BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 62, No. 2, 2014
DOI: 10.2478/bpasts-2014-0033

Embedded control system development for the solution of self-adjusted regulator design problem and its robustness properties estimation

S.V. ZAMYATIN, V.V. KURGANKIN, and V.A. RUDNICKI*

Institute of Cybernetics, Tomsk Polytechnic University, 84 Sovetskaya St., Tomsk, 634034, Russia

Abstract. The possibility of the embedded automatic-control system construction for a self-adjusted regulator design on the basis of dynamic compensation principle is observed. The description of mathematical and algorithmic apparatus of Control Object identification in a digital form resulted in a design of the regulator. Results of natural experiments are given. An analysis of the regulator robustness properties is carried out.

Key words: automatic control system, embedded system, regulator design, dynamic compensation principle, robustness, Real Interpolation Method, discrete real Laplace transformation.

1. Introduction

Design of Automatic Control System regulators is one of key problems in control theory. As a rule, initial data for the design solution are the mathematical description of Control Object and some demands of the designed system functioning. These demands are generally made only with respect to one or two indices of goodness of system's functioning. The regulator design on such initial data is observed in [1–6]. Widespread engineering methods of regulators design are reduced to approximation of the design equation, but not to its exact implementation. Approximation is aimed at simplification of the regulator structure. It leads to accuracy decrease and sometimes to the solution of impossibility. In addition, it is necessary to develop algorithms and methods of design equation approximation, therefore, the problem considerably becomes complicated.

Development of approaches applicable for the embedded control systems construction is one of the modern Control Theory actual problems. The reason is growing demand of the object control quality and, simultaneously, to maintenance of their autonomy. However, the basic trouble of the application embedded control systems is their computing resources limitation. In this connection there is a necessity of the machine-oriented approaches [7, 8]. Undoubtedly, approaches based on digital forms of the systems representation are more preferable. In this connection the approach on the basis of the discrete real Laplace transform is well represented [9].

In this work the Real Interpolation Method allowing to work with system models not only in a continuous form offered in [9] but also in a digital form [3], is used as the base of the embedded system software. Implementation of automatic tuning regulator algorithms of the embedded control system and its experimental researches with a direct-current motor as Control Object are observed.

2. Statement of problem

Assume that Control Object is one-dimensional as shown in Fig. 1.

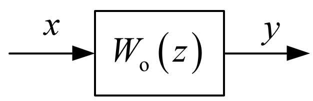

Fig. 1. Model of Control Object

In Fig. 1 the following designations are accepted: x – input (test) discrete signal, y – discrete response of Control Object to input discrete affecting, $W_o(z)$ – the Control Object discrete transfer function.

The control system can be presented as the block diagram shown in Fig. 2.

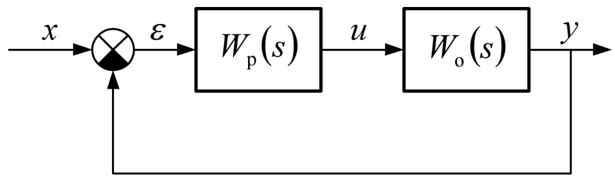

Fig. 2. One-loop automatic control system

In Fig. 2 the following designations are accepted: $W_o(s)$, $W_p(s)$ – continuous Transfer Functions of Control Object and Regulator accordingly; x and y – input and output signals; ε – error signal; u – control signal.

The problem of the self-adjusted regulator's design consists in:

*e-mail: vlad_rudnicki@mail.ru

1. Definition of the mathematical model of Control Object.
2. Requirements to control system elicitation.
3. The regulator design on the basis of Control Object model and requirements to a system.

3. Real Discrete Transformation

Real Discrete Transformation is generalization of Real Integrated Transformation [3] on lattice functions $f(nT_0)$, where $f(t)$ is the transformable time function, T_0 – sampling period, $n = 1, 2, \ldots$.

