Individual Task 5

Fredholm integral equations. Part IV

Case 1

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\,ds = 2x - \pi.$
   1.2. $y(x) - \int_{-1}^{1} \exp(\arcsin x)\, y(s)\,ds = \operatorname{tg} x.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s)ds = 0.$
   2.2. $y(x) - \lambda\int_{-\pi/2}^{\pi/2} (x^2 sin\ s + t^2 \cos x) y(s)ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2s^2; \qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} x(s-1), & 0 \le x \le s \\ s(x-1), & \mathrm{s} \le x \le 1 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 2

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\,ds = 2x - \pi.$
   1.2. $y(x) - \lambda\int_{-\pi/4}^{\pi/4} \operatorname{tg} s\, y(s)\,ds = \operatorname{ctg} x.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s)ds = 0.$
   2.2. $y(x) - \lambda\int_0^{2\pi} sin\ x \cdot \sin s \cdot y(s)ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2s^2; \qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} s(x+1), & 0 \le x \le s \\ x(s+1), & \mathrm{s} \le x \le 1 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 3

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\, ds = 2x - \pi.$
   1.2. $y(x) - \lambda \int_0^1 \arccos(s)\, y(s)\, ds = \frac{1}{\sqrt{1-x^2}}.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda \int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s) ds = 0.$
   2.2. $y(x) - \lambda \int_0^{\pi} \cos(x+s) y(s) ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2 s^2; \qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} (x+1)(s-2), & 0 \le x \le s \\ (s+1)(x-2), & s \le x \le 1 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 4

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\, ds = 2x - \pi.$
   1.2. $y(x) - \lambda \int_0^1 (x \ln s - s \ln x)\ y(s)\, ds = \frac{6}{5}(1-4x).$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda \int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s) ds = 0.$
   2.2. $y(x) - \lambda \int_0^1 (45x^2 \ln s - 9s^2 \ln x)\, y(s) ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2 s^2; \qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} \sin x \cos s, & 0 \le x \le s \\ \sin s \cos x, & s \le x \le \frac{\pi}{2} \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 5

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4 \int_0^{\pi/2} \sin^2 x \cdot y(s)\, ds = 2x - \pi.$
   1.2. $y(x) - \lambda \int_0^{\pi/2} \sin x \cos s\, y(s)\, ds = \sin x.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda \int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s) ds = 0.$
   2.2. $y(x) - \lambda \int_0^{1} (2xs - 4x^2) y(s) ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2 s^2; \qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} \sin x \cos s, & 0 \le x \le s \\ \sin s \cos x, & s \le x \le \pi \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 6

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4 \int_0^{\pi/2} \sin^2 x \cdot y(s)\, ds = 2x - \pi.$
   1.2. $y(x) - \lambda \int_0^{\pi} \sin(x - s)\, y(s)\, ds = \cos x.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda \int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s) ds = 0.$
   2.2. $y(x) - \lambda \int_{-1}^{1} (5xs^3 + 4x^2 s) y(s) ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2 s^2; \qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} \sin x \sin(s-1), & -\pi \le x \le s \\ \sin s \sin(x-1), & s \le x \le \pi \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 7

1. Find the solution of integral equations with degenerate kernels:

   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\, ds = 2x - \pi.$

   1.2. $y(x) - \lambda\int_0^{2\pi}(\sin x \cos s - \sin 2x \cos 2s + \sin 3x \cos 3s) y(s)\, ds = \cos x.$

2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:

   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s) ds = 0.$

   2.2. $y(x) - \lambda\int_{-1}^{1}(5xs^3 + 4x^2 s + 3xs) y(s) ds = 0.$

3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:

   3.1. $K(x,s) = 1 + xs + x^2 s^2; \qquad -1 \le x,\ s \le 1.$

   3.2. $K(x,s) = \begin{cases} \sin\left(x + \frac{\pi}{4}\right) \sin\left(s - \frac{\pi}{4}\right), & 0 \le x \le s \\ \sin\left(s + \frac{\pi}{4}\right) \sin\left(x - \frac{\pi}{4}\right), & s \le x \le \pi \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 8

