

VIII. ВЕКТОРНЫЙ АНАЛИЗ

Задания

Задача 1.

Найти производную скалярного поля $u(x, y, z)$ в точке M по направлению нормали к поверхности S , образующей острый угол с положительным направлением оси Oz .

1.1. $u = 4 \ln(3 + x^2) - 8xyz$, $S: x^2 - 2y^2 + 2z^2 = 1$, $M(1, 1, 1)$.

1.2. $u = x\sqrt{y} + y\sqrt{z}$, $S: 4z + 2x^2 - y^2 = 0$, $M(2, 4, 4)$.

1.3. $u = -2 \ln(x^2 - 5) - 4xyz$, $S: x^2 + 2y^2 - 2z^2 = 1$, $M(1, 1, 1)$.

1.4. $u = \frac{1}{4}x^2y - \sqrt{x^2 + 5z^2}$, $S: z^2 = x^2 + 4y^2 - 4$, $M\left(-2, \frac{1}{2}, 1\right)$.

1.5. $u = xz^2 - \sqrt{x^3y}$, $S: x^2 - y^2 - 3z + 12 = 0$, $M(2, 2, 4)$.

1.6. $u = x\sqrt{y} - yz^2$, $S: x^2 + y^2 = 4z$, $M(2, 1, -1)$.

1.7. $u = 7 \ln(1/13 + x^2) - 4xyz$, $S: 7x^2 - 4y^2 + 4z^2 = 7$, $M(1, 1, 1)$.

1.8. $u = \arctg(y/x) - 8xyz$, $S: x^2 + y^2 - 2z^2 = 10$, $M(2, 2, -1)$.

1.9. $u = \ln(1 + x^2) - xy\sqrt{z}$, $S: 4x^2 - y^2 + z^2 = 16$, $M(1, -2, 4)$.

1.10. $u = \sqrt{x^2 + y^2} - z$, $S: x^2 + y^2 = 24z$, $M(3, 4, 1)$.

1.11. $u = x\sqrt{y} - (z + y)\sqrt{x}$, $S: x^2 - y^2 + z^2 = 4$, $M(1, 1, -2)$.

1.12. $u = \sqrt{xy} - \sqrt{4 - z^2}$, $S: z = x^2 - y^2$, $M(1, 1, 0)$.

1.13. $u = (x^2 + y^2 + z^2)^{3/2}$, $S: 2x^2 - y^2 + z^2 - 1 = 0$, $M(0, -3, 4)$.

1.14. $u = \ln(1 + x^2 + y^2) - \sqrt{x^2 + z^2}$, $S: x^2 - 6x + 9y^2 + z^2 = 4z + 4$, $M(3, 0, -4)$.

Найти производную скалярного поля $u(x, y, z)$ в точке M по направлению вектора \mathbf{l} .

$$u = (x^2 + y^2 + z^2)^{3/2},$$

$$u = x + \ln(z^2 + y^2),$$

1.15. $\mathbf{l} = \mathbf{i} - \mathbf{j} + \mathbf{k}$,

1.16. $\mathbf{l} = -2\mathbf{i} + \mathbf{j} - \mathbf{k}$,

$$M(1, 1, 1).$$

$$M(2, 1, 1).$$

$$u = x^2 y - \sqrt{xy + z^2},$$

$$1.17. \mathbf{l} = 2\mathbf{j} - 2\mathbf{k},$$

$$M(1, 5, -2).$$

$$u = y \ln(1 + x^2) - \operatorname{arctg} z,$$

$$1.18. \mathbf{l} = 2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k},$$

$$M(0, 1, 1).$$

$$u = x(\ln y - \operatorname{arctg} z),$$

$$1.19. \mathbf{l} = 8\mathbf{i} + 4\mathbf{j} + 8\mathbf{k},$$

$$M(-2, 1, -1).$$

$$u = \ln(3 - x^2) + xy^2 z,$$

$$1.20. \mathbf{l} = -\mathbf{i} + 2\mathbf{j} - 2\mathbf{k},$$

$$M(1, 3, 2).$$

$$u = \sin(x + 2y) + \sqrt{xyz},$$

$$1.21. \mathbf{l} = 4\mathbf{i} + 3\mathbf{j},$$

$$M(\pi/2, 3\pi/2, 3).$$

$$u = x^2 y^2 z - \ln(z - 1),$$

$$1.22. \mathbf{l} = 5\mathbf{i} - 6\mathbf{j} + 2\sqrt{5}\mathbf{k},$$

$$M(1, 1, 2).$$

$$u = x^3 + \sqrt{y^2 + z^2},$$

$$1.23. \mathbf{l} = \mathbf{j} - \mathbf{k},$$

$$M(1, -3, 4).$$

$$u = \frac{\sqrt{x}}{y} - \frac{yz}{x + \sqrt{y}},$$

$$1.24. \mathbf{l} = 2\mathbf{i} + \mathbf{k},$$

$$M(4, 1, -2).$$

$$u = \sqrt{xy} + \sqrt{9 - z^2},$$

$$1.25. \mathbf{l} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k},$$

$$M(1, 1, 0).$$

$$u = 2\sqrt{x + y} + y \operatorname{arctg} z,$$

$$1.26. \mathbf{l} = 4\mathbf{i} - 3\mathbf{k},$$

$$M(3, 2, -1).$$

$$u = z^2 + 2 \operatorname{arctg}(x - y),$$

$$1.27. \mathbf{l} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k},$$

$$M(1, 2, -1).$$

$$u = \ln(x^2 + y^2) + xyz,$$

$$1.28. \mathbf{l} = \mathbf{i} - \mathbf{j} + 5\mathbf{k},$$

$$M(1, -1, 2).$$

$$u = xy - \frac{x}{z},$$

$$1.29. \mathbf{l} = 5\mathbf{i} + \mathbf{j} - \mathbf{k},$$

$$M(-4, 3, -1).$$

$$u = \ln\left(x + \sqrt{y^2 + z^2}\right),$$

$$1.30. \mathbf{l} = -2\mathbf{i} - \mathbf{j} + \mathbf{k},$$

$$M(1, -3, 4).$$

$$u = x^2 - \operatorname{arctg}(y + z),$$

$$1.31. \mathbf{l} = 3\mathbf{j} - 4\mathbf{k},$$

$$M(2, 1, 1).$$

Задача 2. Найти угол между градиентами скалярных полей $u(x, y, z)$ и $v(x, y, z)$ в точке M .

$$2.1. v = \frac{x^3}{2} + 6y^3 + 3\sqrt{6}z^3, \quad u = \frac{yz^2}{x^2}, \quad M\left(\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right).$$

$$2.2. v = \frac{4\sqrt{6}}{x} - \frac{\sqrt{6}}{9y} + \frac{3}{z}, \quad u = x^2yz^3, \quad M\left(2, \frac{1}{3}, \sqrt{\frac{3}{2}}\right).$$

$$2.3. v = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} - \frac{4z^3}{\sqrt{3}}, \quad u = \frac{z^3}{xy^2}, \quad M\left(\frac{1}{3}, 2, \sqrt{\frac{3}{2}}\right).$$

$$2.4. v = \frac{3}{x} + \frac{4}{y} - \frac{1}{\sqrt{6}z}, \quad u = \frac{z}{x^3y^2}, \quad M\left(1, 2, \frac{1}{\sqrt{6}}\right).$$

$$2.5. v = \frac{x^3}{2} + 6y^3 + 3\sqrt{6}z^3, \quad u = \frac{x^2}{yz^2}, \quad M\left(\sqrt{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right).$$

$$2.6. v = 3\sqrt{2}x^2 - \frac{y^2}{\sqrt{2}} + 3\sqrt{2}z^3, \quad u = \frac{z^2}{xy^2}, \quad M\left(\frac{1}{3}, 2, \sqrt{\frac{2}{3}}\right).$$

$$2.7. v = 6\sqrt{6}x^3 - 6\sqrt{6}y^3 + 2z^3, \quad u = \frac{xz^2}{y}, \quad M\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, 1\right).$$

$$2.8. v = \frac{\sqrt{6}}{2x} - \frac{\sqrt{6}}{2y} + \frac{2}{3z}, \quad u = \frac{yz^2}{x}, \quad M\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right).$$

$$2.9. v = 3\sqrt{2}x^2 - \frac{y^2}{\sqrt{2}} - 3\sqrt{2}z^2, \quad u = \frac{xy^2}{z^2}, \quad M\left(\frac{1}{3}, 2, \sqrt{\frac{2}{3}}\right).$$

$$2.10. v = \frac{3}{x} + \frac{4}{y} - \frac{1}{\sqrt{6}z}, \quad u = \frac{x^3y^2}{z}, \quad M\left(1, 2, \frac{1}{\sqrt{6}}\right).$$

$$2.11. v = -\frac{4\sqrt{2}}{x} + \frac{\sqrt{2}}{9y} + \frac{1}{\sqrt{3}z}, \quad u = \frac{1}{x^2yz}, \quad M\left(2, \frac{1}{3}, \frac{1}{\sqrt{6}}\right).$$

$$2.12. v = \frac{6}{x} + \frac{2}{y} - \frac{3\sqrt{3}}{2\sqrt{2}z}, \quad u = \frac{x^2}{y^2z^3}, \quad M\left(\sqrt{2}, \sqrt{2}, \frac{\sqrt{3}}{2}\right).$$

$$2.13. v = x^2 + 9y^2 + 6z^2, \quad u = xyz, \quad M\left(1, \frac{1}{3}, \frac{1}{\sqrt{6}}\right).$$

$$2.14. v = \frac{2}{x} + \frac{3}{2y} - \frac{\sqrt{6}}{4z}, \quad u = \frac{y^3}{x^2z}, \quad M\left(\sqrt{\frac{2}{3}}, \sqrt{\frac{3}{2}}, \frac{1}{2}\right).$$

$$2.15. v = \sqrt{2}x^2 - \frac{3y^2}{\sqrt{2}} - 6\sqrt{2}z^2, \quad u = xy^2z, \quad M\left(1, \frac{2}{3}, \frac{1}{\sqrt{6}}\right).$$

