Individual task 1

Volterra integral equations

Case 1

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :

   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2} \varphi(t)dt$

   1.2. $\varphi(x) = e^x(\cos e^x - e^x \sin e^x)$; $\varphi(x) = (1 - xe^{2x}) \cos 1 - e^{2x} \sin 1 + \int_0^x [1 - (x-t)e^{2x}]\varphi(t)dt$

2. Solve the integral equation using differentiation method:

   2.1. $y(x) = x - \int_0^x e^{x-s} y(s)ds$.

   2.2. $\int_0^x e^{x+s} y(s)ds = x$.

3. Find the resolvent for Volterra integral equation using kernel K(x,s):

   3.1. $K(x,s) = x - s$.

   3.2. $K(x,s) = e^{x-s}$.

Individual task 1

Volterra integral equations

Case 2

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :

   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2} \varphi(t)dt$

   1.2. $\varphi(x) = xe^x$; $\varphi(x) = e^x \sin x + 2\int_0^x \cos(x-t)\, \varphi(t)dt$

2. Solve the integral equation using differentiation method:

   2.1. $y(x) = x - \int_0^x e^{x-s} y(s)ds$.

   2.2. $\int_0^x e^{x-s} y(s)ds = x$.

3. Find the resolvent for Volterra integral equation using kernel K(x,s):

   3.1. $K(x,s) = x - s$.

   3.2. $K(x,s) = \frac{1+x^2}{1+s^2}$.

Individual task 1

Volterra integral equations

Case 3

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :

   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2} \varphi(t)dt$

   1.2. $\varphi(x) = x - \frac{x^3}{6}$; $\varphi(x) = x - \int_0^x \sinh(x-t)\, \varphi(t)dt$

2. Solve the integral equation using differentiation method:

   2.1. $y(x) = x - \int_0^x e^{x-s} y(s)ds$.

   2.2. $y(x) = \int_0^x \frac{s}{s+1} y(s)ds + e^x$.

3. Find the resolvent for Volterra integral equation using kernel K(x,s):

   3.1. $K(x,s) = x - s$.

   3.2. $K(x,s) = e^{x^2-s^2}$.

Individual task 1

Volterra integral equations

Case 4

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x)=\frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x)=\frac{3x+2x^3}{3(1+x^2)^2}-\int_0^x\frac{3x+2x^3-t}{(1+x^2)^2}\varphi(t)dt$
   1.2. $\varphi(x)=3$; $x^3=\int_0^x(x-t)^2\varphi(t)dt$
2. Solve the integral equation using differentiation method:
   2.1. $y(x)=x-\int_0^x e^{x-s}y(s)ds.$
   2.2. $y(x)=\int_0^x(x-s)y(s)ds+2\operatorname{sh}x$.
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s)=x-s.$
   3.2. $K(x,s)=\frac{2+\cos x}{2+\cos s}$.

Individual task 1

Volterra integral equations

Case 5

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x)=\frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x)=\frac{3x+2x^3}{3(1+x^2)^2}-\int_0^x\frac{3x+2x^3-t}{(1+x^2)^2}\varphi(t)dt$
   1.2. $\varphi(x)=\frac{1}{2}$; $\int_0^x\frac{\varphi(t)}{\sqrt{x-t}}dt=\sqrt{x}$
2. Solve the integral equation using differentiation method:
   2.1. $y(x)=x-\int_0^x e^{x-s}y(s)ds.$
   2.2. $y(x)=4\int_0^x(s-x)y(s)ds+3\cos x.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s)=x-s.$
   3.2. $K(x,s)=\frac{\cosh x}{\cosh s}$.

Individual task 1

Volterra integral equations

Case 6

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x)=\frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x)=\frac{3x+2x^3}{3(1+x^2)^2}-\int_0^x\frac{3x+2x^3-t}{(1+x^2)^2}\varphi(t)dt$
   1.2. $\varphi(x)=\frac{1}{\pi\sqrt{x}}$; $\int_0^x\frac{\varphi(t)}{\sqrt{x-t}}dt=1$
2. Solve the integral equation using differentiation method:
   2.1. $y(x)=x-\int_0^x e^{x-s}y(s)ds.$
   2.2. $y(x)=\int_0^x\{3(x-s)-(x-s)^2\}y(s)ds+e^{2x}-2x^2-2x-1.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s)=x-s.$
   3.2. $K(x,s)=2-(x-s)$, assuming that $\lambda=1$.

