

КРАТНЫЕ ИНТЕГРАЛЫ

Ответы и указания к упражнениям для самостоятельной работы

1. a) $\int_1^4 dx \int_1^x f(x, y) dy, \int_1^4 dy \int_y^4 f(x, y) dx$
 (рис. 81);

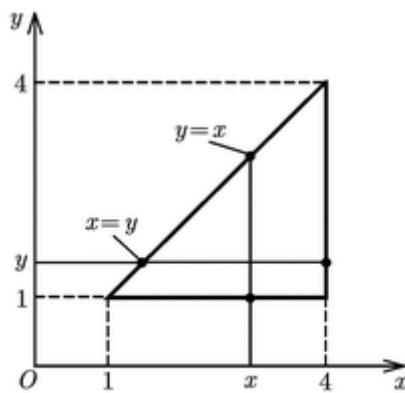


Рис. 81

б)

$$\int_2^3 dx \int_{(x+1)/3}^{6x-11} f(x, y) dy + \int_3^5 dx \int_{(x+1)/3}^{(29-5x)/2} f(x, y) dy, \int_1^2 dy \int_{(y+11)/6}^{3y-1} f(x, y) dx + \int_2^7 dy \int_{(y+11)/6}^{(29-2y)/5} f(x, y) dx$$

(рис. 82);

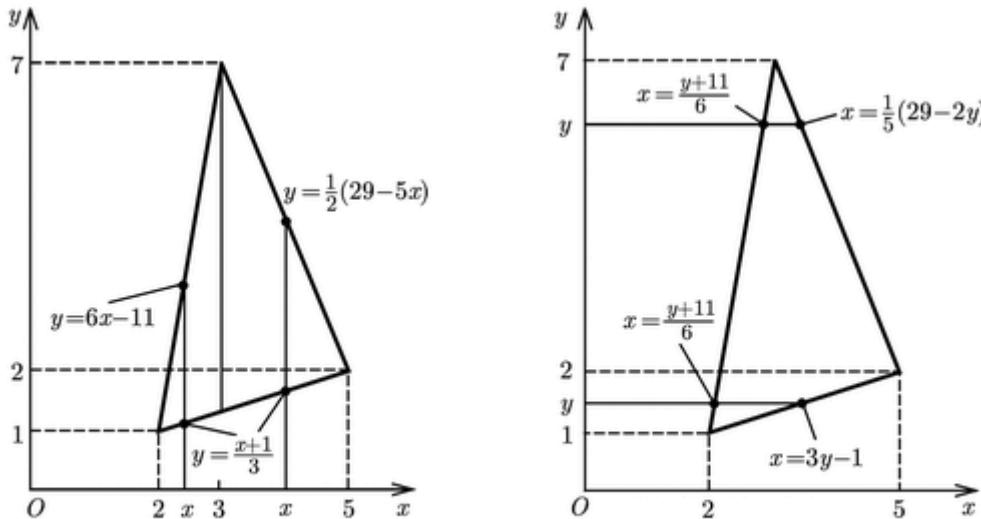


Рис. 82

$$\text{б)} \int_{-2}^1 dx \int_{3x^2}^{6-3x} f(x, y) dy, \quad \int_0^3 dy \int_{-\sqrt{y/3}}^{\sqrt{y/3}} f(x, y) dx + \int_3^{12} dy \int_{-\sqrt{y/3}}^{2-y/3} f(x, y) dx$$

(рис. 83);

$$\text{г)} \int_{-2}^4 dx \int_{-2-\sqrt{8+2x-x^2}}^{-2+\sqrt{8+2x-x^2}} f(x, y) dy, \quad \int_{-5}^1 dy \int_{1-\sqrt{5-4y-y^2}}^{1+\sqrt{5-4y-y^2}} f(x, y) dx$$

;

$$\text{д)} \int_0^4 dx \int_{2x}^{3x} f(x, y) dy, \quad \int_0^8 dy \int_{y/3}^{y/2} f(x, y) dx + \int_8^{12} dy \int_{y/3}^4 f(x, y) dx$$

(рис. 84);

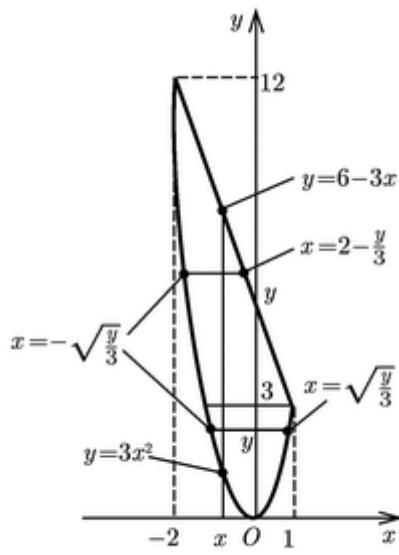


Рис. 83

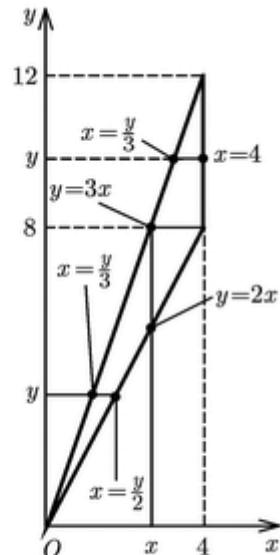


Рис. 84

$$\text{е)} \int_{-1}^1 dx \int_{(5-3x)/2}^4 f(x, y) dy + \int_1^4 dx \int_1^4 f(x, y) dy + \int_4^5 dx \int_{3x-11}^4 f(x, y) dy$$

