

## Theory, that is necessary for calculation

# 1. PROCESSING OF MEASUREMENT RESULTS BY THE LEAST SQUARES METHOD

## 1.1. Measurement errors

In addition to statistical fluctuations, associated with the probabilistic nature of the phenomenon, the results of experiments are influenced by random effects arising in the course of experiment and processing. The totality of external disturbances increases the spread of the results and causes displacement of the mean value. The latter is compounded by the action of a number of systematic reasons (“moved” scale of instruments, bad geometry of the experiment, and so on). Each of accidental reasons is usually subjected to its own distribution. Thus, the measurement results will be described by distribution arising as a superposition of many individual distributions. Eventually, however, its form will approach the Gaussian, if there are no prevailing reasons. This fact is a consequence of the so-called *central limit theorem* of the probability theory, stating that the action of a large number of reasons with the intensity of exposure of about one order leads to the normal distribution of values, arising under the influence of these effects.

In the experiment, the deviation of results from the mean value is interpreted as a measurement error. We differentiate between random and systematic errors due to random and systematic factors, respectively. However, the notion of “measurement error” should be used with certain caution.

Thus, following a one-sided error determination, it is possible to “find” it even under conditions of ideal experiment, whereas the discrepancy between the experimental data will reflect the objective reality of the phenomenon. Although, of course, we can mention a class of experiments on measuring absolute constants (charge, mass, spin of elementary particles, and so on), where the spread of values in the determination of these values, must be apparently connected to the “clean” measurement errors.

Unfortunately, in practice, the errors of measurement techniques are not always possible to assess. Therefore, at present, it is usual to indicate a *confidence* interval instead of error, within which with a certain probability (confidence probability) we can expect the values of the studied quantities under conditions of the proposed method of measurement.

For a random quantity  $x$  a confidence interval  $[\bar{x} - \Delta\xi; \bar{x} + \Delta\xi]$  corresponds to confidence probability  $(1-\alpha)$ , if

$$P(|x - \bar{x}| \leq \Delta\xi) = 1 - \alpha. \quad (1.1)$$

Probability  $(1-\alpha)$  is also called a reliability coefficient, and quantity  $\alpha$  – level of significance. A reliable criterion for assessing the confidence interval for a given level of significance is mean square deviation  $\sigma$ , the square of which is the dispersion that characterizes the spread of values of the random quantity in the vicinity of its mean value (if  $\bar{x}$  and  $\sigma^2$  exist).

Let us assume, that in measurements the results were obtained  $\xi_1, \xi_2, \dots, \xi_n$ . Then, as an estimate of the mean value  $\bar{x}$  and dispersion  $\sigma^2$  the following relations are used.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n \xi_i, \quad \sigma_x^2 = \frac{1}{n-1} \sum_{i=1}^n (\xi_i - \bar{x})^2. \quad (1.2)$$

The average result of a series of measurements is less deviated from the exact value, than individual measurement; the dispersion of mean value is  $n$  times smaller than the dispersion of individual measurements, i.e.

$$\sigma_{\bar{x}}^2 = \frac{1}{n \cdot (n-1)} \sum_{i=1}^n (\xi_i - \bar{x})^2.$$

The larger  $n$ , the more exact the relations (1.2).

## 1.2. Errors of function of measured quantities

Parameters of function distribution  $\Phi = \Phi(x_1, x_2, \dots, x_n)$  of random variables  $x_1, x_2, \dots, x_n$ , independent of each other are obtained in the following way:

If errors of determination of each variable  $\bar{x}_i$  is small enough, then the function  $\Phi(x_i)$  can be decomposed into a Taylor series about mean values, and it is possible to neglect the expansion terms, higher than the first order of smallness, i.e.

$$\Phi(x_1, x_2, \dots, x_n) \cong \Phi(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) + \sum_{i=1}^n (x_i - \bar{x}_i) \frac{\partial \Phi}{\partial \bar{x}_i}.$$

This relation becomes exact for linear functions  $\Phi = \Phi(x_i)$ . Averaging it over  $x_i$ , we have

$$\bar{\Phi} \cong \Phi(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n), \quad (1.3)$$

And dispersion  $\sigma_{\Phi}^2$  is equal to

$$\sigma_{\Phi}^2 = \sum_{i=1}^n \left( \frac{\partial \Phi}{\partial \bar{x}_i} \right)^2 \sigma_{x_i}^2. \quad (1.4)$$

So, for sum or difference of two quantities the absolute error is determined by the formula:

$$\sigma_{x_1 \pm x_2} = \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2},$$

And a relative error will be

$$\delta_{x_1 \pm x_2} = \frac{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}{\bar{x}_1 \pm \bar{x}_2}.$$

Suppose, that during time  $t$ ,  $N$  particles are registered, then the expected intensity of particles is equal to  $\nu = N/t$ . Dispersion of quantity  $\nu$  is determined by expression

$$\sigma_{\nu}^2 = \frac{\sigma_N^2}{t^2} = \frac{\nu t}{t^2} = \frac{\nu}{t},$$

mean square error

$$\sigma_{\nu} = \sqrt{\nu/t},$$

and relative error

$$\delta_{\nu} = \frac{\sqrt{\nu/t}}{\nu} = \frac{1}{\sqrt{\nu t}} = \frac{1}{\sqrt{N}},$$

here we take into account the fact, that in a single measurement the dispersion of quantity  $N$  – is  $N$  itself.

