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An Analytical Model for the Concentration of Particles in a Countercurrent Cylindrical Cyclone Apparatus

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Abstract—Representing a disperse phase as a continuous medium of particles, we consider their concentration in the turbulent flow of an aerosol in a cyclone apparatus. In this work we use the known results of the aerodynamic studies of flows in cyclone apparatuses. Analytical equations, which do not require the use of experimental coefficients on the separation of particles, are derived for the efficiency of the concentration of particles in a cyclone apparatus.

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INTRODUCTION

Cyclone apparatuses are the most important element of the gas dedusting systems in the production of chemicals and other products. However, improvement of the methods for calculating the efficiency of the separation of particles in these apparatuses is still the subject of many studies. Several methods for estimating the fractional separation of particles and the dedusting efficiency are recognized, including the method of particle paths [1–3], the method based on the stochastic model, where the motion of particles is considered as a random process on which a deterministic action is imposed [4, 5], and the turbulent transport model [6], which may be classified with the stochastic model in some cases.

According to the existing hypotheses of the separation process, LIOT cyclones with a long cylindrical part and submerged gas removal pipe were developed by the method of particle paths and found widespread use in the purification of gases in the 1940s. In the coaxial channel, an aerosol makes several revolutions (turns), and particles are subjected to the action of inertial forces for a long time, upon the expiration of which fine particles are expected to be removed from the flow. However, the experience in the operation of such cyclones and subsequent experiments have not confirmed these hypotheses, and the further improvement of cyclones and their embodiment were performed on the basis of other hypotheses about the separation of particles or experimentally [7, 8], depending on the type of problems to be solved. The method of particle paths gives satisfactory results for coarse particles, but in the case of fine particles it is necessary to take into account their turbulent transport [9–12].

The method for estimating the fractional efficiency of separation with the use of the probability integral of a random value, which is the ratio of two logarithms of variables, has received wide practical recognition. The first logarithm is the ratio of the current size of a particle to the size of a particle caught with the efficiency of 50.0%, and the other logarithm is the dispersion logarithm and represents the standard deviation of a random value in the distribution of partial purification coefficients. The size of a particle caught with a 50.0% efficiency and the dispersions are determined experimentally [13].

In [14] it is shown that the injection of dust particles into the near-wall zone does not guarantee their entry into a dust collector and 100% entrapment. Ter Linden [15] plotted the isoefficiency curves for the separation of particles injected at different points of the separation chamber. It turns out that these curves pass through points located on the axis; i.e., the particles injected below the inlet cross section of the outlet channel on the axis prove to be caught with the same efficiency as those injected at the periphery. According to the data obtained by Kizin [14], the particles injected at the top axial point of the conic part are caught more efficiently than those injected into the peripheral zone of the upper part of a cyclone. Our data on the efficiency of the separation of particles injected at different points on the axes of the vortex chamber and a cocurrent cyclone concentrator indicate a similar tendency in the separation of particles [16].

The represented data show that particles are withdrawn from the central zone to the periphery, owing to their diffusion transport towards the area of high centripetal accelerations in the neighborhood of the radius $R_m < R_1$ (outlet pipe radius). The neighborhood of the radius R_m is the zone of the transition from the quasi-solid rotation to the quasi-potential rotation of a gas. It has been established that the central zone is the area of the ejection action on near-axis flows and the intensive withdrawal of particles. Here the flow turbulence intensity reaches a level higher than 40.0%, whereas in straight channels it does not exceed 2.0%. The entry of a high-concentrated flow into the dust collector along the periphery and the egress of a low-concentrated flow from the dust collector to the near-axis zone are also explained by ejection effects. In the present work, we consider the diffusion model of the transport of particles under the assumption that the particle diffusion coefficient is constant in the cross sections of an apparatus and estimate the fractional efficiency on the basis of the balance relationships for the axial flows of particles.