$$2.16. v = -\frac{\sqrt{6}}{2x} + \frac{\sqrt{6}}{2y} - \frac{2}{3z}, \quad u = \frac{x}{yz^2}, \quad M\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right).$$

$$2.17. v = \frac{6}{x} + \frac{2}{y} + \frac{3\sqrt{3}}{2\sqrt{2}z}, \quad u = \frac{y^2z^3}{x^2}, \quad M\left(\sqrt{2}, \sqrt{2}, \frac{\sqrt{3}}{2}\right).$$

$$2.18. v = \frac{1}{\sqrt{2}x} - \frac{2\sqrt{2}}{y} - \frac{3\sqrt{3}}{2z}, \quad u = \frac{y^2z^3}{x}, \quad M\left(\frac{1}{\sqrt{2}}, \sqrt{2}, \frac{\sqrt{3}}{2}\right).$$

$$2.19. v = 6\sqrt{6}x^3 - 6\sqrt{6}y^3 + 2z^3, \quad u = \frac{y}{xz^2}, \quad M\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, ?\right).$$

$$2.20. v = x^2 - y^2 - 3z^2, \quad u = \frac{yz^2}{x}, \quad M\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right).$$

$$2.21. v = \frac{3x^2}{\sqrt{2}} - \frac{y^2}{\sqrt{2}} + \sqrt{2}z^2, \quad u = \frac{z^2}{x^2y^2}, \quad M\left(\frac{2}{3}, 2, \sqrt{\frac{2}{3}}\right).$$

$$2.22. v = \frac{x^3}{\sqrt{2}} - \frac{y^3}{\sqrt{2}} - \frac{8z^3}{\sqrt{3}}, \quad u = \frac{x^2}{y^2z^3}, \quad M\left(\sqrt{2}, \sqrt{2}, \frac{\sqrt{3}}{2}\right).$$

$$2.23. v = \frac{3}{2}x^2 + 3y^2 - 2z^2, \quad u = x^2yz^3, \quad M\left(2, \frac{1}{3}, \sqrt{\frac{3}{2}}\right).$$

$$2.24. v = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} - \frac{4z^3}{\sqrt{3}}, \quad u = \frac{xy^2}{z^3}, \quad M\left(\frac{1}{3}, 2, \sqrt{\frac{3}{2}}\right).$$

$$2.25. v = \sqrt{2}x^2 - \frac{3y^2}{\sqrt{2}} - 6\sqrt{2}z^2, \quad u = \frac{1}{xy^2z}, \quad M\left(1, \frac{2}{3}, \frac{1}{\sqrt{6}}\right).$$

$$2.26. v = x^2 + 9y^2 + 6z^2, \quad u = \frac{1}{xyz}, \quad M\left(1, \frac{1}{3}, \frac{1}{\sqrt{6}}\right).$$

$$2.27. v = \frac{1}{\sqrt{2}x} - \frac{2\sqrt{2}}{y} - \frac{3\sqrt{3}}{2z}, \quad u = \frac{x}{y^2z^3}, \quad M\left(\frac{1}{\sqrt{2}}, \sqrt{2}, \frac{\sqrt{3}}{2}\right).$$

$$2.28. v = -\frac{4\sqrt{2}}{x} + \frac{\sqrt{2}}{9y} + \frac{1}{\sqrt{3}z}, \quad u = x^2yz, \quad M\left(2, \frac{1}{3}, \frac{1}{\sqrt{6}}\right).$$

$$2.29. v = \frac{x^3}{\sqrt{2}} - \frac{y^3}{\sqrt{2}} - \frac{8z^3}{\sqrt{3}}, \quad u = \frac{y^2z^3}{x^2}, \quad M\left(\sqrt{2}, \sqrt{2}, \frac{\sqrt{3}}{2}\right).$$

$$2.30. v = -\frac{3x^3}{\sqrt{2}} + \frac{2\sqrt{2}y^3}{3} + 8\sqrt{3}z^3, \quad u = \frac{x^2z}{x^2}, \quad M\left(\sqrt{\frac{2}{3}}, \sqrt{\frac{3}{2}}, \frac{1}{2}\right).$$

$$2.31. v = x^2 - y^2 - 3z^2, \quad u = \frac{x}{yz^2}, \quad M\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right).$$

Задача 3. Найти векторные линии в векторном поле \mathbf{a} .

3.1. $\mathbf{a} = 4y\mathbf{i} - 9x\mathbf{j}$.

3.2. $\mathbf{a} = 2y\mathbf{i} + 3x\mathbf{j}$.

3.3. $\mathbf{a} = 2x\mathbf{i} + 4y\mathbf{j}$.

3.4. $\mathbf{a} = x\mathbf{i} + 3y\mathbf{j}$.

3.5. $\mathbf{a} = x\mathbf{i} + 4y\mathbf{j}$.

3.6. $\mathbf{a} = 3x\mathbf{i} + 6z\mathbf{k}$.

3.7. $\mathbf{a} = 4z\mathbf{i} - 9x\mathbf{k}$.

3.8. $\mathbf{a} = 2z\mathbf{i} + 3x\mathbf{k}$.

3.9. $\mathbf{a} = 4y\mathbf{j} + 8z\mathbf{k}$.

3.10. $\mathbf{a} = y\mathbf{j} + 3z\mathbf{k}$.

3.11. $\mathbf{a} = 2x\mathbf{i} + 8z\mathbf{k}$.

3.12. $\mathbf{a} = x\mathbf{i} + 3z\mathbf{k}$.

3.13. $\mathbf{a} = 4z\mathbf{j} - 9y\mathbf{k}$.

3.14. $\mathbf{a} = 2z\mathbf{j} + 3y\mathbf{k}$.

3.15. $\mathbf{a} = 5x\mathbf{i} + 10y\mathbf{j}$.

3.16. $\mathbf{a} = 2x\mathbf{i} + 6y\mathbf{j}$.

3.17. $\mathbf{a} = y\mathbf{j} + 4z\mathbf{k}$.

3.18. $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$.

3.19. $\mathbf{a} = 9y\mathbf{i} - 4x\mathbf{j}$.

3.20. $\mathbf{a} = 5y\mathbf{i} + 7x\mathbf{j}$.

3.21. $\mathbf{a} = 6x\mathbf{i} + 12z\mathbf{k}$.

3.22. $\mathbf{a} = 2y\mathbf{j} + 6z\mathbf{k}$.

3.23. $\mathbf{a} = 4x\mathbf{i} + y\mathbf{j}$.

3.24. $\mathbf{a} = 9z\mathbf{i} - 4x\mathbf{k}$.

3.25. $\mathbf{a} = x\mathbf{i} + z\mathbf{k}$.

3.26. $\mathbf{a} = 5z\mathbf{i} + 7x\mathbf{k}$.

3.27. $\mathbf{a} = 7y\mathbf{j} + 14z\mathbf{k}$.

3.28. $\mathbf{a} = 2x\mathbf{i} + 6z\mathbf{k}$.

3.29. $\mathbf{a} = 4x\mathbf{i} + z\mathbf{k}$.

3.30. $\mathbf{a} = 5z\mathbf{j} + 7y\mathbf{k}$.

3.31. $\mathbf{a} = 9z\mathbf{j} - 4y\mathbf{k}$.

Задача 4. Найти поток векторного поля \mathbf{a} через часть поверхности S , вырезаемую плоскостями P_1, P_2 (нормаль внешняя к замкнутой поверхности, образуемой данными поверхностями).

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

$$4.1. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 2.$$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} - z\mathbf{k}.$$

$$4.2. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 4.$$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}.$$

$$4.3. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 3.$$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z^3\mathbf{k}.$$

$$4.4. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 1.$$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + xyz\mathbf{k}.$$

$$4.5. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 5.$$

$$\mathbf{a} = (x - y)\mathbf{i} + (x + y)\mathbf{j} + z^2\mathbf{k}.$$

$$4.6. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 2.$$

$$\mathbf{a} = (x + y)\mathbf{i} - (x - y)\mathbf{j} + xyz\mathbf{k}.$$

$$4.7. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 4.$$

$$\mathbf{a} = (x^3 + xy^2)\mathbf{i} + (y^3 + x^2y)\mathbf{j} + z^2\mathbf{k}.$$

$$4.8. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 3.$$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + \sin z\mathbf{k}.$$

$$4.9. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 5.$$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + \mathbf{k}.$$

$$4.10. S: x^2 + y^2 = 1, \\ P_1: z = 0, P_2: z = 2.$$

Найти поток векторного поля \mathbf{a} через часть поверхности S , вырезаемую плоскостью P (нормаль внешняя к замкнутой поверхности, образуемой данными поверхностями).

$$4.11. \mathbf{a} = (x + xy^2)\mathbf{i} + (y - yx^2)\mathbf{j} + (z - 3)\mathbf{k}, \quad S: x^2 + y^2 = z^2 \quad (z \geq 0), \quad P: z = 1.$$

$$4.12. \mathbf{a} = y\mathbf{i} - x\mathbf{j} + \mathbf{k}, \quad S: x^2 + y^2 = z^2 \quad (z \geq 0), \quad P: z = 4.$$

$$4.13. \mathbf{a} = xy\mathbf{i} - x^2\mathbf{j} + 3\mathbf{k}, \quad S: x^2 + y^2 = z^2 \quad (z \geq 0), \quad P: z = 1.$$

$$4.14. \mathbf{a} = xz\mathbf{i} + yz\mathbf{j} + (z^2 - 1)\mathbf{k}, \quad S: x^2 + y^2 = z^2 \quad (z \geq 0), \quad P: z = 4.$$

$$4.15. \mathbf{a} = y^2x\mathbf{i} - yx^2\mathbf{j} + \mathbf{k}, \quad S: x^2 + y^2 = z^2 \quad (z \geq 0), \quad P: z = 5.$$