## 1.3. Processing of the results by the least squares method

Very often in practice there are problems when numerical values of arguments with their experimental errors are known, and it is necessary to define a function that relates these quantities.

So, let us analyze the dependence of some physical quantity  $y$  on another physical quantity  $x$ :

$$y = f(x),$$

which is unknown and which is necessary to be found.

Fig. 1.1 represents a totality of experimental points  $(x_i, y_i)$ , where  $i = 1, 2, 3, \dots, n$ . At that  $y_i$  – random quantities, each of which deviates from the true value by some random quantity  $\varepsilon_i = y_i - f(x_i)$ .

Fitting and balancing of a curve  $y = f(x)$  through the experimental points concerns to the so-called regression analysis, which is typically based on the method of least squares. At that, the best curve  $y = f(x)$  is considered to be the one, for which the sum of squares of the relation is minimum  $\varepsilon_i/\sigma_i$ , where  $\varepsilon_i$  – the above mentioned deviation of the empirical points  $y_i$  from the supposed ones, a  $\sigma_i$  – mean square error of measurement, i.e.

$$S = \sum_{i=1}^n \left( \frac{\varepsilon_i}{\sigma_i} \right)^2 = \min .$$

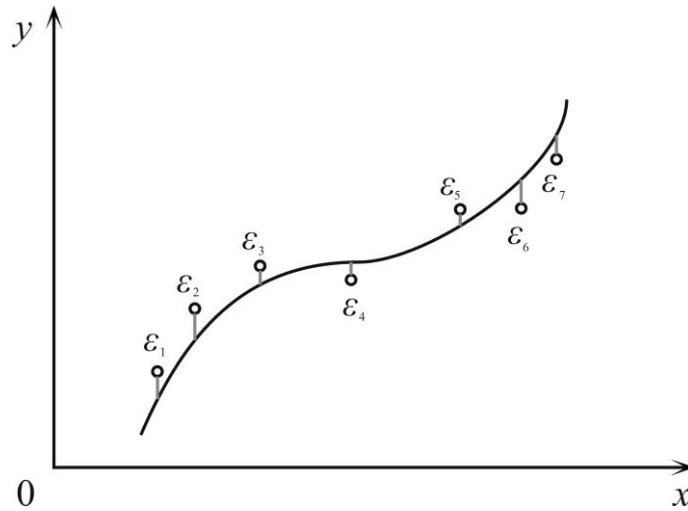


Fig. 1.1. Curve constructed using the experimental points by the method of least squares

Typically, the required function is approximated by some polynomial of finite degree  $m - 1$ , for example,

$$y = f(x) = \sum_{k=0}^{m-1} B_k x^k ,$$

and minimum of a given quadratic form is achieved, varying the sum over coefficients  $B_k$ , i.e.

$$\frac{\partial S}{\partial B_k} = 0, (k = 0, 1, \dots, m - 1).$$

Then the regression coefficients  $B_k$  are determined by linear system of equations

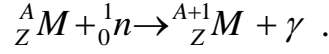
$$\sum_{i=1}^n (y - \sum_{k=0}^{m-1} B_k x_i^k) x_i^{k'} / \sigma_i^2 = 0, k' = 0, 1, \dots, m - 1,$$

and calculated according to the general methods for solving linear equations. Obviously, to find the coefficients of regression curve  $m$ , the number of experimental points is required  $n \geq m$ .

## 2. DETERMINATION OF HALF-LIFE OF ARTIFICIAL RADIONUCLIDES

### 2.1. Basic theoretical information

In the majority of cases, the result of interaction of thermal neutrons with non-fissionable nuclei is the implementation of the radiative capture ( $n, \gamma$ ), which is realized as follows:



At that, a formed isotope  ${}^{A+1}_ZM$ , the result of “overload” in the number of neutrons, is often unstable and later undergoes  $\beta^-$ -decay, i.e. it is artificially radioactive.

Mathematically, the problem of accumulation of radioactive atoms in the irradiated sample may be considered as follows. Let a thin sample (sample is considered to be thin, if the change in the flow of particles, passing through it, is much less than that flow) of the stable isotope containing  $N_{cm}$  atoms is placed in thermal neutron flux with density of  $\Phi$ ,  $\text{cm}^{-2} \cdot \text{s}^{-1}$ . Then, during time  $dt$ ,  $\Phi N_{cm} \sigma_a dt$  new active atoms will appear ( $\sigma_a$  - macroscopic cross section of neutron absorption by a stable nucleus). Along with the formation of active nuclei the process of their decay occurs. If by the moment of time  $t$  there are  $N$  active nuclei, then during time  $dt$ ,  $\lambda N dt$  of them will decay, where  $\lambda$  – is a decay constant. Since the processes of accumulation and decay of active nuclei are going simultaneously, the differential equation for determining the change in the number of active nuclei in time  $N(t)$  is as follows:

$$\frac{dN(t)}{dt} = \Phi N_{cm} \sigma_a - \lambda N(t), \quad (2.1)$$

where  $\Phi N_{cm} \sigma_a$  – the number of radioactive nuclei formed per unit time. Integrating equation (2.1) under the initial condition: at the moment of time  $t=0$   $N(t)=0$ , and assuming that during irradiation time at each moment, the number of formed active nuclei is much smaller than the number of nuclei of the stable isotope, we get:

$$N(t) = \frac{\Phi N_{cm} \sigma_a}{\lambda} [1 - \exp(-\lambda t)] = N_{\max} [1 - \exp(-\lambda t)]. \quad (2.2)$$

