VII. КРАТНЫЕ ИНТЕГРАЛЫ

Теоретические вопросы

1. Определение двойного и тройного интегралов. Их геометрический и физический смысл.

2. Основные свойства двойных и тройных интегралов.

3. Теорема о среднем для двойного и тройного интегралов.

4. Вычисление двойных интегралов двумя последовательными интегрированиями (случай прямоугольной области).

5. Вычисление двойных интегралов двумя последовательными интегрированиями (общий случай).

6. Замена переменных в двойном интеграле.

7. Якобиан, его геометрический смысл.

8. Двойной интеграл в полярных координатах.

9. Тройной интеграл в цилиндрических координатах.

10. Тройной интеграл в сферических координатах.

Теоретические упражнения

1. Пользуясь определением двойного интеграла, доказать, что

$$\iint\limits_{x^2+y^2\leq R^2} x^m y^n dxdy = 0,$$

если m и n - натуральные числа, и, по меньшей мере, одно из них нечетно.

2. С помощью теоремы о среднем найти

$$\lim_{R\to 0}\frac{1}{\pi R^2}\iint\limits_{x^2+y^2\leq R^2} f(x,y)dxdy,$$

где $f(x,y)$ - непрерывная функция.

3. Оценить интеграл

$$\iiint\limits_{x^2+y^2+z^2\leq R^2}\frac{dxdydz}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}},\ x_0^2+y_0^2+z_0^2 > R^2,$$

т.е. указать, между какими значениями заключена его величина.

4. Вычислить двойной интеграл

$$\iint\limits_{D} f(x,y)\,dxdy,$$

если область D - прямоугольник $\{\, a \le x \le b, \;\; c \le y \le d \,\}$, а $f(x,y) = F''_{xy}(x,y)$.

5. Доказать равенство

$$\iint\limits_{D} f(x)g(y)\,dxdy = \int\limits_{a}^{b} f(x)\,dx \int\limits_{c}^{d} g(y)\,dy \, б$$

если область D - прямоугольник $\{\, a \le x \le b, \;\; c \le y \le d \,\}$.

6. Доказать формулу Дирихле

$$\int\limits_{0}^{a} dx \int\limits_{0}^{x} f(x,y)\,dy = \int\limits_{0}^{a} dy \int\limits_{y}^{a} f(x,y)\,dx, \qquad a > 0.$$

7. Пользуясь формулой Дирихле, доказать равенство

$$\int\limits_{0}^{a} dy \int\limits_{0}^{y} f(x)\,dx = \int\limits_{0}^{a} (a-x) f(x)\,dx.$$

8. Какой из интегралов больше

$$\int\limits_{0}^{1} dx \int\limits_{0}^{1} dy \int\limits_{0}^{1} f(x,y,z)\,dz \qquad \text{или} \qquad \int\limits_{0}^{1} dx \int\limits_{0}^{1-x} dy \int\limits_{0}^{1-x-y} f(x,y,z)\,dz,$$

если $f(x,y,z) > 0$?

Расчетные задания

Задача 1. Изменить порядок интегрирования.

1.1. $\displaystyle\int_{-2}^{-1} dy \int_{-\sqrt{2+y}}^{0} f\,dx + \int_{-1}^{0} dy \int_{-\sqrt{-y}}^{0} f\,dx.$

1.2. $\displaystyle\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f\,dx + \int_{1}^{\sqrt{2}} dy \int_{-\sqrt{-y}}^{0} f\,dx.$

1.3. $\displaystyle\int_{0}^{1} dy \int_{0}^{y} f\,dx + \int_{1}^{\sqrt{2}} dy \int_{0}^{\sqrt{2-y^2}} f\,dx.$

1.4. $\displaystyle\int_{0}^{1} dy \int_{0}^{\sqrt{y}} f\,dx + \int_{1}^{2} dy \int_{0}^{\sqrt{2-y}} f\,dx.$

1.5. $\displaystyle\int_{-\sqrt{2}}^{-1} dx \int_{-\sqrt{2-x^2}}^{0} f\,dy + \int_{-1}^{0} dx \int_{x}^{0} f\,dy.$

1.6. $\displaystyle\int_{0}^{1/\sqrt{2}} dy \int_{0}^{\arcsin y} f\,dx + \int_{1/\sqrt{2}}^{1} dy \int_{0}^{\arccos y} f\,dx.$

1.7. $\displaystyle\int_{-2}^{-1} dy \int_{0}^{\sqrt{2+y}} f\,dx + \int_{-1}^{0} dy \int_{0}^{\sqrt{-y}} f\,dx.$

1.8. $\displaystyle\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f\,dx + \int_{1}^{e} dy \int_{-1}^{-\ln y} f\,dx.$

1.9. $\displaystyle\int_{-\sqrt{2}}^{-1} dx \int_{0}^{\sqrt{2-x^2}} f\,dy + \int_{-1}^{0} dx \int_{0}^{x^2} f\,dy.$

1.10. $\displaystyle\int_{-2}^{-\sqrt{3}} dx \int_{-\sqrt{4-x^2}}^{0} f dy + \int_{-\sqrt{3}}^{0} dx \int_{\sqrt{4-x^2}-2}^{0} f dy.$

1.11. $\displaystyle\int_{0}^{1} dx \int_{1-x^2}^{1} f\,dy + \int_{1}^{e} dx \int_{\ln x}^{1} f\,dy.$

1.12. $\displaystyle\int_{0}^{1} dy \int_{0}^{\sqrt[3]{y}} f\,dx + \int_{1}^{2} dy \int_{0}^{2-y} f\,dx.$

1.13. $\displaystyle\int_{0}^{\pi/4} dy \int_{0}^{\sin y} f\,dx + \int_{\pi/4}^{\pi/2} dy \int_{0}^{\cos y} f\,dx..$

1.14. $\displaystyle\int_{-2}^{-1} dx \int_{-(2+x)}^{0} f\,dy + \int_{-1}^{0} dx \int_{\sqrt[3]{x}}^{0} f\,dy.$

1.15. $\displaystyle\int_{0}^{1} dy \int_{0}^{\sqrt{y}} f\,dx + \int_{1}^{e} dy \int_{\ln y}^{1} f\,dx.$

1.16. $\displaystyle\int_{0}^{1} dy \int_{-\sqrt{y}}^{0} f\,dx + \int_{1}^{2} dy \int_{-\sqrt{2-y}}^{0} f\,dx.$

