Mathematical modeling

Mikhail E. Semenov

2012

About me

- 3rd year at TPU
- Graduated from TSU in 2000
  - PhD Math
  - Thesis title: *Math model and software for research features of plastic deformation in FCC materials*

  Research areas:
  - Ordinary differential equations
  - Scientific computing and numerical methods
  - Mathematical modeling in material science

Lectures and Notes

- Lectures are electronic and will be available online after class.
- All homework, and most other lecture material will be available online

http://portal.tpu.ru/SHARED/s/SME/work/MathMod

Mathematical Modeling

- What is Mathematical Modeling

  …and how do you spell it?

  "Mathematics consists of the study and development of methods for prediction"

  Modeling = The use of mathematics as a tool to explain and make predictions of natural phenomena

One-variable optimization

One-variable optimization problems, sometimes called **maximum-minimum** problems, are typically discussed in first-semester calculus. A wide variety of practical applications can be handled using just these techniques.

One-variable optimization

One-variable optimization problems, sometimes called **maximum-minimum** problems, are typically discussed in first-semester calculus. A wide variety of practical applications can be handled using just these techniques.

One-variable optimization

The purpose of this lecture 1, aside from a review of these basic techniques, is to introduce the fundamentals of mathematical modeling in a familiar setting.

1.1. The Five-Step Method

In this section we outline a general procedure that can be used to solve problems using mathematical modeling. We will illustrate this procedure, called the *five-step method,* by using it to solve a one-variable maximum-minimum problem.

1.1. The Five-Step Method

Example. A pig weighing 200 pounds gains 5 pounds per day and costs 45 cents a day to keep. The market price for pigs is 65 cents per pound, but is falling 1 cent per day. When should the pig be sold?

1.1. The Five-Step Method

The mathematical modeling approach to problem solving consists of five steps:

- Ask the question.
- Select the modeling approach.
- Formulate the model.
- Solve the model.
- Answer the question.

1.1. The Five-Step Method

The question must be phrase in Math terms. We need to define our terms.

Go through the problem and make list of **variables**.

Next make a list of **assumptions** about these variables.

Write down in explicit math language the **objective** of this problem.

1.1. The Five-Step Method

In Example the weight w of the pig (in lbs), the number of days t until we sell the pig, the cost C of keeping the pig t days (in dollars), the market price p for pigs ($/lb), the revenue R obtained when we sell to pig ($), and out resulting net profit P ($) are all **variables**.

1.1. The Five-Step Method

The **assumption** inherent in our problem are as follows:

Our **objective** in this problem is to maximize our net profit, *P* dollars.

1.1. The Five-Step Method

Step 2 is to select the modeling approach. Now that we have a question stated in mathematical language, we need to select a mathematical approach to use to get an answer.

Our example problem will be modeled as a **one-variable optimization** problem, or maximum-minimum problem.

1.1. The Five-Step Method

Step 3 is to formulate the model.

Step 4 is to solve the model.

1.1. The Five-Step Method

Theorem. If f attains its maximum or minimum at an interior point x in subset S of real line, then $f'(x)\neq 0$, assuming that f is differentiable at x. This allows us to rule out any interior point x in S at which $f'(x)\neq 0$ as a candidate for max/min.

1.1. The Five-Step Method

Step 5 is to answer the questions.

1.2. Sensitivity Analysis

The preceding section outlines the five-step approach to math. modeling.

We are rarely certain enough about things to be able to expect all of these assumptions to be exactly valid. Therefore, we need to consider how **sensitive** our conclusions are to each of the assumptions we have made.

1.2. Sensitivity Analysis

Some data are naturally known with much more certainty than others.

The current weight of the pig, the current price for pigs, and the cost per day of keening the pig are easy to measure and are known to a great degree of certainty.

1.2. Sensitivity Analysis

The rate of growth of the pig is a bit less certain, and the rate at which the price is falling is even less certain.

1.2. Sensitivity Analysis

It is most natural and most useful to interpret sensitivity data in t of relative change or percent change, rather than in absolute terms.

1.2. Sensitivity Analysis

If x changes by an amount Δx, the relative change in x is given by Δx/x, and the percent change in x is

100 Δx/x.

If r changes by Δr, resulting in the change Δx in x, then the rst£ between the relative changes is Δx/x divided by Δr/r.

1.2. Sensitivity Analysis

Letting Δr->0 using the definition of the derivative, we obtain

Δx/x: Δr/r = dx/dr * (x/r).

We call this limiting quantity the sensitivity of x to r, and we will denote it by S(x, r).

1.2. Sensitivity Analysis

In the our Example we have….

Homework

An automobile manufacturer makes a profit of $1,500 on the sale of a certain model. It is estimated that for every $100 of rebate, sales increase by 15%.

(a) What amount of rebate will maximize profit? Use the five-step method, and model as a one-variable optimization problem.

(b) Compute the sensitivity of your answer to the 15% assumption. Consider both the amount of rebate and the resulting profit.

(c) Suppose that rebates actually generate only a 10% increase in sales per $100. What is the effect? What if the response is somewhere between 10 and 15% per $100 of rebate?

(d) Under what circumstances would a rebate offer cause a reduction in profit?

Further Reading

Meerschaert, Mark M. Mathematical Modeling / M. M. Meerschaert. — Second edition. — San Diego: Academic Press, 1999. — 351 p.

00-1219, 00-1220 519 M47