Ответы и указания к упражнениям для самостоятельной работы

1. $2\sqrt{x} + \ln|x|$.

2. $(1/12)(x^3 + 1)^4$.

3. $2\sqrt{3}\,\mathrm{arctg}\,(x/\sqrt{3}) - x$.

4. $27x - 9x^3 + 9x^5/5 - x^7/7$.

5. $(-1/57)(3x + 2)^{-19}$.

6. $15^x/\ln 15$.

7. $4(x^2 + 7)/(7\sqrt{4x})$

8. $\ln|x| - 1/(4x^4)$.

9. $x + 2\ln\left|\frac{x-1}{x+1}\right|$.

10. $-\mathrm{ctg}\,x - x$.

11. $3x/2 - (5/4)\ln|x + 3/2|$.

12. $-\frac{2}{\ln 5}\left(\frac{1}{5}\right)^x + \frac{1}{5\ln 2}\left(\frac{1}{2}\right)^x$.

13. $x - \mathrm{th}\,x$.

14. $0{,}5\,\mathrm{ch}\,2x$.

15. $0{,}5\,\mathrm{sh}\,(2x + 3)$.

16. $\frac{4^x}{\ln 4} + 2\frac{6^x}{\ln 6} + \frac{9^x}{\ln 9}$.

17. $\arcsin x + \ln(x + \sqrt{1 + x^2})$.

18. $(1/22)(2x - 3)^{11}$.

19. $(-\cos x + \sin x)\,\mathrm{sgn}\,(\cos x + \sin x)$.

20. $(-2/5)\sqrt{2-5x}$.

21. $\frac{1}{\sqrt{6}}\operatorname{arctg}(x\sqrt{3/2})$.

22. $(1/\sqrt{3})\ln|x\sqrt{3}+\sqrt{3x^2-2}|$.

23. $(-1/2)\operatorname{ctg}(2x+\pi/4)$.

24. $\operatorname{tg}(x/2)$.

25. $\operatorname{tg}(x/2-\pi/4)$.

26. $(1/4)(1+x^3)^{4/3}$.

27. $-0,5\text{cos}(2x+3)$.

28. $(1/4)\operatorname{arctg}(x^2/2)$.

29. $2\operatorname{arctg}\sqrt{x-}$. *Указание*: Воспользуйтесь тем, что $dx/\sqrt{x-}=2d(\sqrt{x-})$.

30. $1,5(1+2x)^{1/3}$.

31. $-\arcsin(1/|x|)$.

32. $2\operatorname{sgn}x\ \ln(\sqrt{|x|}+\sqrt{|x-1|})$, $x(x-1)>0$.

33. $2\sqrt{\ln x}$.

34. $(-1/2)e^{-x2}$.

35. $(1/3)\ln^3 x$.

36. $\ln|2+\ln x|$.

37. $(1/30)(1+27x)^{10/9}$.

38. $0,75(\ln x)^{4/3}$.

39. $(1/6)\sin^6 x$.

40. $(3/2)\sqrt[3]{1-\sin 2x}$.

41. $(-1/\sqrt{2})\ln|\sqrt{2}\cos x+\sqrt{\cos(2x)}|$.

42. $\ln|\operatorname{tg}(x/2)|$.

43. $\ln|\text{th}\,(x/2)|$.

44. $0{,}5(\text{arctg}\,x)^2$.

45. $\frac{1}{4}\ln^2\frac{1+x}{1-x}$.

46. $-\frac{1+55x^2}{6600}(1-5x^2)^{11}$.

47. $-\frac{1}{15}(8+4x^2+3x^4)\sqrt{1-x^2}$.

48. $\left(\frac{2}{3}-\frac{4}{7}\sin^2 x+\frac{2}{11}\sin^4 x\right)\sqrt{\sin^3 x}$.

49. $-x-2e^{-x/2}+2\ln(1+e^{x/2})$.

50. $x-2\ln(1+\sqrt{1+e^x})$.

51. $(\text{arctg}\,\sqrt{x}-)^2$.

52. $0{,}5[x\sqrt{a^2-x^2}+a^2\arcsin(x/a)]$.

53. $-\sqrt{a^2-x^2}+a\arcsin(x/a)$.

54. $0{,}5\times[x\sqrt{a^2+x^2}-a^2\ln(x+\sqrt{a^2+x^2})]$.

55. $\sqrt{x^2-a^2}-2a\ln(\sqrt{x-a}+\sqrt{x+a})$, если $x>a$;
$-\sqrt{x^2-a^2}+2a\ln(\sqrt{-x+a}+\sqrt{-x-a})$, если $x<-a$.

56. $\ln|x+\sqrt{x^2+a^2}|$.

57. $\ln|x+\sqrt{x^2-a^2}|$.

58. $x(\ln x-1)$.

