INTRODUCTION

The computer simulation of a random phenomenon consists in generation of sequences of random numbers which are statistically equivalent to the sequences obtained in the real experiment with a random outcome which can be repeated many times under the same conditions.

Example 1. An experiment with multiple tossing of a fair coin gives the sequence of random numbers $S = 0$ (”tails”) and $S = 1$ (”heads”) with probabilities $P(0) = P(1) = 1/2$. In a computer simulation of this experiment such numbers can be obtained by the formula

$$S = [2\gamma_i],$$

γ_i being the random numbers uniformly distributed on the interval $(0, 1)$. The symbol $[q]$ means the integer part of a number q.

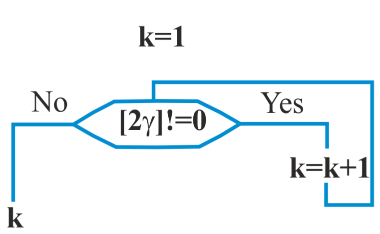
Example 2. The random numbers $1, 2, ...6$ with equal probabilities which can be observed in the experiment consisting in rolling of a fair die can be obtained by the formula

$$S = [6\gamma] + 1.$$

Example 3. If the range (a, b) where the random numbers x_i take values is divided into n equal parts then index s_i of the interval where x_i appears can be calculated by the formula

$$S = [n\frac{(x_i - a)}{b - a}] + 1.$$

Example 4. The random number s which is equal to the number of tosses of a fair coin until the first ”tail” appears can be obtained by the flowchart and listing are given in Fig. 1.



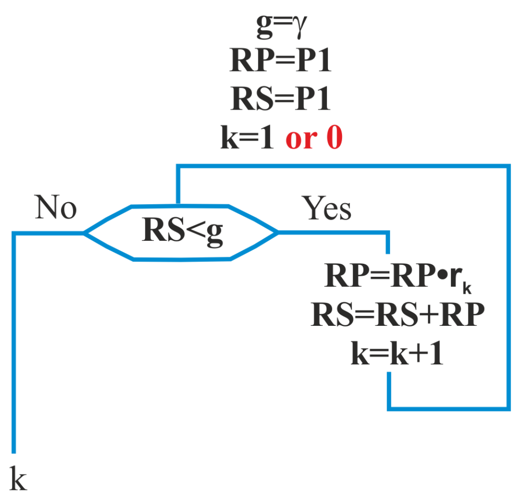

```
S:=(k=1;

  While[IntegerPart[2 RandomReal[]]!=0,
        k=k+1];

  Return[k];)
```

Figure 1: The flowchart and code describing the subroutine for the simulation of a coin tossing until the first ”tail” appears.

Example 5. Let the range $(0, 1)$ where the uniformly distributed random numbers γ vary is divided into unequal parts with the lengths P_k which are determined by the recurrent formula

$$P_{k+1} = r_k P_k, \ (\sum_k P_k = 1). \tag{1}$$

The index k of the interval where γ appears can be determined using the flowchart given in Fig. 2

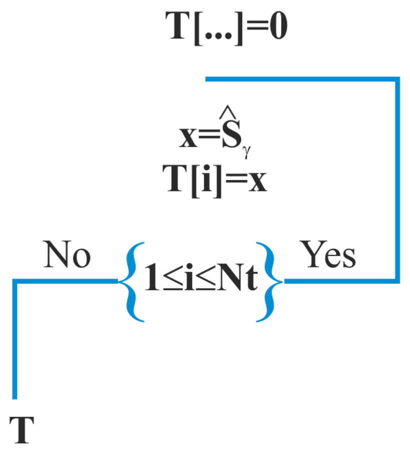



```
P1=?;
r[k_]:=?;
.........................
S:=(g=Random[];
   RP=P1;
   RS=P1;
   k=1;

   While[RS<g,
        RP*=r[k];
        RS+=RP;
        k+=1];

  Return[k];)
```

Figure 2: The flowchart and code describing the subroutine for the simulation of the interval index for the probabilities given by the recurrent formula (1).

The problems of statistical simulation considered below can be classified into 3 groups.

1. One needs to get Nt random numbers T_i and print them. The code of the program which solves this problem is given in Fig. 3




```
Nt=?;
S:=?;
.........................

T=Table[0,{Nt}];

Do[x=S;
   T[[i]]=x,

   {i,1,Nt}];

T
```

Figure 3: The code describing the program for the simulation of Nt random numbers.

