

УДК 517

Высшая математика. Определенный интеграл. Методические указания
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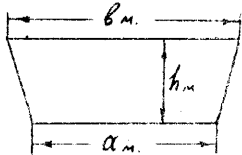
Методические указания рассмотрены и рекомендованы
методическим семинаром кафедры высшей математики - I
4 сентября 1987 г.

Зав. кафедрой



К.П. Артюхин

- 1-7. Вычислить определенный интеграл.
 8-10. Вычислить площадь фигуры, ограниченной указанными линиями.
 11-13. Вычислить длину дуги кривой.
 14. Вычислить объем тела, ограниченного указанными поверхностями.
 15. Вычислить объем тела, образованного вращением плоской фигуры, ограниченной указанными линиями. В вариантах 1-15 ось вращения Ox , в вариантах 16-30 ось вращения Oy .
 16-17. Вычислить несобственные интегралы или установить их расходимость.
 18. Варианты 1-10. Вычислить силу, с которой вода давит на плотину.



сечение которой имеет форму равнобедренной трапеции. Плотность воды

$$\rho = 1000 \text{ кг/м}^3,$$

ускорение свободного падения $g = 10 \text{ м/с}^2$

Давление на глубине x равно $\rho g x$

Варианты 11-20. Определить работу (в джоулях), совершаемую при подъеме спутника с поверхности Земли на высоту H км. Масса спутника равна m т., радиус Земли $R = 6380$ км. Ускорение свободного падения g у поверхности Земли положить равным 10 м/с^2 .

Варианты 21-30. Цилиндр наполнен газом под атмосферным давлением $103,3$ кПа. Считая газ

идеальным, определить работу (в

джоулях) при изотермическом сжа-

тии газа поршнем, переместившимся

внутрь цилиндра на h м. Урав-

нение состояния газа $pV = \text{const}$,

где p - давление, V - объем.



ВАРИАНТ I

$$1. \int_{-2}^0 (x^2 + 5x + 6) \cos 2x \, dx = \frac{5}{4} - \frac{1}{4} \cos 4 - \frac{1}{4} \sin 4 = 1,6026$$

$$2. \int_{e+1}^{e^2+1} \frac{1 + \ln(x-1)}{x-1} \, dx = 2,5$$

$$3. \int_{\pi/2}^{2 \operatorname{arctg} 2} \frac{dx}{\sin^2 x (1 - \cos x)} = 0,5729$$

$$4. \int_{\pi/4}^{\operatorname{arctg} 3} \frac{dx}{(3 \operatorname{tg} x + 5) \sin 2x} = 0,0539$$

$$5. \int_{\pi/2}^{\pi} 2^{\sin x} \sin^8 x \, dx = 35\pi = 109,9557$$

$$6. \int_0^1 \frac{4\sqrt{1-x} - \sqrt{3x+1}}{(\sqrt{3x+1} + 4\sqrt{1-x})(3x+1)^2} \, dx = \frac{1}{16} \ln 5 = 0,1006$$

$$7. \int_0^{16} \sqrt{256 - x^2} \, dx = 64\pi = 201,0619$$

8. $y = (x-2)^3,$ $y = 4x - 8. \quad S = 8.$	9. $x = 4\sqrt{2} \cos^3 t, \quad x = 2 (x=2).$ $y = 2\sqrt{2} \sin^2 t, \quad S = 2,7124.$
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10. $z = 4 \cos 3\varphi,$ $z = 2 (z=2). \quad S = 7,6529$	11. $y = \ln x,$ $\sqrt{3} \leq x \leq \sqrt{15}. \quad l = 2,2939$
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12. $x = 5(t - \sin t), \quad 0 \leq t \leq \pi.$ $y = 5(1 - \cos t). \quad l = 20$	13. $\rho = 3e^{3\varphi/4}$ $-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}. \quad l = 14,7016$
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14. $\frac{x^2}{9} + y^2 = 1, \quad (y \geq 0).$ $z = y, \quad z = 0. \quad V = 2$	15. $y = -x^2 + 5x - 6,$ $y = 0. \quad V = 0,1047$
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$$16. \int_{-\infty}^0 \frac{dx}{x^2 + x + 1} = 2,4184$$

$$17. \int_0^3 \frac{dx}{x^2 - 5x^2}$$

расходится

$$18. a=4,5; b=6,6;$$

$$h=3,0. p=23,4 \cdot 10^4$$

Вариант 2

$$1 \int_{-2}^0 (x^2 - 4) \cos 3x \, dx = \frac{2}{27} (6 \cos 6 - \sin 6) = 0,4474$$

$$2 \int_0^1 \frac{(x^2 + 1) dx}{(x^3 + 3x + 1)^2} = 0,2667$$

$$3 \int_0^{\pi/2} \frac{\cos x \, dx}{2 + \cos x} = \frac{\pi}{2} \left(1 - \frac{4}{3\sqrt{3}}\right) = 0,3616$$

$$4 \int_{\arccos(1/\sqrt{17})}^{\pi/4} \frac{2 \cot x + 1}{(2 \sin x + \cos x)^2} dx = 0,8863$$

$$5 \int_0^{\pi} 2 \sin^6 x \cos^2 x \, dx = \frac{5\pi}{8} = 1,9635$$

$$6 \int_1^{e^4} \frac{1 - \sqrt{x} + 2\sqrt[3]{x}}{x + 2\sqrt{x^3} + \sqrt[3]{x^9}} dx = 6 \ln \frac{4}{3} = 1,7261$$

$$7 \int_0^1 x^2 \sqrt{1-x^2} \, dx = \frac{\pi}{16} = 0,1963$$

$$8 \quad y = x\sqrt{9-x^2}, \quad y=0 \\ 0 \leq x \leq 3 \quad S = 9$$

$$9 \quad x = \sqrt{2} \cos t, \quad y = 2 (y \geq 2) \\ y = 2\sqrt{2} \sin t \quad S = 1,1416$$

$$10 \quad z = \cos 2\varphi \quad S = 1,5708$$

$$11 \quad y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2 \\ l = 1,0966$$

$$12 \quad x = 3(2 \cos t - \cos 2t), \\ y = 3(2 \sin t - \sin 2t), \\ 0 \leq t \leq 2\pi \quad l = 48$$

$$13 \quad p = 2e^{4\varphi/3} \\ -\frac{\sqrt{2}}{2} \leq \varphi \leq \frac{\sqrt{2}}{2} \quad l = 19,9934$$

$$14 \quad z = x^2 + 4y^2, \quad z = 2 \\ V = 3,1416$$

$$15 \quad 2x - x^2 - y = 0, \\ 2x^2 - 4x + y = 0 \quad V = 10,0531$$

$$16 \int_1^{\infty} \frac{dx}{x^2 + x} = 0,6931$$

$$17 \int_3^6 \frac{dx}{x^2 - 7x + 10} \text{ расходятся}$$

$$18 \quad a = 4,8, \quad b = 7,2 \\ h = 3,0 \quad p = 25,2 \cdot 10^4$$

Вариант 3

$$1 \int_{-1}^0 (x^2 + 4x + 3) \cos x \, dx = 4 - 2 \sin 1 - 2 \cos 1 = 1,2364$$

$$2 \int_0^1 \frac{4x \operatorname{arctg} x - x}{1+x^2} \, dx = \frac{\pi^2}{8} - 0,5 \ln 2 = 0,8871$$

$$3 \int_{\frac{\pi}{2}}^{2 \operatorname{arctg} 2} \frac{dx}{\sin^2 x (1 + \cos x)} = 1,2083$$

$$4 \int_0^{\operatorname{arccos}(1/\sqrt{7})} \frac{3 + 2 \operatorname{tg} x}{2 \sin^2 x + 3 \cos^2 x - 1} \, dx = \frac{\pi}{\sqrt{2}} + \ln 4 = 3,6077$$

$$5 \int_0^{2\pi} \sin^4 x \cos^4 x \, dx = 0,1473$$

$$6 \int_{-14/25}^{-2/5} \frac{6\sqrt{x+2}}{(x+2)^2 \sqrt{x+1}} \, dx = 1$$

$$7 \int_0^5 \frac{dx}{(25+x^2)^{3/2}} = 0,0283$$

$$8. \begin{cases} y = 4 - x^2 \\ y = x^2 - 2x \end{cases} \quad S = 9$$

$$9. \begin{cases} x = 4(1 - \sin t), & 0 < x < 8\pi \\ y = 4(1 - \cos t), & S = 57,1327 \\ y = 4(y \geq 4) \end{cases}$$

$$10. \begin{cases} z = \sqrt{3} \cos \varphi, & (0 \leq \varphi \leq \frac{\pi}{2}) \\ z = \sin \varphi \end{cases} \quad S = 0,2215$$

$$11. \begin{cases} y = \sqrt{1-x^2} + \operatorname{arcsin} x, \\ 0 \leq x \leq \frac{7}{9} \end{cases} \quad \ell = 0,9428$$

$$12. \begin{cases} x = 4(\cos t + t \sin t), \\ y = 4(\sin t - t \cos t), \\ 0 \leq t \leq 2. \end{cases} \quad \ell = 8.$$

$$13. \begin{cases} \rho = \sqrt{2} e^{\varphi}, \\ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}. \end{cases} \quad \ell = 9,2052$$

$$14. \begin{cases} \frac{x^2}{9} + \frac{y^2}{4} - z^2 = 1, \\ z = 0, z = 3 \end{cases} \quad V = 226,1947$$

$$15. \begin{cases} y = 3 \sin x, & 0 \leq x \leq \pi \\ y = \sin x, & V = 39,4784 \end{cases}$$

$$16 \int_1^{\infty} \frac{\ln x}{x^2} \, dx = 1$$

$$17 \int_{-2}^2 \frac{x \, dx}{x^2 - 4} \text{ расходящаяся}$$

$$18. \begin{cases} a = 5,1, & b = 7,8, \\ h = 3,0 & p = 27 \cdot 10^4 \end{cases}$$

Вариант 4

$$1. \int_{-2}^0 (x+2)^2 \cos 3x dx = \frac{2}{27} (6 - \sin 6) = 0,4651$$

$$2. \int_0^2 \frac{x^3 dx}{x^2 + 4} = 2 - 2 \ln 2 = 0,6137$$

$$3. \int_{2 \arctg \frac{1}{2}}^{\pi/2} \frac{\cos x dx}{(1 - \cos x)^2} = 1,3$$

$$4. \int_{\pi/4}^{\arctg 3} \frac{4 \operatorname{tg} x - 5}{1 - \sin 2x + 4 \cos^2 x} dx = 2 \ln 2 - \frac{\pi}{8} = 0,9936$$

$$5. \int_0^{2\pi} \sin^2 \frac{x}{4} \cos^6 \frac{x}{4} dx = 0,2454$$

$$6. \int_6^9 \sqrt{\frac{9-2x}{2x-21}} dx = 3,1416$$

$$7. \int_0^3 \frac{dx}{(9+x^2)^{3/2}} = 0,0786$$

$$8. \begin{cases} y = \sin x \cos^2 x, & y = 0 \\ 0 \leq x \leq \pi/2 \end{cases} \quad S = 0,3333$$

$$9. \begin{cases} x = 16 \cos^3 t, & x = 2 \quad (x \geq 2) \\ y = 2 \sin^3 t. \end{cases} \quad S = 12,5664$$

$$10. \begin{cases} z = 4 \sin 3\varphi, \\ z = 2 \quad (z \geq 2). \end{cases} \quad S = 2,4567$$

$$11. \begin{cases} y = \ln \frac{5}{2x}, \\ \sqrt{3} \leq x \leq \sqrt{6}. \end{cases} \quad l = 1,2027$$

$$12. \begin{cases} x = (t^2 - 2) \sin t + 2t \cos t, \\ y = (2 - t^2) \cos t + 2t \sin t, \\ 0 \leq t \leq \pi \end{cases} \quad l = 10,3354$$

$$13. \begin{cases} p = 5e^{5\varphi/12}, \\ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}. \end{cases} \quad l = 18,2582$$

$$14. \begin{cases} \frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{36} = -1, \\ z = 12 \end{cases} \quad V = 75,3982$$

$$15. \begin{cases} y = 5 \cos x, & x = 0 \quad (x \geq 0) \\ y = \cos x. \end{cases} \quad V = 59,2176$$

$$16. \int_{-\infty}^{-1} \frac{dx}{2x^2 + 6x + 5} = 2,3562$$

$$17. \int_0^1 \ln x dx = -1$$

$$18. \begin{cases} a = 5,4; & b = 8,4, \\ h = 3,0 \end{cases} \quad p = 28,8 \cdot 10^4$$