DIFFUSION MODEL OF THE TURBULENT FLOW OF AN AEROSOL

In the phenomenological approach to the investigation of a disperse flow with a low concentration of particles the concept of the conditional continuum of components in a medium is employed, thus allowing the application of the techniques of continuum mechanics [6, 17]. The continuity equation [6] in the form $\iint\limits_{\Sigma}\left(\overline{CV_n} + \overline{C'V_n'}\right)d\sigma - q = 0$, is valid for the turbulent flow of a medium of particles, and the turbulent flow of particles is $\overline{C'V_n'} = -D_n\frac{dC}{dn}$, where D_n is the coefficient of the turbulent diffusion of particles in the center of the surface element $d\sigma$; $\overline{V}_n$, V_n' are the projections of the averaged and fluctuating particle velocities, respectively, onto the normal to the surface $d\sigma$, q is the intensity of the source of a medium of particles in the volume, and C is the concentration of particles with the ith size. Depending on the conditions of transport, the flows of a medium of particles at the boundaries of the volume are represented by the value $\overline{CV}_n|_\sigma$ on the specified surface area σ. When particles are deposited on enclosing surfaces, the summary flow is equilibrated by the flow in the normal direction, and the boundary condition has the form $\overline{CV}_n = \overline{CV}_n - D_n\frac{dC}{dn}|_\sigma = 0$. In a thin layer, where the attenuation of turbulence takes place, D_n tends to zero. In the absence of transport (deposition) $\overline{C}\,\overline{V}_n - D_n\frac{\partial C}{\partial n}|_\sigma = 0$[12].

The equations of transport for a swirl flow have the following form

$$\overline{V}_\varphi^2/R = -(\overline{W}_r - \overline{V}_r)/\tau,\quad \overline{W}_\varphi = \overline{V}_\varphi,\quad \overline{W}_z = \overline{V}_z,$$

$$\frac{\partial}{\partial Z}R\left(\overline{C}\overline{V}_z + \overline{C}'\overline{V}_z'\right) + \frac{\partial}{\partial \varphi}\left(\overline{C}\overline{V}_\varphi + \overline{C}'\overline{V}_\varphi'\right) + \frac{\partial}{\partial R}R\left(\overline{C}\overline{V}_r + \overline{C}'\overline{V}_r'\right) = 0.$$

Here V_r, V_φ, and V_z are the radial, tangential, and axial components of the velocity of particles (averaging sign is omitted). The relaxation time for the Stocks flow of an external gas over spherical particles is $\tau = \rho_\delta\delta^2/\rho \times 18\nu$.

At the boundary of a swirl flow near an enclosing surface, the tangential velocity of a gas flow is reduced and becomes zero on the very wall of the flow. Inertial (centrifugal) forces acting on fine particles also decrease to zero in the boundary layer. Particles are entrained by turbulent pulsations near the wall and withdrawn from it, but returned back by inertial (centrifugal) forces. Therefore, particles are in dynamic equilibrium near the wall, and their transport at the flow boundary in the radial direction is absent in the average (particle nonadhesion condition). This circumstance may also be formulated as follows: owing to the impermeability of the wall, the summary particle flow produced by centrifugal forces and diffusion transport must be zero. In most cases the flows in the averaged motion in the axial and circumferential directions are much greater than the diffusion flows in the turbulent motion in the same directions, i.e., $\overline{C}\overline{V}_z \gg \overline{C}'\overline{V}_z'$, and $\overline{C}\overline{V}_\varphi \gg \overline{C}'\overline{V}_\varphi'$, so the diffusion flow in the radial direction has a determinative importance. According to the theory of turbulent transport [6], $\overline{C}'\overline{V}_r' = = -D_n(\partial C/\partial R)$. It will be shown below that the pulsation radial component proves to be higher than the average one, so $\Delta U = \left(V_\varphi^2/R\right)\tau$.

We shall consider "coarse" particles to be the particles that, moving in a motionless gas under the action of the gravity force, gain the velocities (soaring velocities) that equal or exceed the gas pulsation velocities typical for the energy spectrum of a turbulent swirl flow. By "medium" particles we mean the particles whose soaring velocities are several tens lower in comparison with the pulsation velocities of the energy spectrum, and, similarly, by "fine" particles we mean the particles that have the soaring velocities several hundreds lower than the pulsation velocities of this spectrum. The diffusion coefficient for particles is estimated in [11]. The average degree of the entrainment of a particle by turbulent pulsations is $\overline{\mu}^2 = \overline{u_p^2}/\overline{u^2} = 1/(1 + \omega_E\tau)$, where u_p and u are the pulsation velocities of particles and a gas, respectively, and ω_E is the pulsation frequency of energy-intensive vortexes of a medium. The turbulent diffusion coefficient of fine particles is equal to the turbulent diffusion coefficient of a gas. In a turbulent flow, the transport mechanisms for momentum and mass are identical, so the turbulent diffusion coefficient of a gas is assumed to be equal to the turbulent viscosity coefficient ε For coarse particles, it is necessary to introduce the correction for the inertia of particles $D_n = \varepsilon/(1 + \omega_E\tau)$ [11].