The real image-function $F^*(\delta)$ of original-function $f(kT_0)$ is defined by the formula of direct transformation

$$F^*(\delta) = \sum_{k=0}^{\infty} f(nT_0)e^{-\delta kT_0}, \quad \delta \in [c_\delta,\ \infty), \quad c_\delta \geq 0. \quad (1)$$

A value of the parameter c_δ is chosen according to a condition of convergence of series. In practice the following transformation is more convenient to use than the formula (1):

$$F(v) = \sum_{k=0}^{\infty} f(kT_0)v^{-k}, \qquad \nu \in [c_v, \infty). \quad (2)$$

Expression (1) is converted to expression (2) by substitution $\nu = \exp(\delta T_0)$. Besides, it is necessary to provide the convergence of series, that is to impose restriction on the lower bound of the variable ν change: $c_\nu \geq 1$.

Formulas (1) and (2) can be considered as special cases of the discrete Laplace transform and z-transformation accordingly. It allows to use correspondence tables of originals and their Laplace-images $F^*(p)$ and z-images handling real images $F^*(\delta)$ and $F(\nu)$ to find images $F^*(\delta)$ and $F(\nu)$ on functions $F^*(p)$ and $F(z)$ by formal substitutions $p \to \delta$ or $z \to \nu$, and to solve other certain problems.

For representation of real images in computing systems the models in the form of Numerical Characteristics are involved. In case of the use of the $F(\nu)$ function, a set $\{F(\nu_i)\}_\eta = \{F(\nu_1);\ F(\nu_2); \ldots; F(\nu_\eta)\}$, defined by values of function $F(\nu)$ on a grid $c \leq \nu_1 < \nu_2 < \ldots < \nu_\eta$, is a Numerical Characteristic. This Characteristic formation is usually carried out within the limits of an analytical grid, therefore, a definition of Interpolation Points ν_i is reduced to the definition of values of the first and last knots ν_1 and ν_η. Others Interpolation Points can be defined due to a condition of their uniform distribution

$$\nu_i = \nu_1 + \frac{\nu_\eta - \nu_1}{\eta - 1}(i - 1), \qquad i = \overline{2,\ \eta - 1}. \quad (3)$$

In the general case it is preferable to accept the value of the first Interpolation Point equal to unit (for stable systems). There are two reasons for this purpose, at least. Firstly, for the accepted class of systems at $\nu_1 = 1$ convergence of series in (2) remains. Secondly, such option ν_1 provides an equation formation. For definition of a value of the last point ν_η it is recommended to take advantage of the design formula [3]

$$(0.1...0.2)\left[F(\nu_1) - F(\infty)\right] + F(\infty) = F(\nu_\eta). \quad (4)$$

The resulted data allow finding elements of the Numerical Characteristic $\{F(\nu_i)\}_\eta$ on the set in the analytic form (2)

$$F(\nu_i) = \sum_{k=0}^{\infty} f(kT_0)\nu_i^k, \qquad i = 1, 2, ..., \eta. \quad (5)$$

Using Numerical Characteristics it is necessary to solve not only a direct problem – formation of these Characteristics, but also an inverse problem – obtaining the fractional-rational expression $F(\nu)$ under Numerical Characteristics. For the inverse problem solution the equations system is formed

$$F(\nu_i) = \frac{b_m\nu_i^m + b_{m-1}\nu_i^{m-1} + ... + b_1\nu_i + b_0}{a_n\nu_i^n + a_{n-1}\nu_i^{n-1} + ... + a_1\nu_i + 1}, \quad i = \overline{1, \eta}, \quad (6)$$

here the number of the equations η, defined by dimension of Numerical Characteristic, should be equal to a number of unknown factors: $\eta = m + n + 1$.

Real Discrete Transformation and real images have properties creating some advantages in problems of the automatic control systems calculation. The most significant merits are the following:

- transfer to the real form is carried out much easier in comparison, for example, with the frequency approach;
- there is a simple interconnection between real images and z-forms;
- mathematical models in the form of real functions and Numerical Characteristics are oriented at application of numerical methods;
- Numerical Characteristics reception is possible both on real functions-images and with their originals.

