Fredholm Integral Equations

Fredholm Integral Equations. Overview.

The equation

$$\varphi(x) - \lambda \int_a^b K(x,t)\varphi(t)dt = f(x), \qquad (1)$$

where $\varphi(x)-$ unknown, but $K(x,t)$ and $f(x)$ - known functions, x и t - real variables, varying in the range of (a,b), $\lambda-$ some numerical factor, called *linear Fredholm integral equations of 2nd kind*.

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Funtion $K(x,t)$ is named *kernel of integral equation (1)*. It is assumed that kernel $K(x,t)$ is defined in square $\Omega\{a \leq x \leq b, a \leq t \leq b,\}$ on a plane (x,t) and continous in Ω or with gaps that equation

$$\int_a^b \int_a^b |K(x,t)|^2 dxdt$$

has a finite value.

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If $f(x) \not\equiv 0$ the equation (1) called *inhomogenius*. When $f(x) \equiv 0$ the equation (1) becomes

$$\varphi(x) - \lambda \int_a^b K(x,t)\varphi(t)dt = 0 \tag{2}$$

and named *homogenius*.

If in eq. (1) take of the unknown function $\varphi(x)$ above integral, than equation becames

$$\lambda \int_a^b K(x,t)\varphi(t)dt = f(x), \tag{3}$$

and named *Fradholm integral equation of 1st kind*.

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It should be noted that limits in integral in equations (1,2,3) it can be either finite or infinite.

Solution of integral equations (1,2,3) will be considered as any function $\varphi(x)$ substitution into equation turns it into identity for $x \in (a,b)$.