Вариант 5

$$1. \int_{-4}^0 (x^2 + 7x + 12) \cos x \, dx = 7 + \cos 4 - 2 \sin 4 = 7,86$$

$$2. \int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} \, dx = 0,6931$$

$$3. \int_0^{\pi/2} \frac{\cos x - \sin x}{(1 + \sin x)^2} \, dx = 0,1667$$

$$4. \int_0^{\arctg(1/3)} \frac{8 + \tan x}{18 \sin^2 x + 2 \cos^2 x} \, dx = \frac{\ln 2}{36} + \pi/3 = 1,0664$$

$$5. \int_0^{\pi} 2^y \cos^8 \frac{x}{2} \, dx = 13,7445$$

$$6. \int_0^5 e^{\sqrt{\frac{5-x}{5+x}}} \frac{dx}{(5+x)\sqrt{25-x^2}} = 0,2(e-1) = 0,3437$$

$$7. \int_0^{\sqrt{5}/2} \frac{dx}{(5-x^2)^{3/2}} = 0,1155$$

$$8. \begin{cases} y = \sqrt{4-x^2}, & y=0, \\ x=0, & x=1. \end{cases} \quad S = 1,9132$$

$$9. \begin{cases} x = 2 \cos t, & y = 3 \quad (y \geq 3) \\ y = 6 \sin t. & S = 7,3702 \end{cases}$$

$$10. \begin{cases} z = 2 \cos \varphi, & 0 \leq \varphi \leq \pi/2 \\ z = 2\sqrt{3} \sin \varphi. & S = 0,8859 \end{cases}$$

$$11. \begin{cases} y = -\ln \cos x, \\ 0 \leq x \leq \pi/6. \end{cases} \quad l = 0,5493$$

$$12. \begin{cases} x = 10 \cos^3 t, & 0 \leq t \leq \pi/2. \\ y = 10 \sin^3 t, & l = 15 \end{cases}$$

$$13. \begin{cases} p = 6e^{12\varphi/5}, \\ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}. \end{cases} \quad l = 281,7948$$

$$14. \begin{cases} \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1, \\ z=0, & z=1 \end{cases} \quad V = 34,5575$$

$$15. \begin{cases} y = \sin^2 x, \\ y=0, & x = \pi/2. \end{cases} \quad V = 1,8505$$

$$16. \int_1^{\infty} \frac{\sqrt{1+x^2}}{x^2} \, dx$$

расходится

$$17. \int_2^3 \frac{x+3}{\sqrt{x^2-4}} \, dx = 5,1233$$

$$18. \begin{cases} a=5,7; & b=9,0; \\ h=4,0. & p=54,4 \cdot 10^9 \end{cases}$$

Вариант 6

$$1. \int_0^{\pi} (2x^2 + 4x + 7) \cos 2x dx = 3,1416$$

$$2. \int_0^{\pi/4} \frac{2 \cos x + 3 \sin x}{(2 \sin x - 3 \cos x)^3} dx = -0,9444$$

$$3. \int_{\arctan 3}^{\arctan 2} \frac{dx}{\cos x (1 - \cos x)} = \frac{1}{6} + \ln \frac{2}{3} = -0,2388$$

$$4. \int_0^{\arccos \sqrt{2/3}} \frac{tg x + 2}{\sin^2 x + 2 \cos^2 x - 3} dx = -0,25(\ln 2 + \sqrt{2} \pi) = -1,284$$

$$5. \int_{-\sqrt{2}}^0 2^{\sin^2 x} dx = 109,9557$$

$$6. \int_5^{12} \sqrt{\frac{6-x}{x-14}} dx = 4,1888$$

$$7. \int_1^2 \frac{\sqrt{x^2-1}}{x^4} dx = 0,2165$$

$$8. y = x^2 \sqrt{4-x^2}, \\ y=0 (0 \leq x \leq 2). \quad S = 3,1416$$

$$x = 2(t - \sin t), y = 3(0 \leq x < 4\pi, y \geq 3) \\ 9. y = 2(1 - \cos t). \quad S = 5,1961$$

$$10. z = \sin 3\varphi \\ S = 0,7854$$

$$11. y = e^x + 6, \\ \ln \sqrt{8} \leq x \leq \ln \sqrt{15}. \quad l = 1,0912$$

$$12. x = e^t (\cos t + \sin t), \quad 0 \leq t \leq \pi. \\ y = e^t (\cos t - \sin t). \quad l = 44,2812$$

$$13. \rho = 3e^{3\varphi/4}, \quad 0 \leq \varphi \leq \pi/3 \\ l = 5,9664$$

$$14. x^2 + y^2 = 9, \quad (y \geq 0) \\ z = y, z=0 \quad V = 18$$

$$15. x = \sqrt[3]{y-2}, \\ x=1, y=1. \quad V = 19,7471$$

$$16. \int_{-\infty}^{\infty} \frac{dx}{x^2 + 4x + 9} = 1,405$$

$$17. \int_0^1 \frac{dx}{\sqrt{e^x - 1}} = 1,8382$$

$$18. a=6,0; b=9,6; \\ h=4,0. \quad p = 57,6 \cdot 10^4$$

Вариант 7

$$1. \int_0^{\pi} (9x^2 + 9x + 11) \cos 3x dx = -2(\pi + 1) = -8,2832$$

$$2. \int_0^{1/2} \frac{8x + \operatorname{arctg} 2x}{1 + 4x^2} dx = \ln 2 - \frac{\pi^2}{64} = 0,5389$$

$$3. \int_{2 \operatorname{arctg} 1/3}^{2 \operatorname{arctg} 1/2} \frac{dx}{\sin x (1 - \sin x)} = 1 + \ln 1,5 = 1,4055.$$

$$4. \int_{\operatorname{arcsin}(1/\sqrt{37})}^{\pi/4} \frac{6 \operatorname{tg} x dx}{3 \sin 2x + 5 \cos^2 x} = \frac{5}{6} + \frac{5}{6} \ln \frac{6}{11} = 0,3282$$

$$5. \int_{\pi/2}^{\pi} 2^{\sin^6 x} \cos^2 x dx = 15,708$$

$$6. \int_0^1 \frac{\sqrt{1-x}}{1+x} \frac{dx}{(1+x)\sqrt{1-x^2}} = 1,7183$$

$$7. \int_0^{\sqrt{2}/2} \frac{x^4 dx}{(1-x^2)^{3/2}} = 0,0719$$

$$8. y = \cos x \cdot \sin^2 x, y=0 \\ (0 \leq x \leq \pi/2) \quad S = 0,3333$$

$$9. x = 16 \cos^2 t, x = 6\sqrt{3} (x \geq 6\sqrt{3}) \\ y = \sin^2 t. \quad S = 0,5435$$

$$10. z = 6 \sin 3\varphi, \\ z = 3 (z \geq 3). \quad S = 17,219$$

$$11. y = 2 + \operatorname{arcsin} \sqrt{x} + \sqrt{x-x^2} \\ 1/4 \leq x \leq 1. \quad l = 1$$

$$12. x = 3(t - \sin t), \pi \leq t \leq 2\pi. \\ y = 3(1 - \cos t). \quad l = 12$$

$$13. p = 4e^{4\varphi/3}, \\ 0 \leq \varphi \leq \frac{\pi}{3}. \quad l = 15,2003$$

$$14. z = x^2 + 9y^2, \\ z = 3 \quad V = 4,7124$$

$$15. y = x e^x, \\ y = 0, x = 1. \quad V = 5,0179$$

$$16. \int_0^{\infty} x e^{-2x} dx = 0,25$$

$$17. \int_0^{\pi/4} \frac{x dx}{\sin x^2} \text{ расхождение}$$

$$18. h = 4,0. p = 60,8 \cdot 10^4$$

Вариант 8

$$1 \int_0^{\pi} (8x^2 + 16x + 17) \cos 4x dx = 3,1416$$

$$2 \int_1^4 \frac{\sqrt[4]{2\sqrt{x}} + 1}{(\sqrt{x} + x)^2} dx = 0,3333$$

$$3. \int_{\arctan 1/2}^{\pi} \frac{dx}{(1 + \sin x - \cos x)^2} = 5/3 - \ln 3 = 0,5681$$

$$4. \int_0^{\pi/4} \frac{2tg^2 x - 11tg x - 22}{4 - tg x} dx = 2 \ln 3/8 - 5\pi/4 = -5,8886$$

$$5. \int_0^{\pi} 2^y \sin^y x \cos^y x dx = 1,1781$$

$$6. \int_{5/2}^{10/3} \frac{\sqrt{x+2} + \sqrt{x-2}}{(\sqrt{x+2} - \sqrt{x-2})(x-2)^2} dx = 2,9431$$

$$7 \int_0^{\sqrt{3}} \frac{dx}{(4-x^2)^{3/2}} = 0,433$$

$$8. \begin{cases} y = \sqrt{e^x - 1}, y = 0, \\ x = \ln 2 \end{cases} S = 0,4292$$

$$9. \begin{cases} x = 6 \cos t, y = \sqrt{3} (y \geq \sqrt{3}) \\ y = 2 \sin t. \end{cases} S = 1,087$$

$$10. \begin{cases} z = \cos 3\varphi \\ S = 0,7854 \end{cases}$$

$$11. \begin{cases} y = \ln(x^2 - 1), \\ 2 \leq x \leq 3. \end{cases} l = 1,4055$$

$$12. \begin{cases} x = \frac{1}{2} \cos t - \frac{1}{4} \cos 2t, \pi/2 \leq t \leq 2\pi/3 \\ y = \frac{1}{2} \sin t - \frac{1}{4} \sin 2t. \end{cases} l = 0,4142$$

$$13. \begin{cases} \rho = \sqrt{2} e^{\varphi}, \\ 0 \leq \varphi \leq \pi/3. \end{cases} l = 3,6993$$

$$14. \begin{cases} \frac{x^2}{4} + y^2 - z^2 = 1, \\ z = 0, z = 3. \end{cases} V = 75,3982$$

$$15. \begin{cases} y = 2x - x^2, \\ y = -x + 2, \\ x = 0. \end{cases} V = 5,6548$$

$$16. \int_4^{\infty} x \cos x dx$$

расходится

$$17. \int_2^{\sqrt{3}} \frac{3x dx}{2\sqrt{x^2 - 4}} = 6,4474$$

$$18. \begin{cases} a = 6,6; b = 10,8 \\ h = 4,0 \end{cases} p = 64 \cdot 10^4$$

Вариант 9

$$1 \int_0^{2\pi} (3x^2 + 5) \cos 2x dx = 9,4248$$

$$2 \int_0^1 \frac{x dx}{x^y + 1} = 0,3927$$

$$3 \int_0^{\pi/2} \frac{\cos x dx}{5 + 4 \cos x} = \frac{\pi}{8} - \frac{5}{6} \operatorname{arctg} \frac{1}{3} = 0,1246$$

$$4 \int_0^{\pi/2} \frac{3 \operatorname{tg} x + 1}{2 \sin 2x - 5 \cos 2x + 1} dx = -0,0385$$