In the inlet cross section of an apparatus, the concentration of particles of each size has the same value. Therefore, $C(R, 0) = \text{const}$. Near an enclosing surface

$$-\varepsilon\partial C(R, Z)/\partial R + \Delta UC = 0. \quad (1)$$

Let us call the transport of particles in the radial direction along the radius R an equilibrium process and, respectively, the distribution of particles an equilibrium distribution, if Eq. (1) is valid.

We shall consider an aerosol as consisting of fine particles with the same size. The concentration of particles is small, so their influence on the motion of the gas phase may be neglected. Particles do not interact with each other. The turbulent mixing coefficients of the gas and disperse phases are equal. The axially symmetric motion of an aerosol is described by the following equation

$$r\frac{\partial}{\partial z}(CW_z) + \frac{\partial}{\partial R}R\left(C\Delta U - \varepsilon\frac{\partial C}{\partial R}\right) = 0. \quad (2)$$

where W_z is the particle axial velocity equal to the gas axial velocity, ΔU is the velocity of a particle relative to a gas under the action of the centrifugal force, ε is the turbulent mixing coefficient, and C is the concentration. Multiplying both items in Eq. (2) by dR and integrating from 0 to R_2, we obtain

$$\frac{\partial}{\partial Z}\int_0^{R_2} RCWdR + \left|R\left(C\Delta U - \varepsilon\frac{\partial C}{\partial R}\right)\right|_{R_2} = 0.$$

If $R = R_2$, i.e., the wall radius (boundary), then $\frac{\partial}{\partial Z}\int_0^{R_2} RCWdR = 0$ (condition of the nonaccumulation of particles in the separation chamber of an apparatus).

At the boundary near the wall, the flow of particles in the radial direction is equal to zero owing to diffusion transport and centrifugal forces, i.e.,

$$C\Delta U - \varepsilon\frac{\partial C}{\partial R}\bigg|_{R = R_2} = 0. \quad (3)$$

The solution of Eq. (1) with the boundary conditions (3) for curvilinear and coaxial channels, and also for a cocurrent concentrator, shows that the redistribution of particles close to the equilibrium process of transport occurs within a separation apparatus [18, 19]. Taking into account that the redistribution of particles close to equilibrium [ok?] occurs upon the tangential injection into the upper part of a cyclone, we may suppose that the distribution of particles in the main separation volume is also close to equilibrium [ok?] [18, 19]; i.e.,

$$C\Delta U = \varepsilon\frac{\partial C}{\partial R}. \quad (4)$$

In a countercurrent cyclone, each mole of a gas passes a path in descending, radial, and ascending flows. The transport of angular momentum in the radial direction encourages the generation of turbulence in a swirl flow [20]. Let us consider the motion of a swirl flow in the core of radial outflow on the assumption that the flow is symmetrical and the velocities are changed slightly in the axial direction. The equation of the motion for the averaged components has the following form [20]

$$\rho\left(W_r\frac{\partial W_\varphi}{\partial R} + \frac{W_\varphi W_r}{R}\right) = \frac{\partial\tau_{r\varphi}}{\partial R} + \frac{2}{R}\tau_{r\varphi}, \quad (5)$$

$$\tau_{r\varphi} = \mu_T R\frac{d}{dR}\left(\frac{W_\varphi}{R}\right),\ \frac{\mu_T}{\rho} = \varepsilon = l_{\text{т}}^2 R\left|\frac{d}{dR}\left(\frac{W_\varphi}{R}\right)\right|,\ l_{\text{т}} = \chi R.$$

The continuity equation for the considered mode of gas flow has the form $W_r R = \text{const} = K_1$, where W_r is the radial velocity of a gas. After rearrangements, Eq. (5) can be written in the following form:

$$\rho\frac{1}{R}W_r\frac{\partial(W_\varphi R)}{\partial R} = \frac{1}{R^2}\frac{\partial\left(\tau_{r\varphi}R^2\right)}{\partial R}, \quad (6)$$

and, with consideration for the continuity equation, the integral of the equation of angular momentum (6) will be written as follows:

$$\frac{1}{\rho}\left(R^2\tau_{r\varphi}\right) - K_1 RW_\varphi = K_2. \quad (7)$$

