Chinese Postman Problem for Mixed Graphs

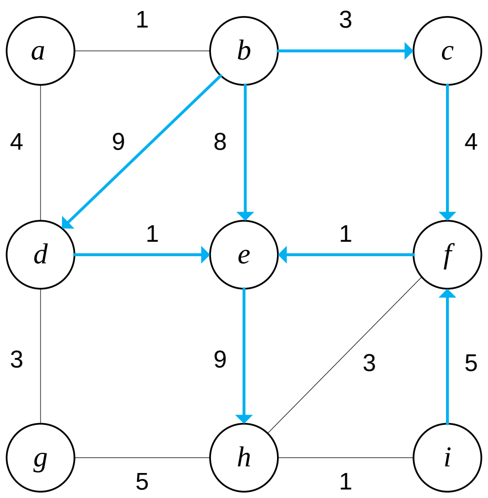
Start. Even non-symmetric mixed graph $G(V, A, B, D)$ is given.

V – vertices;

A – undirected edges;

B – directed edges (arcs);

D – costs.

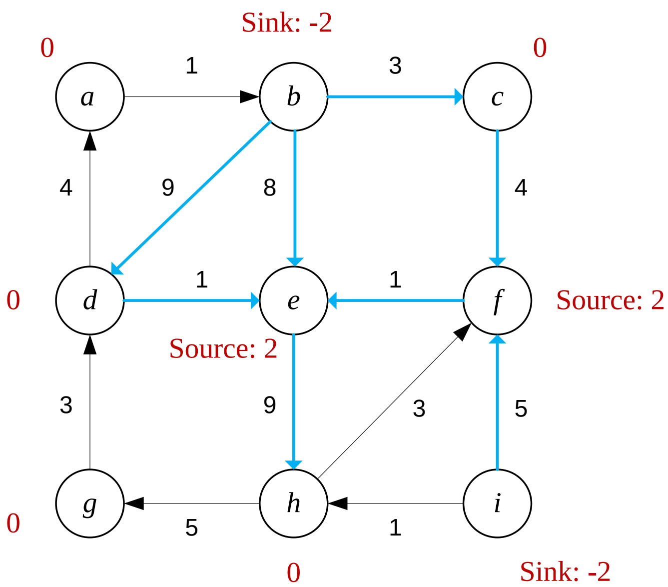


Step 1. Give any direction every edge from A; obtain directed graph $G'(V, A', B, D)$. For every vertex, calculate the value $D(v) = d^-(v) - d^+(v)$ (the number of arcs **going into** the vertex minus the number of arcs **going from** the vertex).

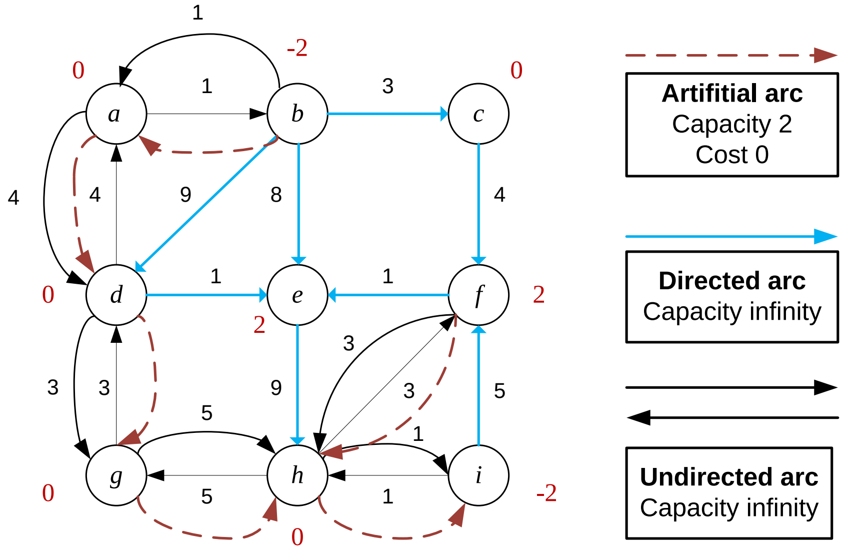
$D(v) > 0$: v is a source with supply $D(v)$;

$D(v) < 0$: v is a sink with demand $-D(v)$;

$D(v) = 0$: v is a transshipment vertex.