1. Find the solution of integral equations with degenerate kernels:

   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\, ds = 2x - \pi.$

   1.2. $y(x) - \frac{1}{2}\int_{-1}^{1}\left(x - \frac{1}{2}(3s^2 - 1) + \frac{1}{2}s(3x^2 - 1)\right) y(s)\, ds = 1.$

2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:

   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s) ds = 0.$

   2.2. $y(x) - \lambda\int_{-1}^{1}(x \cosh s - s \sinh x) y(s) ds = 0.$

3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:

   3.1. $K(x,s) = 1 + xs + x^2 s^2; \qquad -1 \le x,\ s \le 1.$

   3.2. $K(x,s) = \begin{cases} -e^{-s} \sinh x, & 0 \le x \le s \\ -e^{-x} \sinh s, & s \le x \le 1 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 9

1. Find the solution of integral equations with degenerate kernels:

   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\,ds = 2x - \pi.$

   1.2. $y(x) - \lambda\int_0^{\pi} |\pi - s| \sin x\, y(s)\,ds = x.$

2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:

   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s)ds = 0.$

   2.2. $y(x) - \lambda\int_{-1}^{1} (19\,x - 19x \cdot s + 35s^2x^2)y(s)ds = 0.$

3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:

   3.1. $K(x,s) = 1 + xs + x^2s^2; \qquad -1 \le x,\ s \le 1.$

   3.2. $K(x,s) = \begin{cases} \sin\left(3s - \frac{\pi}{4}\right)(\cos 3x + 6\sin 3x), & \pi/6 \le x \le s \\ \sin\left(3x - \frac{\pi}{4}\right)(\cos 3s + 6\sin 3s), & s \le x \le \pi/3 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 10

1. Find the solution of integral equations with degenerate kernels:

   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\,ds = 2x - \pi.$

   1.2. $y(x) - \lambda\int_0^{1} \mathrm{x}\sin(2\pi s)\, y(s)\,ds = x.$

2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:

   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s)ds = 0.$

   2.2. $y(x) - \lambda\int_{-1}^{1} (x\cosh s - s^2\cosh x)y(s)ds = 0.$

3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:

   3.1. $K(x,s) = 1 + xs + x^2s^2; \qquad -1 \le x,\ s \le 1.$

   3.2. $K(x,s) = \begin{cases} \sinh(x-1)\cosh s, & 0 \le x \le s \\ \sinh(s-1)\cosh x, & s \le x \le 1 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 11

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\,ds = 2x - \pi.$
   1.2. $y(x) - \int_0^1 s\ \arcsin x\, y(s)\,ds = \arccos x.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s)ds = 0.$
   2.2. $y(x) - \lambda\int_0^{2\pi} sin\ x \cdot \cos s \cdot y(s)ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2s^2;\qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} e^{(x-s)/4}, & -4 \le x \le s \\ e^{(s-x)/4}, & s \le x \le 4 \end{cases}.$

Individual Task 5

Fredholm integral equations. Part IV

Case 12

1. Find the solution of integral equations with degenerate kernels:
   1.1. $y(x) - 4\int_0^{\pi/2} \sin^2 x \cdot y(s)\,ds = 2x - \pi.$
   1.2. $y(x) - \lambda\int_0^{2\pi} \sin^2 x \sin^2 s\, y(s)\,ds = \cos x.$
2. Find the characteristic numbers and eigenfunctions for equations with degenerate kernel:
   2.1. $y(x) - \lambda\int_0^{\frac{\pi}{4}} \sin^2 x \cdot y(s)ds = 0.$
   2.2. $y(x) - \lambda\int_{-\pi}^{\pi} (s^2\, sin\ x + x\sin^2 s)y(s)ds = 0.$
3. Find the characteristic values and eigenfunctions of homogeneous integral equations with symmetric kernels, if the kernel are as follows:
   3.1. $K(x,s) = 1 + xs + x^2s^2;\qquad -1 \le x,\ s \le 1.$
   3.2. $K(x,s) = \begin{cases} (x+1)(s-3), & 0 \le x \le s \\ (s+1)(x-3), & s \le x \le 1 \end{cases}.$