SYNTHESIS OF MULTI-LOOP AUTOMATIC CONTROL SYSTEMS

Goncharov V, Emelyanova T., Rudnitsky V
National Research Tomsk Polytechnic University, Russian Federation

Key words: multi-loop control system, regulator, real interpolation method (RIM).

Abstract. The paper outlines the problem of the synthesis of multi-loop automatic control systems (ACS). The possibility to solve the problem directly by the general equation of controller synthesis in difference with the traditional sequential calculation method for enclosed loops is discussed. To solve the equation, the numerical method to convert the original equation in a system of nonlinear equations is used and capabilities and difficulties of the method are considered.

Introduction

Control systems with several internal circuits are widely used in order to achieve high dynamic precision control. However, to calculate them is not an easy task. Traditional controller synthesis method uses sequential scheme of loops calculation, starting with the internal one. The well known disadvantage of the method is related to the appearance of additional errors, since we use sequential loops calculation, some errors appear as we distribute desired quality indicators and accuracy according to internal loops. This decomposition can be done only approximately, thus leading to errors. The latter statement is a significant negative factor, even if an exact solution exists, it may be not calculated.

Obviously, to eliminate this disadvantage we need to replace two-stage procedure, having decomposition of desired properties of loops and calculation of regulators, by a single-stage one, where decomposition is excluded. Such variant of our proposed method was earlier presented in (Barkovsky A. et al, 2002). However, while implementing it some fundamental and computational difficulties were discovered. The aim of the given paper is to consider them.

1 Preliminary stage

The use of our method outlined in (Barkovsky A. et al, 2002), shows that it has some limitations, particularly in the number of coefficients to be determined. Therefore, the desire to implement the method requires research of its positive features and constraints.

For simplicity let us consider the dual-loop ACS, the diagram of which is shown in Figure 1, having the following notation, where $W_{OU}(p)$ is the transfer function of control object, and $W_{reg1}(p)$, $W_{reg2}(p)$ is the transfer functions of regulators, and K_1, K_2 is feedback coefficients.

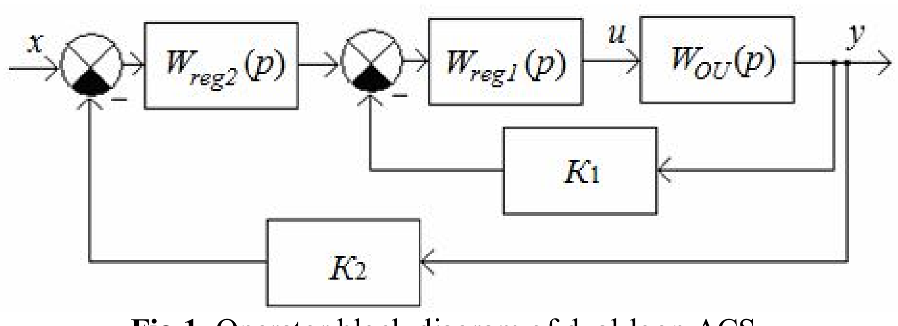

Fig.1. Operator block diagram of dual-loop ACS

Synthesis equation generally determines the ratio of the desired transfer function $W_{des}(p)$ and synthesized transfer function $W_{sint}(p)$ according to the equation:

$$W_{des}(p) \approx F[W_{reg1}(p), W_{reg2}(p), K_1, K_2, W_{OU}(p),]. \tag{1}$$

While solving equation (1), we meet two obvious difficulties. The first one is due to its non-linearity resulting in difficulty of calculation. The second one is the necessity to split the equation into the system of equations allowing us to calculate all unknown coefficients.

The conventional way to perform the splitting of the equation above is the method of frequency, where the transition from the image function $F(p)$ to the function $F(j\omega)$ is carried out by the change of variable $p=\delta+j\omega$ to the imaginary variable $j\omega$, where $\delta=0$. To perform further numerical calculations, we need to use digital models by setting ω_i, $i=1,2,\ldots,\eta$ (Kozlov O.V and Skvortsov L.M., 2015), (Ganchev I., 2004), (Petrkov N. A., 2008). It is also needed to note the negative side of such a method, as it is accompanied by twice increase of the amount of calculation due to the need to reform the real and imaginary components.

Another way to split the equation into the system of equations is the substitution in the image function $F(p)$ of the variable $p=\delta+j\omega$ to the real variable $\delta \in [C, \infty]$, $C \geq 0$ by setting $\omega=0$ (Aleksandrov I., 2014). As a result, we get the image function $F(\delta)$, which is real with a real argument. The next step is to move from a continuous function $F(\delta)$ to a numerical form $F(\delta_i)$, $i=1,2,\ldots,\eta$ based on digitization. The attraction of this method is that it allows us to use well-developed numerical techniques and algorithms, as well as digital hardware and software resources for their implementation. In the paper we consider the features of the synthesis method of multi-loop ACS.

