

Problems for Assignment 1

1. Sports competition is held on a circular system. This means that each pair of sportsmen meets each other exactly once. Prove that at any given time there will be at least two sportsmen who have the same number of meetings.
2. In a company consisting of five people, among any three people there are two acquaintances and two unfamiliar with each other. Prove that the company can seat at a round table so that each person had his acquaintances on both sides.
3. Each of the pupils of class A is friends with three pupils of class B, and each pupil of class B is friends with three pupils of class A. Prove that the number of pupils in these classes is the same.
4. In some country, the airline system is arranged so that any city is connected by airlines with no more than three others, and you can fly from any city in any other one, making no more than one transfer. What is the largest number of cities in this country?
5. Ten candidates are preparing for two space expeditions to Mars. Since the expeditions will continue for several years, and their participants will be in a small confined space, the psychological compatibility of crew members becomes important. By testing, pairs of candidates were established, whose presence in one expedition would be undesirable. Test results are reflected in the table (if there is a cross mark at the intersection of the row and column, candidates with the corresponding numbers are incompatible). Divide the candidates into two groups to participate in the expeditions.

| | | | | | | | | | | |
|----|---|---|---|---|---|---|---|---|---|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | | + | + | + | | | | | | |
| 2 | + | | | | + | | | | | |
| 3 | + | | | | | | + | | | |
| 4 | + | | | | | + | | | | |
| 5 | | + | | | | | | + | | |
| 6 | | | | + | | | | | | + |
| 7 | | | + | | | | | + | | + |
| 8 | | | | | + | | + | | + | |
| 9 | | | | | | | | + | | + |
| 10 | | | | | | + | + | | + | |

6. Seven lectures are held one day, some of them can not be delivered at the same time (in the table such pairs of lectures are marked with a cross). Each lecture takes one hour. Determine the minimum time for which all lectures can be delivered.

| | | | | | | | |
|-------------|---------|---------|-----------|-------------|----------|-----------|---------|
| | algebra | physics | chemistry | informatics | geometry | economics | English |
| algebra | | + | | + | | | + |
| physics | + | | + | | + | + | |
| chemistry | | + | | | + | + | + |
| informatics | + | | | | + | + | |
| geometry | | + | + | + | | + | |
| economics | | + | + | + | + | | |
| English | + | | + | | | | |

7. Several airline companies have to link 100 cities so that two conditions are met: 1) any two cities are connected by a direct line of no more than one company; 2) any company, using its lines, could deliver a passenger from any city to any other (possibly with transfers). What is the largest number of airline companies to satisfy these conditions?
8. The state of the Philippines is located on the islands. Between some of the islands, motor ships cruise daily (one passage in one direction, another in the opposite direction). From any island you can get to any other, possibly with transfers. The Philippine police invited Walker, Texas Ranger, to help catch a dangerous criminal. The criminal is superstitious and does not use the boat on the 13th of every month and every Monday. Walker is not superstitious. In addition, he always knows, with the help of agents, on which island the criminal is. Prove that if Walker and the criminal use only ships, they will end up on the same island.
9. There are two countries: Ordinary and Through the Looking Glass. Every city in the Ordinary country has a double in the Looking Glass, and vice versa. If in an Ordinary country some two cities are connected by an airline, then in the Looking-Glass, they are not connected, and if in the Ordinary country some two cities are not connected by an airline, then in the Looking-Glass they are connected. In the Ordinary country, Alice can not get from city A to city B, making less than two transfers. Prove that Alice will be able to fly in the Looking Glass from any city to any other, with no more than two transfers.
10. Some of the country's 40 cities are pair wise connected by airlines belonging to one of the ten companies. From each city, you can fly to any other without a transfer, and each airline company operates in both directions. Prove that there is a company that can provide a journey with a start and an end in the same city, with a number of flights of at least three, and every transitional city on the road will be visited exactly once.
11. In the city from any metro station you can drive to any other. Prove that one of the stations can be closed for repairs without the right to travel through it so that from any other station you can still drive to any other.
12. In a seaside resort, the streets are so narrow that a one-way traffic is established in the city. Nevertheless, from every point of the city you can drive to any other. Prove that it is possible to offer such a patrolling route for a police car that starts and ends in the same place and passes through each part of streets between two crossroads at least once.
13. In a seaside resort, after the establishment of one-way traffic it turned out that the number of streets by which you can enter every crossroads is equal to the number of streets by which you can leave it. Prove that it is possible to offer such a patrolling route, which begins and ends in one place and passes through each part of the streets exactly once.
14. A city has a two-way traffic. For two years, all roads the city have being repaired. So, in the first year, a one-way traffic was introduced on some roads. The following year, two-way traffic was restored on these roads, and one-way traffic was introduced on the other roads. It is known that at any moment of repair it was possible to drive from any point in the city to any other. Prove that in the city you can establish one-way traffic so that from any point in the city you can go to any other.

15. Prove that it is possible to establish one-way traffic through the streets of any city in such a way, that the difference between the number of streets by which you can enter any crossroads and the number of streets by which you can leave it is not more than one.
16. A saturated hydrocarbon is a compound of carbon C of the valence 4 and hydrogen H of the valence 1. For a given number of carbon atoms, it contains the largest number of hydrogen atoms. Find the formula for a saturated hydrocarbon containing n carbon atoms. Draw molecules of saturated hydrocarbons for $n = 3, 4, 5, 6$.