Besides, it is necessary to note that Control Object identification in a digital form has a number of advantages in relation to continuous models. Firstly, in embedded microprocessor systems the characteristic received from the Control Object is discrete. Secondly, numerical calculation of an identification procedure becomes simpler.

4. Control Object's parametric identification

Let the Control Object output signal characteristic $y(kT_0)$ for the input signal $x(kT_0)$ be set. The form of the Control Object model is

$$W_o(z) = \frac{b_m z^m + b_{m-1}z^{m-1} + ... + b_1 z + b_0}{a_n z^n + a_{n-1}z^{n-1} + ... + a_1 z + 1}, \quad m \leq n, \quad (7)$$

parameters m and n are known. It is required to find factors $b_m, \ldots, b_0,\ a_n, \ldots, a_1$, which provide performance of the set criterion of function $y_M\ (kT_0) = W_o(z) \cdot x(kT_0)$ approach to initial function $y(kT_0)$. For this purpose the estimation

$$\Delta y = \max \frac{|y_M\ (kT_0) - y\ (kT_0)|}{y_\infty} \quad (8)$$

is used, where y_∞ is a steady-state value of characteristic $y(kT_0)$.

It is important to find the solution which satisfies the condition

$$\Delta y \to \min . \tag{9}$$

For the solution of the problem the expression (7) is represented in the real-valued form

$$W_o(\nu) = \frac{b_m\nu^m + b_{m-1}\nu^{m-1} + ... + b_1\nu + b_0}{a_n\nu^n + a_{n-1}\nu^{n-1} + ... + a_1\nu + 1}, \quad m \le n. \tag{10}$$

It is necessary to obtain values of factors in expression (10). Preliminary, it is necessary to define dimensions of Numerical Characteristics quantity η and values of Interpolation Points ν_i, $i = 1, 2, \ldots, \eta$. For definition of dimensions of a quantity η the design formula is used: $\eta = m+n+1$. In case if stable Control Object is observed, it is possible to accept $\nu \in [1, \infty)$ and to define value of the first Interpolation Point as $\nu_1 = 1$ [3]. Interpolation Point ν_η is evaluated according to expression (4) and values of the others Points are defined by formula (3).

Calculation of elements $W_o(\nu_i)$ is a final operation. The design formula results from expression (2) and it has a form of

$$W_o(\nu_i) = \frac{\sum\limits_{k=0}^{\infty} y(kT_0)\,\nu_i^{-k}}{\sum\limits_{k=0}^{\infty} x(kT_0)\,\nu_i^{-k}} \cong \frac{\sum\limits_{k=0}^{N} y(kT_0)\,\nu_i^{-k}}{\sum\limits_{k=0}^{N} x(kT_0)\,\nu_i^{-k}}, \quad \nu_i = \overline{1, \eta}. \tag{11}$$

Here a parameter N defines a time of the transient process termination.

Then, it is necessary to find transfer function factors under the Numerical Characteristic $\{W_o(\nu_i)\}$. This problem is reduced to the solution of the linear algebraic equations system. System of the equations has a form of

$$W_o(\nu_i) = \frac{b_m\nu_i^m + b_{m-1}\nu_i^{m-1} + ... + b_1\nu_i + b_0}{a_n\nu_i^n + a_{n-1}\nu_i^{n-1} + ... + a_1\nu_i + 1}, \quad i = 1, 2, ..., \quad \eta = m+n+1. \tag{12}$$

Existence and uniqueness of the equations system solution are ensured by the choice of Points ν_i and the expression form (10).

The factors of the Transfer Function (10) obtained as a result of the solution are at the same time factors of z-form $W_o(z)$. It allows to generate Transfer Function $W_o(z)$ and to find reaction of the Control Object model $y_M(kT_0)$ on input signal $x(kT_0)$. It is possible to assume that it will not precisely coincide with the initial characteristic $y(kT_0)$.