$$\int_1^4 dy \int_{(5-2y)/3}^{(y+11)/3} f(x, y) dx$$

; ж)

$$\int_{-2}^{-1} dx \int_0^{3x+6} f(x, y) dy + \int_{-1}^0 dx \int_{3x+3}^{3x+6} f(x, y) dy, \quad \int_0^3 dy \int_{y/3-2}^{y/3-1} f(x, y) dx + \int_3^6 dy \int_{y/3-2}^0 f(x, y) dx$$

;

3)

$$\int_{-2}^0 dx \int_{-3x-3}^{1,5x+6} f(x, y) dy + \int_0^3 dx \int_{-3}^{6-3x} f(x, y) dy, \quad \int_{-3}^3 dy \int_{-y/3-1}^{2-y/3} f(x, y) dx + \int_3^6 dy \int_{(2/3)y-4}^{2-y/3} f(x, y) dx$$

;

и)

$$\iint_G f(x, y) dx dy = \iint_{G_1} f(x, y) dx dy + \iint_{G_2} f(x, y) dx dy + \iint_{G_3} f(x, y) dx dy + \iint_{G_4} f(x, y) dx dy + \dots$$

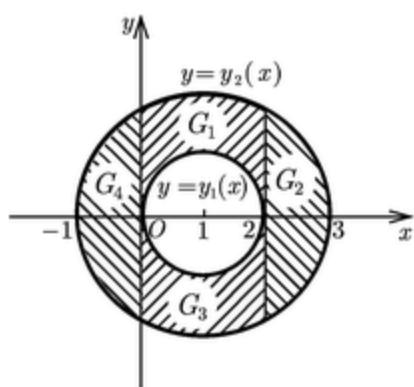


Рис. 85

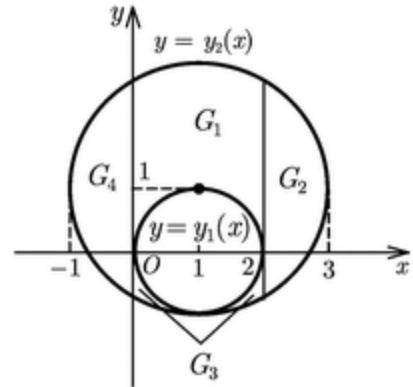


Рис. 86

$$+ \int_2^3 dx \int_{-y_2(x)}^{y_2(x)} f(x, y) dy + \int_0^2 dx \int_{-y_2(x)}^{-y_1(x)} f(x, y) dy + \int_{-1}^0 dx \int_{-y_2(x)}^{y_2(x)} f(x, y) dy , \text{ где}$$

$y_1(x) = \sqrt{2x - x^2}$, $y_2(x) = \sqrt{3 + 2x - x^2}$ (рис. 85);

$$\int_1^2 dy \int_{1-x_2(y)}^{1+x_2(y)} f(x, y) dx + \int_{-1}^1 dy \int_{1+x_1(y)}^{1+x_2(y)} f(x, y) dx + \int_{-2}^{-1} dy \int_{1-x_2(y)}^{1+x_2(y)} f(x, y) dx + \int_{-1}^1 dy \int_{1-x_2(y)}^{1-x_1(y)} f(x, y) dx$$

где $x_1(y) = \sqrt{1 - y^2}$, $x_2(y) = \sqrt{4 - y^2}$; к)

$$\iint_G f(x, y) dx dy = \iint_{G_1} f(x, y) dx dy + \iint_{G_2} f(x, y) dx dy +$$

$$+ \iint_{G_3} f(x, y) dx dy + \iint_{G_4} f(x, y) dx dy = \int_0^2 dx \int_{y_1(x)}^{1+y_2(x)} f(x, y) dy + \int_2^3 dx \int_{1-y_2(x)}^{1+y_2(x)} f(x, y) dy + \int_0^2 dx \int_{1-y_2(x)}^{-y_1(x)} f(x, y) dy$$

где $y_1(x) = \sqrt{2x - x^2}$, $y_2(x) = \sqrt{3 + 2x - x^2}$ (рис. 86);

$$\int_1^3 dy \int_{1-x_2(y)}^{1+x_2(y)} f(x, y) dx + \int_{-1}^1 dy \int_{1+x_1(y)}^{1+x_2(y)} f(x, y) dx + \int_{-1}^1 dy \int_{1-x_2(y)}^{1-x_1(y)} f(x, y) dx$$

где $x_1(y) = \sqrt{1 - y^2}$, $x_2(y) = \sqrt{3 + 2y - y^2}$.