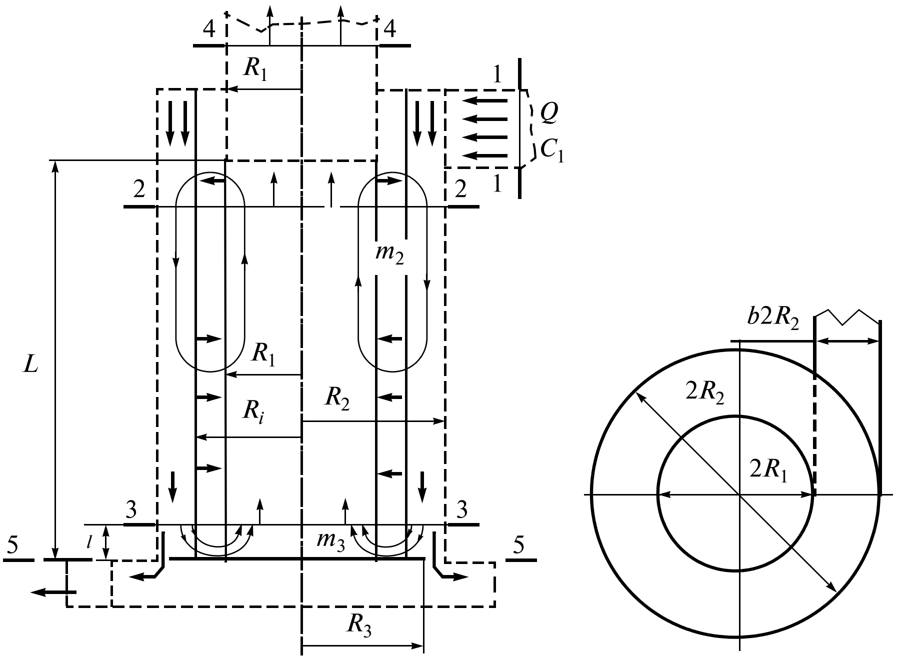
If $W_\varphi R = W_{\phi\text{н}}R_{\text{н}} = K_3$ and $\tau_{r\varphi} = \mu_T R\frac{d}{dR}\left(\frac{W_\varphi}{R}\right)$, $\mu_T = \rho\varepsilon$, Eq. (7) assumes the form $\left[-(K_1/\varepsilon) + 2\right]K_3 = K_2$,, and the values εunequivocally depends on K_1 at $K_1/\varepsilon \to 2$, $K_2 \to 0$, and $K_1/\varepsilon = 2$. Over the entire core zone and at the inlet of a gas from the core $K_2 = 0$. Then Eq. (7) will be written as follows

$$\tau_{r\varphi}/\rho = W_*^2 = W_r W_\varphi, \quad (8)$$

where W_* is the dynamic velocity (friction velocity), which determines the value of turbulent pulsations. Therefore, the friction velocity is determined by the value $(W_r W_\varphi)^{0.5}$, i.e., by the product of the velocities in the radial and circumferential directions. These results can also be obtained directly from the equations given in [20]. From Eq. (8) it follows that the pulsation radial velocity exceeds the averaged radial velocity. The turbulence constant χ is related with the exponent n in the equation of the distribution of circumferential velocities $W_\varphi R^n = W_{\varphi\Delta}R_{\text{н}}^n$ by the equation $\chi = [1/(n+1)]\left(|W_{r1}|/W_{\varphi 1}\right)^{0.5}$, and the exponent n is determined by the constructional ratio M as follows [20]:

$$n = \frac{M}{0.01 + 0.56M} - 1, \quad (9)$$

$$M = \frac{F_1}{\left(2\pi R_2 L\xi\right)^{0.5}}. \quad (10)$$



Scheme of flows in a cylindrical countercurrent cyclone concentrator.