$-\operatorname{arctg}(1/3)$

$$5 \int_0^{2\pi} \sin^2 x \cos^6 x dx = 0,2454$$

$$6 \int_1^8 \frac{5\sqrt{x+24}}{(x+24)^2 \sqrt{x}} dx = 0,125$$

$$7 \int_0^1 \frac{x^y dx}{(2-x^2)^{3/2}} = 2,5 - 0,75\pi = 0,1438$$

$$8. y = \frac{1}{x\sqrt{1+\ln x}}, \quad y=0, x=1, x=e^3, \quad S=2$$

$$9. x=3(t-\sin t), y=3(0 \leq x < 6\pi, y \geq 3) \\ y=3(1-\cos t) \quad S=32,1372$$

$$10. z = \cos \varphi, \quad -\pi/4 \leq \varphi \leq \pi/2. \\ z = \sqrt{2} \cos(\varphi - \pi/4), \quad S=0,5354$$

$$11. y = \sqrt{1-x^2} + \operatorname{arccos} x, \\ 0 \leq x \leq 8/9, \quad l=1,8856$$

$$12. x = 3(\cos t + t \sin t), \quad 0 \leq t \leq \pi/3 \\ y = 3(\sin t - t \cos t), \quad l=1,6449$$

$$13. p = 5e^{5\varphi/12}, \\ 0 \leq \varphi \leq \pi/3, \quad l=7,1113$$

$$14. \frac{x^2}{9} + \frac{y^2}{16} - \frac{z^2}{64} = -1, \\ z=16, \quad V=402,1238$$

$$15. y = 2x - x^2, \\ y = -x + 2, \quad V=0,6283$$

$$16 \int_1^{\infty} \frac{dx}{(x+1)\sqrt{x}} = 1,5708$$

$$17 \int_0^1 \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = 0,8411$$

$$18. a=6,9, \quad b=14,4; \\ h=5,0 \quad P=105 \cdot 10^4$$

Вариант 10

$$1. \int_0^{2\pi} (2x^2 - 15) \cos 3x dx = 2,7925$$

$$2. \int_{\sqrt{3}}^{\sqrt{5}} \frac{x + 1/x}{\sqrt{x^4 + 1}} dx = 1 + 0,5 \ln 1,5 = 1,2024$$

$$3. \int_0^{2\pi/3} \frac{1 + \sin x}{1 + \cos x + \sin x} dx = \pi/3 + \ln 2 = 1,7403$$

$$4. \int_{\pi/4}^{\arctan 3} \frac{1 + \operatorname{ctg} x}{(\sin x + 2 \cos x)^2} dx = 0,2136$$

$$5. \int_0^{2\pi} \cos^3 \frac{x}{4} dx = 1,7181$$

$$6. \int_1^2 \frac{x + \sqrt{3x-2} - 10}{\sqrt{3x-2} + 7} dx = -0,8148$$

$$7. \int_0^2 \frac{x^2 dx}{\sqrt{16-x^2}} = \frac{4}{3}\pi - 2\sqrt{3} = 0,7247$$

$$8. \begin{cases} y = \arccos x, \\ y = 0, x = 0. \end{cases} \quad S = 1$$

$$9. \begin{cases} x = 8\sqrt{2} \cos^3 t, \\ x = 4 (x \geq 4) \end{cases} \quad S = 2,7124$$

$$10. \begin{cases} z = \sin \varphi, \\ z = \sqrt{2} \cos(\varphi - \pi/4) \end{cases} \quad (0 \leq \varphi \leq 3\pi/4). \quad S = 0,5354$$

$$11. y = \ln(1-x^2), \quad 0 \leq x \leq 1/4 \quad l = 0,2608$$

$$12. \begin{cases} x = (t^2 - 2) \sin t + 2t \cos t, \\ y = (2-t^2) \cos t + 2t \sin t. \end{cases} \quad 0 \leq t \leq \pi/3. \quad l = 0,3822$$

$$13. \rho = 12^{12\varphi/5}, \quad 0 \leq \varphi \leq \pi/3. \quad l = 147,4889$$

$$14. \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{16} = 1, \quad z = 0, z = 2. \quad V = 69,115$$

$$15. y = e^{1-x}, y = 0, \quad x = 0, x = 1. \quad V = 10,0359$$

$$16. \int_1^{\infty} \frac{dx}{x^2(1+x)} = 0,3068$$

$$17. \int_0^{\pi/12} \operatorname{ctg} x dx$$

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$$18. \begin{cases} a = 7,4; \\ b = 12,2; \\ h = 6,0. \end{cases} \quad p = 162 \cdot 10^4$$

$$1. \int_0^{\pi/2} (3-7x^2) \cos 2x dx = -21,9911$$

$$2. \int_{\sqrt{3}}^{\sqrt{8}} \frac{x-1/x}{\sqrt{x^2+1}} dx = 1 + 0,5 \ln(2/3) = 0,7973$$

$$3. \int_{\pi/3}^{\pi/2} \frac{\cos x dx}{1 + \sin x - \cos x} = 0,5 \ln 2 - \frac{\pi}{12} = 0,0848$$

$$4. \int_{\pi/4}^{\arccos(1/\sqrt{3})} \frac{\operatorname{tg} x}{\sin^2 x - 5 \cos^2 x + 4} dx = 0,0811$$

$$5. \int_0^{\pi} 2^4 \sin^3 \frac{x}{2} dx = 13,7445$$

$$6. \int_6^{10} \sqrt{\frac{4-x}{x-12}} dx = 4,1888$$

$$7. \int_0^2 \sqrt{4-x^2} dx = 3,1416$$

$$8. \begin{cases} y = (x+1)^2 \\ y^2 = x+1 \end{cases} \quad S = 0,3333$$

$$9. \begin{cases} x = 2\sqrt{2} \cos t, & y = 3 (y \geq 3) \\ y = 3\sqrt{2} \sin t \end{cases} \quad S = 3,4248$$

$$10. \begin{cases} z = 6 \cos 3\varphi \\ z = 3 (z \geq 3) \end{cases} \quad S = 17,219$$

$$11. \begin{cases} y = 2 + \operatorname{ch} x \\ 0 \leq x \leq 1 \end{cases} \quad l = 1,1752$$

$$12. \begin{cases} x = 6 \cos^3 t, & 0 \leq t \leq \pi/3 \\ y = 6 \sin^3 t \end{cases} \quad l = 6,75$$

$$13. \begin{cases} \rho = 1 - \sin \varphi \\ -\pi/2 \leq \varphi \leq -\pi/6 \end{cases} \quad l = 2$$

$$14. \begin{cases} \frac{x^2}{3} + \frac{y^2}{4} = 1, & z = y\sqrt{3} \\ z = 0 (y \geq 0) \end{cases} \quad V = 8$$

$$15. \begin{cases} y = x^2 \\ y^2 - x = 0 \end{cases} \quad V = 0,9425$$

$$16. \int_{-1}^{\infty} \frac{dx}{x^2+x+1} = 2,4184$$

$$17. \int_0^1 \frac{dx}{\sqrt[3]{x}} = 1,5$$

$$18. \begin{cases} m = 7,0, & H = 200 \\ A = 13,6 \cdot 10^9 \end{cases}$$

Вариант 12

$$1. \int_0^{2\pi} (1 - 8x^2) \cos 4x \, dx = -6,2832$$

$$2. \int_0^{\sqrt{3}} \frac{\arctg x + x}{1+x^2} \, dx = \ln 2 + \frac{\pi^2}{18} = 1,2415$$

$$3. \int_0^{\pi/2} \frac{(1+\cos x) \, dx}{1+\cos x + \sin x} = \frac{\pi}{4} + 0,5 \ln 2 = 1,132$$

$$4. \int_0^{\pi/4} \frac{6 \sin^2 x}{3 \cos 2x - 4} \, dx = \frac{1}{\sqrt{7}} \arctg \sqrt{7} - \frac{\pi}{4} = -0,3283$$

$$5. \int_{-\pi}^{\pi} 2 \sin^4 x \cos^2 x \, dx = 31,4159$$

$$6. \int_0^2 \frac{(4\sqrt{2-x} - \sqrt{2x+2}) \, dx}{(\sqrt{2x+2} + 4\sqrt{2-x})(2x+2)^2} = 0,0447$$

$$7. \int_0^1 \frac{dx}{(16+x^2)^{3/2}} = 0,0442$$

$$8. \begin{cases} y = 2x - x^2 + 3, \\ y = x^2 - 4x + 3 \end{cases} \quad S = 9$$

$$9. \begin{cases} x = 6(t - \sin t), \quad y = 9 \quad (0 < x < 12\pi, \quad y \geq 9) \\ y = 6(1 - \cos t) \end{cases} \quad S = 46,7654$$

$$10. \begin{cases} z = 5 \cos 3\varphi, \\ z = 3 \quad (z \geq 3). \end{cases} \quad S = 5,4978$$

$$11. \begin{cases} y = 1 - \ln \cos x, \\ 0 \leq x \leq \pi/6 \end{cases} \quad l = 0,5493$$

$$12. \begin{cases} x = e^t (\cos t + \sin t), \quad \pi/2 \leq t \leq \pi \\ y = e^t (\cos t - \sin t) \end{cases} \quad l = 36,6603$$

$$13. \begin{cases} \rho = 2(1 - \cos \varphi) \\ -\pi \leq \varphi \leq -\pi/2 \end{cases} \quad l = 5,6568$$

$$14. \begin{cases} z = 2x^2 + 8y^2, \\ z = 4 \end{cases} \quad V = 6,2832$$

$$15. \begin{cases} x^2 + (y-2)^2 = 1 \end{cases} \quad V = 39,4784$$

$$16. \int_0^{\infty} \frac{x \, dx}{(x+1)^3} = 0,5$$

$$17. \int_0^5 \frac{5 \, dx}{\sqrt{25-x^2}} = 7,854$$

$$18. \begin{cases} m = 7,0, \quad H = 250 \\ A = 16,8 \cdot 10^9 \end{cases}$$

Вариант 13

$$1. \int_{-1}^0 (x^2 + 2x + 1) \sin 3x dx = -\frac{7}{27} - \frac{2}{27} \cos 3 = -0,1859$$

$$2. \int_0^{\sqrt{3}} \frac{x - (\arctg x)^2}{1+x^2} dx = \ln 2 - 0,2 \left(\frac{\pi}{3}\right)^2 = 0,4413$$