The parametric identification problem solution comes to the end with check of the solution accuracy via estimation (8). If the error is intolerably large, it can be reduced by a displacement of the Point ν_η, which in turn defines proportional changes of other Interpolation Points.

5. Structure-parametrical identification

The structure-parametric identification problem is to find a structure and parameters of Control Object's discrete Transfer Function in a fractional-rational form

$$W_o(z) = \frac{b_m z^m + b_{m-1}z^{m-1} + ... + b_1 z + b_0}{a_n z^n + a_{n-1}z^{n-1} + ... + a_1 z + a_0} = \frac{\sum\limits_{i=0}^{m} b_i z^i}{\sum\limits_{i=0}^{n} a_i z^i}. \tag{13}$$

Assume that degree of polynomials is equal in a numerator and a denominator. This assumption allows to reduce the computation cost.

The problem of structure-parametric identification is as follows: for Object model in form of

$$W_o(z) = \frac{b_m z^m + b_{m-1}z^{m-1} + ... + b_1 z + b_0}{a_m z^m + a_{m-1}z^{m-1} + ... + a_1 z + 1} \tag{14}$$

it is required to find parameter m and factors $b_m, \ldots, b_0$, $a_m, \ldots, a_1$, providing performance of the function $y_M(kT_0)$ set approach criterion to initial function $y(kT_0)$. The estimation (8) is an indicator of identification success.

The first identification stage is based on simple search of model structures, i.e. $m = 1, 2, \ldots, M$, where M is the maximum observed order of model. Then, parametric identification is made for each value m. If there is the best solution the obtained model is checked on stability. If the model is stable the solution is remembered. Stability of the model is estimated on an arrangement of the characteristic equation roots on the complex plane. The result of identification is that model of stable Object for which the condition (9) is satisfied.

6. Regulator design

For the stationary systems the design problem can be divided into two stages [1]:

- The choice of a standard transfer function of the system satisfying technical requirements.
- The definition of the control system elements parameters on the basis of a closeness condition between standard and real transfer functions.

The regulator design problem is solved in the field of continuous transfer functions and it consists in an automatic definition of the embedded control system regulator transfer function $W_p(s)$.

For the system shown in Fig. 2 the problem consists in formulation and the solution of the design equation

$$W_d(s) \cong \frac{W_p(s)W_o(s)}{1 + W_p(s)W_o(s)}, \tag{15}$$

here $W_d(s)$ is the desired transfer function of a closed loop control system. The equation is generated on the basis of equality between the desired transfer function and the designed system model.

For the solution it is necessary to have the Control Object transfer function $W_o(s)$ and the close loop systems desired transfer function $W_d(s)$. The first function is known or can be found on the base of Control Object information. The second transfer function should be defined on the basis of the technical requirements.

Thus, in formula (15) transfer functions $W_d(s)$ and $W_o(s)$ are known, therefore, calculation of operator $W_p(s)$ which is reduced to definition of regulator structure and numerical values of its parameters, is possible:

$$W_p(s) = \frac{W_d(s)}{W_o(s)(1 - W_d(s))}. \quad (16)$$

In the paper, an observed approach is applied to full neutralization of Control Object dynamic characteristics impact on the operator of the closed system. It is based on a principle of dynamic compensation [1, 5]. The principle is not widespread because of computational troubles for an automatic control system with a complex structure. However, rapid development of electronics and microprocessor techniques has allowed to remove the computing restrictions at implementation of complex control algorithms also based on a principle of dynamic compensation. Algorithms of desired transfer functions, applied in an embedded control system, are based on the Konovalov-Orurk method [11].

7. Results of experiments

System functional diagram is shown in Fig. 3.