The ratio $W_{\varphi\Delta}/W_1 = \xi$ is the velocity drop (retention) coefficient and depends on the parameter $F_1/2\pi R_2 L$ [20]. The figure illustrates a cylindrical countercurrent concentrator. According to the research data [22–24], the scheme of flows looks as follows. A dust-laden gas with the flow rate Q enters through the cross section 1 into the annular space with the radii R_i and R_2. The recycling gas with the flow rate m_2Q comes from the cylindrical chamber with the radius R_1 into the same space. The volume of the gas passing through the peripheral annular cross section 2 is $(1 + m_2)Q$. The greater fraction (60.0–80.0%) of the gas swirled in the annular space with the radii R_1 and R_i moves between the cross sections 2 and 3 in the radial direction towards the axis, and its smaller fraction with the flow rates $(m_3 + K)Q$ passes through the peripheral annular cross section 3 and is splitted into two flows between the cross sections 3 and 5. After being purified from dust, the flow with the flow rate KQ is withdrawn from the separation chamber, and the flow with the flow rate m_3Q passes between the cross sections 3 and 5 into the cylindrical part with the radius R_1. The parameter $r_i = R_i/R_2$ is determined from the relationship $r_i = 0.8 + 0.2\exp(-20b)$ [21], where b is the relative width of the inlet pipe. The parameters m_2, m_3, n, and r_m are determined by the values of constructional parameters [22–26]. Taking into account backward flows and assuming that the radial transport of a swirl flow is uniform along the height of an apparatus, we can determine the turbulent mixing coefficient from the relationship similar to that given in [20] as follows:

$$\varepsilon = \frac{W_r r}{n+1} = \frac{Q(1-K+2m_2)r}{2\pi r L(n+1)} = \frac{U_0 R_2(1-K+2m_2)}{(n+1)(L/2R_2)}. \quad (11)$$

Let us denote the parameters average over a cross section in the peripheral and central zones, respectively, by the indexes "+" and "–" as follows:

$$Q_k^- = 2\pi\int_0^{R_1} W_k R dR, \quad Q_k^+ = 2\pi\int_{R_i}^{R_2} W_k R dR,$$

$$Q_5^+ = 2\pi\int_{R_3}^{R_2} W_5 R dR, \quad G_k^- = 2\pi\int_0^{R_1} C_k W_k R dR,$$

$$G_k^+ = 2\pi\int_{R_i}^{R_2} C_k W_k R dR, \quad G_5^+ = 2\pi\int_{R_3}^{R_2} C_5 W_5 R dR,$$

Here the index k is the number of the cross sections 2, 3, and 4, and

$Q_2^+/Q = (1 + m_2)$, $Q_3^+/Q = (m_3 + K)$, $Q_5^+/Q = K$; $Q_3^-/Q = m_3$, $Q_2^-/Q = (1 + m_2 - K)$, $Q_4^-/Q = (1 - K)$, $g_k^- = G_k^-/G$, $g_k^+ = G_k^+/G$, $g_5^+ = G_5^+/G$, $C_k^- = G_k^-/Q_k^-$, $C_k^+ = G_k^+/Q_k^+$, $C_5^+ = G_5^+/Q_5^+$, $c = C/C_1$.

The mass flows of the gas and disperse media are constant in the cross sections 2–2 and 3–3. In every cross section of a countercurrent apparatus the algebraic sum of gas phase flows is equal to the gas flow in the cross section of the withdrawal of separated dust (see figure). Similarly, the algebraic sum of disperse phase flows is equal to the disperse phase flow in the cross section of the withdrawal of separated dust; i.e.,

$$g_2^+ - g_2^- = g_3^+ - g_3^- = g_5^+. \qquad (12)$$

The efficiency of the separation of particles with the size [delta] equals

$$\eta = 1 - g_4^- = g_5^+ = 1 - c_4^-(1-K) = c_5^+ K. \qquad (13)$$

Note that at $K = 0$ dust is withdrawn by the inertial ejection of particles on the section length l into the annular space of the cross section 5. In the case of the inertial path of particles, the axial velocity of their ejection in the annular space can be calculated by the formula $V_{x5} = W_3^+ - l/2\tau)$. Estimates show that the axial velocity of the inertial ejection of fine particles with the size of less than 15 μm is negligible.