$$3. \int_0^{\pi/2} \frac{\sin x dx}{1 + \cos x + \sin x} = \frac{\pi}{4} - 0,5 \ln 2 = 0,4388$$

$$4. \int_0^{\arctg 3} \frac{4 + \tg x}{2 \sin^2 x + 18 \cos^2 x} dx = \frac{\pi}{6} + 0,25 \ln 2 = 0,6969$$

$$5. \int_{\pi/2}^{\pi} 2^{\sin^2 x} \cos^4 x dx = 9,4225$$

$$6. \int_{-1/2}^0 \frac{x dx}{2 + \sqrt{2x+1}} = \frac{7}{6} - 3 \ln 1,5 = -0,0497$$

$$7. \int_0^4 x^2 \sqrt{16-x^2} dx = 50,2655$$

$$8. \begin{cases} y = x\sqrt{36-x^2}, y=0 \\ (0 \leq x \leq 6) \end{cases} \quad S = 72$$

$$9. \begin{cases} x = 32 \cos^3 t, x=4 (x \geq 4) \\ y = \sin^3 t, \end{cases} \quad S = 7,3702$$

$$10. \begin{cases} z = \cos \varphi, z = \sin \varphi \\ 0 \leq \varphi \leq \pi/2 \end{cases} \quad S = 0,1427$$

$$11. \begin{cases} y = e^x + 13, \\ \ln \sqrt{15} \leq x \leq \ln \sqrt{24}. \end{cases} \quad L = 1,0527$$

$$12. \begin{cases} x = 2,5(t - \sin t), \pi/2 \leq t \leq \pi, \\ y = 2,5(1 - \cos t). \end{cases} \quad L = 7,0711$$

$$13. \begin{cases} \rho = 3(1 + \sin \varphi) \\ -\pi/6 \leq \varphi \leq 0 \end{cases} \quad L = 1,907$$

$$14. \begin{cases} \frac{x^2}{81} + \frac{y^2}{25} - z^2 = 1, \\ z=0, z=2. \end{cases} \quad V = 659,7344$$

$$15. \begin{cases} y = 1-x^2, x = \sqrt{y-2} \\ x=0, x=1. \end{cases} \quad V = 15,708$$

$$16. \int_0^{\infty} x e^{-x^2} dx = 0,5$$

$$17. \int_0^2 \frac{\sqrt{2e}}{\sqrt{x}} dx = 7,6885$$

$$18. \begin{cases} m=8,0; H=300 \\ A=17,2 \cdot 10^9 \end{cases}$$

ВАРИАНТ 14

$$1. \int_0^3 (x^2 - 3x) \sin 2x \, dx = 0,25(3\sin 6 + \cos 6 - 1) = -0,2195$$

$$2. \int_0^1 \frac{x^2}{x^2+1} \, dx = 0,5 - 0,5 \ln 2 = 0,1534$$

$$3. \int_0^{\arctan 1/2} \frac{1 + \sin x}{(1 - \sin x)^2} \, dx = 8,6667$$

$$4. \int_0^{\arctan 2} \frac{12 + \tan x}{3 \sin^2 x + 12 \cos^2 x} \, dx = \pi/2 + \frac{1}{3} \ln 2 = 1,6863$$

$$5. \int_0^{\pi/2} 2^y \sin^2 x \cos^6 x \, dx = 1,9635$$

$$6. \int_0^1 e^{\sqrt{\frac{4-x}{4+x}}} \frac{dx}{(4+x)\sqrt{16-x^2}} = 0,25(e-1) = 0,4296$$

$$7. \int_0^{3/2} \frac{x^2 \, dx}{\sqrt{25-x^2}} = 1,1323$$

8. $x = \arccos y$ $x=0, y=0 \quad S=1$	9. $x = 3 \cos t, y = 4 (y \geq 4)$ $y = 8 \sin t. \quad S = 14,7404$
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10. $z = \sqrt{2} \cos(\varphi - \pi/4), \pi/4 \leq \varphi \leq 3\pi/4$ $z = \sqrt{2} \sin(\varphi - \pi/4) \quad S = 0,2854$	11. $y = -\arccos \sqrt{x} + \sqrt{x-x^2},$ $0 \leq x \leq 1/4 \quad l = 1$
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12. $x = 3,5(2 \cos t - \cos 2t),$ $y = 3,5(2 \sin t - \sin 2t)$ $0 \leq t \leq \pi/2 \quad l = 8,201$	13. $\rho = 4(1 - \sin \varphi)$ $0 \leq \varphi \leq \pi/6 \quad l = 2,5427$
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14. $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{36} = -1,$ $Z = 12 \quad V = 150,7964$	15. $y = x^2, y = 1,$ $x = 2 \quad V = 16,3363$
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16. $\int_1^{\infty} \frac{\arctan x}{x^2} \, dx = 1,132$	17. $\int_1^5 \frac{dx}{x \ln x}$ расхождение	18. $m = 6,0; H = 350$ $A = 19,9 \cdot 10^9$
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Вариант 15

1. $\int_0^{\pi} (x^2 - 3x + 2) \sin x dx = \pi^2 - 3\pi = 0,4448$

2. $\int_0^{\sin 1} \frac{(\arcsin x)^2 + 1}{\sqrt{1-x^2}} dx = 1,3333$

3. $\int_0^{\frac{\pi}{2}} \frac{\cos x dx}{1 + \cos x + \sin x} = 1 + \ln \cos 1 = 0,3844$

4. $\int_0^{\arctan(2/3)} \frac{6 + \tan x}{9 \sin^2 x + 4 \cos^2 x} dx = \pi/4 + \frac{\ln 2}{75} = 0,8239$

5. $\int_0^{2\pi} \cos^3 x dx = 1,7181$

6. $\int_{1/6}^1 \frac{15\sqrt{x+3}}{(x+3)^2 \sqrt{x}} dx = 3$

7. $\int_0^5 x^2 \sqrt{25-x^2} dx = 122,7185$

8. $y = x \arctan x,$
 $x = \sqrt{3}, y = 0. S = 1,2284$

9. $x = 6(1 - \sin t), y = 6.$
 $y = 6(1 - \cos t),$
 $(0 < x < 12\pi, y = 6). S = 128,5487$

10. $z = \cos \varphi,$
 $z = 2 \cos \varphi. S = 2,3562$

11. $y = 2 - e^x,$
 $\ln \sqrt{3} \leq x \leq \ln \sqrt{8}. l = 1,2027$

12. $x = 6(\cos t + t \sin t), 0 \leq t \leq \pi$
 $y = 6(\sin t - t \cos t). l = 29,6088$

13. $\rho = 5(1 - \cos \varphi),$
 $-\pi/3 \leq \varphi \leq 0. l = 2,6795$

14. $\frac{x^2}{76} + \frac{y^2}{9} + \frac{z^2}{36} = 1,$
 $z = 0, z = 3 V = 103,6725$

15. $y = x^3,$
 $y = \sqrt{x} V = 1,122$

16. $\int_{-\infty}^{\infty} \frac{e^{-x}}{e^x} dx$
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17. $\int_0^1 \frac{e^{\sqrt{x}} dx}{\sqrt{x}} = 3,4366$

18. $m = 5,0, H = 400$
 $A = 18,8 \cdot 10^3$

ВАРИАНТ 16

$$1. \int_0^{\pi/2} (x^2 - 5x + 6) \sin 3x dx = 2,1324$$

$$2. \int_1^3 \frac{1 - \sqrt{x}}{\sqrt{x}(x+1)} dx = \frac{\pi}{6} - \ln 2 = -0,1695$$

$$3. \int_0^{\arctan 1/3} \frac{\cos x dx}{(1 + \cos x)(1 - \sin x)} = -\frac{1}{3} + 2 \ln 1,5 = 0,4776$$

$$4. \int_0^{\arcsin \sqrt{3}/4} \frac{\operatorname{tg}^2 x dx}{3 \sin^2 x + 4 \cos^2 x - 7} = 0,5154$$

$$5. \int_0^{\pi/4} \sin^8 \frac{x}{4} dx = 1,7181$$

$$6. \int_{-5/3}^1 \frac{\sqrt[3]{3x+5} + 2}{1 + \sqrt[3]{3x+5}} dx = 3,7653$$

$$7. \int_0^4 \sqrt{16 - x^2} dx = 12,5664$$

$$8. y = x^2 \cdot \sqrt{8 - x^2}, y = 0. \\ (0 \leq x \leq 2\sqrt{2}) \quad S = 12,5664$$

$$9. x = 8 \cos^3 t, x = 3\sqrt{3} (x \geq 3\sqrt{3}) \\ y = 4 \sin^3 t, \quad S = 1,087$$

$$10. z = \sin \varphi, \\ z = 2 \sin \varphi. \quad S = 2,3562$$

$$11. y = \arcsin x - \sqrt{1 - x^2}, \\ 0 \leq x \leq \frac{15}{16}. \quad l = 2,1213$$

$$12. x = (t^2 - 2) \sin t + 2t \cos t, \\ y = (2 - t^2) \cos t + 2t \sin t. \\ 0 \leq t \leq \frac{\pi}{2} \quad l = 1,2919$$

$$13. \rho = 6(1 + \sin \varphi) \\ -\frac{\pi}{2} \leq \varphi \leq 0. \quad l = 7,0294$$

$$14. \frac{x^2}{3} + \frac{y^2}{16} = 1, z = y\sqrt{3}, \\ z = 0 (y \geq 0) \quad V = 32$$

$$15. y = \arccos(x/3), \\ y = \arccos x, y = 0. \quad V = 19,7392$$

$$16. \int_1^{\infty} x \sin x dx$$

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$$17. \int_{-1}^1 \frac{dx}{\sqrt{1-x^2}} = 3,1416$$

$$18. m = 5,0; H = 450 \\ A = 21 \cdot 10^9$$

Вариант I7

$$1. \int_{-3}^0 (x^2 + 6x + 9) \sin 2x dx = -0,25(\cos 6 + 17) = -4,49$$

$$2. \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{x^2+1}} = 0,2027$$

$$3. \int_{-\pi/3}^0 \frac{\cos x dx}{1 + \cos x - \sin x} = \pi/3 - \ln 2 = 0,354$$

$$4. \int_0^{\pi/4} \frac{7 + 3 \operatorname{tg} x}{(\sin x + 2 \cos x)^2} dx = 1,3831$$

$$5. \int_0^{\pi} 2^y \sin^6 \frac{x}{2} \cos^2 \frac{x}{2} dx = 1,9635$$

$$6. \int_2^3 \sqrt{\frac{3-2x}{2x-7}} dx = 1,0472$$

$$7. \int_0^{\sqrt{13}} \frac{dx}{(64-x^2)^{3/2}} = 0,0271$$

$$8. \begin{cases} x = \sqrt{e^y - 1}, \\ x = 0, y = \ln 2. \end{cases} S = 0,4292$$

$$9. \begin{cases} x = 6 \cos t, y = 2\sqrt{3} (y \geq 2\sqrt{3}) \\ y = 4 \sin t. \end{cases} S = 2,1741$$

$$10. z = 1 + \sqrt{2} \cos \varphi \quad S = 0,1416$$

$$11. \begin{cases} y = 1 - \ln \sin x, \\ \pi/3 \leq x \leq \pi/2 \end{cases} l = 0,5493$$