1.17. $\displaystyle\int_{0}^{1} dy \int_{-y}^{0} f\,dx + \int_{1}^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^{0} f\,dx.$

1.18. $\displaystyle\int_{0}^{1} dy \int_{0}^{y^2} f\,dx + \int_{1}^{2} dy \int_{0}^{2-y} f\,dx.$

1.19. $\displaystyle\int_{0}^{\sqrt{3}} dx \int_{\sqrt{4-x^2}-2}^{0} f\,dy + \int_{\sqrt{3}}^{2} dx \int_{-\sqrt{4-x^2}}^{0} f\,dy.$

1.20. $\displaystyle\int_{-2}^{-1} dy \int_{-(2+y)}^{0} f\,dx + \int_{-1}^{0} dy \int_{\sqrt[3]{y}}^{0} f\,dx.$

1.21. $\displaystyle\int_{0}^{1} dy \int_{0}^{y} f\,dx + \int_{1}^{e} dy \int_{\ln y}^{1} f\,dx.$

1.22. $\displaystyle\int_{0}^{1} dx \int_{0}^{x^2} f\,dy + \int_{1}^{\sqrt{2}} dx \int_{0}^{\sqrt{2-x^2}} f\,dy.$

3

1.23. $\displaystyle\int\limits_{0}^{\pi/4} dx \int\limits_{0}^{\sin x} f\,dy + \int\limits_{\pi/4}^{\pi/2} dx \int\limits_{0}^{\cos x} f\,dy$.

1.24. $\displaystyle\int\limits_{-\sqrt{2}}^{-1} dy \int\limits_{-\sqrt{2-y^2}}^{0} f\,dx + \int\limits_{-1}^{0} dy \int\limits_{y}^{0} f\,dx$.

1.25. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{0}^{x^2} f\,dy + \int\limits_{1}^{2} dx \int\limits_{0}^{2-x} f\,dy$.

1.26. $\displaystyle\int\limits_{0}^{\sqrt{3}} dx \int\limits_{0}^{2-\sqrt{4-x^2}} f\,dy + \int\limits_{\sqrt{3}}^{2} dx \int\limits_{0}^{\sqrt{4-x^2}} f\,dy$.

1.27. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{-\sqrt{x}}^{0} f\,dy + \int\limits_{1}^{2} dx \int\limits_{-\sqrt{2-x}}^{0} f\,dy$.

1.28. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{0}^{x} f\,dy + \int\limits_{1}^{\sqrt{2}} dx \int\limits_{0}^{\sqrt{2-x^2}} f\,dy$.

1.29. $\displaystyle\int\limits_{0}^{1} dy \int\limits_{0}^{\sqrt{y}} f\,dx + \int\limits_{1}^{\sqrt{2}} dy \int\limits_{0}^{\sqrt{2-y^2}} f\,dx$.

1.30. $\displaystyle\int\limits_{0}^{1} dx \int\limits_{0}^{\sqrt{x}} f\,dy + \int\limits_{1}^{2} dx \int\limits_{0}^{\sqrt{2-x}} f\,dy$.

1.31. $\displaystyle\int\limits_{-2}^{-\sqrt{3}} dx \int\limits_{0}^{\sqrt{4-x^2}} f\,dy + \int\limits_{-\sqrt{3}}^{0} dx \int\limits_{0}^{2-\sqrt{4-x^2}} f\,dy$.

Задача 2. Вычислить.

