are paid at the end of the year at a rate of $5 \%$ per annum of the nominal value of the bond. Calculate the profitability of the acquisition of this bond.
2. The nominal value of the bond is 5000 rubles. The three-month bond, bought for $90 \%$, was sold in 40 days for $94 \%$ to a second buyer. The second buyer held it until maturity. Who has secured a higher profitability?
3. Determine at what price you should buy the bond, if you want to secure a profit of $60 \%$ per annum. Until maturity is 73 days.

## Topic \#4: Inflation in financial calculation .

 Mathematical Elements of Financial Management
## Content

- The essence of inflation and the need to be considered when carrying out financial transactions;
- Rate of inflation and inflation index;
- The definition of the real rate of return of deposit and lending operations;
- A comparison of real and nominal interest rates for simple and compound interest;
- The effect of inflation on financial operations and various options of charge of percent taking into account inflation.


## Inflation and methods of its measurement

Inflation is characterized by depreciation of the national currency, reduction of its purchasing power and the general increase in prices in the country (or deficit)

All the finance indicators can be divided into two groups: nominal (calculated at current prices), and real (taking into account the impact of inflation and calculated at comparable prices of the base period).

## The inflation index and the inflation rate

Index

$$
I_{i}=\frac{S_{1}}{S_{0}}
$$

Rate

$$
=I_{i} \quad 1
$$

Measured: in\% or parts

## The growth rate and the rate of increase

With rates of increase (inflation rates) none of the arithmetic operations cannot be performed!

Growth rates (inflation indexes) can be multiplied to obtain the growth rate over a longer period.

## Calculations

$$
T_{p 1}=\frac{y_{2}}{y_{1}} \cdot 100 \% ; \quad T_{p 2}=\frac{y_{3}}{y_{2}} \cdot 100 \% ; \quad T_{p 3}=\frac{y_{4}}{y_{3}} \cdot 100 \% ;
$$

$$
T_{p \text { за з месяиа }}=\frac{y_{4}}{y_{1}} \cdot 100 \%=\frac{y_{2}}{y_{1}} \cdot \frac{y_{3}}{y_{2}} \cdot \frac{y_{4}}{y_{3}} \cdot 100 \%
$$

$$
\overline{T_{p}}=\sqrt[4]{\frac{y_{2}}{y_{1}} \cdot \frac{y_{3}}{y_{2}} \cdot \frac{y_{4}}{y_{3}} \cdot \frac{y_{5}}{y_{4}}} \cdot 100 \%=\sqrt[4]{\frac{y_{5}}{y_{1}}} \cdot 100 \%=\sqrt[4]{T_{p}} \cdot 100 \%
$$

## The effect of inflation

Due to the interest calculation there is an increase in monies, but its value is decreasing because of inflation.

As each unit of money is devaluated due to inflation, further devaluation occurs over already discounted money.

## Calculating the effect of inflation

$$
I_{И}=\frac{S_{\text {иои }}}{S_{\text {реал }}} \Rightarrow S_{\text {реал }}=\frac{S_{\text {иои }}}{I_{И}}=\frac{S_{\text {иом }}}{(1+\alpha)}
$$

The formula for calculating the inflation-adjusted accreted amount takes the following form:

$$
F V=P V(1+i)^{n} /(1+\alpha)^{n}
$$

Accretion can be done by simple or compound interest, but inflation is always measured by compound interest.

## Example

Determine the actual results for 5000 rub. deposit, placed on six months at $8 \%$ per annum, if the monthly inflation rate is $2 \%$.

## Decision:

Accreted deposit amount
$F V=P V(1+n i)=5000(1+0,5 \cdot 0,08)=5200,00$ rub.
The inflation index for the length of deposit is
$I_{\tau}=(1+0,02)^{6}=1,126$

The real amount of the deposit
$F V_{\tau}=5200 / 1,126=4618,11$ rub.

## Example

Determine the expected annual inflation rate if the level of the month inflation rate is $8 \%$.

## Decision:

The inflation index for the whole period is defined by the formula:

$$
I_{U}=\prod_{i=1}^{N}\left(1+\alpha_{i-1, i}\right)
$$

The inflation index and the inflation rate are linked by the expression $\quad I_{\Lambda}=1+\alpha$
In case of equal values of inflation rates at all time intervals

$$
\alpha_{0,1}=\alpha_{1,2}=\alpha_{2,3}=\alpha_{3,4}=\ldots=\alpha_{i-1, i}=\alpha
$$

The inflation index is defined by the formula: $\quad I_{u}=(1+\alpha)^{N}$.
Then the inflation index for the year is:

$$
I_{u}=1,08^{12}=2,51817
$$

The inflation rate for the year: $\quad \alpha=I_{u}-1=2,5182-1=1,5182=151,82 \%$

## Options for inflation-adjusted charging interest

For accrued interest without capitalization denote $i_{\text {nom }}$ as the inflation-adjusted interest rate.

Accreted amount: $\quad S_{\text {nom }}=P\left(1+n i_{\text {nom }}\right)$.
Using the equation of connection: $\quad I_{И}=\frac{S_{\text {ном }}}{S_{\text {реал }}} \Rightarrow S_{\text {реал }}=\frac{S_{\text {нои }}}{I_{И}}$
Obtain the equation:

$$
P\left(1+n i_{\text {нои }}\right)=P\left(1+n i_{\text {реал }}\right) \cdot I_{u},
$$

## The connection between nominal and real interest rates

When calculating simple interest

$$
i_{\text {ном }}=\frac{\left(1+n i_{\text {реал }}\right) I_{u}-1}{n}
$$

$$
i_{\text {реал }}=\frac{n i_{\text {ном }}+1-I_{u}}{n I_{u}}
$$

When calculating compound interest

$$
i_{\text {ном }}=\left(1+i_{\text {реал }}\right) \cdot \sqrt[n]{I_{u}}-1 . \quad i_{\text {реал }}=\frac{1+i_{\text {нои }}}{\sqrt[n]{I_{u}}}-1
$$

Nominal inflation-adjusted interest rate in case of several times a year calculation of interest:

$$
j_{\text {ном }}=m\left[\left(1+\frac{j_{\text {реал }}}{m}\right) \sqrt[m n]{I_{u}}-1\right],
$$

## Example

What is the real return, if the inflation-adjusted bank interest rate is $25 \%$, inflation index over 4 years was $170 \%$ ?

## Decision:

Real return is determined by the formula:

$$
i_{p}=\frac{1+i_{H}}{\sqrt[n]{I_{u}}}-1
$$

For this task: $\quad i_{p}=\frac{1+i_{H}}{\sqrt[n]{I_{u}}}-1=\frac{1+0,25}{\sqrt[4]{1,7}}-1$
Answer: $\quad i_{p}=9,5 \%$

## Example

What inflation-adjusted interest rate taking into account capitalization a bank have to establish to provide a real return on their clients $18 \%$ per annum? The inflation rate over 4 years was 1.7 .

## Decision:

The nominal interest rate is determined by the formula:

$$
i_{\text {ном }}=\left(1+i_{\text {реал }}\right) \cdot \sqrt[n]{I_{u}}-1
$$

For this task: $\quad i_{\text {ном }}=(1+0,18) \cdot \sqrt[4]{1,7}-1$.
Answer: $\quad i_{\text {ном }}=34,74 \%$

## СПАСИБО ЗА ВНИМАНИЕ!

к.т.н., доц. Калашникова Т.В. tvkalash@tpu.ru