- 4.16. $\mathbf{a} = (xz + y)\mathbf{i} + (yz - x)\mathbf{j} + (z^2 - 2)\mathbf{k}$, $S: x^2 + y^2 = z^2$ ($z \geq 0$), $P: z = 3$.
- 4.17. $\mathbf{a} = xyz\mathbf{i} - x^2z\mathbf{j} + 3\mathbf{k}$, $S: x^2 + y^2 = z^2$ ($z \geq 0$), $P: z = 2$.
- 4.18. $\mathbf{a} = (x + xy)\mathbf{i} + (y - x^2)\mathbf{j} + (z - 1)\mathbf{k}$, $S: x^2 + y^2 = z^2$ ($z \geq 0$), $P: z = 3$.
- 4.19. $\mathbf{a} = (x + y)\mathbf{i} + (y - x)\mathbf{j} + (z - 2)\mathbf{k}$, $S: x^2 + y^2 = z^2$ ($z \geq 0$), $P: z = 2$.
- 4.20. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + (z - 2)\mathbf{k}$, $S: x^2 + y^2 = z^2$ ($z \geq 0$), $P: z = 1$.
- 4.21. $\mathbf{a} = (x + xz)\mathbf{i} + y\mathbf{j} + (z - x^2)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 4$ ($z \geq 0$), $P: z = 0$.
- 4.22. $\mathbf{a} = x\mathbf{i} + (y + yz^2)\mathbf{j} + (z - zy^2)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 4$, $P: z = 0$ ($z \geq 0$).
- 4.23. $\mathbf{a} = (x + z)\mathbf{i} + (y + z)\mathbf{j} + (z - x - y)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 4$, $P: z = 0$ ($z \geq 0$).
- 4.24. $\mathbf{a} = (x + xy)\mathbf{i} + (y - x^2)\mathbf{j} + z\mathbf{k}$, $S: x^2 + y^2 + z^2 = 1$, $P: z = 0$ ($z \geq 0$).
- 4.25. $\mathbf{a} = (x + z)\mathbf{i} + y\mathbf{j} + (z - x)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 1$, $P: z = 0$ ($z \geq 0$).
- 4.26. $\mathbf{a} = x\mathbf{i} + (y + yz)\mathbf{j} + (z - y^2)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 1$, $P: z = 0$ ($z \geq 0$).
- 4.27. $\mathbf{a} = (x - y)\mathbf{i} + (x + y)\mathbf{j} + z\mathbf{k}$, $S: x^2 + y^2 + z^2 = 1$, $P: z = 0$ ($z \geq 0$).
- 4.28. $\mathbf{a} = (x + xz^2)\mathbf{i} + y\mathbf{j} + (z - zx^2)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 9$, $P: z = 0$ ($z \geq 0$).
- 4.29. $\mathbf{a} = (x + y)\mathbf{i} + (y - x)\mathbf{j} + z\mathbf{k}$, $S: x^2 + y^2 + z^2 = 4$, $P: z = 0$ ($z \geq 0$).
- 4.30. $\mathbf{a} = (x + xy^2)\mathbf{i} + (y - yx^2)\mathbf{j} + z\mathbf{k}$, $S: x^2 + y^2 + z^2 = 9$, $P: z = 0$ ($z \geq 0$).
- 4.31. $\mathbf{a} = x\mathbf{i} + (y + z)\mathbf{j} + (z - y)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 9$, $P: z = 0$ ($z \geq 0$).

Задача 5. Найти поток векторного поля \mathbf{a} через часть плоскости P , расположенную в первом октанте (нормаль образует острый угол с осью Oz).

5.1. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
 $P: x + y + z = 1.$

5.2. $\mathbf{a} = y\mathbf{j} + z\mathbf{k}$
 $P: x + y + z = 1.$

5.3. $\mathbf{a} = 2x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
 $P: x + y + z = 1.$

5.4. $\mathbf{a} = x\mathbf{i} + 3y\mathbf{j} + 2z\mathbf{k}$
 $P: x + y + z = 1.$

5.5. $\mathbf{a} = 2x\mathbf{i} + 3y\mathbf{j}$
 $P: x + y + z = 1.$

5.6. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
 $P: x/2 + y + z = 1.$

5.7. $\mathbf{a} = x\mathbf{i} + 2y\mathbf{j} + z\mathbf{k}$
 $P: x/2 + y + z = 1.$

5.8. $\mathbf{a} = y\mathbf{j} + 3z\mathbf{k}$
 $P: x/2 + y + z = 1.$

5.9. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
 $P: x + y/2 + z/3 = 1.$

5.10. $\mathbf{a} = 2x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
 $P: x + y/2 + z/3 = 1.$

5.11. $\mathbf{a} = 3x\mathbf{i} + 2z\mathbf{k}$
 $P: x + y/2 + z/2 = 1.$

5.12. $\mathbf{a} = 2x\mathbf{i} + 3y\mathbf{j} + z\mathbf{k}$
 $P: x/3 + y + z/2 = 1.$

5.13. $\mathbf{a} = x\mathbf{i} + 3y\mathbf{j} - z\mathbf{k}$
 $P: x/3 + y + z/2 = 1.$

5.14. $\mathbf{a} = -2x\mathbf{i} + y\mathbf{j} + 4z\mathbf{k}$
 $P: x/3 + y + z/2 = 1.$

5.15. $\mathbf{a} = x\mathbf{i} - y\mathbf{j} + 6z\mathbf{k}$
 $P: x/2 + y/3 + z = 1.$

5.16. $\mathbf{a} = 2x\mathbf{i} + 5y\mathbf{j} + 5z\mathbf{k}$
 $P: x/2 + y/3 + z = 1.$

5.17. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
 $P: 2x + y/2 + z = 1.$

5.18. $\mathbf{a} = 2x\mathbf{i} + y\mathbf{j} - 2z\mathbf{k}$
 $P: 2x + y/2 + z = 1.$

5.19. $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}$
 $P: 2x + y/2 + z = 1.$

5.20. $\mathbf{a} = -x\mathbf{i} + y\mathbf{j} + 12z\mathbf{k}$
 $P: 2x + y/2 + z = 1.$

5.21. $\mathbf{a} = x\mathbf{i} + 3y\mathbf{j} + 8z\mathbf{k}$
 $P: x + 2y + z/2 = 1.$

5.22. $\mathbf{a} = x\mathbf{i} - y\mathbf{j} + 6z\mathbf{k}$
 $P: x + 2y + z/2 = 1.$

$$\mathbf{a} = x\mathbf{i} + 2y\mathbf{j} + 5z\mathbf{k}$$

5.23. $P: x + 2y + \frac{z}{2} = 1.$

$$\mathbf{a} = x\mathbf{i} + 4y\mathbf{j} + 5z\mathbf{k}$$

5.24. $P: x + 2y + \frac{z}{2} = 1.$

$$\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

5.25. $P: 2x + 3y + z = 1.$

$$\mathbf{a} = 2x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

5.26. $P: 2x + 3y + z = 1.$

$$\mathbf{a} = 2x\mathbf{i} + 3y\mathbf{j} + z\mathbf{k}$$

5.27. $P: 2x + 3y + z = 1.$

$$\mathbf{a} = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$$

5.28. $P: 2x + 3y + z = 1.$

$$\mathbf{a} = x\mathbf{i} + 9y\mathbf{j} + 8z\mathbf{k}$$

5.29. $P: x + 2y + 3z = 1.$

$$\mathbf{a} = 8x\mathbf{i} + 11y\mathbf{j} + 17z\mathbf{k}$$

5.30. $P: x + 2y + 3z = 1.$

$$\mathbf{a} = -x\mathbf{i} + 2y\mathbf{j} + z\mathbf{k}$$

5.31. $P: x + 2y + 3z = 1.$

Задача 6. Найти поток векторного поля \mathbf{a} через часть плоскости P , расположенную в 1 октанте (нормаль образует острый угол с осью Oz).

6.1. $\mathbf{a} = 7x\mathbf{i} + (5\pi y + 2)\mathbf{j} + 4\pi z\mathbf{k}$,
 $P: x + y/2 + 4z = 1$.

6.2. $\mathbf{a} = 2\pi x\mathbf{i} + (7y + 2)\mathbf{j} + 7\pi z\mathbf{k}$,
 $P: x + y/2 + z/3 = 1$.

6.3. $\mathbf{a} = 9\pi x\mathbf{i} + \mathbf{j} - 3z\mathbf{k}$,
 $P: x/3 + y + z = 1$.

6.4. $\mathbf{a} = (2x + 1)\mathbf{i} - y\mathbf{j} + 3\pi z\mathbf{k}$,
 $P: x/3 + y + 2z = 1$.

6.5. $\mathbf{a} = 7x\mathbf{i} + 9\pi y\mathbf{j} + \mathbf{k}$,
 $P: x + y/3 + z = 1$.

6.6. $\mathbf{a} = \mathbf{i} + 5y\mathbf{j} + 11\pi z\mathbf{k}$,
 $P: x + y + z/3 = 1$.

6.7. $\mathbf{a} = x\mathbf{i} + (\pi z - 1)\mathbf{k}$,
 $P: 2x + y/2 + z/3 = 1$.

6.8. $\mathbf{a} = 5\pi x\mathbf{i} + (9y + 1)\mathbf{j} + 4\pi z\mathbf{k}$,
 $P: x/2 + y/3 + z/2 = 1$.

6.9. $\mathbf{a} = 2\mathbf{i} - y\mathbf{j} + \frac{3\pi}{2}z\mathbf{k}$,
 $P: x/3 + y + z/4 = 1$.

6.10. $\mathbf{a} = 9\pi x\mathbf{i} + (5y + 1)\mathbf{j} + 2\pi z\mathbf{k}$,
 $P: 3x + y + z/9 = 1$.

6.11. $\mathbf{a} = 7\pi x\mathbf{i} + 2\pi y\mathbf{j} + (7z + 2)\mathbf{k}$,
 $P: x + y + z/2 = 1$.

6.12. $\mathbf{a} = \pi y\mathbf{i} + (4 - 2z)\mathbf{k}$,
 $P: 2x + y/3 + z/4 = 1$.