The distribution of particle concentrations is found by Eq. (4). For any cross section, the distribution of the circumferential velocity of particles $V_\varphi = W_\varphi$ has the following form:

$$V_\varphi = V_\Delta \left(\frac{R_2}{R}\right)^n \quad \text{at } R_2 > R > R_m;$$

$$V_\varphi = V_\Delta \frac{R}{R_m}\left(\frac{R_2}{R_m}\right)^n \quad \text{at } R < R_m,$$

where $V_\Delta = \xi W_1$ is the circumferential velocity of particles at the boundary of the near-wall zone. The value of r_m can be determined from the relationships given in [24] as follows:

$$r_m = 0.35\frac{r_1^{1.5}}{f_1^{0.5}}. \qquad (14)$$

Let us introduce the following notations $f_1 = \frac{F_1}{\pi R_2^2}$, $U_0 = \frac{Q}{\pi R_2^2}$, $Stk = \frac{U_0\tau}{R_2}$, $r = \frac{R}{R_2}$, $\Delta u = \frac{\Delta U}{U_0}$, $\upsilon_\Delta = \frac{V_\Delta}{U_0}$, $\Delta u = \frac{\upsilon^2}{r}Stk$, $\upsilon = \frac{\upsilon_\Delta r}{r_m^{n+1}}$ at $r < r_m$, $\upsilon = \frac{\upsilon_\Delta}{r^n}$ at $r > r_m$. and derive the approximation dependence for the value of $\frac{\upsilon^2}{\mathrm{r}}$ from the relationships $\int_{r_*}^{1}\frac{A}{r}rdr = \; = \int_0^1\frac{\upsilon^2}{r}rdr$, $A = \frac{\int_0^1 \upsilon^2 dr}{1 - r_*}$.. The value of r_* is chosen from the condition $\int_0^{r_*}\upsilon^2 dr = \int_{r_*}^{r_m}\upsilon^2 dr$. Performing some calculations, we obtain

$$A = \upsilon_\Delta^2 \frac{3 - 2r_m^{1-2n}(n+1)}{3(1-2n)(1-r_*)}, \quad r_* = r_m/\sqrt[3]{2} \approx 0.8 r_m. \qquad (15)$$

At $r < r_*$ $A = 0$. In the general case of the change in the circumferential velocity of a gas along the height of a cyclone $A = A_z$; i.e., depends on z. The equation of transport (4) has the form $c\frac{\Delta U R_2}{\varepsilon} = \frac{\partial c}{\partial r}$, $\frac{\Delta U R_2}{\varepsilon} = \frac{\alpha_z}{r} = \Delta u \frac{(n+1)(L/2R_2)}{1 - K + 2m_2}$, $\Delta u = \frac{A}{r}Stk$ where the parameter α_z is the generalized parameter, which characterizes the ratio between the velocity of particles relative to a gas in the averaged motion under the action of the centrifugal force and the velocity of the turbulent mixing of particles in the radial direction, i.e.,

$$c\frac{\alpha_z}{r} = \frac{\partial c}{\partial r}, \qquad (16)$$

where $\alpha_z = A_z Stk \frac{(n+1)(L/2R_н)}{1 - K + 2m_2}$..

The solution of Eq. (16) has the form

$$c = c_{03} = c_{*3}\,|_{r<r_*}, \quad c_3 = c_{03}\left(\frac{r}{r_{*3}}\right)^{\alpha_3}\Bigg|_{r>r_*}, \qquad (17)$$

for the cross section 3–3, and

$$c = c_{02} = c_{*2}\,|_{r<r_*} \quad c_2 = c_{02}\left(\frac{r}{r_{*2}}\right)^{\alpha_2}\Bigg|_{r>r_*}. \qquad (18)$$

for the cross section 2–2.

Taking into consideration Eqs. (17) and (18), we obtain

$$g_2^+ = \frac{2c_{02}}{1 - r_i^2}\frac{1 - r_2^{\alpha_2+2}}{r_{*2}^{\alpha_2}(\alpha_2+2)}(1+m_2) = c_{02}\left[2^+\right](1+m_2),$$

$$g_2^- = \frac{2c_{02}}{r_1^2}\left[\frac{r_{*2}^2}{2} + \frac{r_1^{\alpha_2+2} - r_{*2}^{\alpha_2+2}}{r_{*2}^{\alpha_2}(\alpha_2+2)}\right](1+m_2-K) =$$