$$12. \begin{cases} x = 8 \cos^3 t, & 0 \leq t \leq \pi/6 \\ y = 8 \sin^3 t. \end{cases} l = 3$$

$$13. \begin{cases} \rho = 7(1 - \sin \varphi) \\ -\pi/6 \leq \varphi \leq \pi/6. \end{cases} l = 10,2487$$

$$14. \begin{cases} z = x^2 + 5y^2, \\ z = 5 \end{cases} V = 17,562$$

$$15. \begin{cases} y = \arcsin \frac{x}{5}, y = \pi/2, \\ y = \arcsin x. \end{cases} V = 59,2176$$

$$16. \int_{\pi/6}^{\infty} \frac{1}{x^2} \sin \frac{1}{x} dx = 1$$

$$17. \int_0^{\sqrt{e}} \frac{\sqrt{e}}{\sqrt{x}} dx = 10,8731$$

$$18. \begin{cases} m = 4,0, H = 500 \\ A = 18,5 \cdot 10^9 \end{cases}$$

Вариант 18		
1. $\int_0^{\pi/4} (x^2 + 17,5) \sin 2x dx = 8,8927$		
2. $\int_1^e \frac{1 + \ln x}{x} dx = 1,5$		
3. $\int_{-\pi/2}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)^2} = 0,1931$		
4. $\int_{\arcsin(3/\sqrt{10})}^{\arcsin(3/\sqrt{10})} \frac{5 + 2 \tan x}{(5 - \tan x) \sin 2x} dx = 1,6219$ $\arcsin(2/\sqrt{5})$		
5. $\int_{-\pi/2}^0 2^6 \sin^4 x \cos^4 x dx = 9,4248$		
6. $\int_0^7 \frac{\sqrt{x+25}}{(x+25)^2 \sqrt{x+1}} dx = 0,025$		
7. $\int_{\sqrt{2}}^{2\sqrt{2}} \frac{\sqrt{x^2-2}}{x^2} dx = -0,1082$		
8. $y = x\sqrt{4-x^2}, y=0$ $(0 \leq x \leq 2) \quad S = 2,6667$	$x = 10(t - \sin t), y = 15,$ 9. $y = 10(1 - \cos t),$ $(0 < x < 20\pi, y \geq 15) \quad S = 672,4548$	
10. $z = \frac{1}{2} + \cos \varphi$ $S = 2,0844$	11. $y = 1 - \ln(x^2 - 1)$ $3 \leq x \leq 4 \quad l = 1,1823$	
12. $x = e^t (\cos t + \sin t),$ $y = e^t (\cos t - \sin t),$ $0 \leq t \leq 2\pi \quad l = 1068,9797$	13. $\rho = 8(1 - \cos \varphi)$ $-\frac{2\pi}{3} \leq \varphi \leq 0. \quad l = 16$	
14. $\frac{x^2}{9} + \frac{y^2}{4} - z^2 = 1,$ $z = 0, z = 4 \quad V = 477,522$	15. $y = x^2, x = 2,$ $y = 0. \quad V = 25,1327$	
16. $\int_{-\infty}^0 x^2 2^x dx = 6,0056$	17. $\int_2^6 \frac{dx}{x^2 - 7x + 10}$ расколотая	18. $m = 4,0; H = 550$ $A = 20,2 \cdot 10^9$

Вариант 19

1. $\int_0^{\pi/2} (1-5x^2) \sin x dx = -4,708$		
2. $\int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2-1}} = 0,2618$		
3. $\int_0^{\pi/2} \frac{\cos x dx}{(1+\cos x + \sin x)^2} = 0,25$		
4. $\int_{-\arccos(1/\sqrt{10})}^0 \frac{3\lg^2 x - 50}{2\lg x + 7} dx = -12,0189$		
5. $\int_{\pi/2}^{\pi} 2^8 \sin^2 x \cos^6 x dx = 15,708$		
6. $\int_0^2 \frac{(4\sqrt{2-x} - \sqrt{3x+2}) dx}{(\sqrt{3x+2} + 4\sqrt{2-x})(3x+2)^2} = 0,0503$		
7. $\int_0^{2\sqrt{2}} \frac{x^4 dx}{(16-x^2)^{3/2}} = 1,1504$		
8. $y = x/(1+\sqrt{x})$ $y=0, x=1 \quad S=0,2804$	9. $x = 2\sqrt{2} \cos^3 t, \quad x=1 (x \geq 1)$ $y = \sqrt{2} \sin^3 t. \quad S=0,6781$	
10. $\begin{cases} x = \cos \varphi + \sin \varphi, \\ z = \cos \varphi - \sin \varphi \end{cases} \quad S=0,2854$	11. $y = \sqrt{x-x^2} - \arccos \sqrt{x} + 5$ $1/9 \leq x \leq 1 \quad l=1,3333$	
12. $x = 4(t - \sin t),$ $y = 4(1 - \cos t),$ $\pi/2 \leq t \leq 2\pi/3 \quad l=3,3137$	13. $\rho = 2\varphi$ $0 \leq \varphi \leq 3/4 \quad l=1,6306$	
14. $\frac{x^2}{9} + \frac{y^2}{25} - \frac{z^2}{100} = -1,$ $Z=20 \quad V=628,3185$	15. $y = x^2 + 1, y = x$ $x=0, x=1 \quad V=2,618$	
16. $\int_1^{\infty} \frac{\sqrt{x} dx}{(1+x)^2} = 1,2854$	17. $\int_0^{1/n} \frac{dx}{x \ln^2 x} = 1,4427$	18. $m=3,0; H=600$ $A=16,4 \cdot 10^3$

ВЕРИАНТ 20

1. $\int_{\pi/4}^{\pi/2} (3x - x^2) \sin 2x dx = 0,25(\sqrt{2} - 3 - 3 \sin 6 - \cos 6) = -0,3878$

2. $\int_1^e \frac{x^2 + \ln x^2}{x} dx = 0,5e^2 + 0,5 = 4,1945$

3. $\int_0^{\arctg 1/2} \frac{1 - \sin x}{\cos x (1 + \cos x)} dx = 2 \ln 1,5 - 0,5 = 0,3109$

4. $\int_0^{\pi/4} \frac{5 \operatorname{tg} x + 2}{2 \sin 2x + 5} dx = 0,2504$

5. $\int_0^{\pi} 2^y \cos^2 x dx = 13,7445$

6. $\int_0^2 e^{\sqrt{\frac{2-x}{2+x}}} \frac{dx}{(2+x)\sqrt{4-x^2}} = 0,8591$

7. $\int_{-3}^3 x^2 \sqrt{9-x^2} dx = 31,8086$

8. $y = 1/(1 + \cos x), y = 0.$
 $x = -\pi/2, x = \pi/2 \quad S = 2$

9. $x = \sqrt{2} \cos t, y = 4 (y \geq 4)$
 $y = 4\sqrt{2} \sin t. \quad S = 2,2832$

10. $z = \frac{5}{2} \sin \varphi,$
 $z = \frac{3}{2} \sin \varphi. \quad S = 3,1416$

11. $y = -\arccos x + \sqrt{1-x^2} + 1.$
 $0 \leq x \leq \sqrt{10}. \quad l = 0,7071$

12. $x = 2(2 \cos t - \cos 2t),$
 $y = 2(2 \sin t - \sin 2t). \quad l = 2,1436$
 $0 \leq t \leq \pi/3$

13. $\rho = 2\varphi.$
 $0 \leq \varphi \leq \pi/3 \quad l = 3,3208$

14. $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{64} = 1,$
 $z = 0, z = 4. \quad V = 138,2301$

15. $y = \sqrt{x-1}, y = 0, y = 1.$
 $x = 0,5 \quad V = 5,0789$

16. $\int_1^{\infty} \frac{dx}{x\sqrt{x^2-1}} = 1,5708$

17. $\int_{-1}^2 \frac{dx}{x}$
 параболы

18. $m = 6,0; H = 650$
 $A = 35,4 \cdot 10^9$

ВАРИАНТ 21

1. $\int_1^2 x \ln^2 x \, dx = 2 \ln^2 2 - 2 \ln 2 + 0,75 = 0,3246$		
2. $\int_0^1 \frac{x \, dx}{\sqrt{x^2 + x^2 + 1}} = 0,5 \ln(3 + 2\sqrt{3}) - 0,5 \ln 3 = 0,3838$		
3. $\int_0^{\pi/2} \frac{\sin x \, dx}{(1 + \sin x)^2} = -0,6667$		
4. $\int_{\arcsin(2/\sqrt{5})}^{\pi/4} \frac{(4 \operatorname{tg} x - 5) \, dx}{4 \cos^2 x - \sin 2x + 1} = 2 \ln 5/4 - 0,5 \operatorname{arctg} 0,5 = 0,2145$		
5. $\int_0^{2\pi} \sin^3 x \, dx = 1,7181$		
6. $\int_2^5 \sqrt{\frac{2-x}{x-6}} \, dx = 2,0944$		
7. $\int_1^{\sqrt{3}} \frac{dx}{(1+x^2)^{3/2}} = 0,1589$		
8. $x = (y-2)^3,$ $x = 4y-8. \quad S = 8$	9. $x = t - \sin t, y = 1 - \cos t.$ $y = 1 (0 \leq x \leq 2\pi, y \geq 1). \quad S = 5,5708$	
10. $z = 1,5 \cos \varphi,$ $z = 2,5 \cos \varphi. \quad S = 3,1416$	11. $y = \ln \sin x,$ $\pi/3 \leq x \leq \pi/2. \quad l = 0,5493$	
12. $x = 8(\cos t + t \sin t),$ $y = 8(\sin t - t \cos t)$ $0 \leq t \leq \pi/4 \quad l = 2,4674$	13. $\rho = 2\varphi$ $0 \leq \varphi \leq 5/\pi. \quad l = 0,8568$	
14. $\frac{x^2}{27} + \frac{y^2}{25} = 1, \quad y = \sqrt{3}z,$ $z = 0 (y \neq 0) \quad V = 50$	15. $y = \ln x$ $x = 2, y = 0. \quad V = 3,9979$	
16. $\int_1^{\infty} \frac{dx}{x^2+x} = 0,6931$	17. $\int_{-1}^3 \frac{dx}{x^2}$ расходится	18. $H = 0,4; h = 0,35;$ $R = 0,1$ $A = 2,6993$