6.13. $\mathbf{a} = (3\pi - 1)x\mathbf{i} + (9\pi y + 1)\mathbf{j} + 6\pi z\mathbf{k}$,
 $P: \frac{x}{2} + \frac{y}{3} + \frac{z}{9} = 1$.

6.14. $\mathbf{a} = \pi x\mathbf{i} + \frac{\pi}{2}y\mathbf{j} + (4 - 2z)\mathbf{k}$,
 $P: x + \frac{y}{3} + \frac{z}{4} = 1$.

6.15. $\mathbf{a} = (5y + 3)\mathbf{j} + 11\pi z\mathbf{k}$,
 $P: x + y/3 + 4z = 1$.

6.16. $\mathbf{a} = 9\pi y\mathbf{j} + (7z + 1)\mathbf{k}$,
 $P: x + y + z = 1$.

6.17. $\mathbf{a} = \pi y\mathbf{j} + (1 - 2z)\mathbf{k}$,
 $P: x/4 + y/2 + z = 1$.

6.18. $\mathbf{a} = (27\pi - 1)x\mathbf{i} + (34\pi y + 3)\mathbf{j} + 20\pi z\mathbf{k}$,
 $P: 3x + \frac{y}{9} + z = 1$.

6.19. $\mathbf{a} = \pi x\mathbf{i} + 2\mathbf{j} + 2\pi z\mathbf{k}$,
 $P: x/2 + y/3 + z = 1$.

6.20. $\mathbf{a} = 4\pi x\mathbf{i} + 7\pi y\mathbf{j} + (2z + 1)\mathbf{k}$,
 $P: 2x + y/3 + 2z = 1$.

6.21. $\mathbf{a} = 3\pi x\mathbf{i} + 6\pi y\mathbf{j} + 10\mathbf{k}$,
 $P: 2x + y + z/3 = 1.$

6.22. $\mathbf{a} = \pi x\mathbf{i} - 2y\mathbf{j} + \mathbf{k}$,
 $P: 2x + y/6 + z = 1.$

6.23. $\mathbf{a} = (21\pi - 1)x\mathbf{i} + 62\pi y\mathbf{j} + (1 - 2\pi z)\mathbf{k}$,
 $P: 8x + y/2 + z/3 = 1.$

6.24. $\mathbf{a} = \pi x\mathbf{i} + 2\pi y\mathbf{j} + 2\mathbf{k}$,
 $P: x/2 + y/4 + z/3 = 1.$

6.25. $\mathbf{a} = 9\pi x\mathbf{i} + 2\pi y\mathbf{j} + 8\mathbf{k}$,
 $P: 2x + 8y + z/3 = 1.$

6.26. $\mathbf{a} = 7\pi x\mathbf{i} + (4y + 1)\mathbf{j} + 2\pi z\mathbf{k}$,
 $P: x/3 + 2y + z = 1.$

6.27. $\mathbf{a} = 6\pi x\mathbf{i} + 3\pi y\mathbf{j} + 10\mathbf{k}$,
 $P: 2x + y/2 + z/3 = 1.$

6.28. $\mathbf{a} = (\pi - 1)x\mathbf{i} + 2\pi y\mathbf{j} + (1 - \pi z)\mathbf{k}$,
 $P: \frac{x}{4} + \frac{y}{2} + \frac{z}{3} = 1.$

6.29. $\mathbf{a} = \frac{\pi}{2}x\mathbf{i} + \pi y\mathbf{j} + (4 - 2z)\mathbf{k}$,
 $P: x + \frac{y}{3} + \frac{z}{4} = 1.$

6.30. $\mathbf{a} = 7\pi x\mathbf{i} + 4\pi y\mathbf{j} + 2(z + 1)\mathbf{k}$,
 $P: x/3 + y/4 + z = 1.$

6.31. $\mathbf{a} = 5\pi x\mathbf{i} + (1 - 2y)\mathbf{j} + 4\pi z\mathbf{k}$,
 $P: x/2 + 4y + z/3 = 1.$

Задача 7. Найти поток векторного поля \mathbf{a} через замкнутую поверхность S (нормаль внешняя).

7.1. $\mathbf{a} = (e^z + 2x)\mathbf{i} + e^x \mathbf{j} + e^y \mathbf{k}$, $S: x + y + z = 1$, $x = 0$, $y = 0$, $z = 0$.

7.2. $\mathbf{a} = (3z^2 + x)\mathbf{i} + (e^x - 2y)\mathbf{j} + (2z - xy)\mathbf{k}$, $S: x^2 + y^2 = z^2$, $z = 1$, $z = 4$.

7.3. $\mathbf{a} = (\ln y + 7x)\mathbf{i} + (\sin z - 2y)\mathbf{j} + (e^y - 2z)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 2x + 2y + 2z - 2$.

7.4. $\mathbf{a} = (\cos z + 3x)\mathbf{i} + (x - 2y)\mathbf{j} + (3z + y^2)\mathbf{k}$, $S: z^2 = 36(x^2 + y^2)$, $z = 6$.

7.5. $\mathbf{a} = (e^{-z} - x)\mathbf{i} + (xz + 3y)\mathbf{j} + (z + x^2)\mathbf{k}$, $S: 2x + y + z = 2$, $x = 0$, $y = 0$, $z = 0$.

7.6. $\mathbf{a} = (6x - \cos y)\mathbf{i} - (e^x + z)\mathbf{j} - (2y + 3z)\mathbf{k}$, $S: x^2 + y^2 = z^2$, $z = 1$, $z = 2$.

7.7. $\mathbf{a} = (4x - 2y^2)\mathbf{i} + (\ln z - 4y)\mathbf{j} + (x + 3z/4)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 2x + 3$.

7.8. $\mathbf{a} = (1 + \sqrt{z})\mathbf{i} + (4y - \sqrt{x})\mathbf{j} + xy\mathbf{k}$, $S: z^2 = 4(x^2 + y^2)$, $z = 3$.

7.9. $\mathbf{a} = (\sqrt{z} - x)\mathbf{i} + (x - y)\mathbf{j} + (y^2 - z)\mathbf{k}$, $S: 3x - 2y + z = 6$, $x = 0$, $y = 0$, $z = 0$.

7.10. $\mathbf{a} = (yz + x)\mathbf{i} + (x^2 + y)\mathbf{j} + (xy^2 + z)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 2z$.

7.11. $\mathbf{a} = (e^{2y} + x)\mathbf{i} + (x - 2y)\mathbf{j} + (y^2 + 3z)\mathbf{k}$, $S: x - y + z = 1$, $x = 0$, $y = 0$, $z = 0$.

7.12. $\mathbf{a} = (\sqrt{z} - 2x)\mathbf{i} + (e^x + 3y)\mathbf{j} + \sqrt{y + x}\mathbf{k}$, $S: x^2 + y^2 = z^2$, $z = 2$, $z = 5$.

7.13. $\mathbf{a} = (e^z + x/4)\mathbf{i} + (\ln x + y/4)\mathbf{j} + \frac{z}{4}\mathbf{k}$, $S: x^2 + y^2 + z^2 = 2x + 2y - 2z - 2$.

7.14. $\mathbf{a} = (3x - 2z)\mathbf{i} + (z - 2y)\mathbf{j} + (1 + 2z)\mathbf{k}$, $S: z^2 = 4(x^2 + y^2)$, $z = 2$.

7.15. $\mathbf{a} = (e^y + 2x)\mathbf{i} + (x - y)\mathbf{j} + (2z - 1)\mathbf{k}$, $S: x + 2y + z = 2$, $x = 0$, $y = 0$, $z = 0$.

7.16. $\mathbf{a} = (x + y^2)\mathbf{i} + (xz + y)\mathbf{j} + (\sqrt{x^2 + 1} + z)\mathbf{k}$, $S: x^2 + y^2 = z^2$, $z = 2$, $z = 3$.

7.17. $\mathbf{a} = (e^y + 2x)\mathbf{i} + (xz - y)\mathbf{j} + (1/4)(e^{xy} - z)\mathbf{k}$, $S: x^2 + y^2 + z^2 = 2y + 3$.

7.18. $\mathbf{a} = (\sqrt{z} + y)\mathbf{i} + 3x\mathbf{j} + (3z + 5x)\mathbf{k}$, $S: z^2 = 8(x^2 + y^2)$, $z = 2$.

$$7.19. \mathbf{a} = (8yz - x)\mathbf{i} + (x^2 - 1)\mathbf{j} + (xy - 2z)\mathbf{k}, S: 2x + 3y - z = 6, x = 0, y = 0, z = 0.$$

$$7.20. \mathbf{a} = (y + z^2)\mathbf{i} + (x^2 + 3y)\mathbf{j} + xy\mathbf{k}, S: x^2 + y^2 + z^2 = 2x.$$

$$7.21. \mathbf{a} = (2yz - x)\mathbf{i} + (xz + 2y)\mathbf{j} + (x^2 + z)\mathbf{k}, S: y - x + z = 1, x = 0, y = 0, z = 0.$$

$$7.22. \mathbf{a} = (\sin z + 2x)\mathbf{i} + (\sin x - 3y)\mathbf{j} + (\sin y + 2z)\mathbf{k}, S: x^2 + y^2 = z^2, z = 3, z = 6.$$

$$7.23. \mathbf{a} = (\cos z + x/4)\mathbf{i} + (e^x + y/4)\mathbf{j} + \left(\frac{z}{4} - 1\right)\mathbf{k}, S: x^2 + y^2 + z^2 = 2z + 3.$$

$$7.24. \mathbf{a} = (\sqrt{z} + 1 + x)\mathbf{i} + (2x + y)\mathbf{j} + (\sin x + z)\mathbf{k}, S: \begin{cases} z^2 = x^2 + y^2, \\ z = 1. \end{cases}$$

$$7.25. \mathbf{a} = (5x - 6y)\mathbf{i} + (11x^2 + 2y)\mathbf{j} + (x^2 - 4z)\mathbf{k}, S: \begin{cases} x + y + 2z = 2, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$7.26. \mathbf{a} = (y^2 + z^2 + 6x)\mathbf{i} + (e^z - 2y + x)\mathbf{j} + (x + y - z)\mathbf{k}, S: \begin{cases} x^2 + y^2 = z^2, \\ z = 1, z = 3. \end{cases}$$

$$7.27. \mathbf{a} = \frac{1}{2}(x + z)\mathbf{i} + \frac{1}{4}(x \cdot z + y)\mathbf{j} + (xy - 2)\mathbf{k}, S: x^2 + y^2 + z^2 = 4x - 2y + 4z - 8.$$

$$7.28. \mathbf{a} = (3yz - x)\mathbf{i} + (x^2 - y)\mathbf{j} + (6z - 1)\mathbf{k}, S: \begin{cases} z^2 = 9(x^2 + y^2), \\ z = 3. \end{cases}$$

$$7.29. \mathbf{a} = (yz - 2x)\mathbf{i} + (\sin x + y)\mathbf{j} + (x - 2z)\mathbf{k}, S: \begin{cases} x + 2y - 3z = 6, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$7.30. \mathbf{a} = (8x + 1)\mathbf{i} + (zx - 4y)\mathbf{j} + (e^x - z)\mathbf{k}, S: x^2 + y^2 + z^2 = 2y.$$

$$7.31. \mathbf{a} = (2y - 5x)\mathbf{i} + (x - 1)\mathbf{j} + (2\sqrt{xy} + 2z)\mathbf{k}, S: \begin{cases} 2x + 2y - z = 4, \\ x = 0, y = 0, z = 0. \end{cases}$$