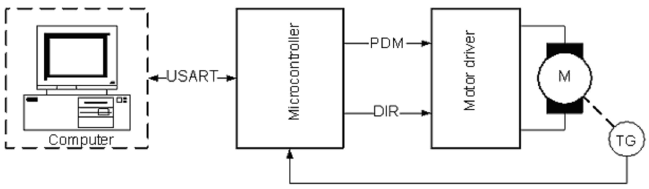

Fig. 3. System functional diagram

The following designations are accepted here: M is a direct current motor (Control Object); TG – tachogenerator (velocity sensor); USART – the interface of communication of the microcontroller and the computer ; DIR – signal defining a direction of engine drive shaft; PDM – pulse-duration modulation (the operating signal setting speed of rotation). Regulator device is the controller STM32F103 [10], Dynamo Sliven PIVT6-25/3A with the built-in tachogenerator is used as a motor and microcircuit chip Pololu High-Power Motor Driver 18v15 is used as a motor driver.

On the basis of aforementioned algorithms the software realized on computing platform STM32F103 is developed.

The procedure of the Automatic Control System regulator design consists in the following:

1. The test signal is given by the Control Object.
2. Arrays of input and output values of Control Object are formed.
3. Structure-parametrical identification of Control Object is realized on the basis of the gained files with help of the Real Interpolation Method in the discrete transfer function form.
4. On the basis of the bilinear transformation the transfer from discrete Object model to its continuous model is performed.
5. Desired transfer function of the system is formed with help of the Konovalov-Orurk method on the basis of the given quality rating.
6. Regulator structure and regulator parameters are defined on the basis of expression (16).

At the moment the device starts the test step signal with level equal to 50% of maximum (PDM duty factor is 50%, and motor armature voltage average value is 15V) is formed. The device remembers an output speed values array generated as a result of the test signal supply. On the basis of the obtained input and output data arrays and algorithm of structure-parametric identification the mathematical description of the Object is formed.

As a result of an input's test step signal the system's response (see Fig. 4, curve Object) is obtained. Considering a monotonous character of the response signal, assume that it is possible to describe an object's model as the first order transfer function.

In consequence of the Real Interpolation Method identification procedure the following model of Control Object is obtained

$$W_{o1}(s) = \frac{5.054 \cdot 10^{-2}}{2.773 \cdot 10^{-2}s + 1}.$$

Step responses of Control Object and its model are shown in Fig. 4.