$$= c_{02}\left[2^-\right](1+m_2-K),$$

$$g_3^+ = \frac{2c_{03}}{1-r_i^2}\frac{1-r_2^{\alpha_3+2}}{r_{*3}^{\alpha_3}(\alpha_3+2)}(K+m_3) =$$

$$= c_{03}\left[3^+\right](1+m_2-K),$$

$$g_3^- = \frac{2c_{03}}{r_1^2}\left[\frac{r_{*3}^2}{2} + \frac{r_1^{\alpha_3+2} - r_{*3}^{\alpha_3+2}}{r_{*3}^{\alpha_3}(\alpha_3+2)}\right]m_3 = c_{03}\left[3^-\right]m_3,$$

$$g_5^+ = \frac{2c_{*3}}{1-r_3^2}\frac{1-r_3^{\alpha_3+2}}{r_{*3}^{\alpha_3}(\alpha_3+2)} = c_{03}\left[5^+\right]K.$$

The balance relationships for flows will be written as follows

Comultipliers and efficiencies calculated by Eq. (19)

a_z	0	1	2	4	8
B_2	1	0.46	0.21	0.049	0.004
B_3	1	0.30	0.27	0.25	0.25
h	$K = 0.1$	0.44	0.66	0.90	0.990

$$c_{02}\,[2^-](1 - K) + c_{*3}\,[5^+]K = 1,$$

$$c_{02}\,[2^+](1 + m_2) - c_{02}\,[2^-](1 + m_2 - K) = c_{03}\,[3^+](K + m_3) - c_{03}\,[3^-]m_3,$$

$$\eta = c_{03}\,[5^+]K.$$

The fractional efficiency of the concentration of fine particles is equal to

$$\eta = \frac{1}{1 + B_1 B_2 B_3}, \qquad (19)$$

where $B_1 = \frac{1-K}{K}$, $B_2 = \frac{[2^+]}{[5^+]}$, and $B_3 = \frac{[3^+](K+m_3) - [3^-]m_3}{[2^+](1+m_2) - [2^-](1+m_2-K)}$.

The values of B_2, B_3, and η obtained at $K = 0.1$, $m_2 = 0.2$, $m_3 = 0.2$, $r_1 = 0.6$, $r_3 = 0.95$, and $f_1 = 0.13$ depending on α_z on condition that they are constant in the axial direction are listed in the table.

According to Eqs. (9), (11), (14), and (15), at $\rho_\delta/\rho = 2300$, $R_2 = 0.3$ m, $L/2R_2 = 2$, $U_0 = 3.5$ m/s, $\xi = 1.0$, $\nu = 1.5 \times 10^{-5}$ m^2/s we obtain that $n = 0.56$, $\varepsilon = 0.44$ m^2/s, $r_m = 0.45$, $A_z = 112$. The value of α_z is determined by the relationship $\alpha_z = A_z Stk \frac{(n+1)(L/2R_2)}{1 - K + 2m_2} = 269 Stk = 269(U_0\tau/R_2) \times 10^{-12} \approx 0.027\delta^2$, where δ is measured in microns.

In [16] it is shown that the efficiency of a cylindrical countercurrent concentrator with a diameter of 0.130 m in purifying gas from [ok?] concrete dust grows from 0.92 to 0.98 with an increase in K from 0.08 to 0.22. The parameter m_2 tends to zero with increasing K [23]. The represented relationships also indicate growth in the efficiency with decreasing m_2 and increasing K and allow the calculation of the efficiency as a function of other determinative parameters.

CONCLUSIONS

The equation of the transport of a medium consisting of particles in the turbulent motion of an aerosol has a simple solution, if the concept of the equilibrium transport of particles, i.e., of the equality of the radial particle flows induced by turbulent transport and centrifugal forces, is introduced. On the analysis of the known solutions for the turbulent motion of an aerosol in curvilinear channels we may suppose that the process of transport is close to an equilibrium one at any point of the separation chamber of a countercurrent cyclone concentrator. This allows us to derive simple relationships for estimating the fractional efficiency of the separation of particles in a cylindrical countercurrent cyclone gas purifier.