2. a) $\int_0^1 dx \int_{x^3}^{x^2} f(x, y) dy$
 (рис. 87);

б) $\int_1^2 dy \int_e^{e^y} f(x, y) dx + \int_2^4 dy \int_{e^{y/2}}^{e^2} f(x, y) dx$
 (рис. 88);

в) $\int_0^2 dy \int_{y/2}^y f(x, y) dx + \int_2^4 dy \int_{y/2}^2 f(x, y) dx$
 ;

г) $\int_{-1}^0 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx + \int_0^1 dy \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x, y) dx$
 (рис. 89);

д) $\int_{-1}^0 dx \int_{-(1+x)/2}^0 f(x, y) dy + \int_0^1 dx \int_{-(1+x)/2}^{-\sqrt{x}} f(x, y) dy$
 (рис. 90);

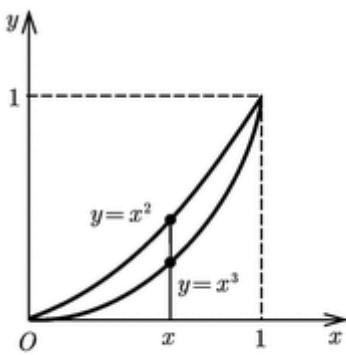


Рис. 87

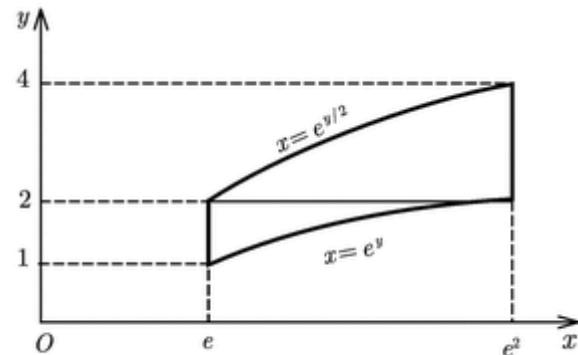


Рис. 88

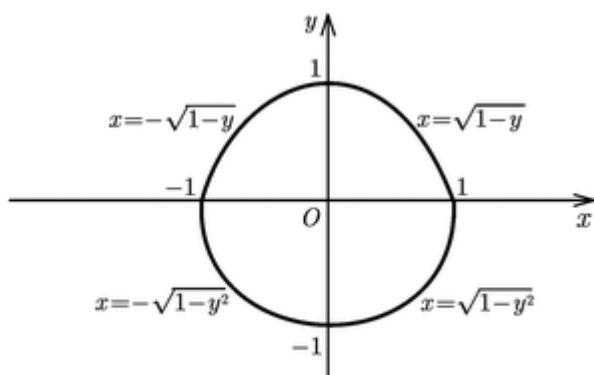


Рис. 89

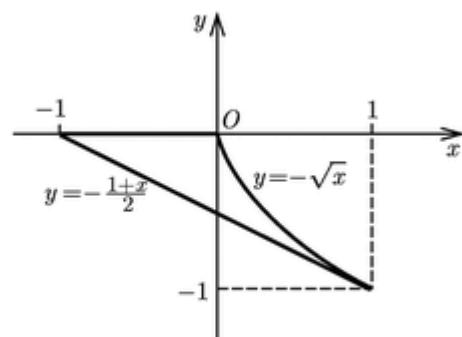


Рис. 90

е) $\int_{0,5\sqrt{3}}^{\sqrt{2}-0,5} dy \int_{\sqrt{1-y^2}}^{0,5} f(x, y) dx + \int_{\sqrt{2}-0,5}^1 dy \int_{\sqrt{1-y^2}}^{\sqrt{2-y}} f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2}-y} f(x, y) dx$
 (рис. 91);

$$\text{жк)} \int_{-1}^{-0,5\sqrt{3}} dx \int_{0,5}^{0,5+\sqrt{1-x^2}} f(x, y) dy + \int_{-0,5\sqrt{3}}^0 dx \int_{\sqrt{1-x^2}}^1 f(x, y) dy \quad (\text{рис. 92}).$$

3. а) 4,5; б) $-27/4$; в) 61.

$$\text{4. а)} \int_0^{2\pi} d\varphi \int_0^a \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho = \int_0^a \rho d\rho \int_0^{2\pi} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi$$

$$\text{б)} \int_0^\pi d\varphi \int_0^{2 \sin \varphi} \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho = \int_0^2 \rho d\rho \int_{\arcsin(\rho/2)}^{\pi - \arcsin(\rho/2)} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi \quad (\text{рис. 93}); \text{ в})$$

$$\int_0^{2\pi} d\varphi \int_a^b \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho =$$

$$= \int_a^b \rho d\rho \int_0^{2\pi} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi \quad ; \Gamma) \int_{-\pi/4}^{\pi/4} d\varphi \int_0^{a\sqrt{\cos 2\varphi}} \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho =$$