2.1. $\iint\limits_{D}\left(12x^2y^2+16x^3y^3\right)dxdy;$

$D: x=1,\ y=x^2,\ y=-\sqrt{x}.$

2.2. $\iint\limits_{D}\left(9x^2y^2+48x^3y^3\right)dxdy;$

$D: x=1,\ y=\sqrt{x},\ y=-x^2.$

2.3. $\iint\limits_{D}\left(36x^2y^2-96x^3y^3\right)dxdy;$

$D: x=1,\ y=\sqrt[3]{x},\ y=-x^3.$

2.4. $\iint\limits_{D}\left(18x^2y^2+32x^3y^3\right)dxdy;$

$D: x=1,\ y=x^3,\ y=-\sqrt[3]{x}.$

2.5. $\iint\limits_{D}\left(27x^2y^2+48x^3y^3\right)dxdy;$

$D: x=1,\ y=x^2,\ y=-\sqrt[3]{x}.$

2.6. $\iint\limits_{D}\left(18x^2y^2+32x^3y^3\right)dxdy;$

$D: x=1,\ y=\sqrt[3]{x},\ y=-x^2.$

2.7. $\iint\limits_{D}\left(18x^2y^2+32x^3y^3\right)dxdy;$

$D: x=1,\ y=x^3,\ y=-\sqrt{x}.$

2.8. $\iint\limits_{D}\left(27x^2y^2+48x^3y^3\right)dxdy;$

$D: x=1,\ y=\sqrt{x},\ y=-x^3.$

2.9. $\iint\limits_{D}\left(4xy+3x^2y^2\right)dxdy;$

$D: x=1,\ y=x^2,\ y=-\sqrt{x}.$

2.10. $\iint\limits_{D}\left(12xy+9x^2y^2\right)dxdy;$

$D: x=1,\ y=\sqrt{x},\ y=-x^2.$

2.11. $\iint\limits_{D}\left(8xy+9x^2y^2\right)dxdy;$

$D: x=1,\ y=\sqrt[3]{x},\ y=-x^3.$

2.12. $\iint\limits_{D}\left(24xy+18x^2y^2\right)dxdy;$

$D: x=1,\ y=x^3,\ y=-\sqrt[3]{x}.$

2.13. $\iint\limits_{D}\left(12xy+27x^2y^2\right)dxdy;$

$D: x=1,\ y=x^2,\ y=-\sqrt[3]{x}.$

2.14. $\iint\limits_{D}\left(8xy+18x^2y^2\right)dxdy;$

$D: x=1,\ y=\sqrt[3]{x},\ y=-x^2.$

2.15. $\iint\limits_{D}\left(\dfrac{4}{5}xy+\dfrac{9}{11}x^2y^2\right)dxdy;$

$D: x=1,\ y=x^3,\ y=-\sqrt{x}.$

2.16. $\iint\limits_{D}\left(\dfrac{4}{5}xy+9x^2y^2\right)dxdy;$

$D: x=1,\ y=\sqrt{x},\ y=-x^3.$

2.17. $\iint\limits_{D}\left(24xy-48x^3y^3\right)dxdy;$

$D: x=1,\ y=x^2,\ y=-\sqrt{x}.$

2.18. $\iint\limits_{D}\left(6xy+24x^3y^3\right)dxdy;$

$D: x=1,\ y=\sqrt{x},\ y=-x^2.$

2.19. $\iint\limits_{D}\left(4xy+16x^3y^3\right)dxdy;$

$D:\ x=1,\ y=\sqrt[3]{x},\ y=-x^3.$

2.20. $\iint\limits_{D}\left(4xy+16x^3y^3\right)dxdy;$

$D:\ x=1,\ y=x^3,\ y=-\sqrt[3]{x}.$

2.21. $\iint\limits_{D}\left(44xy+16x^3y^3\right)dxdy;$

$D:\ x=1,\ y=x^2,\ y=-\sqrt[3]{x}.$

2.22. $\iint\limits_{D}\left(4xy+176x^3y^3\right)dxdy;$

$D:\ x=1,\ y=\sqrt[3]{x},\ y=-x^3.$

2.23. $\iint\limits_{D}\left(xy-4x^3y^3\right)dxdy;$

$D:\ x=1,\ y=x^3,\ y=-\sqrt{x}.$

2.24. $\iint\limits_{D}\left(4xy+176x^3y^3\right)dxdy;$

$D:\ x=1,\ y=\sqrt{x},\ y=-x^3.$

2.25. $\iint\limits_{D}\left(6x^2y^2+\dfrac{25}{3}x^4y^4\right)dxdy;$

$D:\ x=1,\ y=x^2,\ y=-\sqrt{x}.$

2.26. $\iint\limits_{D}\left(9x^2y^2+25x^4y^4\right)dxdy;$

$D:\ x=1,\ y=\sqrt{x},\ y=-x^2.$

2.27. $\iint\limits_{D}\left(3x^2y^2+\dfrac{50}{3}x^4y^4\right)dxdy;$

$D:\ x=1,\ y=\sqrt[3]{x},\ y=-x^3.$

2.28. $\iint\limits_{D}\left(9x^2y^2+25x^4y^4\right)dxdy;$

$D:\ x=1,\ y=x^3,\ y=-\sqrt[3]{x}.$

2.29. $\iint\limits_{D}\left(54x^2y^2+150x^4y^4\right)dxdy;$

$D:\ x=1,\ y=x^2,\ y=-\sqrt[3]{x}.$

2.30. $\iint\limits_{D}\left(xy-9x^5y^5\right)dxdy;$

$D:\ x=1,\ y=\sqrt[3]{x},\ y=-x^2.$

2.31. $\iint\limits_{D}\left(54x^2y^2+150x^4y^4\right)dxdy;$

$D:\ x=1,\ y=x^3,\ y=-\sqrt{x}.$

Задача 3. Вычислить.

