Системы случайных величин. **Задача 64** $\rho(x,y) := 24\cdot x\cdot y$

$F(0.5,2)$ $\int_0^{0.5}\int_0^{1-x}\rho(x,y)\,dy\,dx = 0.6875$ $\frac{11}{16} = 0.6875$

$$\rho 1(x) := \int_0^{1-x}\rho(x,y)\,dy \rightarrow 12\cdot x^3 - 24\cdot x^2 + 12\cdot x \qquad 0 \le x \le 1$$

$$\rho 2(y) := \int_0^{1-y}\rho(x,y)\,dx \rightarrow 12\cdot y^3 - 24\cdot y^2 + 12\cdot y \qquad 0 \le y \le 1$$

$$U1(x,y) := \frac{\rho(x,y)}{\rho 2(y)} \rightarrow 24\cdot x\cdot\frac{y}{12\cdot y^3 - 24\cdot y^2 + 12\cdot y}$$

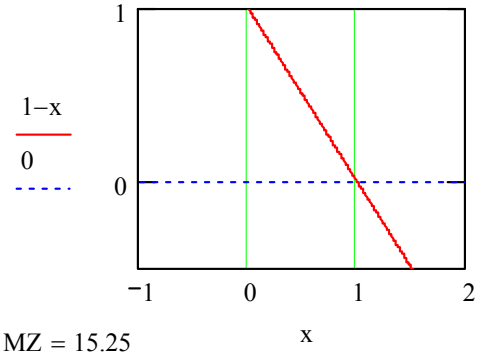
$$U2(y,x) := \frac{\rho(x,y)}{\rho 1(x)} \rightarrow 24\cdot x\cdot\frac{y}{12\cdot x^3 - 24\cdot x^2 + 12\cdot x}$$

Задача 66

$$A := \begin{pmatrix} 0 & -2 & 3 \\ 1 & 0.32 & 0.18 \\ 4 & 0.13 & 0.37 \end{pmatrix}$$

$$MZ := \sum_{j=1}^{2}\sum_{i=1}^{2}\left[\left[\left(A_{0,j}\right)^2 + \left(A_{i,0}\right)^2\right]\cdot A_{i,j}\right]$$

$MZ = 15.25$



Задача 67

$$\rho(x,y,R) := 2\cdot\frac{\left(x^2 + y^2\right)}{\pi\cdot R^4}$$

В полярной системе координат

$$\rho(r,R) := \frac{2\cdot r^2}{\pi\cdot R^4}$$

$$mZ(R) := \int_0^R r^5\cdot\rho(r,R)\,dr\cdot\int_0^{2\cdot\pi}\left(\sin(\phi)\right)^2\left(\cos(\phi)\right)^2 d\phi \rightarrow \frac{1}{16}\cdot R^4$$

Задача 68

$$A := \begin{pmatrix} 0 & 1 & 2 \\ 3 & 0.2 & 0.3 \\ 4 & 0.4 & 0.1 \end{pmatrix}$$

$$AX := \begin{pmatrix} 1 & 2 \\ 0.6 & 0.4 \end{pmatrix} \qquad AY := \begin{pmatrix} 3 & 4 \\ 0.5 & 0.5 \end{pmatrix}$$

$$MAX := \sum_{i=0}^{1} AX_{0,i}\cdot AX_{1,i}$$

$MAX = 1.4$

$$MAY := \sum_{i=0}^{1} AY_{0,i}\cdot AY_{1,i}$$

$MAY = 3.5$

$$cov := \left[\sum_{i=1}^{2}\sum_{j=1}^{2}\left[\left[\left(A_{0,j}\cdot A_{i,0}\right)\cdot A_{i,j}\right]\right]\right] - MAX\cdot MAY$$

$cov = -0.1$

Задача 69 (см. зад. 64) $mx := \int_0^1 x\cdot\rho 1(x)\,dx$ $mx = 0.4$ $my := \int_0^1 y\cdot\rho 2(y)\,dy$ $my = 0.4$

$$COV := \left(\int_0^1\int_0^{1-x} 24x^2\cdot y^2\,dy\,dx\right) - mx\cdot my$$

$COV = -0.027$ $\frac{-2}{75} = -0.027$

Статистическое распределение выборки. Эмпирическая функция распределения. Полигон и гистограмма

Решение задачи 1.

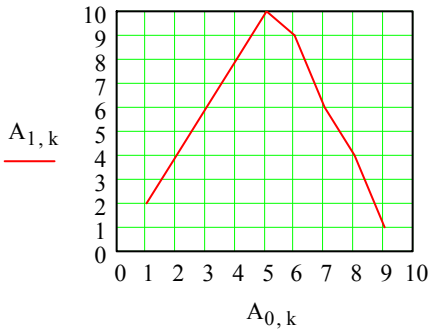
$$A := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 10 & 9 & 6 & 4 & 1 \end{pmatrix} \qquad N := \sum_{i=0}^{8} A_{1,i} \qquad N = 50$$

$$M := \sum_{i=0}^{8} A_{0,i} \cdot \frac{A_{1,i}}{N} \qquad M = 4.94$$

$$D := \left[\sum_{i=0}^{8} \frac{\left[\left(A_{0,i}\right) - M\right]^2 \cdot A_{1,i}}{N} \right] \qquad D = 3.696 \qquad \sqrt{D} = 1.923 \qquad D \cdot \frac{N}{N-1} = 3.772$$

$$\sqrt{D \cdot \frac{50}{49}} = 1.942 \qquad V := \frac{\sqrt{D} \cdot 100}{M} \qquad V = 38.919 \qquad V1 := \frac{\sqrt{D \cdot \frac{N}{N-1}} \cdot 100}{M} \qquad V1 = 39.314$$