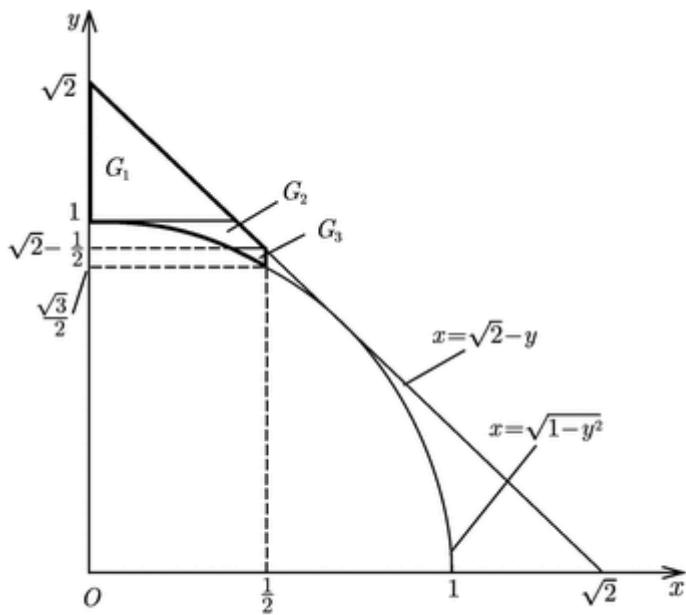


Рис. 91

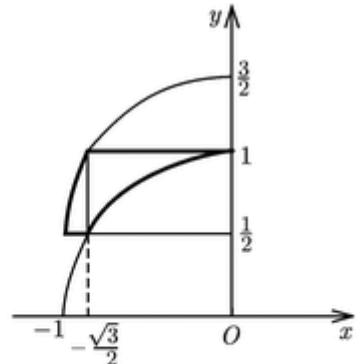


Рис. 92

$$= \int_0^a \rho d\rho \int_{-\arccos(\rho^2/a^2)}^{0,5\arccos(\rho^2/a^2)} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi$$

(рис. 94); д)

$$\int_0^{\pi/2} d\varphi \int_0^{1/(\sin \varphi + \cos \varphi)} \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho =$$

$$= \int_0^{1/\sqrt{2}} \rho d\rho \int_0^{\pi/2} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi + \int_{1/\sqrt{2}}^1 \rho d\rho \left[\int_0^{\alpha - \pi/4} f(\rho \times \right.$$

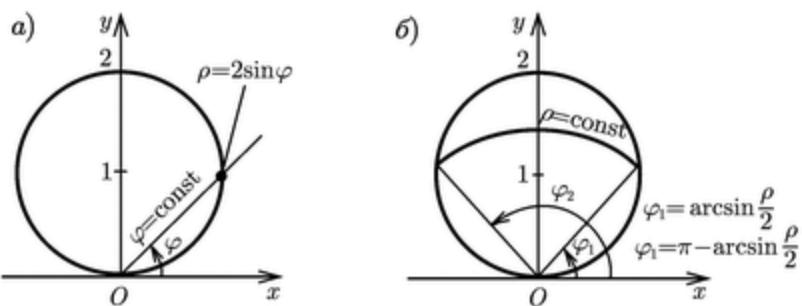


Рис. 93

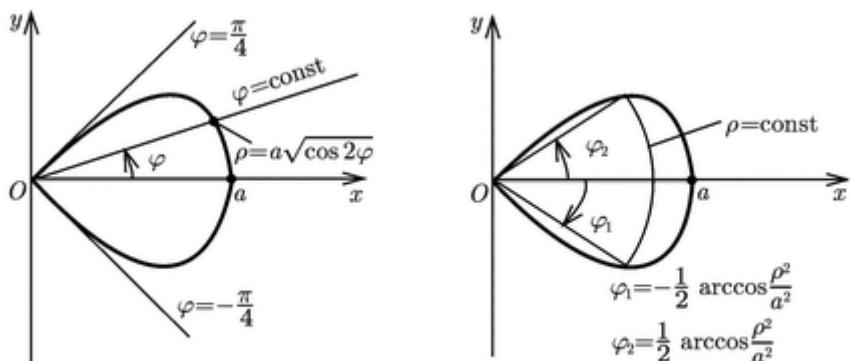


Рис. 94

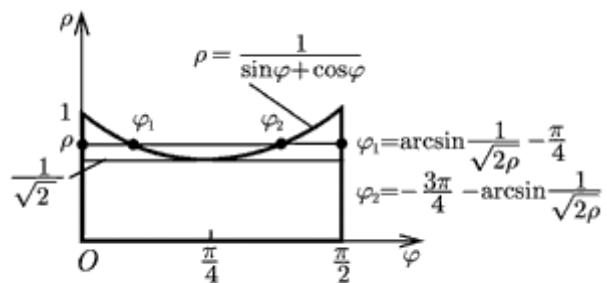


Рис. 95

$$\times \cos \varphi, \rho \sin \varphi) d\varphi + \int_{3\pi/4 - \alpha}^{\pi/2} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi \Big]$$