3.1. $\iint\limits_D y e^{xy/2} dxdy;$

$D: y = \ln 2, \ y = \ln 3, \ x = 2, \ x = 4.$

3.2. $\iint\limits_D y^2 \sin\dfrac{xy}{2} \ dxdy;$

$D: x = 0, \ y = \sqrt{\pi}, \ y = \dfrac{x}{2}.$

3.3. $\iint\limits_D y \cos xy \ dxdy;$

$D: y = \pi/2, \ y = \pi, \ x = 1, \ x = 2.$

3.4. $\iint\limits_D y^2 e^{-xy/4} dxdy;$

$D: x = 0, \ y = 2, \ y = x.$

3.5. $\iint\limits_D y \sin xy \ dxdy;$

$D: y = \pi/2, \ y = \pi, \ x = 1, \ x = 2.$

3.6. $\iint\limits_D y^2 \cos\dfrac{xy}{2} \ dxdy;$

$D: x = 0, \ y = \sqrt{\pi/2}, \ y = x/2.$

3.7. $\iint\limits_D 4 y e^{2xy} dxdy;$

$D: y = \ln 3, \ y = \ln 4, \ x = \dfrac{1}{2}, \ x = 1.$

3.8. $\iint\limits_D 4 y^2 \sin xy \ dxdy;$

$D: x = 0, \ y = \sqrt{\dfrac{\pi}{2}}, \ y = x.$

3.9. $\iint\limits_D y \cos 2xy \ dxdy;$

$D: y = \dfrac{\pi}{2}, \ y = \pi, \ x = \dfrac{1}{2}, \ x = 1.$

3.10. $\iint\limits_D y^2 e^{-xy/8} dxdy;$

$D: x = 0, \ y = 2, \ y = \dfrac{x}{2}.$

3.11. $\iint\limits_D 12 y \sin 2xy \ dxdy;$

$D: y = \dfrac{\pi}{4}, \ y = \dfrac{\pi}{2}, \ x = 2, \ x = 3.$

3.12. $\iint\limits_D y^2 \cos xy \ dxdy;$

$D: x = 0, \ y = \sqrt{\pi}, \ y = x.$

3.13. $\iint\limits_D y e^{xy/4} dxdy;$

$D: y = \ln 2, \ y = \ln 3, \ x = 4, \ x = 8.$

3.14. $\iint\limits_D y^2 \sin 2xy \ dxdy;$

$D: x = 0, \ y = \sqrt{2\pi}, \ y = 2x.$

3.15. $\iint\limits_D 2 y \cos 2xy \ dxdy;$

$D: y = \dfrac{\pi}{4}, \ y = \dfrac{\pi}{2}, \ x = 1, \ x = 2.$

3.16. $\iint\limits_D y^2 e^{-xy/2} dxdy;$

$D: x = 0, \ y = \sqrt{2}, \ y = x.$

3.17. $\iint\limits_D y\sin xy \, dxdy;$

$D: \ y = \pi, \ y = 2\pi, \ x = \dfrac{1}{2}, \ x = 1.$

3.18. $\iint\limits_D y^2\cos 2xy \, dxdy;$

$D: \ x = 0, \ y = \sqrt{\dfrac{\pi}{2}}, \ y = \dfrac{x}{2}.$

3.19. $\iint\limits_D 8ye^{4xy} dxdy;$

$D: \ y = \ln 3, \ y = \ln 4, \ x = \dfrac{1}{4}, \ x = \dfrac{1}{2}.$

3.20. $\iint\limits_D 3y^2\sin\dfrac{xy}{2} \, dxdy;$

$D: \ x = 0, \ y = \sqrt{\dfrac{4\pi}{3}}, \ y = \dfrac{2}{3}x.$

3.21. $\iint\limits_D y\cos xy \, dxdy;$

$D: \ y = \pi, \ y = 3\pi, \ x = 1/2, \ x = 1.$

3.22. $\iint\limits_D y^2 e^{-xy/2} dxdy;$

$D: \ x = 0, \ y = 1, \ y = \dfrac{x}{2}.$

3.23. $\iint\limits_D y\sin 2xy \, dxdy;$

$D: \ y = \pi/2, \ y = 3\pi/2, \ x = 1/2, \ x = 2.$

3.24. $\iint\limits_D y^2\cos xy \, dxdy;$

$D: \ x = 0, \ y = \sqrt{\pi}, \ y = 2x.$

3.25. $\iint\limits_D 6ye^{xy/3} dxdy;$

$D: \ y = \ln 2, \ y = \ln 3, \ x = 3, \ x = 6.$

3.26. $\iint\limits_D y^2\sin\dfrac{xy}{2} \, dxdy;$

$D: \ x = 0, \ y = \sqrt{\pi}, \ y = x.$

3.27. $\iint\limits_D y\cos 2xy \, dxdy;$

$D: \ y = \pi/2, \ y = 3\pi/2, \ x = 1/2, \ x = 2.$

3.28. $\iint\limits_D y^2 e^{-xy/8} dxdy;$

$D: \ x = 0, \ y = 4, \ y = 2x.$

3.29. $\iint\limits_D 3y\sin xy \, dxdy;$

$D: \ y = \pi/2, \ y = 3\pi, \ x = 1, \ x = 3.$

3.30. $\iint\limits_D y^2\cos\dfrac{xy}{2} \, dxdy;$

$D: \ x = 0, \ y = \sqrt{2\pi}, \ y = 2x.$

3.31. $\iint\limits_D 12ye^{6xy} dxdy;$

$D: \ y = \ln 3, \ y = \ln 4, \ x = 1/6, \ x = 1/3.$

Задача 4. Вычислить.

4.1. $\iiint\limits_V 2y^2 e^{xy}\, dx\, dy\, dz;$

$V \begin{cases} x=0,\ y=1,\ y=x, \\ z=0,\ z=1. \end{cases}$

4.2. $\iiint\limits_V x^2 z \sin(xyz)\, dx\, dy\, dz;$

$V \begin{cases} x=2,\ y=\pi,\ z=1, \\ x=0,\ y=1,\ z=0. \end{cases}$

4.3. $\iiint\limits_V y^2 \mathrm{ch}(2xy)\, dx\, dy\, dz;$

$V \begin{cases} x=0,\ y=-2,\ y=4x, \\ z=0,\ z=2. \end{cases}$

4.4. $\iiint\limits_V 8y^2 z\, e^{2xyz}\, dx\, dy\, dz;$

$V \begin{cases} x=-1,\ y=2,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$

4.5. $\iiint\limits_V x^2 \mathrm{sh}(3xy)\, dx\, dy\, dz;$

$V \begin{cases} x=1,\ y=2x,\ y=0, \\ z=0,\ z=36. \end{cases}$

4.6. $\iiint\limits_V y^2 z \cos(xyz)\, dx\, dy\, dz;$

$V \begin{cases} x=1,\ y=2\pi,\ z=2, \\ x=0,\ y=1,\ z=0. \end{cases}$

4.7. $\iiint\limits_V y^2 \cos\left(\dfrac{\pi}{4}xy\right) dx\, dy\, dz;$

$V \begin{cases} x=0,\ y=-1,\ y=x/2, \\ z=0,\ z=-\pi^2. \end{cases}$

4.8. $\iiint\limits_V x^2 z \sin\dfrac{xyz}{4}\, dx\, dy\, dz;$

$V \begin{cases} x=1,\ y=2\pi,\ z=4, \\ x=0,\ y=0,\ z=0. \end{cases}$

4.9. $\iiint\limits_V y^2 e^{-xy}\, dx\, dy\, dz;$

$V \begin{cases} x=0,\ y=-2,\ y=4x, \\ z=0,\ z=1. \end{cases}$

4.10. $\iiint\limits_V 2y^2 z\, e^{2xyz}\, dx\, dy\, dz;$

$V \begin{cases} x=1,\ y=1,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$

4.11. $\iiint\limits_V y^2 \mathrm{ch}(2xy)\, dx\, dy\, dz;$

$V \begin{cases} x=0,\ y=1,\ y=x, \\ z=0,\ z=8. \end{cases}$

4.12. $\iiint\limits_V x^2 z\, \mathrm{sh}(xyz)\, dx\, dy\, dz;$

$V \begin{cases} x=2,\ y=1,\ z=1, \\ x=0,\ y=0,\ z=0. \end{cases}$

4.13. $\iiint\limits_V y^2 e^{xy/2}\, dx\, dy\, dz;$

$V \begin{cases} x=0,\ y=2,\ y=2x, \\ z=0,\ z=-1. \end{cases}$

4.14. $\iiint\limits_V y^2 z \cos\dfrac{xyz}{3}\, dx\, dy\, dz;$

$V \begin{cases} x=3,\ y=1,\ z=2\pi, \\ x=0,\ y=0,\ z=0. \end{cases}$

4.15. $\iiint\limits_{V} y^2\cos\left(\dfrac{\pi xy}{2}\right) dx\, dy\, dz;$

$V \begin{cases} x = 0,\ y = -1,\ y = x, \\ z = 0,\ z = 2\pi^2. \end{cases}$

4.16. $\iiint\limits_{V} x^2 z\ \mathrm{sh}(xyz)\ dx\, dy\, dz;$

$V \begin{cases} x = 1,\ y = -1,\ z = 1, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$

4.17. $\iiint\limits_{V} y^2\cos(\pi xy)\ dx\, dy\, dz;$

$V \begin{cases} x = 0,\ y = 1,\ y = 2x, \\ z = 0,\ z = \pi^2. \end{cases}$

4.18. $\iiint\limits_{V} 2x^2 z\ \mathrm{sh}(2xyz)\ dx\, dy\, dz;$

$V \begin{cases} x = 2,\ y = 1/2,\ z = 1/2, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$

4.19. $\iiint\limits_{V} x^2\mathrm{sh}(2xy)\ dx\, dy\, dz;$

$V \begin{cases} x = -1,\ y = x,\ y = 0, \\ z = 0,\ z = 8. \end{cases}$

4.20. $\iiint\limits_{V} x^2 z\ \sin\dfrac{xyz}{2}\ dx\, dy\, dz;$

$V \begin{cases} x = 1,\ y = 4,\ z = \pi, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$