Вариант 22

1. $\int_1^{e^2} \frac{\ln^2 x}{\sqrt{x}} dx = 5,7462$		
2. $\int_0^1 \frac{x^3 dx}{(x^2+1)^2} = 0,5 \ln 2 - 0,25 = 0,0966$		
3. $\int_0^{\pi/2} \frac{\sin x dx}{(1+\sin x+\cos x)^2} = 0,1931$		
4. $\int_0^{\arcsin \sqrt{7/8}} \frac{6 \sin^2 x}{4+3 \cos 2x} dx = \frac{\sqrt{7} \pi}{4} - \arctg \sqrt{7} = 0,8685$		
5. $\int_0^{\frac{\pi}{4}} \sin^6 \frac{x}{4} \cos^2 \frac{x}{4} dx = 0,2454$		
6. $\int_{1/\sqrt{2}}^{1/3} \frac{5\sqrt{x+1}}{(x+1)^2 \sqrt{x}} dx = 3$		
7. $\int_0^2 \frac{dx}{(16-x^2)^{3/2}} = 0,0361$		
8. $y = \cos^5 x \sin 2x, y=0,$ $0 \leq x \leq \pi/2. S = 0,2857$	9. $x = 8 \cos^3 t, x=1 (x \geq 1)$ $y = 8 \sin^2 t. S = 2,1741$	
10. $z = 4 \cos 4\varphi$ $S = 25,1327$	11. $y = \ln 7 - \ln x$ $\sqrt{3} \leq x \leq \sqrt{8}. l = 1,2027$	
12. $x = (t^2-2) \sin t + 2t \cos t,$ $y = (2-t^2) \cos t + 2t \sin t.$ $0 \leq t \leq 2\pi. l = 82,6834$	13. $\rho = 2\varphi$ $0 \leq \varphi \leq \pi/5. l = 7,8494$	
14. $z = 4x^2 + 9y^2,$ $z = 6. V = 9,4248$	15. $y = (x-1)^2,$ $y = 1. V = 8,3776$	
16. $\int_0^{\infty} x e^{-\frac{x}{2}} dx = 4$	17. $\int_0^1 \frac{dx}{(x-1)(x-3)}$ расходится	18. $H = 0,4; h = 0,3;$ $R = 0,1.$ $A = 1,7995$

Вариант 23

$$1. \int_1^6 \frac{\ln^2 x}{\sqrt{x^2}} dx = 54 \ln^2 2 - 108 \ln 2 + 54 = 5,0846$$

$$2. \int_0^{\pi/4} \operatorname{tg} x \ln \cos x dx = -0,125 \ln^2 2 = -0,0601$$

$$3. \int_{-\pi/8}^0 \frac{\sin x dx}{(1 + \cos x - \sin x)^2} = -0,1931$$

$$4. \int_{-\arccos(1/\sqrt{5})}^{\pi/4} \frac{11 - 3 \operatorname{tg} x}{\operatorname{tg} x + 3} dx = 4,5448$$

$$5. \int_0^{\pi/2} 2^x \sin^y \frac{x}{2} \cos^y \frac{x}{2} dx = 1,1781$$

$$6. \int_3^{15} \sqrt{\frac{6-x}{x-18}} dx = 6,2832$$

$$7. \int_0^2 \frac{x^y dx}{(8-x^2)^{3/2}} = 0,5752$$

$$8. y = \frac{x}{(x^2+1)^2}, y=0, x=1$$

$$S = 0,25$$

$$9. x = 9 \cos t, y = 2 (y \geq 2)$$

$$y = 4 \sin t, S = 22,1106$$

$$10. z = \sin 6\varphi; S = 1,5708$$

$$11. y = ch x + 3$$

$$0 \leq x \leq 1, l = 1,1752$$

$$12. x = 4 \cos^3 t, \pi/6 \leq t \leq \pi/4$$

$$y = 4 \sin^3 t, l = 1,5$$

$$13. \rho = 4\varphi$$

$$0 \leq \varphi \leq 3/4, l = 3,2613$$

$$14. x^2 + \frac{y^2}{4} - z^2 = 1, z=0, z=3, V = 75,3982$$

$$15. y^2 = x - 2, y = 0, y = x^3, y = 1, V = 15,4985$$

$$16. \int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2} = 3,1416$$

$$17. \int_0^1 x^2 \ln x dx = -0,1111$$

$$18. H = 0,4; h = 0,2; R = 0,1; A = 0,8998$$

ВАРИАНТ 24

$$1. \int_0^1 (x+1) \ln^2(x+1) dx = 2 \ln^2 2 - 2 \ln 2 + 0,75 = 0,3246$$

$$2. \int_{-1}^0 \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)} dx = 1,2128$$

$$3. \int_{2\pi/3}^0 \frac{\cos^2 x dx}{(1 + \cos x - \sin x)^2} = \sqrt{3}/2 - 0,5 \ln 4 = 0,1729$$

$$4. \int_0^{\arcsin(3/\sqrt{10})} \frac{2 \operatorname{tg} x - 5}{(4 \cos x - \sin x)^2} dx = -0,5226$$

$$5. \int_{-\pi/2}^0 2^x \sin^2 x \cos^6 x dx = 15,708$$

$$6. \int_0^1 \frac{(4\sqrt{1-x} - \sqrt{2x+1}) dx}{(\sqrt{2x+1} + 4\sqrt{1-x})(2x+1)^2} = 0,1341$$

$$7. \int_3^6 \frac{\sqrt{x^2-9}}{x^2} dx = 0,0241$$

$$8. \begin{cases} x = 4 - y^2, \\ x = y^2 - 2y. \end{cases} \quad S = 9$$

$$9. \begin{cases} x = 8(t - \sin t), \quad y = 12, \\ y = 8(1 - \cos t). \end{cases} \quad S = 83,1384$$

$(0 < x < 16\pi, y \geq 12)$

$$10. \begin{cases} z = 2 \cos \varphi, \\ z = 3 \cos \varphi. \end{cases} \quad S = 3,927$$

$$11. \begin{cases} y = 1 + \arcsin x - \sqrt{1-x^2}, \\ 0 \leq x \leq 3/4. \end{cases} \quad l = 1,4142$$

$$12. \begin{cases} x = e^t (\cos t + \sin t), \\ y = e^t (\cos t - \sin t), \\ 0 \leq t \leq 3\pi/2 \end{cases} \quad l = 220,6340$$

$$13. \begin{cases} \rho = 3\varphi \\ 0 \leq \varphi \leq 4/3 \end{cases} \quad l = 4,9812$$

$$14. \begin{cases} \frac{x^2}{25} + \frac{y^2}{9} - \frac{z^2}{100} = -1, \\ z = 20. \end{cases} \quad V = 628,3185$$

$$15. \begin{cases} y = x^3, \\ y = x^2. \end{cases} \quad V = 0,3142$$

$$16. \int_0^{\infty} e^{-\sqrt{x}} dx = 2$$

$$17. \int_0^1 \frac{dx}{(x-1)^{4/3}} = 6$$

$$18. \begin{cases} H = 0,8; \quad h = 0,7 \\ R = 0,2 \end{cases} \quad A = 21,5947$$

Вариант 25

$$1. \int_2^3 (x-1)^3 \ln^2(x-1) dx = 4 \ln^2 2 - 2 \ln 2 + \frac{15}{32} = 1,0043$$

$$2. \int_0^{\frac{1}{\sqrt{2}}} \frac{(a \operatorname{arccos} x)^3 - 1}{\sqrt{1-x^2}} dx = \frac{\pi}{4} \left(\frac{15\pi^3}{256} - 1 \right) = 0,6415$$

$$3. \int_0^{\frac{\pi}{2}} \frac{\sin^2 x dx}{(1 + \cos x + \sin x)^2} = 0,3068$$

$$4. \int_{\frac{\pi}{4}}^{\operatorname{arccos}(1/\sqrt{2})} \frac{36 dx}{(6 - \lg x) \sin 2x} = 9,6566$$

$$5. \int_{\frac{\pi}{2}}^{\pi} 2^x \cos^2 x dx = 109,9557$$

$$6. \int_1^{64} \frac{(2 + \sqrt[3]{x}) dx}{(\sqrt{x} + 2\sqrt[3]{x} + \sqrt{x})\sqrt{x}} = 6,1639$$

$$7. \int_0^1 \sqrt{4-x^2} dx = \frac{\pi}{3} + \frac{\sqrt{3}}{2} = 1,9132$$

$$8. x = \frac{1}{y\sqrt{1+\ln y}}, \quad x=0, y=1$$

$$S=2$$

$$9. x = 24 \cos^3 t, \quad x = 9\sqrt{3} \quad (x \geq 9\sqrt{3})$$

$$y = 2 \sin^3 t, \quad S = 1,6305$$

$$10. z = \sin \varphi + \cos \varphi$$

$$S = 1,5708$$

$$11. y = \ln \cos x + 2$$

$$0 \leq x \leq \frac{\pi}{6}, \quad l = 0,5493$$

$$12. x = 2(t - \sin t),$$

$$y = 2(1 - \cos t).$$

$$0 \leq t \leq \frac{\pi}{2}, \quad l = 2,3431$$

$$13. \rho = 5\varphi$$

$$0 \leq \varphi \leq \frac{12}{5}, \quad l = 19,6236$$

$$14. \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{100} = 1.$$

$$z=0, z=5, \quad V = 172,7876$$

$$15. y = \operatorname{arccos} \frac{x}{3}, \quad y=0,$$

$$y = \operatorname{arccos} \frac{x}{3}, \quad V = 39,4784$$

$$16. \int_2^{\infty} \frac{dx}{x \ln x}$$

расходится

$$17. \int_2^6 \frac{dx}{(4-x)^{2/3}} = 7,5595$$

$$18. H=0,8; h=0,6;$$

$$R=0,2$$

$$A=14,3964$$

Вариант 26

1. $\int_{-1}^0 (x+2)^3 \ln^2(x+2) dx = 4 \ln^2 2 - 2 \ln 2 + \frac{15}{32} = 1,0043$		
2. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1 - \cos x}{(x - \sin x)^2} dx = 0,1591$		
3. $\int_0^{\frac{2\pi}{3}} \frac{\cos^2 x dx}{(1 + \cos x + \sin x)^2} = 0,1729$		
4. $\int_0^{\frac{\pi}{4}} \frac{4 - 7 \tan x}{2 + 3 \tan x} dx = 0,354$		
5. $\int_0^{\frac{\pi}{2}} 2^4 \sin^8 x dx = 13,7445$		
6. $\int_{\frac{1}{1645}}^{\frac{1}{3}} \frac{4\sqrt{x}}{x^2 \sqrt{x-1}} dx = 2$		
7. $\int_2^4 \frac{\sqrt{x^2-4}}{x^2} dx = 0,0541$		
8. $y = e^{1/x}/x^2, y=0,$ $x=1, x=2. S = 1,0696$	9. $x = 3 \cos t, y = 4\sqrt{3} (y \geq 4\sqrt{3})$ $y = 8 \sin t. S = 2,1741$	
10. $z = 2 \sin 4\varphi$ $S = 6,2832$	11. $y = e^x + 26$ $\ln \sqrt{8} \leq x \leq \ln \sqrt{24}. l = 2,1438$	
12. $x = 4(2 \cos t - \cos 2t),$ $y = 4(2 \sin t - \sin 2t).$ $0 \leq t \leq \pi. l = 32$	13. $\rho = 2 \cos \varphi$ $0 \leq \varphi \leq \frac{\pi}{6}. l = 1,0472$	
14. $\frac{x^2}{27} + y^2 = 1, y = \sqrt{3} z,$ $z = 0 (y \geq 0) V = 2$	15. $y = \arcsin x,$ $y = \arccos x,$ $y = 0. V = 1,5708$	
16. $\int_2^{\infty} \frac{dx}{x \ln^2 x} = 1,4427$	17. $\int_0^1 \frac{x^4 dx}{\sqrt{1-x^5}} = 0,4$	18. $H = 0,8; h = 0,4.$ $R = 0,2.$ $A = 7,1982$