Hence it follows that when irradiation time increases ( $t \rightarrow \infty$ ) the number of active nuclei, accumulated in the sample, tends to its limiting value  $N_{\max} = \frac{\Phi N_{cm} \sigma_a}{\lambda}$ . If irradiation time is 8 ÷ 10 half-lives, the  $N(t)$  will be different from  $N_{\max}$  only by 10<sup>-2</sup>%, and practically we can assume, that the sample has reached saturation, in which the number of radioactive nuclei formed per unit time is equal to the number of decaying nuclei.

In order to estimate the rate of increase of the number of radioactive atoms, it is necessary to monitor the changes in the activity of the sample  $A$  in time:

$$A = \lambda N = \lambda N_{\max} [1 - \exp(-\lambda t)]. \quad (2.3)$$

Denoting  $\lambda N_{\max}$  as  $A_{\max}$  – Activity of sample saturation (at the moment of time  $t \rightarrow \infty$ ), we obtain that the activity of the sample increases according to the exponential law with the same period that the number of radioactive nuclei:  $A = A_{\max} [1 - \exp(-\lambda t)] = A_{\max} \left[ 1 - \exp\left(-\frac{\ln 2}{T_{1/2}} t\right) \right]$ ,

$$(2.4)$$

where  $T_{1/2} = \frac{\ln 2}{\lambda}$  – half-life, i.e. the time during which the activity of the sample is reduced by half.

Suppose that at the moment of time  $t=t_0$  the sample irradiation by neutrons has stopped. Radioactive nuclei accumulated by this moment, will decay according to the exponential law:

$$N=N_0 \exp(-\lambda t), \quad (2.5)$$

where  $N_0$  – number of radioactive nuclei accumulated by this moment of time  $t_0$ ;  $t$  – time since the end of irradiation;  $\lambda$  – decay constant.

Change in the activity  $A$  of the sample in time is determined by the relation:

$$A = \frac{dN}{dt} = \lambda N = \lambda N_0 \exp(-\lambda t). \quad (2.6)$$

Denoting  $\lambda N_0$  as  $A_0$  – activity of the sample after irradiation (at the moment of time  $t=t_0$ ), we obtain that the activity of the sample decreases according to the exponential law with the same period that the number of radioactive nuclei:

$$A = A_0 \exp(-\lambda t) = A_0 \exp\left(-\frac{\ln 2}{T_{1/2}} t\right). \quad (2.7)$$

Fig. 2.1 shows time increase of activity in the sample during irradiation and its decrease during the subsequent de-excitation.

The change in the activity of the sample can be monitored experimentally, since it is equal to the number of particles emitted by the sample per unit time, which can be registered by counters or other devices. Suppose, for example, the source of  $\beta^-$  particles is located about beta counter. Then the activity of the test sample is proportional to the number of pulses registered by a counter per unit time:

$$\text{at decay} - A(t) = \frac{n(t)}{\omega} = \frac{n_0}{\omega} \exp(-\lambda t) = A_0 \exp(-\lambda t), \quad (2.8)$$

$$\text{at activation} - A(t) = \frac{n(t)}{\omega} = \frac{n_{\max}}{\omega} [1 - \exp(-\lambda t)] = A_{\max} [1 - \exp(-\lambda t)]. \quad (2.9)$$

where  $n(t)$  – the number of pulses registered by a counter per unit time, at the moment of time  $t$  (count rate);  $n_0$  – count rate at the initial moment of time  $t = t_0$ ;  $n_{\max}$  – count rate at the final moment of time  $t \rightarrow \infty$  under experimental conditions it is necessary to satisfy the condition  $t > 10T_{1/2}$ ; relations  $A_0 = \frac{n_0}{\omega}$  and  $A_{\max} = \frac{n_{\max}}{\omega}$  – sample activity at the initial and final moment of time, respectively;  $\omega = 0,3$  – mean probability of  $\beta^-$ -particle registration.