4.21. $\iiint\limits_{V} y^2\mathrm{ch}(xy)\ dx\, dy\, dz;$

$V \begin{cases} x = 0,\ y = -1,\ y = x, \\ z = 0,\ z = 2. \end{cases}$

4.22. $\iiint\limits_{V} x^2 z\ \mathrm{ch}(xyz)\ dx\, dy\, dz;$

$V \begin{cases} x = 1,\ y = 1,\ z = 1, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$

4.23. $\iiint\limits_{V} x^2\cos\left(\dfrac{\pi}{2}xy\right) dx\, dy\, dz;$

$V \begin{cases} x = 2,\ y = x,\ y = 0, \\ z = 0,\ z = \pi. \end{cases}$

4.24. $\iiint\limits_{V} y^2 z\cos\dfrac{xyz}{9}\ dx\, dy\, dz;$

$V \begin{cases} x = 9,\ y = 1,\ z = 2\pi, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$

4.25. $\iiint\limits_{V} x^2\cos(\pi xy)\ dx\, dy\, dz;$

$V \begin{cases} x = 1,\ y = 2x,\ y = 0, \\ z = 0,\ z = 4\pi. \end{cases}$

4.26. $\iiint\limits_{V} y^2 z\ \mathrm{ch}\left(\dfrac{xyz}{2}\right) dx\, dy\, dz;$

$V \begin{cases} x = 2,\ y = -1,\ z = 2, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$

4.27.

$$\iiint\limits_V y^2 \mathrm{ch}(3xy)\ dx\ dy\ dz;$$

$$V \begin{cases} x = 0,\ y = 2,\ \mathrm{y} = 6x, \\ z = 0,\ z = -3. \end{cases}$$

4.28.

$$\iiint\limits_V 2y^2 z\ \mathrm{ch}(2xyz)\ dx\ dy\ dz;$$

$$V \begin{cases} x = \dfrac{1}{2},\ y = 2,\ \mathrm{z} = -1, \\ x = 0,\ y = 0,\ z = 0. \end{cases}$$

4.29.

$$\iiint\limits_V x^2 \sin(4\pi xy)\ dx\ dy\ dz;$$

$$V \begin{cases} x = 1,\ y = x/2,\ \mathrm{y} = 0, \\ z = 0,\ z = 8\pi. \end{cases}$$

4.30.

$$\iiint\limits_V 8y^2 z\ e^{-xyz}\ dx\ dy\ dz;$$

$$V \begin{cases} x = 2,\ y = -1,\ \mathrm{z} = 2, \\ x = 0,\ \mathrm{y} = 0,\ z = 0. \end{cases}$$

4.31.

$$\iiint\limits_V x^2 \mathrm{sh}(xy)\ dx\ dy\ dz;$$

$$V \begin{cases} x = 2,\ y = x/2,\ \mathrm{y} = 0, \\ z = 0,\ z = 1. \end{cases}$$

Задача 5. Вычислить.

$$\iiint\limits_{V} x \, dx \, dy \, dz;$$

5.1. $V: \ y = 10x, \ y = 0, \ x = 1,$
$\quad\quad z = xy, \ z = 0.$

$$\iiint\limits_{V} \frac{dx \, dy \, dz}{\left(1 + \dfrac{x}{3} + \dfrac{y}{4} + \dfrac{z}{8}\right)^{4}};$$

5.2. $V: \ 1 + \dfrac{x}{3} + \dfrac{y}{4} + \dfrac{z}{8} = 1,$
$\quad\quad x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_{V} 15\left(y^{2} + z^{2}\right) dx \, dy \, dz;$$

5.3. $V: \ z = x + y, \ x + y = 1,$
$\quad\quad x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_{V} (3x + 4y) \, dx \, dy \, dz;$$

5.4. $V: \ y = x, \ y = 0, \ x = 1,$
$\quad\quad z = 5\left(x^{2} + y^{2}\right), \ z = 0.$

$$\iiint\limits_{V} \left(1 + 2x^{3}\right) dx \, dy \, dz;$$

5.5. $V: \ y = 9x, \ y = 0, \ x = 1,$
$\quad\quad z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_{V} \left(27 + 54y^{3}\right) dx \, dy \, dz;$$

5.6. $V: \ y = x, \ y = 0, \ x = 1,$
$\quad\quad z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_{V} y \, dx \, dy \, dz;$$

5.7. $V: \ y = 15x, \ y = 0, \ x = 1,$
$\quad\quad z = xy, \ z = 0.$

$$\iiint\limits_{V} \frac{dx \, dy \, dz}{\left(1 + \dfrac{x}{16} + \dfrac{y}{8} + \dfrac{z}{3}\right)^{5}};$$

5.8. $V: \ \dfrac{x}{16} + \dfrac{y}{5} + \dfrac{z}{3} = 1,$
$\quad\quad x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_{V} \left(3x^{2} + y^{2}\right) dx \, dy \, dz;$$

5.9. $V: \ z = 10y, \ x + y = 1,$
$\quad\quad x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_{V} (15x + 30z) \, dx \, dy \, dz;$$

5.10. $V: \ z = x^{2} + 3y^{2}, \ z = 0,$
$\quad\quad y = x, \ y = 0, \ z = 0.$

$$\iiint\limits_{V} \left(4 + 8z^{3}\right) dx \, dy \, dz;$$

5.11. $V: \ y = x, \ y = 0, \ x = 1,$
$\quad\quad z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_{V} \left(1 + 2x^{3}\right) dx \, dy \, dz;$$

5.12. $V: \ y = 36x, \ y = 0, \ x = 1,$
$\quad\quad z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_V 21xz \, dx \, dy \, dz;$$

5.13. $V: y = x, \ y = 0, \ x = 2,$
    $z = xy, \ z = 0.$

$$\iiint\limits_V \frac{dx \, dy \, dz}{\left(1 + \dfrac{x}{10} + \dfrac{y}{8} + \dfrac{z}{3}\right)^6};$$

5.14. $V: \dfrac{x}{10} + \dfrac{y}{8} + \dfrac{z}{3} = 1,$
    $x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_V \left(x^2 + 3y^2\right) dx \, dy \, dz;$$

5.15. $V: z = 10x, \ x + y = 1,$
    $x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_V (60y + 90z) \, dx \, dy \, dz;$$