Вариант 27

$$1. \int_0^2 (x+1)^2 \ln^2(x+1) dx = 9 \ln^2 3 - 6 \ln 3 + \frac{52}{27} = 6,1967$$

$$2. \int_0^{\pi/4} \frac{\sin x - \cos x}{(\cos x + \sin x)^5} dx = -0,1875$$

$$3. \int_{\pi/2}^{2 \arctan 2} \frac{dx}{\sin x (1 + \sin x)} = \ln 2 - \frac{1}{3} = 0,3598$$

$$4. \int_{-\arcsin(2/\sqrt{5})}^{\pi/4} \frac{2 - \tan x}{(\sin x + 3 \cos x)^2} dx = 2,3637$$

$$5. \int_0^{2\pi} \sin^4 x \cos^2 x dx = 0,2454$$

$$6. \int_0^6 e^{\sqrt{\frac{6-x}{6+x}}} \frac{dx}{(6+x)\sqrt{36-x^2}} = 0,2864$$

$$7. \int_0^2 \frac{dx}{(4+x^2)^{3/2}} = 0,1768$$

$$8. y = x^2 \sqrt{16-x^2}, y=0, (0 \leq x \leq 4) \quad S = 50,2655$$

$$x = 2(t - \sin t), y = 2$$

$$9. y = 2(1 - \cos t) \quad S = 14,2832$$

$(0 < x < 4\pi, y \geq 2)$

$$10. z = 2 \cos 6\varphi, \quad S = 6,2832$$

$$11. y = (e^x + e^{-x})/2 + 3, \quad l = 3,6269$$

$0 \leq x \leq 2$

$$12. x = 2(\cos t + t \sin t), y = 2(\sin t - t \cos t) \quad l = 2,4674$$

$0 \leq t \leq \pi/2$

$$13. \rho = 8 \cos \varphi, \quad l = 6,2832$$

$0 \leq \varphi \leq \pi/4$

$$14. z = 2x^2 + 18y^2, \quad V = 9,4248$$

$z = 6$

$$15. y = x^2 - 2x + 1, \quad V = 3,6652$$

$x = 2, y = 0$

$$16. \int_1^{\infty} \frac{x^2}{1+x^6} dx = 0,2618$$

$$17. \int_0^2 \frac{dx}{(x-1)^2}$$

расходится

$$18. H = 1,6; h = 1,4$$

$R = 0,3$
 $A = 97,176$

Вариант 28

$1. \int_1^e \sqrt{x} \ln^2 x dx = \frac{1}{27} (10e^{3/2} - 16) = 1,0673$		
$2. \int_{\pi/4}^{\pi/2} \frac{x \cos x + \sin x}{(x \sin x)^2} dx = \frac{2}{\sqrt{e}} (2\sqrt{2} - 1) = 1,164$		
$3. \int_0^{\pi/2} \frac{dx}{(1 + \sin x + \cos x)^2} = 0,3068$		
$4. \int_{\pi/4}^{\arcsin \sqrt{2}/3} \frac{8 \operatorname{tg} x dx}{3 \cos^2 x + 8 \sin^2 x - 7} = -1,6219$		
$5. \int_0^{2\pi} \sin^4 \frac{x}{4} \cdot \cos^4 \frac{x}{4} dx = 0,1473$		
$6. \int_1^{64} \frac{6 - \sqrt{x} + \sqrt[4]{x}}{\sqrt{x^3 - 7x - 6\sqrt{x^3}}} dx = -5,3698$		
$7. \int_0^{\sqrt{2}} \frac{x^4 dx}{(4 - x^2)^{3/2}} = 0,2876$		
$8. \begin{aligned} x &= \sqrt{4 - y^2}, y=0, y=1 \\ x &= 0 \end{aligned} \quad S = 1,9132$	$9. \begin{aligned} x &= 4\sqrt{2} \cos^3 t, x=2 (x \geq 2) \\ y &= \sqrt{2} \sin^3 t. \end{aligned} \quad S = 1,3562$	
$10. \begin{aligned} z &= \cos \varphi - \sin \varphi \\ S &= 1,5708 \end{aligned}$	$11. \begin{aligned} y &= 22 \cos \sqrt{x} - \sqrt{x - x^2} + 4 \\ 0 &\leq x \leq \frac{1}{2}. \end{aligned} \quad L = 1,4142$	
$12. \begin{aligned} x &= (t^2 - 2) \sin t + 2t \cos t, \\ y &= (2 - t^2) \cos t + 2t \sin t. \\ 0 &\leq t \leq 3\pi \end{aligned} \quad L = 279,0565$	$13. \begin{aligned} \rho &= 6 \cos \varphi \\ 0 &\leq \varphi \leq \pi/3 \end{aligned} \quad L = 6,2832$	
$14. \begin{aligned} \frac{x^2}{25} + \frac{y^2}{9} - z^2 &= 1, \\ z &= 0, z=2. \end{aligned} \quad V = 219,9115$	$15. \begin{aligned} y &= x^3, \\ y &= x \end{aligned} \quad V = 0,8378$	
$16. \int_2^{\infty} \frac{1 + \ln(x-1)}{x-1} dx$ расходится	$17. \int_0^1 \frac{e^{2x} dx}{e^x - 1}$ расходится	$\begin{aligned} H &= 1,6; h = 1,2. \\ 18. R &= 0,3 \\ A &= 64,7838 \end{aligned}$

ВАРИАНТ 29

1. $\int_{-1}^1 x^2 e^{-x/2} dx = 10\sqrt{e} - 26/\sqrt{e} = 0,7174$		
2. $\int_0^1 \frac{x^3+x}{x^4+1} dx = 0,25 \ln 2 + 0,25\pi = 0,9587$		
3. $\int_0^{\pi/2} \frac{\sin x dx}{2+\sin x} = 0,3616$		
4. $\int_{\arccos(1/\sqrt{2})}^{\arccos(1/\sqrt{2})} \frac{12 dx}{(6+5 \operatorname{tg} x) \sin 2x} = 0,1214$		
5. $\int_0^{\pi} 2^4 \sin^2 \frac{x}{2} \cos^6 \frac{x}{2} dx = 1,9635$		
6. $\int_0^1 \frac{(4\sqrt{1-x} - \sqrt{x+1}) dx}{(\sqrt{x+1} + 4\sqrt{1-x})(x+1)^2} = 0,2012$		
7. $\int_0^{\sqrt{2}} \frac{1}{(1-x^2)^{3/2}} dx = 1$		
8. $y = (x-1)^2$ $y^2 = x-1$ $S = 0,3333$	9. $x = 2\sqrt{2} \cos t, y = 5 (y \geq 5)$ $y = 5\sqrt{2} \sin t. S = 5,708$	
10. $z = 3 \sin \varphi,$ $z = 5 \sin \varphi. S = 12,5664$	11. $y = (e^{2x} + e^{-2x} + 3)/4,$ $0 \leq x \leq 2 \quad l = 13,6449$	
12. $x = 2 \cos^3 t, \quad 0 \leq t \leq \pi/4$ $y = 2 \sin^3 t$ $l = 1,5$	13. $\rho = 2 \sin \varphi,$ $0 \leq \varphi \leq \pi/6. \quad l = 1,0472$	
14. $\frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{64} = -1,$ $z = 16 \quad V = 402,1238$	15. $y = \arccos x,$ $x = 0. \quad V = 0,8966$	
16. $\int_0^{\infty} \frac{(\arcsin x)^2 + 1}{\sqrt{1-x^2}} dx$ расходится	17. $\int_{-1}^1 \frac{e^x + 1}{e^x - 1} dx$ расходится	18. $H = 1,6; h = 0,8$ $R = 0,3$ $A = 32,392$

ВАРИАНТ 30

$$1. \int_0^1 x^2 e^{3x} dx = \frac{1}{27} (5e^3 - 2) = 3,6455$$

$$2. \int_2^9 \frac{x dx}{\sqrt[3]{x-1}} = 23,1$$

$$3. \int_0^{\pi/4} \frac{dx}{\cos x (1 + \cos x)} = -\operatorname{tg} \frac{\pi}{8} + \ln \frac{1 + \operatorname{tg} \frac{\pi}{8}}{1 - \operatorname{tg} \frac{\pi}{8}} = 0,4672$$

$$4. \int_0^{\pi/3} \frac{\operatorname{tg}^2 x}{4 + 3 \cos 2x} dx = 0,1985$$

$$5. \int_{-\pi/2}^0 2^8 \cos^6 x dx = 109,9557$$

$$6. \int_0^3 e^{\sqrt{\frac{3-x}{3+x}}} \frac{dx}{(3+x)\sqrt{9-x^2}} = 0,5728$$

$$7. \int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}} = 0,1812$$

$$8. \begin{cases} x = 4 - (y-1)^2 \\ x = y^2 - 4y + 3 \end{cases} \quad S = 9$$

$$9. \begin{cases} x = 4(t - \sin t), \quad y = 6 \\ y = 4(1 - \cos t) \end{cases} \quad S = 20,7846 \\ (0 < x < 8\pi, y \geq 6)$$

$$10. \begin{cases} z = 2 \sin \varphi \\ z = 4 \sin \varphi \end{cases} \quad S = 9,4248$$

$$11. \begin{cases} y = (1 - e^x - e^{-x})/2 \\ 0 \leq x \leq 3 \end{cases} \quad l = 10,0178$$

$$12. \begin{cases} x = e^t (\cos t + \sin t) \\ y = e^t (\cos t - \sin t) \\ \pi/8 \leq t \leq \pi/4 \end{cases} \quad l = 1,0104$$

$$13. \begin{cases} \rho = 8 \sin \varphi \\ 0 \leq \varphi \leq \pi/4 \end{cases} \quad l = 6,2832$$

$$14. \begin{cases} \frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{144} = 1 \\ z = 0, z = 6 \end{cases} \quad V = 207,3451$$

$$15. \begin{cases} y = 2 - x^2/2 \\ x + y = 2 \end{cases} \quad V = 4,1888$$

$$16. \int_{-\infty}^0 \frac{x dx}{x^4 + 1} = -0,7854$$

$$17. \int_0^{\pi} \frac{1 - \sin \sqrt{x}}{\sqrt{x}} dx = 1,1403$$

$$18. \begin{cases} H = 2,0; \quad h = 1,2 \\ R = 0,4 \\ A = 95,1554 \end{cases}$$

Указания.

Задача №1. Использовать формулу интегрирования по частям.

Задача №3. Применить универсальную тригонометрическую подстановку

$$\operatorname{tg} \frac{x}{2} = t.$$

Задача №4. Применить подстановку $\operatorname{tg} x = t$.

Задача №5. Использовать формулы удвоения угла

$$\sin^2 t = \frac{1 - \cos 2t}{2}, \quad \cos^2 t = \frac{1 + \cos 2t}{2}.$$

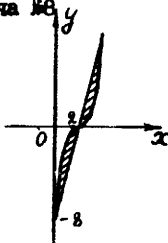
Задача №6. С помощью соответствующей подстановки избавиться от иррациональности в подынтегральной функции.

Задача №7. Использовать одну из подстановок $x = a \sin t$, $x = a \operatorname{tg} t$,

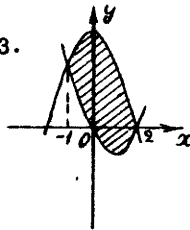
$x = \frac{a}{\sin t}$, где a выбирается в зависимости от условия задачи.