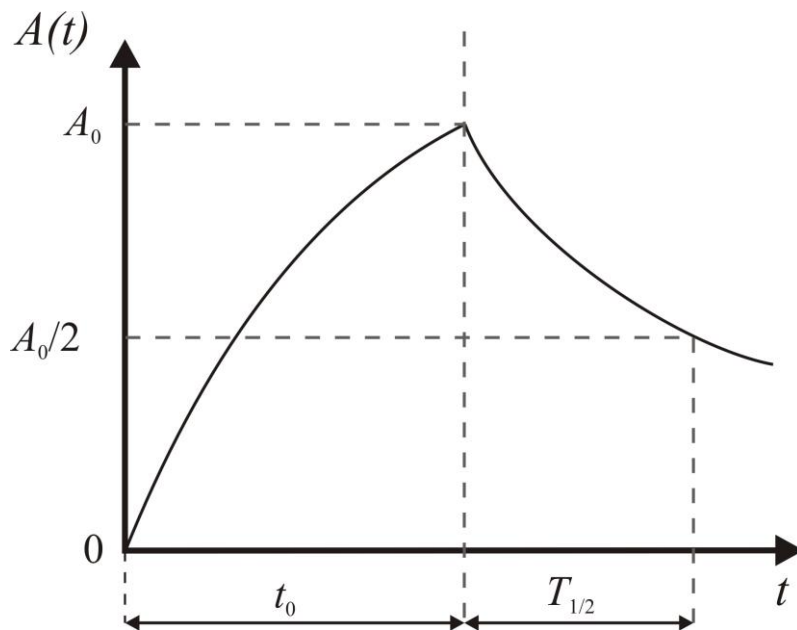


Fig. 2.1. Change in the sample activity in time during its irradiation and radioactive decay

Introduction of  $\omega$  is caused by the following reasons. Firstly, if an active sample is situated outside the sensitive volume of counter, the latter registers only a part of the particles. The smaller the solid angle is, at which the counter can be seen from the radiation source, the lower the part of registered particles. Secondly, a part of particles, flying in the counter direction, can be absorbed in the source itself, in the air on the way to the counter or in the walls of the counter. Finally, the presence of the dead time (it is also called insensitive time, resolution time) of devices, registering radiation, leads to the fact that a part of the particles that have passed through the counter is not registered. In different experiments, these factors can differently influence the measured quantity. For example, when measuring the absolute activity of the sample, all these three factors are important.

Let us assume, that at some moment of time  $t$ , a counting device is switched for time  $dt$ . If measurement time is much shorter than the half-life of the test nuclide, the count rate can be assumed to be constant during measurement time  $dt$ . Then, knowing the efficiency of registration  $\omega$  and count rate, we can construct a curve of increase or decrease in activity of the sample in time.

Taking the logarithm of equation (2.8) or (2.9), we can determine the decay constant of the test nuclide:

$$\text{at decay} - \lambda = \frac{\ln A_0 - \ln A(t)}{t} = \frac{\ln n_0 - \ln n(t)}{t}, \quad (2.10)$$

$$\text{at activation} - \lambda = \frac{\ln A_{\max} - \ln[A_{\max} - A(t)]}{t} = \frac{\ln n_{\max} - \ln[n_{\max} - n(t)]}{t}. \quad (2.11)$$

Hence, plotting value  $\ln n(t)$  or  $\ln[n_{\max} - n(t)]$  on a logarithmic graph, we obtain a straight line, the slope of which is equal to  $\lambda$ . After determination of the slope, a half-life is calculated:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0,693}{\lambda}. \quad (2.12)$$

The considered method of analysis of the decay or activation curve is commonly called a differential method.

If the sample is a mixture of two isotopes, then accumulated radioactive nuclei will decay according to the following law:

$$\begin{aligned}
& \text{for 1}^{\text{st}} \text{ isotope} - N_1 = N_{01} \exp(-\lambda_1 t), \\
& \text{for 2}^{\text{nd}} \text{ isotope} - N_2 = N_{02} \exp(-\lambda_2 t), \\
& \text{for total number of radioactive nuclei} - \\
& N = N_1 + N_2 = N_{01} \exp(-\lambda_1 t) + N_{02} \exp(-\lambda_2 t),
\end{aligned} \tag{2.13}$$

where  $N_{01}$  and  $N_{02}$  – number of radioactive nuclei accumulated by the moment of time  $t_0$  of the 1<sup>st</sup> and 2<sup>nd</sup> isotope, respectively;  $\lambda_1$  and  $\lambda_2$  – decay constants for the 1<sup>st</sup> and 2<sup>nd</sup> isotope, respectively;  $t$  – time since the end of irradiation.

In this case, the activity of the sample will be determined from the relationship:

$$\begin{aligned}
A &= A_1 + A_2 = \frac{dN_1}{dt} + \frac{dN_2}{dt} = \lambda_1 N_1 + \lambda_2 N_2 = \\
&= \lambda_1 N_{01} \exp(-\lambda_1 t) + \lambda_2 N_{02} \exp(-\lambda_2 t)
\end{aligned} \tag{2.14}$$

where  $A_1$  and  $A_2$  – contribution to the activity of the sample due to the 1<sup>st</sup> and 2<sup>nd</sup> isotope, respectively.

If a half-life of the 1<sup>st</sup> isotope is much longer than a half-life of the 2<sup>nd</sup> isotope ( $T_{1/2}^1 \gg T_{1/2}^2$ ), then after time  $t > 10T_{1/2}^2$ , we can assume that the activity of the sample affects only the decay of the 1<sup>st</sup> isotope, i.e.

$$A = A_1 = \frac{dN_1}{dt} = \lambda_1 N_1 = \lambda_1 N'_{01} \exp(-\lambda_1 t) = A'_{01} \exp(-\lambda_1 t). \tag{2.15}$$

where  $A'_{01}$  – contribution to the activity of the sample of the 1<sup>st</sup> isotope at the moment of time  $t_{01} = 10T_{1/2}^2$ ,  $N'_{01}$  – number of radioactive nuclei of the 1<sup>st</sup> isotope at the moment of time  $t_{01}$ .