Or you could use this code:

```
Nt=?
S:=?
T=Table[S,{i,Nt}]
Histogram[T,15,''PDF'']
```

2. One needs to get Nt random numbers T_i from the given distribution and calculate the mean value

$$\bar{s} = \frac{1}{N} \sum_{i=1}^{N} T_i.$$

The code of the program AVERAGE which solves this problem is given in Fig. 4

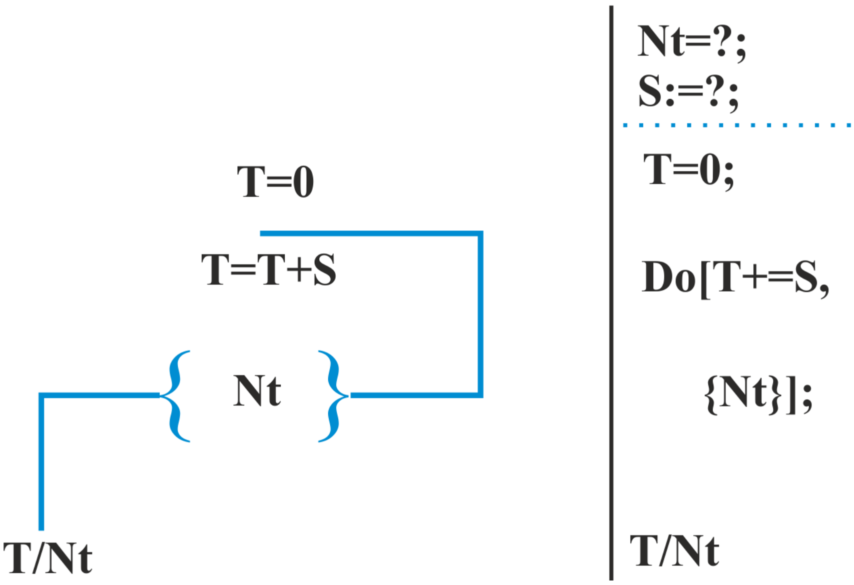


Figure 4: The code of the program for the calculation of the mean value of a random quantity.

Or you could use this simple code:

```
Nt=?
S:=?
T=Table[S,{i,Nt}]
Mean[T]
```

UNIT 2

RANDOM POINTS IN MULTI-DIMENSIONAL SPACE

QUESTIONS

1. Describe the algorithm of the simulation of random points uniformly distributed on a rectangle and in a multi-dimensional parallelepiped.
2. Describe the algorithm of the simulation of random points uniformly distributed on a disc and in a ball.
3. Describe the statistical method for the calculation of bodies areas and volumes.
4. Describe the statistical method for the calculation of the mass, the center of gravity position and the moment of inertia for a specified body.
5. Describe the statistical method for the estimation of single and multiple integrals.

EXERCISES

1. Write the formulas for the simulation of random points uniformly distributed on the rectangle $(b-a) \times b$. Use them to get coordinates of a few points.
2. Write the formulas for the simulation of random points uniformly distributed in the brick with the sides a, b, c. Use them to get coordinates of a few points.
3. Make up the subroutine for the simulation of random points uniformly distributed on the disc of radius R. Use them to get coordinates of a few points.
4. Make up the subroutine for the simulation of the random points uniformly distributed in the ball of radius R. Use them to get coordinates of a few points.
5. Make up the subroutine for the simulation of the random points uniformly distributed on the rectangle $(b-a) \times b$ and appearing under the curve $f(x)$ inscribed in the rectangle. Use it to get a few values of the random x.

PROBLEMS

1. Write the program C-DISTRIBUTION for the simulation of N random points uniformly distributed on the rectangle $(b-a) \times b$. Present

the simulation results of the probability density functions of $x-$ and $y-$coordinates.

2. Write the program C-DISTRIBUTION for the simulation of N random points uniformly distributed in the brick with the sides a, b, c. Present the simulation results of the probability density functions of $x-$, $y-$ and $z-$coordinates.

3. Write the program VIEW for the simulation of the random points uniformly distributed on the disc of radius R. Present the simulation results of coordinates of a few random points.

4. Write the program C-DISTRIBUTION for the simulation of random points uniformly distributed on the disc of radius R. Present the simulation results of the probability density functions of $x-$ and $y-$coordinates.

5. Write the program C-DISTRIBUTION for the simulation of random points uniformly distributed in the ball of radius R. Present the simulation results of the probability density functions of $x-$, $y-$ and $z-$coordinates.

6. Write the program AVERAGE for the statistical calculation of the area of the disc of radius R. Present the calculation results for a few values of N.

7. Write the program AVERAGE for the statistical calculation of the number π. Present the calculation results for a few values of N.

8. Write the program AVERAGE for the statistical calculation of the volume of the ball of radius R. Present the calculation results for a few values of N.

9. Write the program AVERAGE for the statistical calculation of the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Present the calculation results for a few values of N.

10. Write the program AVERAGE for the statistical calculation of the integral $I = \int_0^1 \sqrt{1 - x^2} dx$. Compare the calculation results with the exact value of I for a few values of N.

11. Write the program AVERAGE for the statistical calculation of the mass, the center of gravity and the moment of inertia for the ball of radius r_1 with the spherical cavity of radius r_2. The cavity is shifted by a relative to the ball center. Present the results of the calculations.