5.16. $V: y = x, \ y = 0, \ x = 1,$
    $z = x^2 + y^2, \ z = 0.$

$$\iiint\limits_V \left(\frac{10}{3}x + \frac{5}{3}\right) dx \, dy \, dz;$$

5.17. $V: y = 9x, \ y = 0, \ x = 1,$
    $z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_V (9 + 18z) \, dx \, dy \, dz;$$

5.18. $V: y = 4x, \ y = 0, \ x = 1,$
    $z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_V 3y^2 \, dx \, dy \, dz;$$

5.19. $V: y = 2x, \ y = 0, \ x = 2,$
    $z = xy, \ z = 0.$

$$\iiint\limits_V \frac{dx \, dy \, dz}{\left(1 + \dfrac{x}{2} + \dfrac{y}{4} + \dfrac{z}{6}\right)^6};$$

5.20. $V: \dfrac{x}{2} + \dfrac{y}{4} + \dfrac{z}{6} = 1,$
    $x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_V x^2 \, dx \, dy \, dz;$$

5.21. $V: z = 10(x + 3y), \ x + y = 1,$
    $x = 0, \ y = 0, \ z = 0.$

$$\iiint\limits_V (8y + 12z) \, dx \, dy \, dz;$$

5.22. $V: y = x, \ y = 0, \ x = 1,$
    $z = 3x^2 + 2y^2, \ z = 0.$

$$\iiint\limits_V 63\left(1 + 2\sqrt{y}\right) dx \, dy \, dz;$$

5.23. $V: y = x, \ y = 0, \ x = 1,$
    $z = \sqrt{xy}, \ z = 0.$

$$\iiint\limits_V (x + y) \, dx \, dy \, dz;$$

5.24. $V: y = x, \ y = 0, \ x = 1,$
    $z = 30x^2 + 60y^2, \ z = 0.$

$$\iiint\limits_{V} \frac{dx\,dy\,dz}{\left(1+\dfrac{x}{6}+\dfrac{y}{4}+\dfrac{z}{16}\right)^{6}};$$

5.25. $V:\ \dfrac{x}{6}+\dfrac{y}{4}+\dfrac{z}{16}=1,$

$x=0,\ y=0,\ z=0.$

$$\iiint\limits_{V} xyz\,dx\,dy\,dz;$$

5.26. $V:\ y=x,\ y=0,\ x=2,$

$z=xy,\ z=0.$

$$\iiint\limits_{V} y^{2}\,dx\,dy\,dz;$$

5.27. $V:\ z=10(3x+y),\ x+y=1,$

$x=0,\ y=0,\ z=0.$

$$\iiint\limits_{V}\left(5x+\dfrac{3z}{2}\right)dx\,dy\,dz;$$

5.28. $V:\ y=x,\ y=0,\ x=2,$

$z=x^{2}+15y^{2},\ z=0.$

$$\iiint\limits_{V}\left(x^{2}+4y^{2}\right)dx\,dy\,dz;$$

5.29. $V:\ z=20(2x+y),\ x+y=1,$

$x=0,\ y=0,\ z=0.$

$$\iiint\limits_{V} \frac{dx\,dy\,dz}{\left(1+\dfrac{x}{8}+\dfrac{y}{3}+\dfrac{z}{5}\right)^{6}};$$

5.30. $V:\ \dfrac{x}{8}+\dfrac{y}{3}+\dfrac{z}{5}=1,$

$x=0,\ y=0,\ z=0.$

$$\iiint\limits_{V} x^{2}z\,dx\,dy\,dz;$$

5.31. $V:\ y=3x,\ y=0,\ x=2,$

$z=xy,\ z=0.$

Задача 6. Найти площадь фигуры, ограниченной данными линиями.

6.1. $y = 3/x,\ y = 4e^x,\ y = 3,\ y = 4.$

6.2. $x = \sqrt{36 - y^2},\ x = 6 - \sqrt{36 - y^2}.$

6.3. $x^2 + y^2 = 72,\ 6y = -x^2\ \left(y \le 0\right).$

6.4. $x = 8 - y^2,\ x = -2y.$

6.5. $y = \dfrac{3}{x},\ y = 8e^x,\ y = 3,\ y = 8.$

6.6. $y = \dfrac{\sqrt{x}}{2},\ y = \dfrac{1}{2x},\ \mathrm{x} = 16.$

6.7. $x = 5 - y^2,\ x = -4y.$

6.8. $x^2 + y^2 = 12,\ \ -\sqrt{6}y = x^2\ \left(y \le 0\right).$

6.9. $y = \sqrt{12 - x^2},\ y = 2\sqrt{3} - \sqrt{12 - x^2},\ x = 0\ \left(x \ge 0\right).$

6.10. $y = \dfrac{3}{2}\sqrt{x},\ y = \dfrac{3}{2x},\ \mathrm{x} = 9.$

6.11. $y = \sqrt{24 - x^2},\ 2\sqrt{3}y = x^2,\ x = 0\ \left(x \ge 0\right).$

6.12. $y = \sin x,\ y = \cos x,\ \mathrm{x} = 0,\ \left(x \ge 0\right).$

6.13. $y = 20 - x^2,\ y = -8x.$

6.14. $y = \sqrt{18 - x^2},\ y = 3\sqrt{2} - \sqrt{18 - x^2}.$

6.15. $y = 32 - x^2,\ y = -4x.$

6.16. $y = 2/x,\ y = 5e^x,\ y = 2,\ y = 5.$

6.17. $x^2 + y^2 = 36,\ \ 3\sqrt{2}y = x^2\ \left(y \ge 0\right).$

6.18. $y = 3\sqrt{x},\ y = 3/x,\ x = 4.$

6.19. $y = 6 - \sqrt{36 - x^2},\ y = \sqrt{36 - x^2},\ x = 0\ \left(x \ge 0\right).$

6.20. $y = 25/4 - x^2,\ y = x - 5/2.$

6.21. $y = \sqrt{x},\ y = 1/x,\ x = 16.$

6.22. $y = 2/x$, $y = 7e^x$, $y = 2$, $y = 7$.

6.23. $x = 27 - y^2$, $x = -6y$.

6.24. $x = \sqrt{72 - y^2}$, $6x = y^2$, $y = 0$ $(y \geq 0)$.

6.25. $y = \sqrt{6 - x^2}$, $y = \sqrt{6} - \sqrt{6 - x^2}$.

6.26. $y = \dfrac{3}{2}\sqrt{x}$, $y = \dfrac{3}{2x}$, $x = 4$.

6.27. $y = \sin x$, $y = \cos x$, $x = 0$, $(x \leq 0)$.