Taking the logarithm of equation (2.15), we can determine the decay constant of the 1<sup>st</sup> isotope:

$$\lambda_1 = \frac{\ln A_{01} - \ln A(t)}{t} = \frac{\ln n_{01} - \ln n(t)}{t}, \tag{2.16}$$

where  $n_{01}$  – contribution of the 1<sup>st</sup> isotope to the count rate at the moment of time  $t_{01} = 10T_{1/2}^2$ .

As a result, we can determine the contribution of the 1<sup>st</sup> isotope to the sample activity at any moment of time, and we can determine a decay constant for the 2<sup>nd</sup> isotope from expression 14:

$$A(t) = A_1(t) + A_2(t) = A_{01} \exp(-\lambda_1 t) + A_{02} \exp(-\lambda_2 t), \tag{2.17}$$

where  $A_{01}$  and  $A_{02}$  – contribution of the 1<sup>st</sup> and 2<sup>nd</sup> isotope to the sample activity at the end of irradiation  $t_0$ .

$$A_2(t) = A_{02} \exp(-\lambda_2 t) = A(t) - A_1(t). \tag{2.18}$$

Taking the logarithm of expression (2.18) we obtain:

$$\lambda_2 = \frac{\ln A_{02} - \ln[A(t) - A_1(t)]}{t} = \frac{\ln n_{02} - \ln[n(t) - n_1(t)]}{t}, \tag{2.19}$$

where  $n_{02}$  – contribution of the 2<sup>nd</sup> isotope to the count rate at the moment of time  $t_0$ ;  $n_1(t)$  – contribution of the 1<sup>st</sup> isotope to the count rate at the moment of time  $t$ .

Using expression (2.12) we can determine the half-life for the 1<sup>st</sup> and 2<sup>nd</sup> isotopes.

## 2.2. Description of the experimental installation

Beta-activity of the sample is measured by standard computational devices. Counter of ionizing particles converts ionization from the passing of a charged particle, arising in its

volume, into electrical pulses. The pulses from the counter output are fed to the former, which converts them to the standard in amplitude and duration, required for the operation of the scaling device. If necessary, the formation of pulses is preceded by additional amplification. As beta-counter a scintillation counter is used.

Table № 2.1

Nuclear characteristics of indium activation detectors

Isotopic composition of natural indium	Isotope content, %	Activation cross section, barns	Radioactive product	Half-life of radioactive product
$^{113}_{49}\text{In}$	4,23	58±12	$^{114m}_{49}\text{In}$	49 days
			$^{114}_{49}\text{In}$	72 c
$^{115}_{49}\text{In}$	95,77	197±15	$^{116m}_{49}\text{In}$	54 min.
			$^{116}_{49}\text{In}$	13 c

Note: Activation cross-sections are given for neutrons with velocities 2000 m/s.

As a test sample for laboratory work: decay and activation curve indium plate is used. Table №2.1 contains isotopic composition of natural indium and products of (n,  $\gamma$ )–reaction, occurring during irradiation of natural isotopes with thermal neutrons.

### 2.3. Operational procedure

#### *Determining the half-life according to decay curve*

To study the safety instructions when working in a laboratory and performing the requirements contained therein, to start measurements with the permission of the instructor.

- To assure yourself of operability of counting device in the test mode.
- To measure the background of counting device 2-3 times. Time of one measurement ( $t_{\text{det}}$ ) is 100 s.
- To remove the sample from the container with a source of neutrons and to expose it without measurement for 30-40 s.
- To remove the dependence of count rate on time  $n(t)$ . Measurement is carried out at each moment of time ( $t$ ). Time of one measurement ( $t_{\text{ИЗМ}}$ ) is 100 s. The obtained results ( $n$ ) are recorded in Table № 2.2.
- To repeat the measurement of scaling device background after the measurements.

#### *Determining the half-life according to activation curve*

To study the safety instructions when working in a laboratory and performing the requirements contained therein, to start measurements with the permission of the instructor.

- To assure yourself of operability of counting device in the test mode.
- To measure the background of counting device 2-3 times. Time of one measurement ( $t_{\text{ИЗМ}}$ ) is 100 s.
- To put indium samples in a container with a neutron source, and start the stopwatch.
- At the scheduled time, remove the sample from the container and expose it without measuring for 30-40 seconds.



- e) To measure count rate  $n(t)$  three times and record the value of sample mass ( $m$ ) in table # 2.3. Measurement time is 100 seconds.
- f) To repeat points (c) and (d) for the rest of the foils. The count rate of the last foil is measured the next day to determine  $n_{max}$ . The results of measurements ( $n$ ) are summarized in table # 2.3.
- g) After finishing measurements, repeat the measurement of the counting device background.