Задача 8. Найти поток векторного поля \mathbf{a} через замкнутую поверхность S (нормаль внешняя).

$$\mathbf{a} = (x + z)\mathbf{i} + (z + y)\mathbf{k},$$

$$8.1. \quad S: \begin{cases} x^2 + y^2 = 9, \\ z = x, z = 0 \quad (z \geq 0). \end{cases}$$

$$\mathbf{a} = 2x\mathbf{i} + z\mathbf{k},$$

$$8.2. \quad S: \begin{cases} z = 3x^2 + 2y^2 + 1, \\ x^2 + y^2 = 4, z = 0. \end{cases}$$

$$\mathbf{a} = 2x\mathbf{i} + 2y\mathbf{j} + z\mathbf{k},$$

$$8.3. \quad S: \begin{cases} y = x^2, y = 4x^2, y = 1 \quad (x \geq 0) \\ z = y, z = 0. \end{cases}$$

$$\mathbf{a} = 3x\mathbf{i} - z\mathbf{j},$$

$$8.4. \quad S: \begin{cases} z = 6 - x^2 - y^2, \\ z^2 = x^2 + y^2 \quad (z \geq 0). \end{cases}$$

$$\mathbf{a} = (z + y)\mathbf{i} + y\mathbf{j} - x\mathbf{k},$$

$$8.5. \quad S: \begin{cases} x^2 + y^2 = 2y, \\ y = 2. \end{cases}$$

$$\mathbf{a} = x\mathbf{i} - (x + 2y)\mathbf{j} + y\mathbf{k},$$

$$8.6. \quad S: \begin{cases} x^2 + y^2 = 1, z = 0, \\ x + 2y + 3z = 6. \end{cases}$$

$$\mathbf{a} = 2(z - y)\mathbf{j} + (x - z)\mathbf{k},$$

$$8.7. \quad S: \begin{cases} z = x^2 + 3y^2 + 1, z = 0, \\ x^2 + y^2 = 1. \end{cases}$$

$$\mathbf{a} = x\mathbf{i} + z\mathbf{j} - y\mathbf{k},$$

$$8.8. \quad S: \begin{cases} z = 4 - 2(x^2 + y^2), \\ z = 2(x^2 + y^2). \end{cases}$$

$$\mathbf{a} = z\mathbf{i} - 4y\mathbf{j} + 2x\mathbf{k},$$

$$8.9. \quad S: \begin{cases} z = x^2 + y^2, \\ z = 1. \end{cases}$$

$$\mathbf{a} = 4x\mathbf{i} - 2y\mathbf{j} - z\mathbf{k},$$

$$8.10. \quad S: \begin{cases} 3x + 2y = 12, 3x + y = 6, y = 0, \\ x + y + z = 6, z = 0. \end{cases}$$

$$\mathbf{a} = x\mathbf{i} - 2y\mathbf{j} + x\mathbf{k},$$

$$8.11. \quad S: \begin{cases} x + y = 1, x = 0, y = 0, \\ z = x^2 + y^2, z = 0. \end{cases}$$

$$\mathbf{a} = z\mathbf{i} + x\mathbf{j} - z\mathbf{k},$$

$$8.12. \quad S: \begin{cases} 4z = x^2 + y^2, \\ z = 4. \end{cases}$$

$$\mathbf{a} = 6x\mathbf{i} - 2y\mathbf{j} - z\mathbf{k},$$

$$8.13. \quad S: \begin{cases} z = 3 - 2(x^2 + y^2), \\ z = x^2 + y^2 \quad (z \geq 0). \end{cases}$$

$$\mathbf{a} = (z + y)\mathbf{i} + (x - z)\mathbf{j} + z\mathbf{k},$$

$$8.14. \quad S: \begin{cases} x^2 + 4y^2 = 4, \\ 3x + 4y + z = 12, z = 1. \end{cases}$$

$$\mathbf{a} = (y + 2z)\mathbf{i} - y\mathbf{j} + 3x\mathbf{k},$$

$$8.15. \quad S: \begin{cases} 3z = 27 - 2(x^2 + y^2), \\ z^2 = x^2 + y^2, \quad (z \geq 0). \end{cases}$$

$$\mathbf{a} = (y + 6x)\mathbf{i} + 5(x + z)\mathbf{j} + 4y\mathbf{k},$$

$$8.16. \quad S: \begin{cases} y = x, y = 2x, y = 2, \\ z = x^2 + y^2, z = 0. \end{cases}$$

$\mathbf{a} = y\mathbf{i} + 5y\mathbf{j} + z\mathbf{k},$	$\mathbf{a} = z\mathbf{i} + (3y - x)\mathbf{j} - z\mathbf{k},$
8.17. $S: \begin{cases} x^2 + y^2 = 1, \\ z = x, z = 0 \ (z \geq 0). \end{cases}$	8.18. $S: \begin{cases} x^2 + y^2 = 1, \\ z = x^2 + y^2 + 2, z = 0. \end{cases}$
$\mathbf{a} = y\mathbf{i} + (x + 2y)\mathbf{j} + x\mathbf{k}, \qquad \mathbf{a} = (x + y + z)\mathbf{i} + (2y - x)\mathbf{j} + (3z + y)\mathbf{k},$	
8.19. $S: \begin{cases} x^2 + y^2 = 2x, \\ z = x^2 + y^2, \\ z = 0. \end{cases}$	8.20. $S: \begin{cases} y = x, y = 2x, x = 1, \\ z = x^2 + y^2, \\ z = 0. \end{cases}$
$\mathbf{a} = 7x\mathbf{i} + z\mathbf{j} + (x - y + 5z)\mathbf{k}, \qquad \mathbf{a} = 17x\mathbf{i} + 7y\mathbf{j} + 11z\mathbf{k},$	
8.21. $S: \begin{cases} z = x^2 + y^2, \\ z = x^2 + 2y^2, \\ y = x, y = 2x, x = 1. \end{cases}$	8.22. $S: \begin{cases} z = x^2 + y^2, \\ z = 2(x^2 + y^2), \\ y = x^2, y = x. \end{cases}$
$\mathbf{a} = x\mathbf{i} - 2y\mathbf{j} + 3z\mathbf{k}, \qquad \mathbf{a} = (2x + y)\mathbf{i} + (y + 2z)\mathbf{k},$	
8.23. $S: \begin{cases} x^2 + y^2 = z, \\ z = 2x. \end{cases}$	8.24. $S: \begin{cases} z = 2 - 4(x^2 + y^2), \\ z = 4(x^2 + y^2). \end{cases}$
$\mathbf{a} = (2y - 3z)\mathbf{i} + (3x + 2z)\mathbf{j} + (x + y + z)\mathbf{k}, \qquad \mathbf{a} = -2x\mathbf{i} + z\mathbf{j} + (x + y)\mathbf{k},$	
8.25. $S: \begin{cases} x^2 + y^2 = 1, \\ z = 4 - x - y, z = 0. \end{cases}$	8.26. $S: \begin{cases} x^2 + y^2 = 2y, \\ z = x^2 + y^2, z = 0. \end{cases}$
$\mathbf{a} = (2y - 15x)\mathbf{i} + (z - y)\mathbf{j} - (x - 3y)\mathbf{k}, \qquad \mathbf{a} = (y + z)\mathbf{i} + (x - 2y + z)\mathbf{j} + x\mathbf{k},$	
8.27. $S: \begin{cases} z = 3x^2 + y^2 + 1, z = 0, \\ x^2 + y^2 = \frac{1}{4}. \end{cases}$	8.28. $S: \begin{cases} x^2 + y^2 = 1, \\ z = x^2 + y^2, z = 0. \end{cases}$
$\mathbf{a} = (3x - y - z)\mathbf{i} + 3y\mathbf{j} + 2z\mathbf{k}, \qquad \mathbf{a} = (x + y)\mathbf{i} + (y + z)\mathbf{j} + (z + x)\mathbf{k},$	
8.29. $S: \begin{cases} z = x^2 + y^2, z = 2y. \end{cases}$	8.30. $S: \begin{cases} y = 2x, y = 4x, x = 1, \\ z = y^2, z = 0. \end{cases}$
$\mathbf{a} = (x + z)\mathbf{i} + y\mathbf{k},$	
8.31. $S: \begin{cases} z = 8 - x^2 - y^2, \\ z = x^2 + y^2. \end{cases}$	

Задача 9. Найти поток векторного поля \mathbf{a} через замкнутую поверхность S (нормаль внешняя).

