Lectures on nonlinear equations of mathematical physics

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COURSE DESCRIPTION (Nonlinear Equations of Mathematical Physics)

Course code	Course group	Volume in ECTS credits	Course valid from	Course valid to	Reg. No.
B1	541-546	2	2016 09 01	2016 12 31	

Course type (compulsory or optional)	**Compulsory**
Course level (study cycle)	**3 year(**bachelor level**)**
Semester the course is delivered	**5 term, autumn**
Study form (face-to-face or distant)	**face-to-face**

Course title in English

Nonlinear Equations of Mathematical Physics

Short course annotation in English (up to 500 characters)

The aim of the course is to study the basic concepts and methods of nonlinear mathematical physics. Learning the "Non-linear equations of mathematical physics" course (NEMP) involves the following tasks:
- studying the general properties of nonlinear partial differential equations, they differ from linear ones;
- studying the simple models of nonlinear thermal conductivity, reaction-diffusion phenomena, nonlinear waves;
- studying the foundations of the theory of solitons on the examples simple soliton equations;
- developing the skills of construction and analysis models of nonlinear phenomena.

Prerequisites for entering the course

General mathematics, general physics (at bachelor level), differential and integral calculus, linear algebra, ordinary differential equations, integral equations, the theory of functions of a complex variable, and linear equations of mathematical physics.

Course aim

Provide introduction to Nonlinear Science

Links between study programme outcomes, course outcomes and criteria of learning achievement evaluation

Study programme outcomes	Course outcomes	Criteria of learning achievement evaluation
1. To master basic ideas and specific features of nonlinear partial differential equations, nonlinear phenomena and their models.	1. To explain the role of classification in study of nonlinear partial differential equations.	Description of specific properties of nonlinear partial differential equations.
	2. An overview of revealed characteristic nonlinear phenomena. They include, in particular, nonlinear heat transfer, reaction-diffusion systems, waves in nonlinear media, generation of solitary waves (solitons) in nonlinear dispersive media.	Classification of quasilinear partial differential equations in particular cases.
2. To master basic models of nonlinear heat transfer.	3. To analyze basic models of heat distribution in two-phase medium.	Explanation of the Stephan model results. Test tasks..
	4. To explain basic regularities of the heat propagation from a point-like source through nonlinear medium.	Understanding of heat transfer from an instantaneous point-like source. Test tasks.
3. To study the pattern formation in a nonlinear system of the reaction-diffusion type.	5. To analyze conditions of the steady-state stability in the Turing model.	Understanding of dissipative structures in reaction-diffusion systems.
	6. To analyze types of instability in the Turing model.	Analysis of the stability loss of the steady-state in the Turing type system.
	7. To analyze the Gierer - Meinhardt model and the Brusselator model.	Assessment of the system parameters by examples.

Link between course outcomes and content

Course outcomes	Content (topics)
1. To explain the characteristic features of nonlinear partial differential equations (PDEs).	Nonlinear systems and their models. The canonical forms of the second-order quasi-linear PDE.
2. To use experimental techniques to measure the level of various types of radiation, dosimetry.	Linear and nonlinear heat phenomena. The physical laws of heat propagation. The Fourier Law. Deviation from the linear heat law.
3. Stefan's problem of phase transition.	Self-similar variables and self-similar solutions of nonlinear equations. The heat distribution in the phase change medium. Stefan's problem solution for the phase transition.
4. Distribution of thermal perturbations in the nonlinear medium from the instantaneous point source.	Distribution of thermal perturbations in the nonlinear medium. Heat propagation from the instant heat source. Self-similar solutions of nonlinear heat equations.
5. Systems of the reaction-diffusion type.	Distributed systems with feedback. Positive and negative feedback. The concept of activators and inhibitors. Dissipative structures and self-organization in active media.

Study (teaching and learning) methods

Teaching methods: lectures, discussions

Learning methods: discussions, reading, seminars, consultations

Methods of learning achievement assessment

Individual tasks, testing

Distribution of workload for students (contact and independent work hours)

Lectures	34
Seminars	18
Individual students work	20
Consutations	4
Total:	76

Structure of cumulative score and value of its constituent parts

Midterm – 25 %, practical work – 25 %, exam – 50%.

Recommended reference materials

No.	Publication year	Authors of publication and title	Publishing house	Number of copies in		
				University library	*Self-study rooms*	*Other libraries*
			Basic materials			
1.	1984	Dodd R. K., et. al. Solitons and Nonlinear Wave Equations.	Academic Press	2		
2.	2011	Ablowitz M. Nonlinear Dispersive Waves: Asymptotic Analysis and Solitons (Cambridge Texts in Applied Mathematics)	Cambridge University Press	3		
3.	1987	Newell A. C. Solitons in Mathematics and Physics (CBMS-NSF Regional Conference Series in Applied Mathematics)	Society for Industrial and Applied Mathematics	2		
			Supplementary materials			
		http://www.scholarpedia.org/article/Linear_and_nonlinear_waves	E-resource, Scholarpedia			
		https://www.tu-chemnitz.de/physik/KSND/abb/node6.html	E-resource	1		

Examples of the lectures

Variety of nonlinear phenomena

Nonlinear waves

- Waves occur in most scientific and engineering disciplines, for example: fluid mechanics, optics, electromagnetism, solid mechanics, structural mechanics, quantum mechanics, etc. The waves for all these applications are described by solutions to either linear or nonlinear PDEs.
- Figure below shows a plane wave in ocean that exists far from its source and any physical boundaries so, effectively, it is located within an infinite domain. Its position vector remains perpendicular to a given plane and satisfies the 1D wave equation.

Figure 2: A plain wave in ocean

- Surfing is a surface water sport in which the wave rider, referred to as a surfer, rides on the forward or deep face of a moving wave, which is usually carrying the surfer towards the shore. Surfers uses nonlinear properties of waves.

Figure 3: A surfing in the ocean

Reaction diffusion systems and nonlinear waves

- Reaction-diffusion models have found numerous applications in pattern formation in biology, chemistry, and physics. These systems show that diffusion can produce the spontaneous formation spatio-temporal patterns.
- Figure below gives an example of a spiral wave.

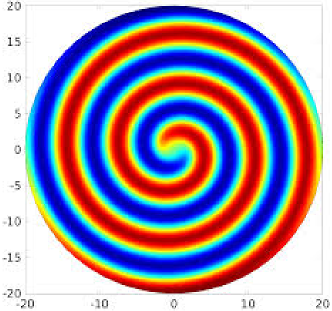

Figure 4: A spiral wave in a reaction diffusion system

Next we go to study mathematical background.

Lecture 4: The two phase Stefan problem

- In mathematics and its applications, particularly to phase transitions in matter, a Stefan problem (also Stefan task) is a particular kind of boundary value problem for a partial differential equation (PDE), adapted to the case in which a phase boundary can move with time.

Figure 7: Josef Stefan, Austrian physicist and mathematician

- The classical Stefan problem aims to describe the temperature distribution in a homogeneous medium undergoing a phase change, for example ice passing to water: this is accomplished by solving the heat equation imposing the initial temperature distribution on the whole medium, and a particular boundary condition, the Stefan condition, on the evolving boundary between its two phases.