An example of the table of experimental results and calculations for a decay curve

$t, \text{ min}$	$n, \text{ pulse/s}$	$\bar{n} \pm \sigma_n, \text{ pulse/s}$	$\bar{N} \pm \sigma_N, \text{ pulse/s}$	$\bar{A} \pm \sigma_A, \text{ decay/s}$	$\overline{\ln A} \pm \sigma_{\ln}$
0					
10					
20					
30					
45					
60					

Note: To determine the count rate, it is necessary to divide all the readings of the scaling device to measurement time.

Table # 2.3

An example of the table of experimental results and calculations for an activation curve

$m, \Gamma$	$t, \text{мин}$	$n, \text{pulse/s}$	$\bar{n} \pm \sigma_n, \text{pulse/s}$	$\bar{N} \pm \sigma_N, \text{pulse/s}$	$\bar{A} \pm \sigma_A, \text{расп/(с}\cdot\Gamma)$	$\bar{A}' \pm \sigma_{A'}, \text{расп/(с}\cdot\Gamma)$	$\overline{\ln A'} \pm \sigma_{\ln}$
$m=$	5						
$m=$	10						
$m=$	20						
$m=$	30						
$m=$	45						
$m=$	60						
$m=$	max					-	-

Note: To determine the count rate, it is necessary to divide all the readings of the scaling device to measurement time.

$m$  – mas of measured sample.

max –irradiation time of indium foil of more than 24 hours.

In cells, where there is a dash (-), the determination of values should not be performed.

## 2.4. Processing of experimental results

### *Determining the half-life according to decay curve*

a) According to Table 2.1, to carry out the analysis in order to select the radioactive isotope of indium, for which the half-life can be calculated according to the results of measurements.

b) To calculate the mean background of a scaling device ( $\bar{n}_\phi$ ) and its measurement error ( $\sigma_\phi$ ) by the relations:

$$\bar{n}_\phi = \frac{1}{I} \sum_{i=1}^I n_{\phi_i}; \quad \sigma_\phi = \sqrt{\frac{1}{I(I-1)} \sum_{i=1}^I (\bar{n}_\phi - n_{\phi_i})^2},$$

where  $i$  – in this case, measurement number can take the values 1, 2, 3;  $I$  – number of background measurements, in this case it is equal to 3;  $n_{\phi_i}$  – count rate of background in  $i$ -measurement.

c) To determine the mean value of count rate ( $\bar{n}$ ) and its measurement error ( $\sigma_n$ ) for all the moments of time  $t$  by the relations:

$$\bar{n} = \frac{1}{I} \sum_{i=1}^I n_i; \quad \sigma_n = \sqrt{\frac{1}{I(I-1)} \sum_{i=1}^I (\bar{n} - n_i)^2},$$

where  $i$  – measurement number at the moment of time  $t$  and in this case it can take the values 1, 2, 3;  $I$  – number of measurements at the moment of time  $t$  and in this case it is equal to 3;  $n_i$  – count rate in  $i$ -measurement.

d) To determine the mean value of count rate at the moment of time  $t$ , caused only by indium sample activity ( $\bar{N}$ ), i.e. to eliminate background and to evaluate its error ( $\sigma_N$ ) from all obtained gaugings, using relations:

$$\bar{N} = \bar{n} - \bar{n}_\phi; \quad \sigma_N = \sqrt{\sigma_n^2 + \sigma_\phi^2}.$$

e) To determine the mean value of the indium sample and its error using relations

$$\bar{A} = \frac{\bar{N}}{\omega}; \quad \sigma_A = \frac{\sigma_N}{\omega}.$$

f) To determine the mean value of the logarithm of the indium sample activity ( $\overline{\ln A}$ ) and its error ( $\sigma_{\ln}$ ) using relations:

$$\overline{\ln A} = \ln \bar{A}; \quad \sigma_{\ln} = \frac{\sigma_A}{\bar{A}}.$$

g) To construct a graph of the dependence of sample activity on time on the semilogarithmic scale, ( $\overline{\ln A} = f(t)$ ). For this purpose, experimental values  $\overline{\ln A}$  with confidence intervals are plotted on a graph, within which two straight lines are constructed. Since a decay constant is determined according to the slope of these lines, it is necessary to construct them in two extreme slope angle positions (“shallow” - 1 and “sharp” - 2 lines in Fig. 2.2, a).

If the values of confidence intervals are low for the construction of straight lines ( $\sigma_{\ln} \ll \overline{\ln A}$ ), then the lines are constructed in such a way, that the points above and below the line balance each other (fig. 2.2, b).

h) According to the slope of the straight lines, to determine two limiting values of the decay constant ( $\lambda_1, \lambda_2$ ) using relations:

$$\lambda_1 = \frac{\ln A_{11} - \ln A_{12}}{\Delta t}; \lambda_2 = \frac{\ln A_{21} - \ln A_{22}}{\Delta t},$$

where  $\ln A_{11}, \ln A_{12}$  – values of the logarithms of activity for the “shallow” line (1 Fig. 2) at the first and second points respectively;  $\ln A_{21}, \ln A_{22}$  – values of the logarithms of activity for the “sharp” line (2 Fig. 2) at the first and second points respectively.

i) To determine the mean value of the decay constant ( $\bar{\lambda}$ ) and its error ( $\sigma_\lambda$ ) using relations:

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2}{2}; \sigma_\lambda = \sqrt{\frac{(\bar{\lambda} - \lambda_1)^2 + (\bar{\lambda} - \lambda_2)^2}{2}}.$$

j) To determine the mean value of the half-life ( $\bar{T}$ ) and its error ( $\sigma_T$ ) using relations:

$$\bar{T} = \frac{\ln 2}{\bar{\lambda}} = \frac{0,693}{\bar{\lambda}}; \sigma_T = \frac{0,693 \cdot \sigma_\lambda}{\bar{\lambda}^2}.$$

k) To construct a decay curve, i.e. the dependence of indium sample activity on time. For this purpose, it is necessary to plot experimental values with confidence intervals ( $\bar{A} \pm \sigma_A$ ) on a graph and to construct the dependence:

$$A(t) = \bar{A}_0 \exp(-\bar{\lambda}t),$$

where  $\bar{A}_0$  – value of indium sample activity at the moment of time  $t=0$  (to take from table № 2.2).

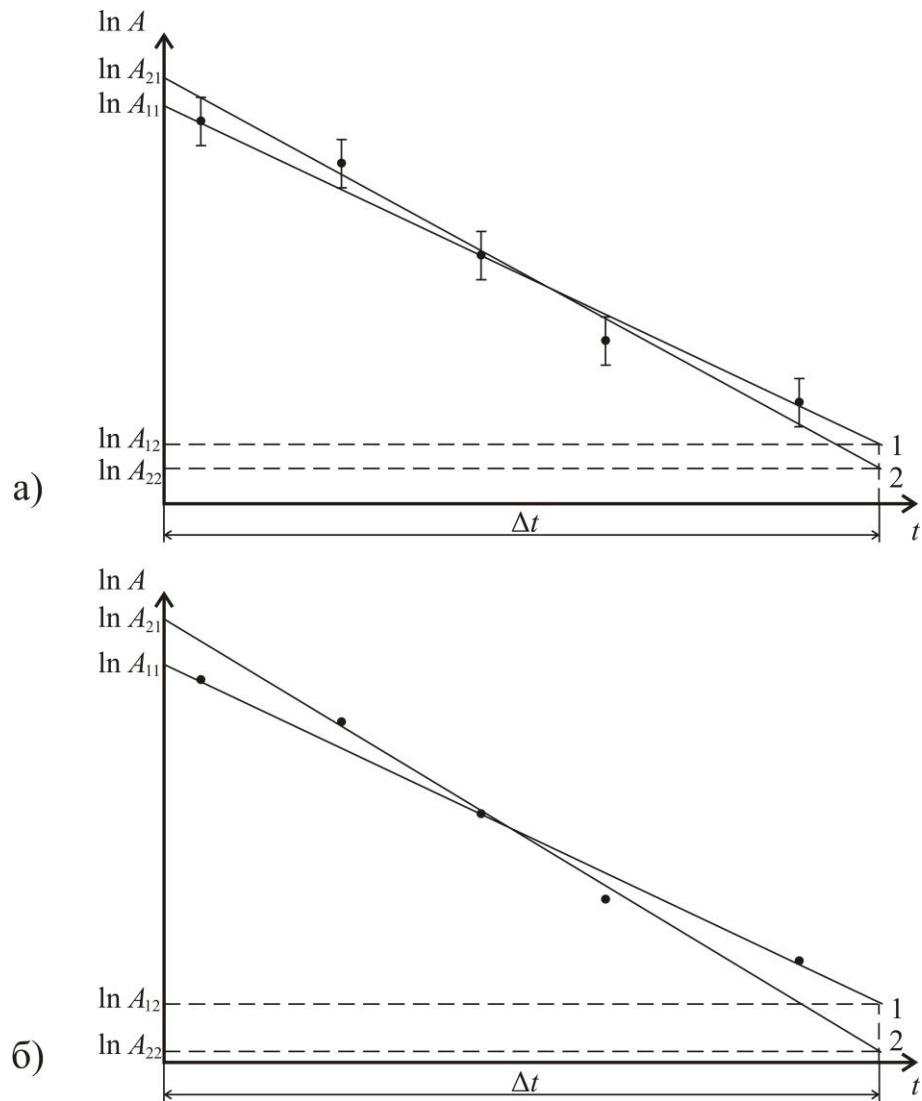


Fig. 2.2. An example of processing of dependence of sample activity on time on a semilogarithmic scale

- 1) To draw up a report on the performed work, which must include the following:
- independently formulated objective of the work;
  - necessary theoretical information;
  - the results of measurements and calculations of required quantities with their errors (table № 2.2);
  - required dependencies;
  - conclusion of the work.