$\alpha = \arcsin \frac{1}{\sqrt{2}\rho}$ (рис. 95);
, где

e)

$$\begin{aligned}
 & \int_0^{\pi/4} d\varphi \int_0^{a \sin \varphi / \cos^2 \varphi} \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho + \int_{\pi/4}^{3\pi/4} d\varphi \int_0^{a / \sin \varphi} \rho f(\rho \cos \varphi, \rho \sin \varphi) d\rho + \int_{3\pi/4}^{\pi} d\varphi \int_0^{a \sin \varphi / \cos^2 \varphi} \\
 & = \int_0^a \rho d\rho \int_{\alpha_1(\rho)}^{\pi - \alpha_1(\rho)} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi + \int_a^{\sqrt{2}a} \rho d\rho \left[\int_{\alpha_1(\rho)}^{\alpha_2(\rho)} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi \right] + \int_{\pi - \alpha_2(\rho)}^{\pi - \alpha_1(\rho)} f(\rho \cos \varphi, \rho \sin \varphi) d\varphi \\
 & = \arcsin \frac{\sqrt{a^2 + 4\rho^2} - a}{2\rho}, \quad \alpha_2(\rho) = \arcsin \frac{a}{\rho}
 \end{aligned}$$

5. a)

$$\alpha = \arcsin \frac{\sqrt{1 + 4\rho^2} - 1}{2\rho}$$

б)

$$\begin{aligned}
 \int_{\pi/6}^{\pi/4} d\varphi \int_0^{2/\sin \varphi} \rho f(\rho^2) d\rho &= \int_0^{2\sqrt{2}} f(\rho^2) \rho d\rho \int_{\pi/6}^{\pi/4} d\varphi + \int_{2\sqrt{2}}^4 f(\rho^2) \rho d\rho \int_{\pi/6}^{\arcsin(2/\rho)} d\varphi \\
 \text{б) } \alpha(\rho) &= \arccos \frac{1}{\sqrt{2}\rho}
 \end{aligned}$$

$$6. \text{ а) } \frac{1}{2} \int_2^3 du \int_{u-6}^u f(u, v) dv; \text{ б) } \int_2^3 u du \int_3^4 f(v) dv = \frac{5}{2} \int_3^4 f(v) dv$$

8. Положите $u = xy$, $v = x - 2y$.

9. а) $2\pi ab/3$; б) 2; в) 0 и $21\pi a^4/2^9$.

10. а) 18π , б) $4/3$; в) 0; г)

11. а) $(15/8 - \ln 4)a^2$; б) π ; в) $0,5a^2 \ln 2$.

12. а) $a^2(\sqrt{3} - \pi/3)$; б) $a^2(4 - \sqrt{15} + \pi/2 - \arccos(\sqrt{15}/4))$.

13. а) $a^2b^2/(2c^2)$; б) $1/35$; в) $ab(a^2 + b^2)\pi/2$; г) 3π , д) $2/3$.

14. а) $\pi(3 \ln 3 - 2)$; б) $2\pi^2$; в) $3/4$.

15. а) $x_0 = -1; y_0 = 3,2$; б) $x_0 = 4,5, y_0 = -1$; в) $x_0 = y_0 = \pi a/8$.

16. $x_0 = -a/5, y_0 = 0$.

17. а) $I_x = ab^3\rho_0/3, I_y = a^3b\rho_0/3$; б) $I_x = \rho_0/6, I_y = 7\rho_0/6$ в) $I_x = I_y = a^4(1 - 5\pi/16)\rho_0$; г) $I_x = I_y = 3\pi a^4\rho_0/4\sqrt{2}$.

18. $I_x = a^2b^4/8, I_y = a^4b^2/8$; б) $I_x = 0,1, I_y = 13/30$.

19. $p_e = \pi a^2 \rho(h - (2/3)a), p_h = \pi a^2 \rho(h + (2/3)a)$.

20. Взяв ось x в качестве оси абсцисс, получаем

$$I_{x'} = \iint_G (y - a)^2 \rho(x, y) dx dy = I_x - 2aM_x + a^2 m$$

. Так как по условию $M_x = 0$, то приходим к равенству $I_{x'} = I_x + ma^2$.

$$\int_{-a}^a dx \int_{-b\sqrt{1-x^2/a^2}}^{b\sqrt{1-x^2/a^2}} dy \int_{-c\sqrt{1-x^2/a^2-y^2/b^2}}^{c\sqrt{1-x^2/a^2-y^2/b^2}} f(x, y, z) dz, \int_{-b}^b dy \int_{-a\sqrt{1-y^2/b^2}}^{a\sqrt{1-y^2/b^2}} dx \int_{-c\sqrt{1-x^2/a^2-y^2/b^2}}^{c\sqrt{1-x^2/a^2-y^2/b^2}} f(x, y, z) dz$$