2 Problem definition

The technology to calculate RIM in the calculation of multi-loop ACS is consecutive interpreting the synthesis equation $W_{des}^{cl}(p) \cong W_{sint}^{cl}(p)$ first in a real form $W_{des}^{cl}(\delta) \cong W_{sint}^{cl}(\delta)$, then in numerical form $W_{des}^{cl}(\delta_i) \cong W_{sint}^{cl}(\delta_i), \quad i=1,2,\ldots$ to solve it. The first step of the generated sequence is performed formally, reflecting the transition 'from general to particular' (Aleksandrov I., 2014)

The transition from continuous functions to discrete ones is a more difficult process. To reduce the difficulties, (Aleksandrov I., 2014) provides guidelines allowing to formalize the transition to numerical model. In this case, we obtain:

$$W_{des}^{cl}(\delta_i) \cong \frac{W_{reg2}(\delta_i)\cdot \dfrac{W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}{1+K_1\cdot W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}}{1+K_2\cdot W_{reg2}(\delta_i)\cdot \dfrac{W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}{1+K_1\cdot W_{reg1}(\delta_i)\cdot W_{OU}(\delta_i)}}, \quad i=\overline{1,\eta}. \tag{2}$$

The only left problem for us is to solve the system of η nonlinear equations.

Generally, to control systems two proportional–integral–derivative (PID) controllers are used. In this case an unknown factors number includes 6 coefficients of these two controllers and 2 feedback coefficients. Thus, it is required to calculate 8 unknowns of equation (2). It can be assumed that the difficulties in solving the problem may be significant, perhaps insurmountable. It is them that we consider further.

3 Testing the method on the example of turning machine control system

Peculiar properties of RIM and difficulties of its implementation in the synthesis of multi-loop ACS are presented on the example of common structure of dual- loop turning machine control system (Richard C. Dorf and Robert H. Bishop, 2008), see Figure 2. To calculate the system, the solution is known; therefore, we can simply conduct a comparative analysis of calculation results.

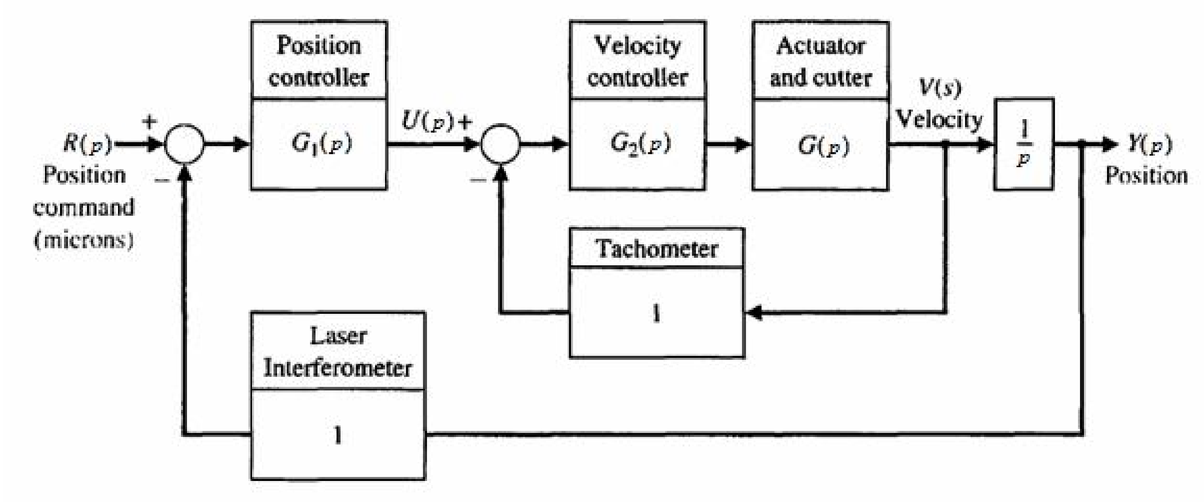

Fig. 2. Turning machine control system

The transfer function of the control object has the following form $G(p) = \dfrac{4500}{p+60}$. Resource (Richard C. Dorf and Robert H. Bishop, 2008) has certain requirements for the control system in accordance with which the controller structures $G_1(p) = K_1 \dfrac{p + b_{1,0}}{p + a_{1,0}}$, $G_2(p) = K_2 \dfrac{p^2 + b_{2,1}p + b_{2,0}}{p(p + a_{2,0})}$ and the desired transfer function $W_{des}^{cl}(p)$ are selected.