$$\mathbf{a} = x^2\mathbf{i} + x\mathbf{j} + xz\mathbf{k},$$

$$\mathbf{a} = (x^2 + y^2)\mathbf{i} + (x^2 + y^2)\mathbf{j} + (x^2 + y^2)\mathbf{k},$$

$$9.1. \quad S: \begin{cases} z = x^2 + y^2, & z = 1, \\ x = 0, & y = 0 \end{cases} \text{ (1 октант).}$$

$$9.2. \quad S: \begin{cases} z = x^2 + y^2, \\ z = 0, & z = 1. \end{cases}$$

$$\mathbf{a} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k},$$

$$\mathbf{a} = x^2\mathbf{i} + y\mathbf{j} + z\mathbf{k},$$

$$9.3. \quad S: \begin{cases} x^2 + y^2 + z^2 = 4, \\ x^2 + y^2 = z^2 \quad (z \geq 0). \end{cases}$$

$$9.4. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ z = 0 \quad (z \geq 0). \end{cases}$$

$$\mathbf{a} = xz\mathbf{i} + z\mathbf{j} + y\mathbf{k},$$

$$\mathbf{a} = 3xz\mathbf{i} - 2x\mathbf{j} + y\mathbf{k},$$

$$9.5. \quad S: \begin{cases} x^2 + y^2 = 1 - z, \\ z = 0. \end{cases}$$

$$9.6. \quad S: \begin{cases} x + y + z = 2, & x = 1, \\ x = 0, & y = 0, & z = 0. \end{cases}$$

$$\mathbf{a} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k},$$

$$\mathbf{a} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k},$$

$$9.7. \quad S: \begin{cases} x^2 + y^2 + z^2 = 2, \\ z = 0 \quad (z \geq 0). \end{cases}$$

$$9.8. \quad S: x^2 + y^2 + z^2 = 1.$$

$$\mathbf{a} = (zx + y)\mathbf{i} + (zy - x)\mathbf{j} - (x^2 + y^2)\mathbf{k},$$

$$\mathbf{a} = y^2x\mathbf{i} + z^2y\mathbf{j} + x^2z\mathbf{k},$$

$$9.9. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ z = 0 \quad (z \geq 0). \end{cases}$$

$$9.10. \quad S: x^2 + y^2 + z^2 = 1.$$

$$\mathbf{a} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k},$$

$$\mathbf{a} = x^2\mathbf{i} + xy\mathbf{j} + 3z\mathbf{k},$$

$$9.11. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ x = 0, & y = 0, & z = 0 \\ \text{(1 октант)}. \end{cases}$$

$$9.12. \quad S: \begin{cases} x^2 + y^2 = z^2, \\ z = 4. \end{cases}$$

$$\mathbf{a} = (zx + y)\mathbf{i} + (xy - z)\mathbf{j} + (x^2 + yz)\mathbf{k},$$

$$\mathbf{a} = xy^2\mathbf{i} + x^2y\mathbf{j} + z\mathbf{k},$$

$$9.13. \quad S: \begin{cases} x^2 + y^2 = 2, \\ z = 0, & z = 1. \end{cases}$$

$$9.14. \quad S: \begin{cases} x^2 + y^2 = 1, & z = 0, & z = 1, \\ x = 0, & y = 0 \\ \text{(1 октант)}. \end{cases}$$

$$\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k},$$

$$\mathbf{a} = 3x^2\mathbf{i} - 2x^2y\mathbf{j} + (2x - 1)z\mathbf{k},$$

$$9.15. \quad S: \begin{cases} x^2 + y^2 + z^2 = 16, \\ x^2 + y^2 = z^2 \quad (z \geq 0). \end{cases}$$

$$9.16. \quad S: \begin{cases} x^2 + y^2 = 1, \\ z = 0, & z = 1. \end{cases}$$

$$\mathbf{a} = x^2\mathbf{i} + y^2\mathbf{j} + 2z\mathbf{k},$$

$$9.17. \quad S: \begin{cases} x^2 + y^2 = \frac{1}{4}, \\ z = 0, z = 2. \end{cases}$$

$$\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k},$$

$$9.18. \quad S: \begin{cases} x^2 + y^2 = 4, \\ z = 0, z = 1. \end{cases}$$

$$\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k},$$

$$9.19. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ x = 0, y = 0, z = 0 \\ (1 \text{ Октант}). \end{cases}$$

$$\mathbf{a} = z\mathbf{i} + yz\mathbf{j} - xy\mathbf{k},$$

$$9.20. \quad S: \begin{cases} x^2 + y^2 = 4, \\ z = 0, z = 1. \end{cases}$$

$$\mathbf{a} = (zx + y)\mathbf{i} - (2y - x)\mathbf{j} - (x^2 + y^2)\mathbf{k},$$

$$9.21. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ z = 0 \ (z \geq 0). \end{cases}$$

$$\mathbf{a} = (x^2 + xy)\mathbf{i} + (y^2 + yz)\mathbf{j} + (z^2 + xz)\mathbf{k},$$

$$9.22. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ x^2 + y^2 = z^2 \ (z \geq 0). \end{cases}$$

$$\mathbf{a} = 3x^2\mathbf{i} - 2x^2y\mathbf{j} - (1 - 2x)\mathbf{k},$$

$$9.23. \quad S: \begin{cases} x^2 + y^2 = 1, \\ z = 0, z = 1. \end{cases}$$

$$\mathbf{a} = x^2\mathbf{i},$$

$$9.24. \quad S: \begin{cases} z = 1 - x - y, \\ x = 0, y = 0, z = 0. \end{cases}$$

$$\mathbf{a} = (y^2 + xz)\mathbf{i} + (yx - z)\mathbf{j} + (yz + x)\mathbf{k},$$

$$9.25. \quad S: \begin{cases} x^2 + y^2 = 1, \\ z = 0, z = \sqrt{2}. \end{cases}$$

$$\mathbf{a} = y\mathbf{i} + y^2\mathbf{j} + yz\mathbf{k},$$

$$9.26. \quad S: \begin{cases} z = x^2 + y^2, z = 1, \\ x = 0, y = 0 \\ (1 \text{ Октант}). \end{cases}$$

$$\mathbf{a} = y\mathbf{i} + 2zy\mathbf{j} + 2z^2\mathbf{k},$$

$$9.27. \quad S: \begin{cases} x^2 + y^2 = 1 - z, \\ z = 0. \end{cases}$$

$$\mathbf{a} = 2xy\mathbf{i} + 2xy\mathbf{j} + z^2\mathbf{k},$$

$$9.28. \quad S: \begin{cases} x^2 + y^2 + z^2 = \sqrt{2}, \\ z = 0 \ (z \geq 0). \end{cases}$$

$$\mathbf{a} = y^2x\mathbf{i} + x^2y\mathbf{j} + z^3\mathbf{k}/3,$$

$$9.29. \quad S: \begin{cases} x^2 + y^2 + z^2 = 1, \\ z = 0, \ (z \geq 0). \end{cases}$$

$$\mathbf{a} = -x\mathbf{i} + 2y\mathbf{j} + yz\mathbf{k},$$

$$9.30. \quad S: \begin{cases} x^2 + y^2 = z^2, \\ z = 4. \end{cases}$$

$$\mathbf{a} = (y^2 + z^2)\mathbf{i} + (xy + y^2)\mathbf{j} + (xz + z)\mathbf{k},$$

$$9.31. \quad S: \begin{cases} x^2 + y^2 = 1, \\ z = 0, z = 1. \end{cases}$$

Задача 10. Найти работу силы \mathbf{F} при перемещении вдоль линии L от точки M к точке N .

$$\mathbf{F} = (x^2 - 2y)\mathbf{i} + (y^2 - 2x)\mathbf{j},$$

10.1. L : отрезок MN ,

$$M(-4,0), N(0,2).$$

$$\mathbf{F} = (x^2 + 2y)\mathbf{i} + (y^2 + 2x)\mathbf{j},$$

10.2. L : отрезок MN ,

$$M(-4,0), N(0,2).$$

$$\mathbf{F} = (x^2 + 2y)\mathbf{i} + (y^2 + 2x)\mathbf{j},$$

10.3. L : $2 - \frac{x^2}{8} = y$,

$$M(-4,0), N(0,2).$$

$$\mathbf{F} = (x + y)\mathbf{i} + 2x\mathbf{j},$$

10.4. L : $x^2 + y^2 = 4$ ($y \geq 0$),

$$M(2,0), N(-2,0).$$

$$\mathbf{F} = x^3\mathbf{i} - y^3\mathbf{j},$$

10.5. L : $x^2 + y^2 = 4$ ($x \geq 0, y \geq 0$),

$$M(2,0), N(0,2).$$

$$\mathbf{F} = (x + y)\mathbf{i} + (x - y)\mathbf{j},$$

10.6. L : $y = x^2$,

$$M(-1,1), N(1,1).$$

$$\mathbf{F} = x^2y\mathbf{i} - y\mathbf{j},$$

10.7. L : отрезок MN ,

$$M(-1,0), N(0,1).$$

$$\mathbf{F} = (2xy - y)\mathbf{i} + (x^2 + x)\mathbf{j},$$

10.8. L : $x^2 + y^2 = 9$ ($y \geq 0$),

$$M(3,0), N(-3,0).$$

$$\mathbf{F} = (x + y)\mathbf{i} + (x - y)\mathbf{j},$$

10.9. L : $x^2 + \frac{y^2}{9} = 1$ ($x \geq 0, y \geq 0$),

$$M(1,0), N(0,3).$$

$$\mathbf{F} = y\mathbf{i} - x\mathbf{j},$$

10.10. L : $x^2 + y^2 = 1$ ($y \geq 0$),

$$M(1,0), N(-1,0).$$

$$\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x^2 - y^2)\mathbf{j},$$

10.11. L : $\begin{cases} x, & 0 \leq x \leq 1; \\ 2 - x, & 1 \leq x \leq 2; \end{cases}$

$$M(2,0), N(0,0).$$

$$\mathbf{F} = y\mathbf{i} - x\mathbf{j},$$

10.12. L : $x^2 + y^2 = 2$ ($y \geq 0$),

$$M(\sqrt{2},0), N(-\sqrt{2},0).$$

$$\mathbf{F} = xy\mathbf{i} + 2y\mathbf{j},$$

10.13. L : $x^2 + y^2 = 1$ ($x \geq 0, y \geq 0$),

$$M(1,0), N(0,1).$$

$$\mathbf{F} = y\mathbf{i} - x\mathbf{j},$$

10.14. L : $2x^2 + y^2 = 1$ ($y \geq 0$),

$$M\left(\frac{1}{\sqrt{2}},0\right), N\left(-\frac{1}{\sqrt{2}},0\right).$$

$$\mathbf{F} = (x^2 + y^2)(\mathbf{i} + 2\mathbf{j}),$$

10.15. L : $x^2 + y^2 = R^2$ ($y \geq 0$),

$$M(R,0), N(-R,0).$$

$$\mathbf{F} = \left(x + y\sqrt{x^2 + y^2}\right)\mathbf{i} + \left(y - x\sqrt{x^2 + y^2}\right)\mathbf{j},$$