Individual task 1

Volterra integral equations

Case 7

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2} \varphi(t)dt$
   1.2. $\varphi(x) = xe^x$; $\varphi(x) = e^x \sin x + 2\int_0^x \cos(x-t)\,\varphi(t)dt$
2. Solve the integral equation using differentiation method:
   2.1. $y(x) = x - \int_0^x e^{x-s} y(s)ds.$
   2.2. $y(x) = \int_0^x \cos(x-s)y(s)ds + x.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s) = x - s.$
   3.2. $K(x,s) = -2 + 3(x-s)$, assuming that $\lambda = 1$.

Individual task 1

Volterra integral equations

Case 8

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2} \varphi(t)dt$
   1.2. $\varphi(x) = \frac{1}{\pi\sqrt{x}}$; $\int_0^x \frac{\varphi(t)}{\sqrt{x-t}} dt = 1$
2. Solve the integral equation using differentiation method:
   2.1. $y(x) = x - \int_0^x e^{x-s} y(s)ds.$
   2.2. $y(x) = \int_0^x \{2e^{2(x-s)} - e^{3(x-s)}\} y(s)ds + 5.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s) = x - s.$
   3.2. $K(x,s) = 2x$, assuming that $\lambda = 1$.

Individual task 1

Volterra integral equations

Case 9

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2} \varphi(t)dt$
   1.1. $\varphi(x) = \frac{1}{\pi\sqrt{x}}$; $\int_0^x \frac{\varphi(t)}{\sqrt{x-t}} dt = 1$
2. Solve the integral equation using differentiation method:
   2.1. $y(x) = x - \int_0^x e^{x-s} y(s)ds.$
   2.2. $y(x) = \int_0^x \frac{s+2}{(x+2)^2} y(s)ds + 2x.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s) = x - s.$
   3.2. $K(x,s) = \frac{1-x^2}{1-s^2}.$

Individual task 1

Volterra integral equations

Case 10

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2}\varphi(t)dt$
   1.2. $\varphi(x) = 3$; $x^3 = \int_0^x (x-t)^2\varphi(t)dt$
2. Solve the integral equation using differentiation method:
   2.1. $y(x) = x - \int_0^x e^{x-s}y(s)ds.$
   2.2. $y(x) = 1 - \int_0^x \frac{(s-1)^2}{s^2+1}e^{x-s}y(s)ds.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s) = x - s.$
   3.2. $K(x,s) = -\frac{4x-2}{2x+1} + \frac{8(x-s)}{2x+1}$, assuming that $\lambda = 1$.

Individual task 1

Volterra integral equations

Case 11

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2}\varphi(t)dt$
   1.1. $\varphi(x) = \frac{1}{2}$; $\int_0^x \frac{\varphi(t)}{\sqrt{x-t}}dt = \sqrt{x}$
2. Solve the integral equation using differentiation method:
   2.1. $y(x) = x - \int_0^x e^{x-s}y(s)ds.$
   2.2. $y(x) = \int_0^x \frac{e^s}{e^x+1}y(s)ds + e^{-x}.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s) = x - s.$
   3.2. $K(x,s) = \frac{\cosh x}{\cosh s}$.

Individual task 1

Volterra integral equations

Case 12

1. Prove that functions $\varphi(x)$ are solutions of corresponding integral equations :
   1.1. $\varphi(x) = \frac{x}{(1+x^2)^{\frac{5}{2}}}$; $\varphi(x) = \frac{3x+2x^3}{3(1+x^2)^2} - \int_0^x \frac{3x+2x^3-t}{(1+x^2)^2}\varphi(t)dt$
   1.2. $\varphi(x) = 3$; $x^3 = \int_0^x (x-t)^2\varphi(t)dt.$
2. Solve the integral equation using differentiation method:
   2.1. $y(x) = x - \int_0^x e^{x-s}y(s)ds.$
   2.2. $y(x) = 2\int_0^x \frac{s}{(x+1)(s+1)}y(s)ds + \frac{\ln(x+1)}{x+1}.$
3. Find the resolvent for Volterra integral equation using kernel K(x,s):
   3.1. $K(x,s) = x - s.$
   3.2. $K(x,s) = \frac{2+\cos x}{2+\cos s}$.