6.28. $y = \dfrac{1}{x}$, $y = 6e^x$, $y = 1$, $y = 6$.

6.29. $y = 3\sqrt{x}$, $y = 3/x$, $x = 9$.

6.30. $y = 11 - x^2$, $y = -10x$.

6.31. $x^2 + y^2 = 12$, $x\sqrt{6} = y^2$ $(x \geq 0)$.

Задача 7. Найти площадь фигуры, ограниченной данными линиями.

7.1.
$$y^2 - 2y + x^2 = 0,$$
$$y^2 - 4y + x^2 = 0,$$
$$y = x/\sqrt{3}, \; y = \sqrt{3}x.$$

7.2.
$$x^2 - 4x + y^2 = 0,$$
$$x^2 - 8x + y^2 = 0,$$
$$y = 0, \; y = x/\sqrt{3}.$$

7.3.
$$y^2 - 6y + x^2 = 0,$$
$$y^2 - 8y + x^2 = 0,$$
$$y = x/\sqrt{3}, \; y = \sqrt{3}x.$$

7.4.
$$x^2 - 2x + y^2 = 0,$$
$$x^2 - 4x + y^2 = 0,$$
$$y = 0, \; y = x.$$

7.5.
$$y^2 - 8y + x^2 = 0,$$
$$y^2 - 10y + x^2 = 0,$$
$$y = x/\sqrt{3}, \; y = \sqrt{3}x.$$

7.6.
$$x^2 - 4x + y^2 = 0,$$
$$x^2 - 8x + y^2 = 0,$$
$$y = 0, \; y = x.$$

7.7.
$$y^2 - 4y + x^2 = 0,$$
$$y^2 - 6y + x^2 = 0,$$
$$y = x, \; x = 0.$$

7.8.
$$x^2 - 2x + y^2 = 0,$$
$$x^2 - 10x + y^2 = 0,$$
$$y = 0, \; y = \sqrt{3}x.$$

7.9.
$$y^2 - 6y + x^2 = 0,$$
$$y^2 - 10y + x^2 = 0,$$
$$y = x, \; x = 0.$$

7.10.
$$x^2 - 2x + y^2 = 0,$$
$$x^2 - 4x + y^2 = 0,$$
$$y = x/\sqrt{3}, \; y = \sqrt{3}x.$$

7.11.
$$y^2 - 2y + x^2 = 0,$$
$$y^2 - 4y + x^2 = 0,$$
$$y = \sqrt{3}x, \; x = 0.$$

7.12.
$$x^2 - 2x + y^2 = 0,$$
$$x^2 - 6x + y^2 = 0,$$
$$y = x/\sqrt{3}, \; y = \sqrt{3}x.$$

7.13.
$$y^2 - 4y + x^2 = 0,$$
$$y^2 - 6y + x^2 = 0,$$
$$y = \sqrt{3}x, \; x = 0.$$

7.14.
$$x^2 - 2x + y^2 = 0,$$
$$x^2 - 8x + y^2 = 0,$$
$$y = x/\sqrt{3}, \; y = \sqrt{3}x.$$

7.15.
$$y^2 - 2y + x^2 = 0,$$
$$y^2 - 6y + x^2 = 0,$$
$$y = x/\sqrt{3}, \; y = 0.$$

7.16.
$$x^2 - 2x + y^2 = 0,$$
$$x^2 - 4x + y^2 = 0,$$
$$y = 0, \; y = x/\sqrt{3}.$$

7.17. $\begin{aligned} y^2 - 2y + x^2 &= 0, \\ y^2 - 10y + x^2 &= 0, \\ y &= x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.18. $\begin{aligned} x^2 - 2x + y^2 &= 0, \\ x^2 - 6x + y^2 &= 0, \\ y &= 0, \ y = x/\sqrt{3}. \end{aligned}$

7.19. $\begin{aligned} y^2 - 4y + x^2 &= 0, \\ y^2 - 10y + x^2 &= 0, \\ y &= x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.20. $\begin{aligned} x^2 - 2x + y^2 &= 0, \\ x^2 - 6x + y^2 &= 0, \\ y &= 0, \ y = x. \end{aligned}$

7.21. $\begin{aligned} y^2 - 2y + x^2 &= 0, \\ y^2 - 4y + x^2 &= 0, \\ y &= x, \ x = 0. \end{aligned}$

7.22. $\begin{aligned} x^2 - 2x + y^2 &= 0, \\ x^2 - 4x + y^2 &= 0, \\ y &= 0, \ y = \sqrt{3}x. \end{aligned}$

7.23. $\begin{aligned} y^2 - 6y + x^2 &= 0, \\ y^2 - 8y + x^2 &= 0, \\ y &= x, \ x = 0. \end{aligned}$

7.24. $\begin{aligned} x^2 - 4x + y^2 &= 0, \\ x^2 - 8x + y^2 &= 0, \\ y &= 0, \ y = \sqrt{3}x. \end{aligned}$

7.25. $\begin{aligned} y^2 - 4y + x^2 &= 0, \\ y^2 - 8y + x^2 &= 0, \\ y &= x, \ x = 0. \end{aligned}$

7.26. $\begin{aligned} x^2 - 4x + y^2 &= 0, \\ x^2 - 8x + y^2 &= 0, \\ y &= x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.27. $\begin{aligned} y^2 - 4y + x^2 &= 0, \\ y^2 - 8y + x^2 &= 0, \\ y &= \sqrt{3}x, \ x = 0. \end{aligned}$

7.28. $\begin{aligned} x^2 - 4x + y^2 &= 0, \\ x^2 - 6x + y^2 &= 0, \\ y &= x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.29. $\begin{aligned} y^2 - 2y + x^2 &= 0, \\ y^2 - 10y + x^2 &= 0, \\ y &= x/\sqrt{3}, \ x = 0. \end{aligned}$

7.30. $\begin{aligned} x^2 - 6x + y^2 &= 0, \\ x^2 - 10x + y^2 &= 0, \\ y &= x/\sqrt{3}, \ y = \sqrt{3}x. \end{aligned}$

7.31. $\begin{aligned} y^2 - 4y + x^2 &= 0, \\ y^2 - 8y + x^2 &= 0, \\ y &= x/\sqrt{3}, \ x = 0. \end{aligned}$

Задача 8. Пластинка D задана ограничивающими ее кривыми, μ - поверхностная плотность. Найти массу пластинки.