Задача №8

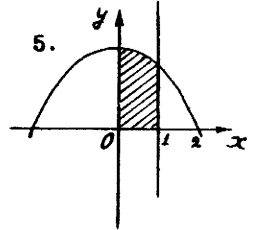
I.



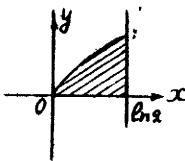
3.



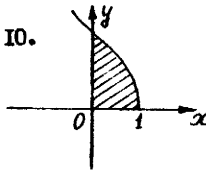
5.



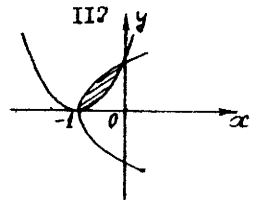
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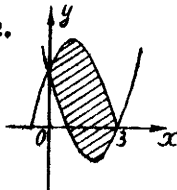
10.



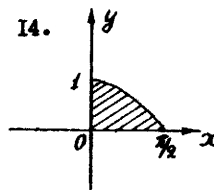
11?



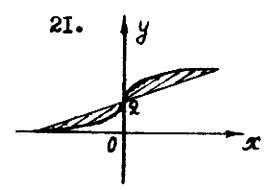
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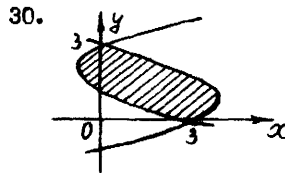
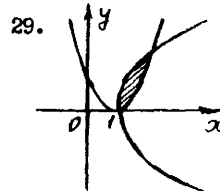
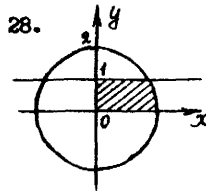
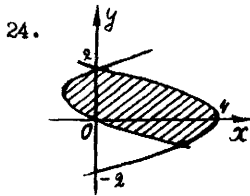


14.

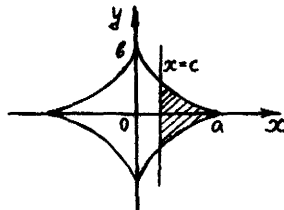


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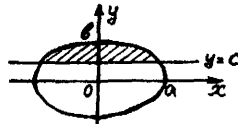




Задача №9. В вариантах 1, 4, 7, 10, 13, 16, 19, 22, 25, 28 уравнения $x = a \cos^3 t$, $y = a \sin^3 t$ определяют астроидальную кривую



В вариантах 2, 5, 8, 11, 14, 17, 20, 23, 26, 29 уравнения $x = a \cos t$, $y = b \sin t$ определяют эллипс

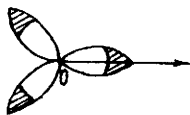


В вариантах 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 уравнения $x = a(t - \sin t)$, $y = a(1 - \cos t)$ определяют циклоиду

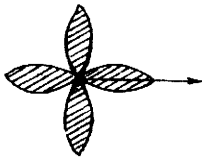


Задача №10.

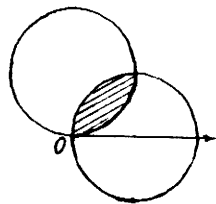
1, 11, 12.



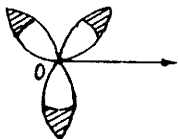
2.



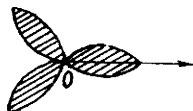
3, 5, 13.



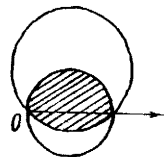
4, 6, 7.



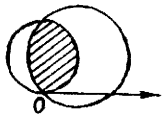
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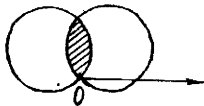
9.



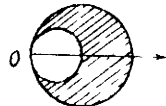
10.



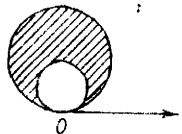
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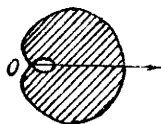
15, 21, 24.



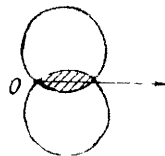
16, 20,
29, 30.



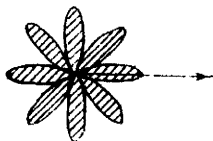
17, 18.



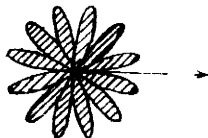
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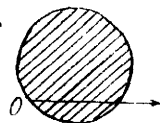
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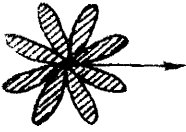


23.

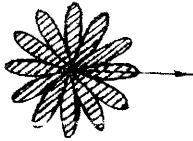


25.

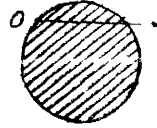




27.

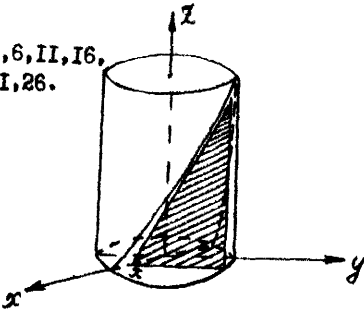


28.

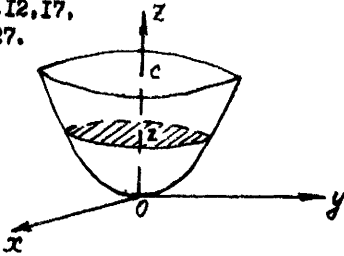


Задача #14.

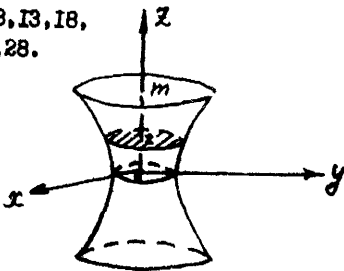
1, 6, 11, 16,
21, 26.



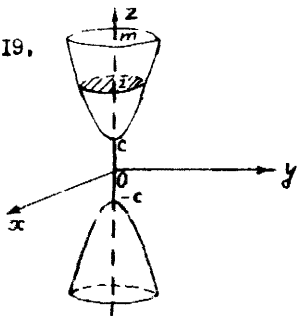
2, 7, 12, 17,
22, 27.



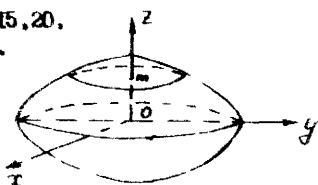
3, 8, 13, 18,
23, 28.



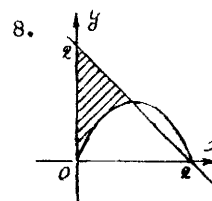
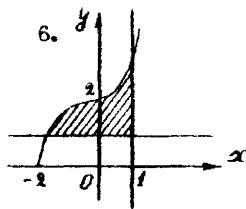
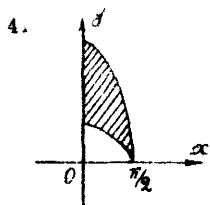
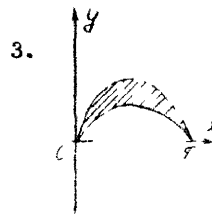
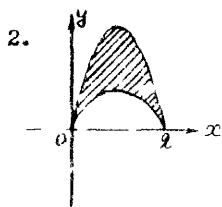
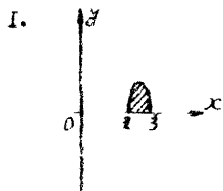
4, 9, 14, 19,
24, 29.

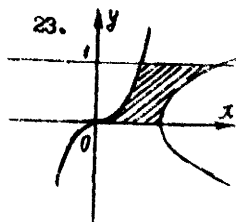
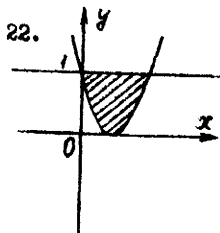
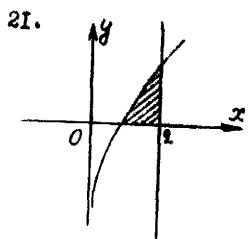
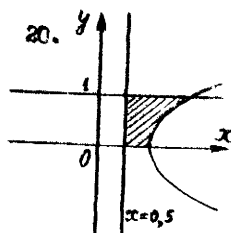
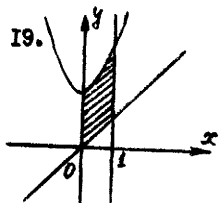
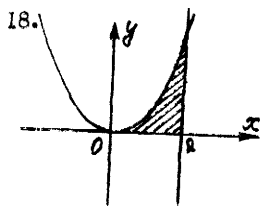
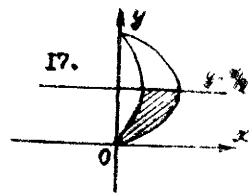
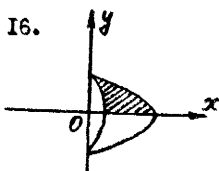
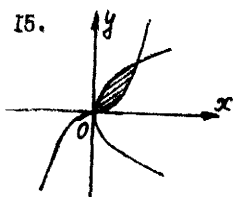
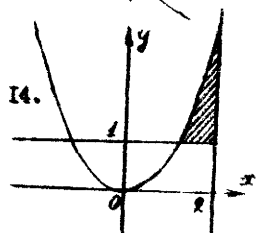
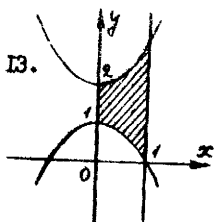
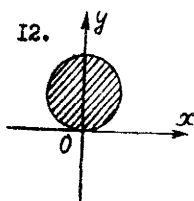
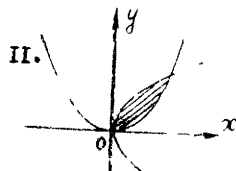
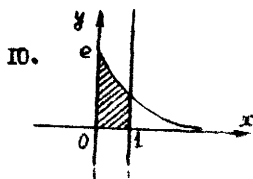
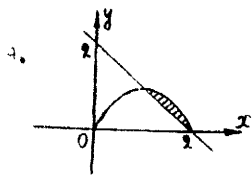


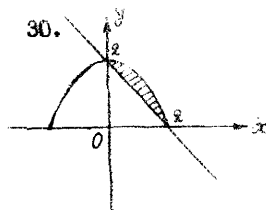
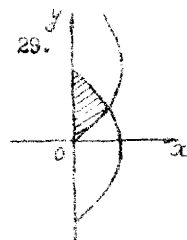
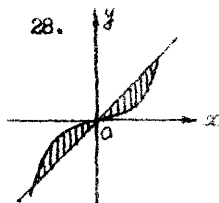
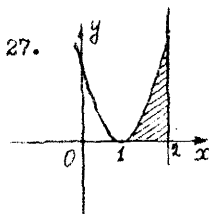
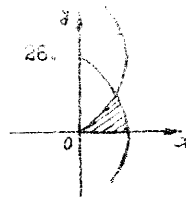
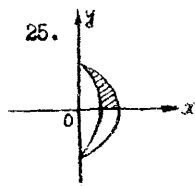
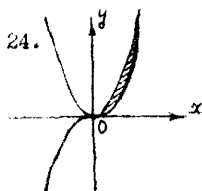
5, 10, 15, 20,
25, 30.



Задача №15.







ОПРЕДЕЛЕННЫЙ ИНТЕГРАЛ

Методические указания и контрольные задания

Составители: Анатолий Алексеевич Лучинин

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Валентина Ивановна Рыжкова

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