21. а)

$$\int_{-a}^a dx \int_{-c\sqrt{1-x^2/a^2}}^{c\sqrt{1-x^2/a^2}} dz \int_{-b\sqrt{1-x^2/a^2-z^2/c^2}}^{b\sqrt{1-x^2/a^2-z^2/c^2}} f(x, y, z) dy, \int_{-c}^c dz \int_{-a\sqrt{1-z^2/c^2}}^{a\sqrt{1-z^2/c^2}} dx \int_{-b\sqrt{1-x^2/a^2-z^2/c^2}}^{b\sqrt{1-x^2/a^2-z^2/c^2}} f(x, y, z) dy$$

$$\int_{-b}^b dy \int_{-a\sqrt{1-y^2/b^2}}^{c\sqrt{1-y^2/b^2}} dz \int_{-a\sqrt{1-y^2/b^2-z^2/c^2}}^{a\sqrt{1-y^2/b^2-z^2/c^2}} f(x, y, z) dx, \int_{-c}^c dz \int_{-b\sqrt{1-z^2/c^2}}^{a\sqrt{1-z^2/c^2}} dy \int_{-a\sqrt{1-y^2/b^2-z^2/c^2}}^{b\sqrt{1-y^2/b^2-z^2/c^2}} f(x, y, z) dx$$

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_{\sqrt{1-x^2-y^2}}^{\sqrt{16-x^2-y^2}} f(x, y, z) dz + \int_0^1 dx \int_0^{\sqrt{1-x^2}} dy \int_0^{\sqrt{16-x^2-y^2}} f(x, y, z) dz + \int_1^4 dx \int_0^{\sqrt{16-x^2}} dy \int_0^{\sqrt{16-x^2-y^2}} f(x, y, z) dz$$

б)

$$\int_0^1 dy \int_0^{\sqrt{1-y^2}} dx \int_{\sqrt{1-x^2-y^2}}^{\sqrt{16-x^2-y^2}} f(x, y, z) dz + \int_0^1 dy \int_0^{\sqrt{16-y^2}} dx \int_0^{\sqrt{16-x^2-y^2}} f(x, y, z) dz + \int_1^4 dy \int_0^{\sqrt{16-y^2}} dx \int_0^{\sqrt{16-x^2-y^2}} f(x, y, z) dz$$

$$\int_0^1 dy \int_0^{\sqrt{1-y^2}} dz \int_{\sqrt{1-y^2-z^2}}^{\sqrt{16-y^2-z^2}} f(x, y, z) dx + \int_0^1 dy \int_0^{\sqrt{16-y^2}} dz \int_0^{\sqrt{16-y^2-z^2}} f(x, y, z) dx + \int_1^4 dy \int_0^{\sqrt{16-y^2}} dz \int_0^{\sqrt{16-y^2-z^2}} f(x, y, z) dx$$

$$\int_0^1 dz \int_0^{\sqrt{1-z^2}} dy \int_{\frac{\sqrt{16-y^2-z^2}}{\sqrt{1-y^2-z^2}}}^{f(x,y,z)} dx + \int_0^1 dz \int_0^{\sqrt{16-z^2}} dy \int_{\frac{\sqrt{16-y^2-z^2}}{\sqrt{1-z^2}}}^{f(x,y,z)} dx + \int_1^4 dz \int_0^{\sqrt{16-z^2}} dy \int_{\frac{\sqrt{16-y^2-z^2}}{0}}^{f(x,y,z)} dx$$

,

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} dz \int_{\frac{\sqrt{16-x^2-z^2}}{\sqrt{1-x^2-z^2}}}^{f(x,y,z)} dy + \int_0^1 dx \int_0^{\sqrt{16-x^2}} dz \int_{\frac{\sqrt{16-x^2-z^2}}{\sqrt{1-x^2}}}^{f(x,y,z)} dy + \int_1^4 dx \int_0^{\sqrt{16-x^2}} dz \int_{\frac{\sqrt{16-x^2-z^2}}{0}}^{f(x,y,z)} dy$$

,

$$\int_0^1 dz \int_0^{\sqrt{1-z^2}} dx \int_{\frac{\sqrt{16-x^2-z^2}}{\sqrt{1-x^2-z^2}}}^{f(x,y,z)} dy + \int_0^1 dz \int_0^{\sqrt{16-z^2}} dx \int_{\frac{\sqrt{16-x^2-z^2}}{\sqrt{1-z^2}}}^{f(x,y,z)} dy + \int_1^4 dz \int_0^{\sqrt{16-z^2}} dx \int_{\frac{\sqrt{16-x^2-z^2}}{0}}^{f(x,y,z)} dy$$