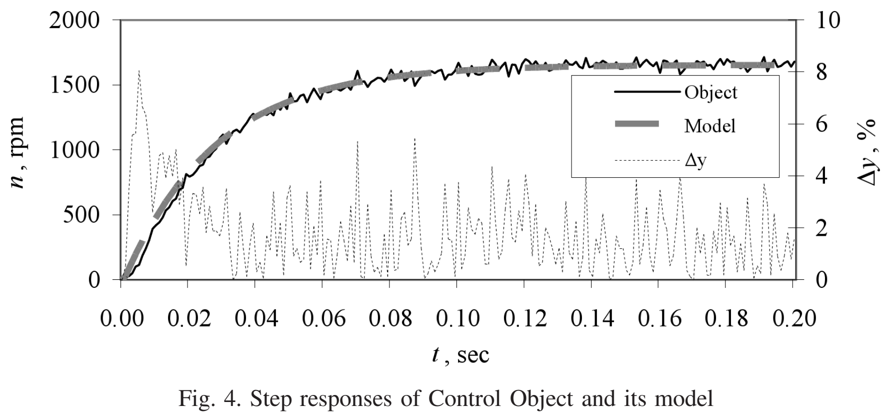

Fig. 4. Step responses of Control Object and its model

Table 1
Requirements to systems

No	t_c, c	σ, %	Desired transfer function	Regulator's transfer functions
1	0.2	0	$\frac{6.67 \cdot 10^{-2}s + 1}{4.44 \cdot 10^{-3}s^2 + 1.33 \cdot 10^{-1}s + 1}$	$\frac{1.85 \cdot 10^{-3}s^2 + 9.44 \cdot 10^{-2}s + 1}{2.25 \cdot 10^{-4}s^2 + 3.37 \cdot 10^{-3}s}$
2	0.2	20	$\frac{1.38 \cdot 10^{-2}s + 1}{9.24 \cdot 10^{-4}s^2 + 2.77 \cdot 10^{-2}s + 1}$	$\frac{3.84 \cdot 10^{-4}s^2 + 4.16 \cdot 10^{-2}s + 1}{4.67 \cdot 10^{-5}s^2 + 7.01 \cdot 10^{-4}s}$
3	4.0	0	$\frac{1.33s + 1}{1.78s^2 + 2.67s + 1}$	$\frac{8.28 \cdot 10^{-2}s^3 + 1.57s^2 + 2.46s + 1}{7.68 \cdot 10^{-3}s^3 + 9.69 \cdot 10^{-2}s^2 + 6.84 \cdot 10^{-2}s}$
4	4.0	20	$\frac{2.77 \cdot 10^{-1}s + 1}{3.70 \cdot 10^{-1}s^2 + 5.54 \cdot 10^{-1}s + 1}$	$\frac{1.72 \cdot 10^{-2}s^3 + 3.75 \cdot 10^{-1}s^2 + 1.41s + 1}{1.60 \cdot 10^{-3}s^3 + 2.01 \cdot 10^{-2}s^2 + 1.42 \cdot 10^{-2}s}$

here t_c – desired control time; σ – desired overshoot.

Here the identification error according to the set criterion is equal to 8.04% and normal control time is 0.081 sec.

The next step is formation of a desirable model of the designed system by direct quality rating (control time and overshoot). In Table 1 two sets of requirements to the system are given. Desirable transfer functions of systems and regulators are calculated on their base.

Then transfer from a regulator continuous model to a discrete model on the basis of the bilinear transformation is carried out. It is caused by specificity of the microcontroller's functioning. Therefore, formation of a control signal of the microcontroller is realized by the discrete regulator.

Reactions of systems' models No 1 and No 2 on step affecting are shown in Figs. 5 and 6.

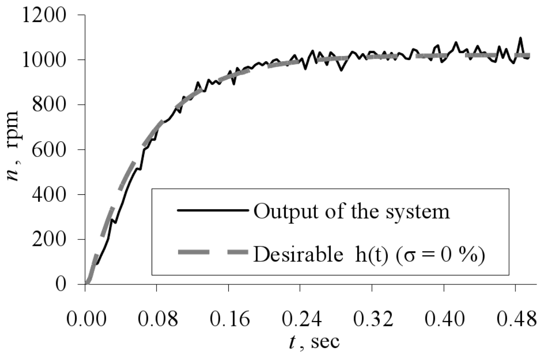

Fig. 5. Step response of System No 1

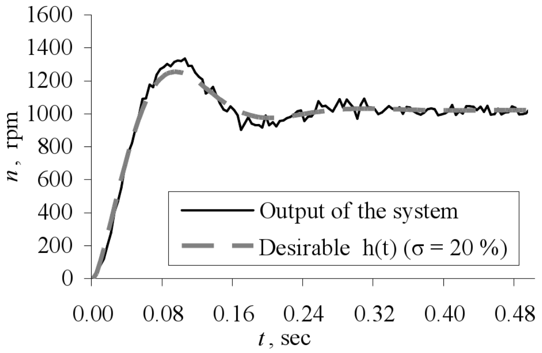

Fig. 6. Step response of System No 2

Graphs show that used algorithms of initial autotuning on the basis of the dynamic compensation method give satisfactory results for the Control Object. The designed regulator can provide not only the stability of the system, but also the satisfactory quality rating of the transient process.