***Determining the half-life according to an activation curve***

a) According to Table 2.1, to carry out the analysis in order to select the radioactive isotope of indium, for which the half-life can be calculated according to the results of measurements.

b) To calculate the mean background of a scaling device ( $\bar{n}_\phi$ ) and its measurement error ( $\sigma_\phi$ ) by the relations:

$$\bar{n}_\phi = \frac{1}{I} \sum_{i=1}^I n_{\phi_i} ; \sigma_\phi = \sqrt{\frac{1}{I(I-1)} \sum_{i=1}^I (\bar{n}_\phi - n_{\phi_i})^2} ,$$

where  $i$  – in this case, measurement number can take the values 1, 2, 3;  $I$  – number of background measurements, in this case it is equal to 3;  $n_{\phi_i}$  – count rate of background in  $i$ -measurement.

c) To determine the mean value of count rate ( $\bar{n}$ ) and its measurement error ( $\sigma_n$ ) for all the moments of time  $t$  by the relations:

$$\bar{n} = \frac{1}{I} \sum_{i=1}^I n_i ; \sigma_n = \sqrt{\frac{1}{I(I-1)} \sum_{i=1}^I (\bar{n} - n_i)^2} ,$$

where  $i$  – measurement number at the moment of time  $t$  and in this case it can take the values 1, 2, 3;  $I$  – number of measurements at the moment of time  $t$  and in this case it is equal to 3;  $n_i$  – count rate in  $i$ -measurement.

d) To determine the mean value of count rate at the moment of time  $t$ , caused only by indium sample activity ( $\bar{N}$ ), i.e. to eliminate background and to evaluate its error ( $\sigma_N$ ) from all obtained gaugings, using relations:

$$\bar{N} = \bar{n} - \bar{n}_\phi ; \sigma_N = \sqrt{\sigma_n^2 + \sigma_\phi^2} .$$

e) To determine the mean value of specific activity of the indium sample and its error using relations:

$$\bar{A} = \frac{\bar{N}}{\omega \cdot m} ; \sigma_A = \frac{\sigma_N}{\omega \cdot m} .$$

f) To determine the mean value of the difference between specific activities of the indium samples in saturation ( $A_{\max}$  when  $t = \max$ ) and at the moment of time  $t$  ( $A$ ) and their error ( $\sigma_{A'}$ ) by the relations:

$$\bar{A}' = \bar{A}_{\max} - \bar{A} ; \sigma_{A'} = \sqrt{\sigma_{\max}^2 + \sigma_A^2} ,$$

where  $\sigma_{\max}$  – error of determination of the value  $A_{\max}$ .

g) To determine the mean value of the logarithm of difference between indium sample activities ( $\overline{\ln A'}$ ) and its error ( $\sigma_{\ln}$ ) using relations:

$$\overline{\ln A'} = \ln \bar{A}' ; \sigma_{\ln} = \frac{\sigma_{A'}}{\bar{A}'} .$$

h) To construct a graph of the dependence of difference between sample activities on time on the semilogarithmic scale, ( $\overline{\ln A'} = f(t)$ ). For this purpose, experimental values  $\overline{\ln A'}$  with confidence intervals are plotted on a graph, within which two straight lines are constructed. Since a decay constant is determined according to the slope of these lines, it is necessary to construct them in two extreme slope angle positions (“shallow” - 1 and “sharp” - 2 lines in Fig. 2.2, a).

If the values of confidence intervals are low for the construction of straight lines ( $\sigma_{\ln} \ll \overline{\ln A'}$ ), then the lines are constructed in such a way, that the points above and below the line balance each other (fig. 2.2, b).

i) According to the slope of the straight lines, to determine two limiting values of the decay constant ( $\lambda_1, \lambda_2$ ) using relations:

$$\lambda_1 = \frac{\ln A'_{11} - \ln A'_{12}}{\Delta t}; \lambda_2 = \frac{\ln A'_{21} - \ln A'_{22}}{\Delta t},$$

where  $\ln A'_{11}, \ln A'_{12}$  – values of the logarithms of difference between activities for the “shallow” line (1 Fig. 2) at the first and second points respectively;  $\ln A'_{21}, \ln A'_{22}$  – values of the logarithms of difference between activities for the “sharp” line (2 Fig. 2) at the first and second points respectively.

j) To determine the mean value of the decay constant ( $\bar{\lambda}$ ) and its error ( $\sigma_{\lambda}$ ) using relations:

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2}{2}; \sigma_{\lambda} = \sqrt{\frac{(\bar{\lambda} - \lambda_1) + (\bar{\lambda} - \lambda_2)}{2}}.$$

k) To determine the mean value of the half-life ( $\bar{T}$ ) and its error ( $\sigma_T$ ) using relations:

$$\bar{T} = \frac{\ln 2}{\bar{\lambda}} = \frac{0,693}{\bar{\lambda}}; \sigma_T = \frac{0,693 \cdot \sigma_{\lambda}}{\bar{\lambda}^2}.$$

l) To construct an activation curve, i.e. the dependence of indium sample activity on irradiation time. For this purpose, it is necessary to plot experimental values with confidence intervals ( $\bar{A} \pm \sigma_A$ ) on a graph and to construct the dependence:

$$A(t) = \bar{A}_{\max}[1 - \exp(-\bar{\lambda}t)],$$

where  $\bar{A}_{\max}$  – value of activity of indium sample at the moment of time  $t = \max$  (take from table # 2.3).

m) To draw up a report on the performed work, which must include the following:

- independently formulated objective of the work;
- necessary theoretical information;
- the results of measurements and calculations of required quantities with their errors (table № 2.3);
- required dependencies;
- conclusion of the work.