10.16. L : $x^2 + y^2 = 1$ ($y \geq 0$),

$$M(1,0), N(-1,0).$$

- | | |
|--|--|
| $\mathbf{F} = x^2 y \mathbf{i} - xy^2 \mathbf{j},$ | $\mathbf{F} = \left(x + y\sqrt{x^2 + y^2}\right) \mathbf{i} + \left(y - \sqrt{x^2 + y^2}\right) \mathbf{j},$ |
| 10.17. $L: x^2 + y^2 = 4 \ (x \geq 0, y \geq 0),$
$M(2,0), N(0,2).$ | 10.18. $L: x^2 + y^2 = 16 \ (x \geq 0, y \geq 0),$
$M(4,0), N(0,4).$ |
| $\mathbf{F} = y^2 \mathbf{i} - x^2 \mathbf{j},$ | $\mathbf{F} = (x + y)^2 \mathbf{i} - (x^2 + y^2) \mathbf{j},$ |
| 10.19. $L: x^2 + y^2 = 9 \ (x \geq 0, y \geq 0),$
$M(3,0), N(0,3).$ | 10.20. $L: \text{отрезок } MN,$
$M(1,0), N(0,1).$ |
| $\mathbf{F} = (x^2 + y^2) \mathbf{i} + y^2 \mathbf{j},$ | $\mathbf{F} = x^2 \mathbf{j},$ |
| 10.21. $L: \text{отрезок } MN,$
$M(2,0), N(0,2).$ | 10.22. $L: x^2 + y^2 = 9 \ (x \geq 0, y \geq 0),$
$M(3,0), N(0,3).$ |
| $\mathbf{F} = (y^2 - y) \mathbf{i} + (2xy + x) \mathbf{j},$ | $\mathbf{F} = xy \mathbf{i},$ |
| 10.23. $L: x^2 + y^2 = 9 \ (y \geq 0),$
$M(3,0), N(-3,0).$ | 10.24. $L: y = \sin x,$
$M(\pi, 0), N(0,0).$ |
| $\mathbf{F} = (xy - y^2) \mathbf{i} + x \mathbf{j},$ | $\mathbf{F} = x \mathbf{i} + y \mathbf{j},$ |
| 10.25. $L: y = 2x^2,$
$M(0,0), N(1,2).$ | 10.26. $L: \text{отрезок } MN,$
$M(1,0), N(0,3).$ |
| $\mathbf{F} = (xy - x) \mathbf{i} + \frac{x^2}{2} \mathbf{j},$ | $\mathbf{F} = -x \mathbf{i} + y \mathbf{j},$ |
| 10.27. $L: y = 2\sqrt{x},$
$M(0,0), N(1,2).$ | 10.28. $L: x^2 + \frac{y^2}{9} = 1 \ (x \geq 0, y \geq 0),$
$M(1,0), N(0,3).$ |
| $\mathbf{F} = -y \mathbf{i} + x \mathbf{j},$ | $\mathbf{F} = (x^2 - y^2) \mathbf{i} + (x^2 + y^2) \mathbf{j},$ |
| 10.29. $L: y = x^3,$
$M(0,0), N(2,8).$ | 10.30. $L: \frac{x^2}{9} + \frac{y^2}{4} = 1 \ (y \geq 0),$
$M(3,0), N(-3,0).$ |
| $\mathbf{F} = (x - y) \mathbf{i} + \mathbf{j},$ | |
| 10.31. $L: x^2 + y^2 = 4 \ (y \geq 0),$
$M(2,0), N(-2,0).$ | |

Задача 11. Найти циркуляцию векторного поля \mathbf{a} вдоль контура Γ (в направлении, соответствующем возрастанию параметра t).

- | | |
|---|---|
| $\mathbf{a} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k},$ $11.1. \quad \Gamma: \begin{cases} x = \frac{\sqrt{2}}{2} \cos t, & y = \frac{\sqrt{2}}{2} \cos t, \\ z = \sin t. \end{cases}$ | $\mathbf{a} = -x^2 y^3 \mathbf{i} + \mathbf{j} + z\mathbf{k},$ $11.2. \quad \Gamma: \begin{cases} x = \sqrt[3]{4} \cos t, & y = \sqrt[3]{4} \sin t, \\ z = 3. \end{cases}$ |
| $\mathbf{a} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k},$ $11.3. \quad \Gamma: \begin{cases} x = \cos t, & y = \sin t, \\ z = 2(1 - \cos t). \end{cases}$ | $\mathbf{a} = x^2 \mathbf{i} + y\mathbf{j} - z\mathbf{k},$ $11.4. \quad \Gamma: \begin{cases} x = \cos t, & y = (\sqrt{2} \sin t)/2, \\ z = (\sqrt{2} \cos t)/2. \end{cases}$ |
| $\mathbf{a} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k},$ $11.5. \quad \Gamma: \begin{cases} x = 4 \cos t, & y = 4 \sin t, \\ z = 1 - \cos t. \end{cases}$ | $\mathbf{a} = 2y\mathbf{i} - 3x\mathbf{j} + x\mathbf{k},$ $11.6. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 2 \sin t, \\ z = 2 - 2 \cos t - 2 \sin t. \end{cases}$ |
| $\mathbf{a} = 2z\mathbf{i} - x\mathbf{j} + y\mathbf{k},$ $11.7. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 2 \sin t, \\ z = 1. \end{cases}$ | $\mathbf{a} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k},$ $11.8. \quad \Gamma: \begin{cases} x = \cos t, & y = \sin t, \\ z = 3. \end{cases}$ |
| $\mathbf{a} = x\mathbf{i} + z^2 \mathbf{j} + y\mathbf{k},$ $11.9. \quad \Gamma: \begin{cases} x = \cos t, & y = 2 \sin t, \\ z = 2 \cos t - 2 \sin t - 1. \end{cases}$ | $\mathbf{a} = 3y\mathbf{i} - 3x\mathbf{j} + x\mathbf{k},$ $11.10. \quad \Gamma: \begin{cases} x = 3 \cos t, & y = 3 \sin t, \\ z = 3 - 3 \cos t - 3 \sin t. \end{cases}$ |
| $\mathbf{a} = -x^2 y^3 \mathbf{i} + 2\mathbf{j} + xz\mathbf{k},$ $11.11. \quad \Gamma: \begin{cases} x = \sqrt{2} \cos t, & y = \sqrt{2} \sin t, \\ z = 1. \end{cases}$ | $\mathbf{a} = 6z\mathbf{i} - x\mathbf{j} + xy\mathbf{k},$ $11.12. \quad \Gamma: \begin{cases} x = 3 \cos t, & y = 3 \sin t, \\ z = 3. \end{cases}$ |
| $\mathbf{a} = z\mathbf{i} + y^2 \mathbf{j} - x\mathbf{k},$ $11.13. \quad \Gamma: \begin{cases} x = \sqrt{2} \cos t, & y = 2 \sin t, \\ z = \sqrt{2} \cos t. \end{cases}$ | $\mathbf{a} = x\mathbf{i} + 2z^2 \mathbf{j} + y\mathbf{k},$ $11.14. \quad \Gamma: \begin{cases} x = \cos t, & y = 3 \sin t, \\ z = 2 \cos t - 3 \sin t - 2. \end{cases}$ |
| $\mathbf{a} = x\mathbf{i} - \frac{1}{3} z^2 \mathbf{j} + y\mathbf{k},$ $11.15. \quad \Gamma: \begin{cases} x = (\cos t)/2, & y = (\sin t)/3, \\ z = \cos t - (\sin t)/3 - 1/4. \end{cases}$ | $\mathbf{a} = 4y\mathbf{i} - 3x\mathbf{j} + x\mathbf{k},$ $11.16. \quad \Gamma: \begin{cases} x = 4 \cos t, & y = 4 \sin t, \\ z = 4 - 4 \cos t - 4 \sin t. \end{cases}$ |

$$\mathbf{a} = -z\mathbf{i} - x\mathbf{j} + xz\mathbf{k},$$

$$11.17. \quad \Gamma: \begin{cases} x = 5 \cos t, & y = 5 \sin t, \\ z = 4. \end{cases}$$

$$\mathbf{a} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k},$$

$$11.18. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 2 \sin t, \\ z = 0. \end{cases}$$

$$\mathbf{a} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k},$$

$$11.19. \quad \Gamma: \begin{cases} x = 3 \cos t, & y = 3 \sin t, \\ z = 2(1 - \cos t). \end{cases}$$

$$\mathbf{a} = 2y\mathbf{i} - z\mathbf{j} + x\mathbf{k},$$

$$11.20. \quad \Gamma: \begin{cases} x = \cos t, & y = \sin t, \\ z = 4 - \cos t - \sin t. \end{cases}$$

$$\mathbf{a} = xz\mathbf{i} + x\mathbf{j} + z^2\mathbf{k},$$

$$11.21. \quad \Gamma: \begin{cases} x = \cos t, & y = \sin t, \\ z = \sin t. \end{cases}$$

$$\mathbf{a} = -x^2y^3\mathbf{i} + 3\mathbf{j} + y\mathbf{k},$$

$$11.22. \quad \Gamma: \begin{cases} x = \cos t, & y = \sin t, \\ z = 5. \end{cases}$$