However, there is possibility to create the system, which does not possess robustness property at regulator design process besides of regulator's complex structure. Experiments with the designed control system are made to estimate such properties. For change of Control Object parameters the variable load (metal disks) on the motor shaft is used. Moment of inertia of one disk makes $0.458 \cdot 10^{-3}$ kg·m^2. For robustness properties estimation the identification experiment was performed for "nominally loaded" object, where the total moment of inertia makes $2.068 \cdot 10^{-3}$ kg·m^2. As a result of the identification the following model of the Object is obtained:

$$W_{o2}(s) = \frac{4.321 \cdot 10^{-3}s + 5.126 \cdot 10^{-2}}{6.206 \cdot 10^{-2}s^2 + 1.130s + 1}.$$

The results of identification are shown in Fig. 7.

Here the identification error by the set criterion (9) is equal to 2.44% and estimated control time is 3.431 sec.

For the robustness analysis of the realized embedded control system 2 sets of requirements to a system presented in the table (systems' models ą3 and ą4) are set within the experiment. On the basis of these demands desirable systems' Transfer Functions are generated and regulators are calculated. Then, the reaction of a system to a step signal is given to the calculated regulator. These steps are carried out automatically at each start of a system in the controller that leads to realization of a complete system's self-adjustment procedure. Further, the moment of inertia on the motor shaft is increased and decreased for 44% concerning "nominally loaded" Object's moment of inertia and the system transient response is obtained. Responses of systems' models No 3 and No 4 (see Table 1) to step excitation are shown in Figs. 8 and 9.