To test the features and possibilities to solve the systems of nonlinear equations (2), let us select the variant where the problem computationally is simplified to the limit, but the main feature, the system of equations being nonlinear, is retained. For this purpose, we assume unknown coefficients K_1 and K_2. For this case, the solution is obtained by Newton's method after four iterations. The found coefficients K_1 = 941,9, K_2 = 2,323 sufficiently are close to the solution of (Richard C. Dorf and Robert H. Bishop, 2008).

For further research we increased the number of unknown coefficients to six. It turned out that the increase in dimension of the equations system led to a significant deterioration in the terms of problem solving, in particular, the convergence domain greatly decreased while using Newton's method, and the number of iterations increased, and computational difficulties grew up as well due to the ill-posed problem.

Moreover, in practice the ill-posed factor is extremely significant. To determine the degree of its influence functional connection of condition number from the number of unknown variables was found. Condition measure function was determined by formula

$k=\|A\|\cdot\|A^{-1}\|$, where $\|A\|$ is the second norm of matrix A according to $\|A\| = \sqrt{\sum_{i=1}^{N} \left| x_i^2 \right|}$. The values of the numbers and the numbers of unknown coefficients η are depicted in Table 1.

Table 1

η	2	3	4	5	6
k	965	$6{,}5\cdot10^6$	$1{,}2\cdot10^8$	$9{,}0\cdot10^8$	$4{,}2\cdot10^9$

These results show the limitations of the method according to the number of unknown coefficients and simultaneously determine the area of its implementation.

Besides, we tested the condition numbers for different classes of control systems. The results of two coefficients calculations are presented further. For minimum-phase system we got k=13,605, for single-loop system with time delay the condition number was k=96,038, and for multi-loop system it was k=965,725.

The above results and some other data show that matrix condition gets worse if the control system has time delay, or distributed parameter in control object, or if the control system is multi-loop or multi-dimensional system. This information can serve as a guide to prepare recommendations to implement our method.

RIM method can be used to calculate multi-loop ACS to find 2-4 customizable regulators coefficients. On the one hand, such a restriction can be considered significant. On the other hand, the same result can be assumed as positive and promising, for example, for adaptive controllers of multi-loop systems which have not practically been implemented yet due to the absence of algorithms to auto-tuning controllers in two loops.

The results of the research have also demonstrated that it is possible to increase the number of coefficients calculated by attracting regularization methods (Kahaner, 1989). There is one more possibility to improve the results, i.e. to replace Newton's method by more suitable ones. Experiments have shown that these ways are practical and effective methods to improve synthesis multi-loop ACS.

References

Aleksandrov I. A., Goncharov V.I., Rudnitsky V. A., Liepinsh A. V. Real Interpolation Method for Automatic Control Problem Solution/ Saarbrucken : LAP LAMBERT Academic Publishing GmbH & Co. KG, 2014 - 291 p.

Avriel, Mordecai. Nonlinear Programming: Analysis and Methods. Dover Publishing. 2003.

Altmann W. Practical Process Control for Engineers and Technician. Amsterdam: Newnes, 2005 - 304 p.

Barkovsky A., Blinova N., Goncharov V., Datsko O. Multi-loop Linear Control Systems Design. System Science, Vol. 28, № 4, 2002.

Ganchev I. Auto-tuning of cascade systems with auxiliary corrector // Proc. Of the 18th Intern. Conf. on SAER. – Varva, 2004. – Sofia. 2004. – P. 46-50.

Henry B. Fine. On Newton's Method of Approximation. Proc Natl Acad Sci U S A. Sep 1916; 2(9): 546–552.

Kahaner, Cleve Moler, Stephen. Numerical Methods and Software. Nash, Prentice Hall, Englewood Cliffs, 1989.

Kessler, C.: Das symmetrische Optimum. Teil 1, rt 6 (1958), Nr.11, S. 395-400 und Teil 2, rt (1958), Nr. 12, S. 432-436.

Kozlov O.V., Skvortsov L.M. The Software package "Bauman" in research and applied research / Kozlov O.V., Skvortsov L.M. // Math modeling. – 2015, V. 27, №11. – p. 32-46.

Petrkov N. A. New approach for adaptive tuning of PI controllers. Application in cascade systems // Inf. Technol. And Contr. – 2008. – V.6. - №1. – P. 19-26.

Richard C. Dorf, Robert H. Bishop. Modern Control Systems, 11th ed. Prentice Hall, 2008.

Vrancic D., Strmenic S., Hanus R. Improving disturbance rejection of PI controllers by means of the magnitude optimum method // ISA Transactions. – 2004. – V. 43. – № 1. – P. 73–74.