NOTATION

A_z —maximum value of the approximation function for the dimensionless centrifugal acceleration (see Eq. (15));

b —inlet pipe relative width;

C —current concentration of particles with the size δ in a gas, g/m^3;

C_1 —concentration of particles with the size δ in a gas at the inlet of a cyclone, g/m^3;

$c = C/C_1$—dimensionless concentration of particles with the size δ;

D_n —particle diffusion coefficient, m^2/s;

F_1—inlet pipe surface area, m^2;

$f_1 = \frac{F_1}{\pi R_2^2}$ —dimensionless inlet surface area;

G —flow rate of dust particles with the diameter δ at the inlet of a cyclone, g/s;

$G_k^- = 2\pi \int_0^{R_1} C_k W_k R dR$ —flow rate of particles with the diameter δ in the central zone and the kth cross section, g/s;

$G_k^+ = 2\pi \int_{R_i}^{R_2} C_k W_k R dR$ —flow rate of particles with the diameter δ in the peripheral zone and the kth cross section, g/s;

$K = Q_5/Q$ —coefficient of the withdrawal of the gas separated from particles;

L —cyclone body length;

l —distance between the cross sections 3–3 and 5–5 in the figure, m;

n —exponent in the equation for the distribution of circumferential velocities;

Q —inlet gas flow rate, m^3/s;

Q_5—flow rate of the gas separated from particles in the cross section 5–5, m^3/s;

$Q_k^- = 2\pi \int_0^{R_1} W_k R dR$ —gas flow rate in the central zone and the kth cross section, m^3/s;

$Q_k^+ = 2\pi \int_{R_i}^{R_2} W_k R dR$—gas flow rate in the peripheral zone and the kth cross section, m^3/s;

R_1—central gas withdrawal pipe radius, m;

R_2 —cyclone body radius, m;

R_3 —cyclone bottom radius, m;

R_i —radial gas outflow core boundary radius, m;

R_m —maximum circumferential velocity radius, m;

$r = \frac{R}{R_2}$, $r_3 = \frac{R_3}{R_2}$, $r_1 = \frac{R_1}{R_2}$, $r_i = \frac{R_i}{R_2}$, $r_m = \frac{R_m}{R_2}$, $r_* = \frac{R_*}{R_2} \approx 0.8 r_m$ —dimensionless radii;

$Stk = \frac{U_0 \tau}{R_2}$ —inertial parameter;

$U_0 = \frac{Q}{\pi R_2^2}$ —average flow rate velocity of a gas in a cyclone, m/s;

ΔU —velocity of a particle relative to a gas under the action of the centrifugal force, m/s;

$\Delta u = \frac{\Delta U}{U_0}$ —dimensionless velocity of a particle relative to a gas under the action of the centrifugal force;

V_r, V_φ, V_z—radial, tangential, and axial components of the velocity of particles, m/s;

V_Δ —circumferential velocity of particles at the near-wall zone boundary, m/s;

W_z, W_φ, W_r—axial, circumferential, and radial gas velocities, m/s;

$W_{\varphi\Delta}$ —circumferential velocity of a gas at the near-wall zone boundary, m/s;

$\alpha_z = r\frac{\Delta U R_2}{\varepsilon} = r\Delta u \frac{(n+1)(L/2R_2)}{1 - K + 2m_2}$ —generalized parameter;

δ —particle diameter, m or μm;

ε —turbulent gas mixing coefficient, m^2/s;

η—separation efficiency of particles with the diameter δ;

μ_T —dynamic turbulent gas mixing coefficient, Pa s;

ν —kinematic viscosity coefficient, m^2/s;

$\upsilon = \frac{V_\varphi}{U_0}$ —dimensionless circumferential velocity of a particle;

$\upsilon_\Delta = \frac{V_\Delta}{U_0}$ —dimensionless circumferential velocity of particles at the near-wall zone boundary;

$\xi = W_{\varphi\Delta}/W_1$ —circumferential gas velocity drop coefficient at the near-wall zone boundary;

ρ, ρ_δ —gas and particle densities, kg/m^3;

$\tau_{r\varphi}$ —friction stress component, Pa;

$\tau = \rho_\delta \delta^2/\rho \times 18\nu$ —relaxation time of a particle, s;

φ —angular coordinate;

χ —turbulence constant;‘

ω_E —pulsation frequency of energy-intensive vortexes of a medium, 1/s.

SUBSCRIPTS AND SUPERSCRIPTS

+—peripheral area;

−—central area;

k —cross sections 2, 3, and 4;

r, z, φ —coordinates.

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