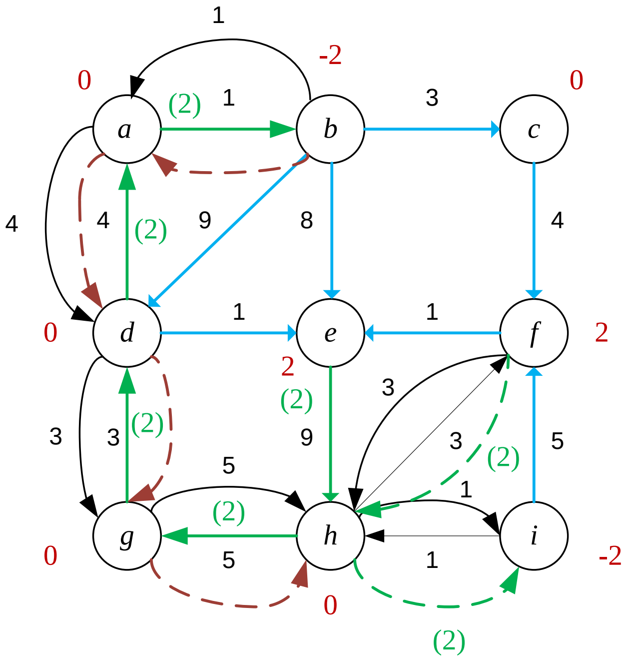
Step 2. Replace every arc from A' by three arcs: two **real** arcs, with the same cost as in the original arc, and with opposite directions; and one **artificial** arc, with zero cost and with the direction opposite to the direction of the arc chosen at Step 1. We use artificial arcs for possibility to change the direction chosen at Step 1. Capacities of all non-artificial arcs equal to infinity, capacities of artificial arcs equal to 2. Obtain graph $G^*(V, A^*, B, C)$.



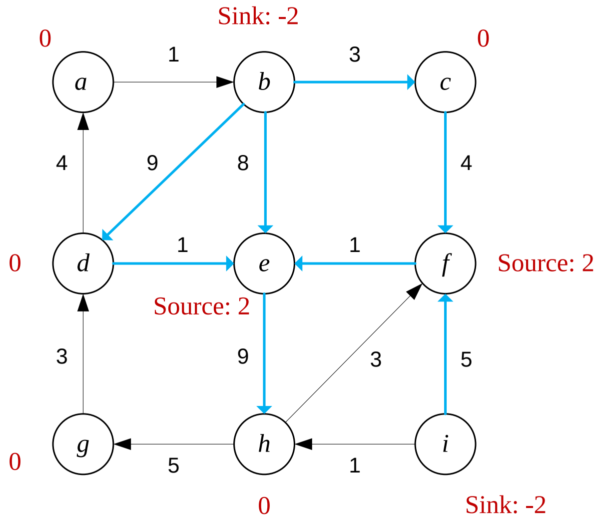
Step 3. In $G^*(V, A^*, B, C)$, find the minimum-cost flow satisfying all the demands of the sinks and all the supplies of the sources. Here the arcs carrying flow are indicated by green color; the value of the flow $f(x, y)$ is written in parentheses (2), zero flow is not indicated. To find the flow, we search for the shortest paths from sources to sinks; in fact, we can use any algorithm for minimum-cost flow. Here we combine two shortest paths:

$\langle e, b \rangle : ehgdab; 9 + 5 + 3 + 4 + 1 = 22;$

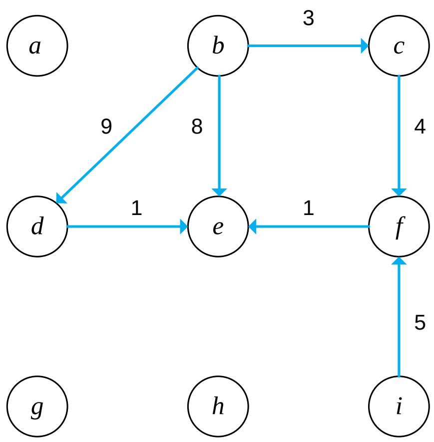
$\langle f, i \rangle : fhi; 0 + 0 = 0.$



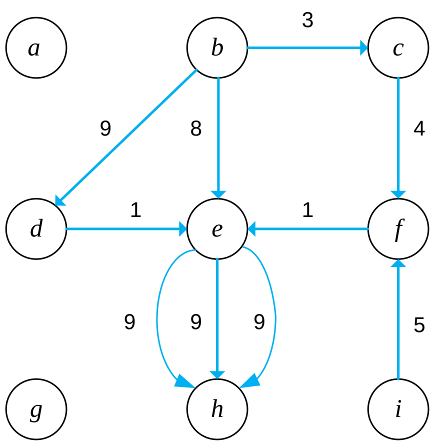
Step 4. Using the flow, construct an Eulerian graph from $G'(V, A', B, D)$.



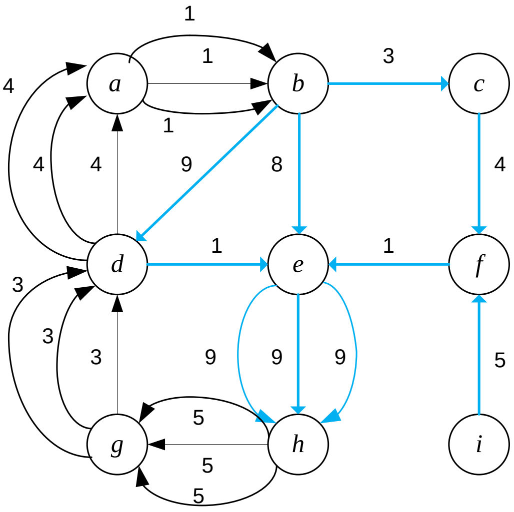
1) If the flow in a directed arc from B is zero, we leave it as it is. The same with arcs from A' if we have zero flow in all corresponding arcs from A^*.



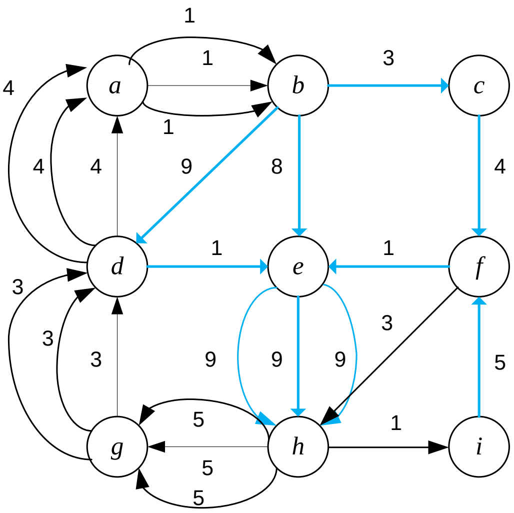
2) If $f(x, y) \neq 0$ for $(x, y) \in B$ (they were directed in the original graph), we add $f(x, y)$ copies of the arc (x, y).



3) If $f(x,y) \neq 0$ for $(x,y) \in A'$ (they were undirected in the original graph), and (x,y) is non-artificial, and the flow is zero in the corresponding artificial arc, we add $f(x,y)$ copies of the arc (x,y), without changing the direction chosen at Step 1.



4) If $f(x,y) = 2$ for an artificial arc $(x,y) \in A'$ we change the direction chosen at Step 1, remove (x,y) and add $f(y,x) + 1$ copies of arc (y,x).



The obtained graph is Eulerian.

Step 5. Construct an Eulerian path $abcfhifehgdabehgdabdehgda$. It corresponds to the postman path in the first graph.