Полигон частот $k := 0..8$

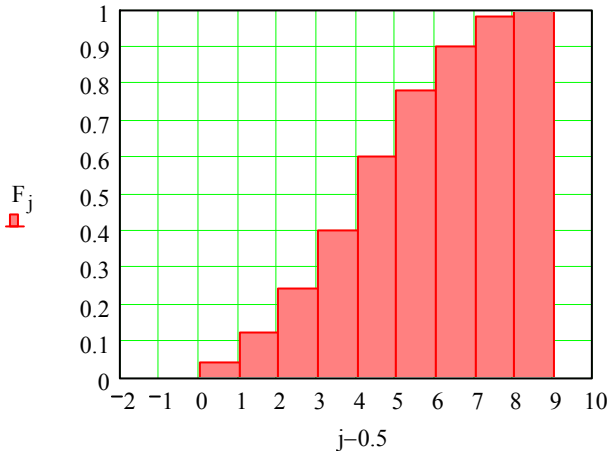


$$F := \left| \begin{array}{l} EFR_0 \leftarrow 0 \\ \text{for } j \in 0..8 \\ \quad \text{for } k \in 0..8 \\ \quad\quad EFR_{k+1} \leftarrow EFR_k + \frac{A_{1,k}}{N} \\ EFR \end{array} \right.$$

Эмпирическая функция распределения

$F^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.04	0.12	0.24	0.4	0.6	0.78	0.9	0.98	1



Решение задачи 2.

$$Z := \begin{pmatrix} 250 & 750 & 1500 & 2500 & 3500 \\ 27 & 11 & 8 & 8 & 2 \end{pmatrix} \qquad n := \sum_{i=0}^{4} Z_{1,i} \qquad n = 56$$

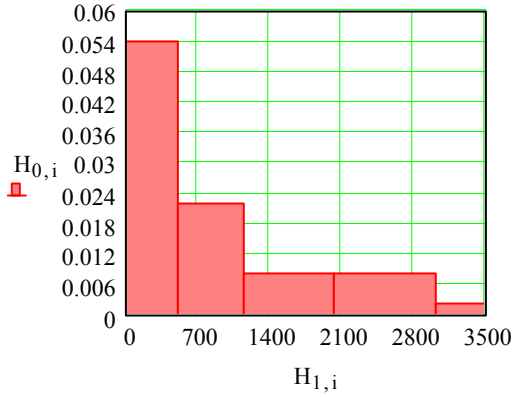
$$i := 0..4$$

$$\rho := \begin{pmatrix} \frac{Z_{1,0}}{500} & \frac{Z_{1,1}}{500} & \frac{Z_{1,2}}{1000} & \frac{Z_{1,3}}{1000} & \frac{Z_{1,4}}{1000} \end{pmatrix}$$

$$\rho = \begin{pmatrix} 0.054 & 0.022 & 8\times 10^{-3} & 8\times 10^{-3} & 2\times 10^{-3} \end{pmatrix}$$

$$H := \begin{pmatrix} 0.054 & 0.022 & 8\times 10^{-3} & 8\times 10^{-3} & 2\times 10^{-3} \\ 250 & 750 & 1500 & 2500 & 3500 \end{pmatrix}$$

Гистограмма



ЭФР

$$FZ := \left| \begin{array}{l} EFRZ_0 \leftarrow 0 \\ \text{for } j \in 0..4 \\ \quad \text{for } k \in 0..4 \\ \quad\quad EFRZ_{k+1} \leftarrow EFRZ_k + \frac{Z_{1,k}}{n} \\ EFRZ \end{array} \right.$$

$$FZ^T = (0 \quad 0.482 \quad 0.679 \quad 0.821 \quad 0.964 \quad 1)$$

$$MZ := \sum_{i=0}^{4} Z_{0,i} \cdot \frac{Z_{1,i}}{n}$$

$$MZ = 964.286$$

$$DZ := \left[\sum_{i=0}^{4} \frac{\left[\left(Z_{0,i}\right) - MZ\right]^2 \cdot Z_{1,i}}{n} \right]$$

$$DZ = 8.6256378\times 10^{5}$$

$$\sqrt{DZ} = 928.743$$

$$DZ \cdot \frac{n}{n-1} = 8.7824675\times 10^{5}$$

$$\sqrt{DZ \cdot \frac{n}{n-1}} = 937.148$$

$$VZ := \frac{\sqrt{DZ}\cdot 100}{MZ} \qquad VZ = 96.314$$

$$VZ1 := \frac{\sqrt{DZ \cdot \frac{n}{n-1}}\cdot 100}{MZ} \qquad VZ1 = 97.186$$

График ЭФР

$$FR := \begin{pmatrix} 0.482 & 0.679 & 0.821 & 0.964 & 1 \\ 250 & 750 & 1500 & 2500 & 3500 \end{pmatrix}$$