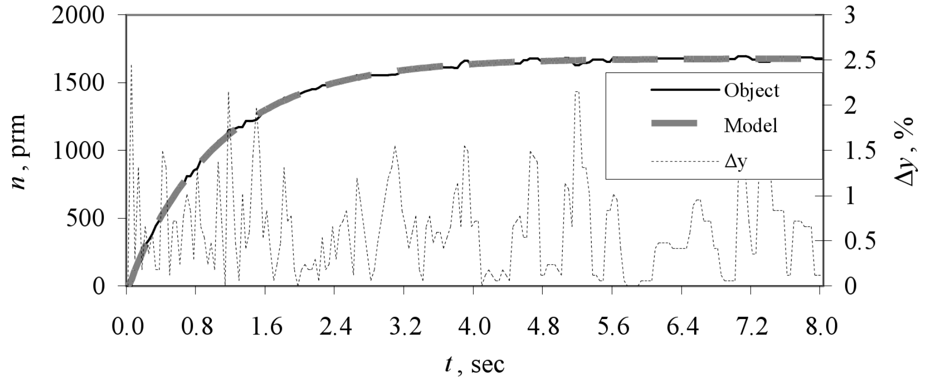

Fig. 7. Results of the "nominally loaded" Object's identification

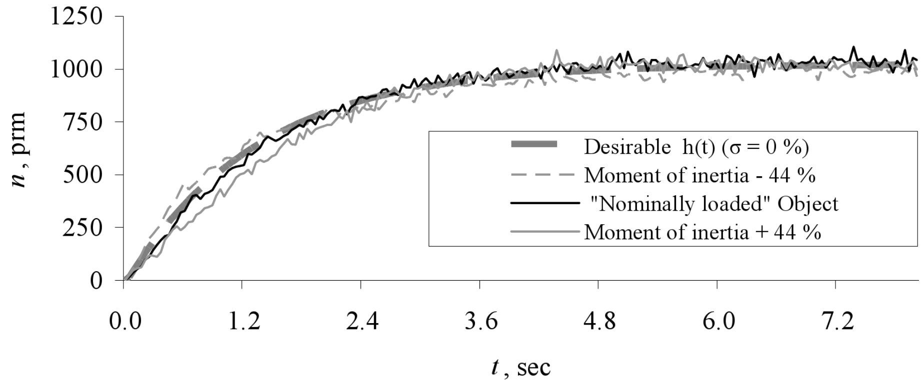

Fig. 8. Results of the designed regulator of system's model No 3 work

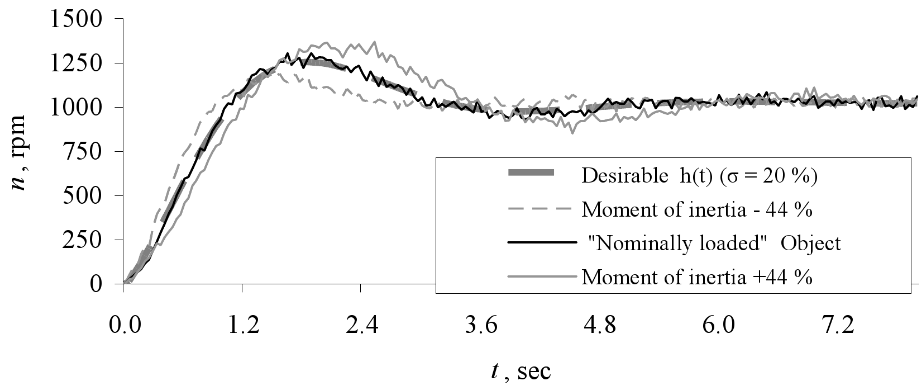

Fig. 9. Results of the designed regulator of system's model No 4 work

The analysis of schedules (Figs. 8 and 9) has shown that the developed system and the underlying algorithms are capable of solving not only the problem of systems' initial autotuning, but also the control problem of Objects with variable parameters.

The developed embedded control system with function of the initial autotuning implementing the dynamic compensation principle can be successfully applied to the wide class of technical systems control.

8. Conclusions

As a result of a series of natural experiments with the control system, where the direct-current motor is the Control Object, it is possible to conclude that the regulator realized on the basis of the microcontroller, is able to solve successfully a problem of dynamic properties stabilization in systems with constant and variable parameters of Control Objects. These parameters can vary in a wide range of values (±44% of nominal).

REFERENCES

[1] K.A. Pupkov and N.D. Pupkova, *Methods of Classical and Modern Automatic Control Theory*, Bauman MGTU Publishing House, Moscow, 2004.

[2] A.S. Kljuev and V.S. Karpov, *Design of Fast-acting Regulators for Objects with Delay*, Energoatomizdat Press, Moscow, 1990.

[3] A.S. Alekseev, A.A. Antropov, V.I. Goncharov, S.V. Zamyatin, and V.A. Rudnitskiy, *Real Interpolation Method in Automatic Control problems*, TPU Publishing House, Tomsk, 2008.

[4] M.J. Belihmajer and V.I. Goncharov, "Design of automatic-control systems' correcting devices on the basis of uniform approach", *Automatics and Telemechanics* 5, 3–11 (1997).

[5] V.J. Rotach, *Control System Theory*, MEI Publishing House, Moscow, 2008.

[6] N. Sarasiri, A. Srikaew, and S. Sujitjorn, "Dynamic compensation of hard-disk R/W hHead and head-stack", *WSEAS Trans. on Systems* 7 (9), 764–773 (2010).

[7] M. Longhua and X. Zhe, "Integrated design and implementation of embedded control systems with scilab", *Sensors* 9 (8), 5501–5515 (2008).

[8] H. Varsakelis and W. Levine, *Handbook of Networking and Embedded Control Systems*, Springer, Berlin, 2008.

[9] A. Alexeev, S. Zamyatin, and M. Pushkarev, "Structure and parametric definition of linear dynamic object via identification based on real interpolation method", *Proc. 18th Int. Conf. on Process Contr. – Tatransk – Lomnica* 1, 216–220 (2011).

[10] *STM32F103 Datasheet//STM32F103 Datasheet And Application Note*, data sheet, circuit, pdf, cross. 2011. URL: http://datasheetarchive.com/STM32F103 datasheet.html (reference date: 12.08.2011).

[11] A.A. Voronov and I.A. Orurk, *Control Systems' Computer Analysis and Optimum Design*, Science Press, Moscow, 1984.