59. $\frac{2}{3}x^{3/2}\left(\ln^2 x-\frac{4}{3}\ln x+\frac{8}{9}\right)$.

60. $-0{,}5(x^2+1)e^{-x2}$.

61. $\frac{1-2x^2}{4}\cos 2x+\frac{x}{2}\sin 2x$.

62. $x\arcsin x+\sqrt{1-x^2}$.

63. $-\frac{\arcsin x}{x} - \ln\left|\frac{1+\sqrt{1-x^2}}{x}\right|$.

64. $\ln \operatorname{tg}(x/2) - \cos x \ln \operatorname{tg} x$.

65. $0{,}5[(1+x^2)(\operatorname{arctg} x)^2 - 2x \operatorname{arctg} x + \ln(1+x^2)]$

66. $\sqrt{1+x^2}\ln(x+\sqrt{1+x^2}) - x$.

67. $0{,}5[e^x\sqrt{e^{2x}+1} + \ln(e^x+\sqrt{e^{2x}+1})]$.

68. $0{,}5x[\sin(\ln x) + \cos(\ln x)]$.

69. $\frac{a\sin bx - b\cos bx}{a^2+b^2}e^{ax}$.

70. $(1/8)e^{2x}(2 - \sin 2x - \cos 2x)$.

71. $\ln|x-2| + \ln|x+5|$.

72. $x + (1/6)\ln|x| - (9/2)\ln|x-2| + (28/3)\ln|x-3|$.

73. $-\frac{5x-6}{x^2-3x+2} + 4\ln\left|\frac{x-1}{x-2}\right|$.

74. $\operatorname{arctg} x + \frac{5}{6}\ln\frac{x^2+1}{x^2+4}$.

75. $-\frac{1}{x-2} - \operatorname{arctg}(x-2)$.

76. $\frac{1}{6}\ln\frac{(x+1)^2}{x^2-x+1} + \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}$.

77. $\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\operatorname{arctg} x$.

78. $\frac{1}{4\sqrt{2}}\ln\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1} + \frac{1}{2\sqrt{2}}\operatorname{arctg}\frac{x\sqrt{2}}{1-x^2}$.

79. $\frac{1}{4}\ln\frac{x^2+x+1}{x^2-x+1} + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{x^2-1}{x\sqrt{3}}$.

80. $\frac{2}{5}\ln\frac{x^2+2x+2}{x^2+x+1/2} + \frac{8}{5}\operatorname{arctg}(x+1) - \frac{2}{5}\operatorname{arctg}(2x+1)$.

81. $\frac{1}{4\sqrt{3}}\ln\frac{1+x\sqrt{3}+x^2}{1-x\sqrt{3}+x^2} + \frac{1}{2}\operatorname{arctg} x + \frac{1}{6}\operatorname{arctg} x^3$.

82. $-\frac{1}{96(x-1)^{96}} - \frac{3}{97(x-1)^{97}} - \frac{3}{98(x-1)^{98}} - \frac{1}{99(x-1)^{99}}$.

83. $\frac{1}{12}\ln\frac{(x^2+1)^2}{x^4-x^2+1} + \frac{1}{3}\operatorname{arctg} x^3 + \frac{1}{2\sqrt{3}}\operatorname{arctg}\frac{2x^2-1}{\sqrt{3}}$.

84. $\frac{x^4}{4} + \frac{1}{4}\ln\frac{x^4+1}{(x^4+2)^4}$.

85. $\frac{1}{7}\ln\frac{|x|^7}{(1+x^7)^2}$.

86. $\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x^2-1}{x\sqrt{3}}$.

87. $\frac{1}{4\sqrt{2}}\ln\frac{x^4-x^2\sqrt{2}+1}{x^4+x^2\sqrt{2}+1}$.

88. $\operatorname{arctg} x + \frac{1}{3}\operatorname{arctg} x^3$.

89. $\frac{3}{4}\ln\frac{x\sqrt[3]{x}}{(1+\sqrt[6]{x})^2(1-\sqrt[6]{x}+2\sqrt{x})^3} - \frac{3}{2\sqrt{7}}\operatorname{arctg}\frac{4\sqrt[6]{x}-1}{\sqrt{7}}$.

90. $0{,}5(x^2 - x\sqrt{x^2-1} + \ln|x+\sqrt{x^2-1}|)$.

91. $-\frac{3}{2}\sqrt[3]{\frac{x+1}{x-1}}$.

92. $\frac{2x-3}{4}\sqrt{x^2+x+1} - \frac{1}{8}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x+1}\right)$.

93. $-\ln\left|\frac{1-x+2\sqrt{x^2+x+1}}{x+1}\right|$.

94. $R + \ln(x+1+R) - \sqrt{2}\ln\left|\frac{x+2+\sqrt{2}R}{x}\right|$, где $R = \sqrt{x^2+2x+2}$.