,

$$\begin{aligned}
& \int_0^2 dx \int_0^{2-x} dy \int_0^{4-2x-2y} f(x,y,z) dz + \int_0^2 dx \int_0^{x-2} dy \int_0^{4-2x+2y} f(x,y,z) dz + \int_{-2}^0 dx \int_{-x-2}^0 dy \int_0^{4+2x+2y} f(x,y,z) dz + \int_{-2}^0 dx \int_0^{x+2} dy \times \\
& \times \int_0^{4+2x-2y} f(x,y,z) dz, \int_0^2 dy \int_0^{2-y} \int_0^{4-2x-2y} f(x,y,z) dz + \int_0^2 dy \int_0^{y-2} \int_0^{4+2x-2y} f(x,y,z) dz + \int_{-2}^0 dy \int_0^{y+2} \int_0^{4-2x+2y} f(x,y,z) dz + \\
& + \int_{-2}^0 dy \int_0^{y+2} \int_0^{4-2x+2y} f(x,y,z) dz + \int_{-2}^0 dy \int_{-2-y}^0 \int_0^{4+2x+2y} f(x,y,z) dz, \int_0^4 dz \int_0^{2-z/2} \int_{y+z/2-2}^{2-y-z/2} f(x,y,z) dx + \\
& + \int_0^4 dz \int_{z/2-2}^0 dy \int_{-y+z/2-2}^{y-z/2+2} f(x,y,z) dx, \int_{-2}^0 dy \int_0^{2y+4} \int_{y+z/2-2}^{2-y-z/2} f(x,y,z) dx + \int_0^2 dy \int_0^{4-2y} \int_{z/2-y-2}^{y-z/2+2} f(x,y,z) dx \\
& \int_0^4 dz \int_0^{2-z/2} dx \int_{x+z/2-2}^{2-x-z/2} f(x,y,z) dy + \int_0^4 dz \int_{z/2-2}^0 dx \int_{-x+z/2-2}^{x-z/2+2} f(x,y,z) dy, \int_{-2}^0 dx \int_0^{2x+4} dz \times \\
& \times \int_{x+z/2-1}^{2-x-z/2} f(x,y,z) dy + \int_0^2 dx \int_0^{4-2x} dz \int_{z/2-x-2}^{y-z/2+2} f(x,y,z) dy
\end{aligned}$$

22. a)

$$\int_0^1 dy \int_0^{(1-y)/a} dx \int_0^{ax+y} f(x,y,z) dz = \int_0^1 dz \int_0^z dy \int_{(z-y)/a}^{(1-y)/a} f(x,y,z) dx + \int_0^1 dz \int_z^1 dy \int_0^{(1-y)/a} f(x,y,z) dx =$$

$$\begin{aligned}
&= \int_0^1 dy \left[\int_y^1 dz \int_{(z-y)/a}^{(1-y)/a} f(x, y, z) dx + \int_0^y dz \int_0^{(1-y)/a} f(x, y, z) dx \right] = \int_0^{1/a} dx \left[\int_0^{ax} dz \int_0^{1-ax} f(x, y, z) dy + \int_{ax}^1 dz \int_{z-ax}^{1-ax} f(x, y, z) dy \right] = \\
&= \int_0^1 dz \left[\int_{z/a}^{1/a} dx \int_0^{1-ax} f(x, y, z) dy + \int_0^{z/a} dx \int_{z-ax}^{1-ax} f(x, y, z) dy \right], \\
&\text{б) } \int_0^1 dx \int_0^1 dy \int_0^{2x^2+3y^2} f(x, y, z) dz = \int_0^1 dy \left[\int_0^{3y^2} dz \int_0^1 f(x, y, z) dx + \int_{3y^2}^{\infty} dz \int_{\sqrt{(z-3y^2)/2}}^1 f(x, y, z) dx \right] = \int_0^3 dz \int_{\sqrt{z/3}}^1 dy \int_0^1 f(x, y, z) dx + \\
&+ \int_0^2 dz \int_0^{\sqrt{z/3}} dy \int_{\sqrt{(z-3y^2)/2}}^1 f(x, y, z) dx + \int_2^3 dz \int_{\sqrt{(z-2)/3}}^{\sqrt{z/3}} dy \int_{\sqrt{(z-3y^2)/2}}^1 f(x, y, z) dx + \int_3^5 dz \times \\
&\times \int_{\sqrt{(z-2)/3}}^1 dy \int_{\sqrt{(z-3y^2)/2}}^1 f(x, y, z) dx = \int_0^1 dx \left[\int_0^{2x^2} dz \int_0^1 f(x, y, z) dy + \int_{2x^2}^{\infty} dz \int_{\sqrt{(z-2x^2)/3}}^1 f(x, y, z) dy \right] = \\
&= \int_0^2 dz \int_{\sqrt{z/2}}^1 dx \int_0^1 f(x, y, z) dy + \int_0^2 dz \int_0^{\sqrt{z/2}} dx \int_{\sqrt{(z-2x^2)/3}}^1 f(x, y, z) dy + \int_2^3 dz \int_0^1 dx \int_{\sqrt{(z-2x^2)/3}}^1 f(x, y, z) dy + \int_3^5 dz \int_{\sqrt{(z-3)/2}}^1 dx \times \\
&\times \int_{\sqrt{(z-2x^2)/3}}^1 f(x, y, z) dy, \\
&\text{в) например, } \int_{-1}^1 dz \int_{|z|}^1 dx \int_{-\sqrt{x^2-z^2}}^{\sqrt{x^2-z^2}} f(x, y, z) dy = \int_0^1 dx \int_{-x}^x dy \int_{-\sqrt{x^2-y^2}}^{\sqrt{x^2-y^2}} f(x, y, z) dz
\end{aligned}$$

г) например,

$$\int_0^1 dy \left[\int_0^{y^2} dz \int_y^1 f(x, y, z) dx + \int_{y^2}^y dz \int_{z/y}^1 f(x, y, z) dx \right]$$

23. а) 0; б) 1/8.