$$\mathbf{a} = 7z\mathbf{i} - x\mathbf{j} + yz\mathbf{k},$$

$$11.23. \quad \Gamma: \begin{cases} x = 6 \cos t, & y = 6 \sin t, \\ z = 1/3. \end{cases}$$

$$\mathbf{a} = xy\mathbf{i} + x\mathbf{j} + y^2\mathbf{k},$$

$$11.24. \quad \Gamma: \begin{cases} x = \cos t, & y = \sin t, \\ z = \sin t. \end{cases}$$

$$\mathbf{a} = x\mathbf{i} - z^2\mathbf{j} + y\mathbf{k},$$

$$11.25. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 3 \sin t, \\ z = 4 \cos t - 3 \sin t - 3. \end{cases}$$

$$\mathbf{a} = (y - z)\mathbf{i} + (z - x)\mathbf{j} + (x - y)\mathbf{k},$$

$$11.26. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 2 \sin t, \\ z = 3(1 - \cos t). \end{cases}$$

$$\mathbf{a} = -2z\mathbf{i} - x\mathbf{j} + x^2\mathbf{k},$$

$$11.27. \quad \Gamma: \begin{cases} x = (\cos t)/3, & y = (\sin t)/3, \\ z = 8. \end{cases}$$

$$\mathbf{a} = x\mathbf{i} - 3z^2\mathbf{j} + y\mathbf{k},$$

$$11.28. \quad \Gamma: \begin{cases} x = \cos t, & y = 4 \sin t, \\ z = 2 \cos t - 4 \sin t + 3. \end{cases}$$

$$\mathbf{a} = x\mathbf{i} - 2z^2\mathbf{j} + y\mathbf{k},$$

$$11.29. \quad \Gamma: \begin{cases} x = 3 \cos t, & y = 4 \sin t, \\ z = 6 \cos t - 4 \sin t + 1. \end{cases}$$

$$\mathbf{a} = -x^2y^3\mathbf{i} + 4\mathbf{j} + x\mathbf{k},$$

$$11.30. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 2 \sin t, \\ z = 4. \end{cases}$$

$$\mathbf{a} = y\mathbf{i}/3 - 3x\mathbf{j} + x\mathbf{k},$$

$$11.31. \quad \Gamma: \begin{cases} x = 2 \cos t, & y = 2 \sin t, \\ z = 1 - 2 \cos t - 2 \sin t. \end{cases}$$

Задача 12. Найти модуль циркуляции векторного поля \mathbf{a} вдоль контура Γ .

$$\mathbf{a} = (x^2 - y)\mathbf{i} + x\mathbf{j} + \mathbf{k},$$

$$\mathbf{a} = xz\mathbf{i} - \mathbf{j} + y\mathbf{k},$$

$$12.1. \quad \Gamma: \begin{cases} x^2 + y^2 = 1, \\ z = 1. \end{cases}$$

$$12.2. \quad \Gamma: \begin{cases} z = 5(x^2 + y^2) - 1, \\ z = 4. \end{cases}$$

$$\mathbf{a} = yz\mathbf{i} + 2xz\mathbf{j} + xy\mathbf{k},$$

$$\mathbf{a} = x\mathbf{i} + yz\mathbf{j} - x\mathbf{k},$$

$$12.3. \quad \Gamma: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 9 \quad (z > 0). \end{cases}$$

$$12.4. \quad \Gamma: \begin{cases} x^2 + y^2 = 1, \\ x + y + z = 1. \end{cases}$$

$$\mathbf{a} = (x - y)\mathbf{i} + x\mathbf{j} - z\mathbf{k},$$

$$\mathbf{a} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k},$$

$$12.5. \quad \Gamma: \begin{cases} x^2 + y^2 = 1, \\ z = 1. \end{cases}$$

$$12.6. \quad \Gamma: \begin{cases} z = 3(x^2 + y^2) + 1, \\ z = 4. \end{cases}$$

$$\mathbf{a} = yz\mathbf{i} + 2xz\mathbf{j} + y^2\mathbf{k},$$

$$\mathbf{a} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k},$$

$$12.7. \quad \Gamma: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 16 \quad (z > 0). \end{cases}$$

$$12.8. \quad \Gamma: \begin{cases} x^2 + y^2 = 9, \\ x + y + z = 1. \end{cases}$$

$$\mathbf{a} = y\mathbf{i} + (1 - x)\mathbf{j} - z\mathbf{k},$$

$$\mathbf{a} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k},$$

$$12.9. \quad \Gamma: \begin{cases} x^2 + y^2 + z^2 = 4, \\ x^2 + y^2 = 1 \quad (z > 0). \end{cases}$$

$$12.10. \quad \Gamma: \begin{cases} x^2 + y^2 = 1, \\ z = 4. \end{cases}$$

$$\mathbf{a} = 4x\mathbf{i} + 2\mathbf{j} - xy\mathbf{k},$$

$$\mathbf{a} = 2y\mathbf{i} - 3x\mathbf{j} + z^2\mathbf{k},$$

$$12.11. \quad \Gamma: \begin{cases} z = 2(x^2 + y^2) + 1, \\ z = 7. \end{cases}$$

$$12.12. \quad \Gamma: \begin{cases} x^2 + y^2 = z, \\ z = 1. \end{cases}$$

$$\mathbf{a} = -3z\mathbf{i} + y^2\mathbf{j} + 2y\mathbf{k},$$

$$\mathbf{a} = 2y\mathbf{i} + 5z\mathbf{j} + 3x\mathbf{k},$$

$$12.13. \quad \Gamma: \begin{cases} x^2 + y^2 = 4, \\ x - 3y - 2z = 1. \end{cases}$$

$$12.14. \quad \Gamma: \begin{cases} 2x^2 + 2y^2 = 1, \\ x + y + z = 3. \end{cases}$$

$$\mathbf{a} = 2y\mathbf{i} + \mathbf{j} - 2yz\mathbf{k},$$

$$\mathbf{a} = (x - y)\mathbf{i} + x\mathbf{j} + z^2\mathbf{k},$$

$$12.15. \quad \Gamma: \begin{cases} x^2 + y^2 - z^2 = 0, \\ z = 2. \end{cases}$$

$$12.16. \quad \Gamma: \begin{cases} x^2 + y^2 - 4z^2 = 0, \\ z = \frac{1}{2}. \end{cases}$$

$$\mathbf{a} = xz\mathbf{i} - \mathbf{j} + y\mathbf{k},$$

$$12.17. \Gamma: \begin{cases} x^2 + y^2 + z^2 = 4, \\ z = 1. \end{cases}$$

$$\mathbf{a} = 2yzi + xzj - x^2\mathbf{k},$$

$$12.18. \Gamma: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 9 \quad (z > 0). \end{cases}$$

$$\mathbf{a} = 4x\mathbf{i} - yz\mathbf{j} + x\mathbf{k},$$

$$12.19. \Gamma: \begin{cases} x^2 + y^2 = 1, \\ x + y + z = 1. \end{cases}$$

$$\mathbf{a} = -y\mathbf{i} + 2\mathbf{j} + \mathbf{k},$$

$$12.20. \Gamma: \begin{cases} x^2 + y^2 - z^2 = 0, \\ z = 1. \end{cases}$$

$$\mathbf{a} = y\mathbf{i} + 3x\mathbf{j} + z^2\mathbf{k},$$

$$12.21. \Gamma: \begin{cases} z = x^2 + y^2 - 1, \\ z = 3. \end{cases}$$

$$\mathbf{a} = 2yzi + xzj + y^2\mathbf{k},$$

$$12.22. \Gamma: \begin{cases} x^2 + y^2 + z^2 = 25, \\ x^2 + y^2 = 16 \quad (z > 0). \end{cases}$$

$$\mathbf{a} = (2 - xy)\mathbf{i} - yz\mathbf{j} - xz\mathbf{k},$$

$$12.23. \Gamma: \begin{cases} x^2 + y^2 = 4, \\ x + y + z = 1. \end{cases}$$

$$\mathbf{a} = -y\mathbf{i} + x\mathbf{j} + 3z^2\mathbf{k},$$

$$12.24. \Gamma: \begin{cases} x^2 + y^2 + z^2 = 9, \\ x^2 + y^2 = 1 \quad (z > 0). \end{cases}$$

$$\mathbf{a} = y\mathbf{i} - x\mathbf{j} + 2z\mathbf{k},$$

$$12.25. \Gamma: \begin{cases} x^2 + y^2 - \frac{z^2}{4} = 0, \\ z = 2. \end{cases}$$

$$\mathbf{a} = x^2\mathbf{i} + yz\mathbf{j} + 2z\mathbf{k},$$

$$12.26. \Gamma: \begin{cases} x^2 + y^2 + z^2 = 25, \\ z = 4. \end{cases}$$

$$\mathbf{a} = y\mathbf{i} - 2x\mathbf{j} + z^2\mathbf{k},$$

$$12.27. \Gamma: \begin{cases} z = 4(x^2 + y^2) + 2, \\ z = 6. \end{cases}$$

$$\mathbf{a} = 3z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k},$$

$$12.28. \Gamma: \begin{cases} x^2 + y^2 = 4, \\ 2x - 3y - 2z = 1. \end{cases}$$

$$\mathbf{a} = (x + y)\mathbf{i} - x\mathbf{j} + 6\mathbf{k},$$

$$12.29. \Gamma: \begin{cases} x^2 + y^2 = 1, \\ z = 2. \end{cases}$$

$$\mathbf{a} = 4\mathbf{i} + 3x\mathbf{j} + 3xz\mathbf{k},$$

$$12.30. \Gamma: \begin{cases} x^2 + y^2 - z^2 = 0, \\ z = 3. \end{cases}$$

$$\mathbf{a} = yz\mathbf{i} - xz\mathbf{j} + xy\mathbf{k},$$

$$12.31. \Gamma: \begin{cases} x^2 + y^2 + z^2 = 9, \\ x^2 + y^2 = 9. \end{cases}$$