95. $-\frac{19+5x+2x^2}{6}\sqrt{1+2x-x^2} - 4\arcsin\frac{1-x}{\sqrt{2}}$.

96. $\left(\frac{63}{256}x - \frac{21}{128}x^3 + \frac{21}{160}x^5 - \frac{9}{80}x^7 + \frac{x^9}{10}\right)\sqrt{1+x^2} - \frac{63}{256}\ln(x+\sqrt{1+x^2})$.

97. $-\frac{1}{2x^2}\sqrt{x^2+1} + \frac{1}{2}\ln\frac{1+\sqrt{x^2+1}}{|x|}$.

98. $\frac{2x^2+1}{3x^3}\sqrt{x^2-1}$.

99. $\arcsin\frac{x-1}{\sqrt{3}}-\frac{\sqrt{2}}{3}\operatorname{arctg}\frac{\sqrt{2+2x-x^2}}{(1-x)\sqrt{2}}-\frac{1}{\sqrt{6}}\ln\frac{\sqrt{6}+\sqrt{2+2x-x^2}}{\sqrt{6}-\sqrt{2+2x-x^2}}$.

100. $\frac{x}{2\sqrt{1+x^2}}+\frac{1}{4\sqrt{2}}\ln\left|\frac{\sqrt{1+x^2}+x\sqrt{2}}{\sqrt{1+x^2}-x\sqrt{2}}\right|$.

101. $\frac{1}{2}\arcsin\frac{x-3}{|x-1|\sqrt{5}}-\frac{1}{2}\ln\left|\frac{3x+1-2\sqrt{x^2-x-1}}{x+1}\right|$.

102. $\frac{2(x-1)}{3\sqrt{x^2+x+1}}$.

103. $\frac{3}{2(2z+1)}+\frac{1}{2}\ln\frac{z^4}{|2z+1|^3}$, где $z=x+\sqrt{x^2+x+1}$.

104. $\ln\left|\frac{z-1}{z}\right|-2\operatorname{arctg}z$, где $z=\frac{1+\sqrt{1-2x+x^2}}{x}$.

105. $\frac{2(3-4z)}{5(1-z-z^2)}+\frac{2}{5\sqrt{5}}\ln\left|\frac{\sqrt{5}+1+2z}{\sqrt{5}-1-2z}\right|$, где $z=-x+\sqrt{x(x+1)}$.

106. $-\frac{1}{\sqrt{2}}\arcsin\frac{x\sqrt{2}}{x^2+1}$. *Указание*: Положите $t=x+\frac{1}{x}$.

107. $-\frac{1}{\sqrt{2}}\ln\left|\frac{x\sqrt{2}+\sqrt{x^4+1}}{x^2-1}\right|$. *Указание*: Положите $t=x-\frac{1}{x}$.

108. $(5/16)x-(1/4)\sin 2x+(3/64)\sin 4x+(1/48)\sin^3 2x$.

109. $(1/16)x-(1/64)\sin 4x+(1/48)\sin^3 2x$.

110. $1/(3\cos^3 x)-1/\cos x$.

111. $(1/4)\operatorname{tg}^4 x-(1/2)\operatorname{tg}^2 x-\ln|\cos x|$.

112. $-2\sqrt{\operatorname{ctg}x}+(2/3)\sqrt{\operatorname{tg}^3 x}$.

113. $\frac{1}{2\sqrt{2}}\ln\frac{z^2+z\sqrt{2}+1}{z^2-z\sqrt{2}+1}-\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{z\sqrt{2}}{z^2-1}$, где $z=\operatorname{tg}x$.

114. $1/4x+1/8\sin 2x+1/(16)\sin 4x+1/(14)\sin 6x$.

115. $-3/(16)\cos 2x+3/(64)\cos 4x+1/(48)\cos 6x-3/(128)\cos 8x+1/(192)\cos 12x$.

116. $\frac{1}{\sqrt{5}}\operatorname{arctg}\frac{3\operatorname{tg}(x/2)+1}{\sqrt{5}}$.

117. а) $\frac{2}{\sqrt{1-\varepsilon^2}}\operatorname{arctg}\left(\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\operatorname{tg}\frac{x}{2}\right)$; б) $\frac{1}{\sqrt{\varepsilon^2-1}}\ln\frac{\varepsilon+\cos x+\sqrt{1-\varepsilon^2}\sin x}{1+\varepsilon\cos x}$.

118. $x - \frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2} \operatorname{tg} x)$.

119. $-\frac{1}{6} \ln \frac{(\sin x + \cos x)^2}{1 - \sin x \cos x} - \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{2\cos x - \sin x}{\sqrt{\sin x}}\right)$.

120. $\operatorname{arctg}\left(\frac{1}{2} \operatorname{tg} 2x\right)$.