24. Например,

$$\int_0^{\pi/4} d\varphi \int_{\arctg(1/\cos\varphi)}^{\pi/2} \sin\theta d\theta \int_{\cos\theta/\sin^2\theta}^{1/(\cos\varphi\sin\theta)} f(r) r^2 dr = \int_{\arctg(1/\sqrt{2})}^{\pi/4} \sin\theta d\theta \int_{\arccos(\tg\theta)}^{\pi/4} d\varphi \int_{\cos\theta/\sin^2\theta}^{1/(\cos\varphi\sin\theta)} f(r) r^2 dr +$$

$$+ \int_{\pi/4}^{\pi/2} \sin \theta d\theta \int_0^{\pi/4} d\varphi \int_{\cos \theta / \sin^2 \theta}^{1/(\cos \varphi \sin \theta)} f(r) r^2 dr$$

25. a) $\pi/10$; б) $3\sqrt{3}\pi/2^5$.

26. a) $4\pi/21$; б) $81/40\pi$.

27. a) $3\sqrt{3}-\pi/5$; б) $8\pi abc/5$; в) $9/4$.

28. a) $2/3$; б) 1; в) $\pi ab(a+b)/8$.

29. a) $21\pi/32$; б) $\pi(b-a)(b^2+ab-2a^2)/3$.

30. a) $21\pi/(4\sqrt{2})$; б) $\pi^2 a^3/(4\sqrt{2})$.

31. a) $\pi abc/60$; б) $81/40$; в) 6.

32. a) $4\pi\rho_0$; б) $128\rho_0/15$; в) $1408\rho_0/105$.

33. a) $x_0 = 0, y_0 = 2, z_0 = 4a/3\pi$, б) $x_0 = y_0 = 16\sqrt{3}/(15\pi), z_0 = 2$; в) $x_0 = 7p/18, y_0 = 0, z_0 = 7p/176$; г) $x_0 = y_0 = 0, z_0 = 7/20$.

34. a) $x_0 = -2, y_0 = 1, z_0 = 4/3$; б) $x_0 = -2, y_0 = 1, z_0 = 2/3$; в) $x_0 = 32/13, y_0 = 16/13, z_0 = 33/65$.

35. a) $I_{xy} = (4/15)\pi\rho_0 abc^3, I_{xz} = (4/15)\pi\rho_0 ab^3 c, I_{yz} = (4/15)\pi\rho_0 a^3 bc$; б) $I_{xy} = \pi\rho_0/10, I_{xz} = I_{yz} = \pi\rho_0/20$; в) $I_{xy} = 81\pi\rho_0/16, I_{yz} = I_{xz} = 9\pi\rho_0/16$.

36. a) $I_{xy} = (4/9)\pi\rho_0 abc^3, I_{xz} = (4/9)\pi\rho_0 ab^3 c, I_{yz} = (4/9)\pi\rho_0 a^3 bc$ б) $I_{xy} = 75\pi, I_{xz} = 62\pi, I_{yz} = 50\pi$.

37. a) $I_x = I_y = 0, 15\pi\rho_0, I_z = 0, 1\pi\rho_0, I_0 = 0, 2\pi\rho_0$; б) $I_x = I_y = \pi\rho_0 a^5(16 - 7\sqrt{2})/60, I_z = \pi\rho_0 a^5(8 - 5\sqrt{2})/30, I_0 = \pi\rho_0 a^5(2 - \sqrt{2})/5$.

38. $5\pi^2\rho_0/64$.

39.

$$F_x = \gamma \iiint_T \frac{\rho_0 m_0 x}{r^3} dx dy dz = \gamma \rho_0 m_0 \iint_{\substack{-R \leqslant y \leqslant R \\ 0 \leqslant z \leqslant h}} dy dz \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \frac{x dx}{(x^2+y^2+z^2)^{3/2}} = 0$$

$$= 0; \quad ; F_y$$

$$F_z = \gamma \rho_0 m_0 \iiint \frac{z}{r^3} dx dy dz = \gamma \rho_0 m_0 \iint_{x^2+y^2 \leq R^2} dx dy \int_0^h \frac{z dz}{(x^2+y^2+z^2)^{3/2}} = \gamma \rho_0 m_0 \iint_{x^2+y^2 \leq R^2} \left(\frac{1}{\sqrt{x^2+y^2}} - \right.$$

$$\left. - \frac{1}{\sqrt{x^2+y^2+h^2}} \right) dx dy = 2\pi \gamma \rho_0 m_0 (R + h - \sqrt{R^2 + h^2})$$

40. a) $\frac{m}{3}$; 6) $\frac{2}{(m-1)!(2m+1)}$

41. $\frac{2^m h_1 h_2 \dots h_m}{|\Delta|}$

42. $\frac{a_1 a_2 \dots a_m}{m!}$

43. $\frac{\frac{8\pi^2 R^5}{15}}{15}$