8.1. $D: x = 1, \ y = 0, \ y^2 = 4x \ \ (y \geq 0);$
$\mu = 7x^2 + y.$

8.2. $D: x^2 + y^2 = 1, \ x^2 + y^2 = 4,$
$x = 0, \ y = 0 \ (x \geq 0, \ y \geq 0);$
$\mu = (x + y)/(x^2 + y^2).$

8.3. $D: x = 1, \ y = 0, \ y^2 = 4x \ \ (y \geq 0);$
$\mu = 7x^2/2 + 5y.$

8.4. $D: x^2 + y^2 = 9, \ x^2 + y^2 = 16,$
$x = 0, \ y = 0 \ (x \geq 0, \ y \geq 0);$
$\mu = (2x + 5y)/(x^2 + y^2).$

8.5. $D: x = 2, \ y = 0, \ y^2 = 2x \ \ (y \geq 0);$
$\mu = 7x^2/8 + 2y.$

8.6. $D: x^2 + y^2 = 1, \ x^2 + y^2 = 16,$
$x = 0, \ y = 0 \ (x \geq 0, \ y \geq 0);$
$\mu = (x + y)/(x^2 + y^2).$

8.7. $D: x = 2, \ y = 0, \ y^2 = x/2 \ \ (y \geq 0);$
$\mu = 7x^2/2 + 6y.$

8.8. $D: x^2 + y^2 = 4, \ x^2 + y^2 = 25,$
$x = 0, \ y = 0 \ (x \geq 0, \ y \leq 0);$
$\mu = (2x - 3y)/(x^2 + y^2).$

8.9. $D: x = 1, \ y = 0, \ y^2 = 4x \ \ (y \geq 0);$
$\mu = x + 3y^2.$

8.10. $D: x^2 + y^2 = 1, \ x^2 + y^2 = 9,$
$x = 0, \ y = 0 \ (x \geq 0, \ y \leq 0);$
$\mu = (x - y)/(x^2 + y^2).$

8.11. $D: x = 1, \ y = 0, \ y^2 = x \ \ (y \geq 0);$
$\mu = 3x + 6y^2.$

8.12. $D: x^2 + y^2 = 9, \ x^2 + y^2 = 25,$
$x = 0, \ y = 0 \ (x \leq 0, \ y \geq 0);$
$\mu = (2y - x)/(x^2 + y^2).$

8.13. $D: x = 2, \ y = 0, \ y^2 = x/2 \ \ (y \geq 0);$
$\mu = 2x + 3y^2.$

8.14. $D: x^2 + y^2 = 4, \ x^2 + y^2 = 16,$
$x = 0, \ y = 0 \ (x \leq 0, \ y \geq 0);$
$\mu = (2y - 3x)/(x^2 + y^2).$

8.15. $D: x = \dfrac{1}{2}, \ y = 0, \ y^2 = 8x \ \ (y \geq 0);$
$\mu = 7x + 3y^2.$

8.16. $D: x^2 + y^2 = 9, \ x^2 + y^2 = 16,$
$x = 0, \ y = 0 \ (x \leq 0, \ y \geq 0);$
$\mu = (2y - 5x)/(x^2 + y^2).$

8.17. $D: x=1,\ y=0,\ y^2=4x\ \ (y\geq 0);$
$\mu=7x^2+2y.$

8.18. $D: x^2+y^2=1,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\geq 0,\ y\geq 0);$
$\mu=(x+3y)\big/(x^2+y^2).$

8.19. $D: x=2,\ y^2=2x,\ y=0\ \ (y\geq 0);$
$\mu=7x^2/4+y/2.$

8.20. $D: x^2+y^2=1,\ x^2+y^2=4,$
$x=0,\ y=0\ (x\geq 0,\ y\geq 0);$
$\mu=(x+2y)\big/(x^2+y^2).$

8.21. $D: x=2,\ y=0,\ y^2=2x\ \ (y\geq 0);$
$\mu=7x^2/4+y.$

8.22. $D: x^2+y^2=1,\ x^2+y^2=9,$
$x=0,\ y=0\ (x\geq 0,\ y\leq 0);$
$\mu=(2x-y)\big/(x^2+y^2).$

8.23. $D: x=2,\ y=0,\ y^2=x/2\ \ (y\geq 0);$
$\mu=7x^2/2+8y.$

8.24. $D: x^2+y^2=1,\ x^2+y^2=25,$
$x=0,\ y=0\ (x\geq 0,\ y\leq 0);$
$\mu=(x-4y)\big/(x^2+y^2).$

8.25. $D: x=1,\ y=0,\ y^2=4x\ \ (y\geq 0);$
$\mu=6x+3y^2.$

8.26. $D: x^2+y^2=4,\ x^2+y^2=16,$
$x=0,\ y=0\ (x\geq 0,\ y\leq 0);$
$\mu=(3x-y)\big/(x^2+y^2).$

8.27. $D: x=2,\ y=0,\ y^2=x/2\ \ (y\geq 0);$
$\mu=4x+6y^2.$

8.28. $D: x^2+y^2=4,\ x^2+y^2=9,$
$x=0,\ y=0\ (x\leq 0,\ y\geq 0);$
$\mu=(y-4x)\big/(x^2+y^2).$

8.29. $D: x=\dfrac{1}{2},\ y=0,\ y^2=2x\ \ (y\geq 0);$
$\mu=4x+9y^2.$

8.30. $D: x^2+y^2=4,\ x^2+y^2=9,$
$x=0,\ y=0\ (x\leq 0,\ y\geq 0);$
$\mu=(y-2x)\big/(x^2+y^2).$

8.31. $D: x=\dfrac{1}{4},\ y=0,\ y^2=16x\ \ (y\geq 0);$
$\mu=16x+9y^2/2.$

Задача 9. Пластинка D задана неравенствами, μ - поверхностная плотность. Найти массу пластинки.

9.1.
$$D: \ x^2 + y^2\big/4 \le 1;$$
$$\mu = y^2.$$

9.2.
$$D: \ 1 \le x^2\big/9 + y^2\big/4 \le 2;$$
$$y \ge 0, \ \ y \le \frac{2}{3}x;$$
$$\mu = y\big/x.$$

9.3.
$$D: \ x^2\big/9 + y^2\big/25 \le 1;$$
$$y \ge 0;$$
$$\mu = x^2 y.$$

9.4.
$$D: \ x^2\big/9 + y^2\big/25 \le 1;$$
$$y \ge 0;$$
$$\mu = 7x^2 y\big/18.$$

9.5.
$$D: \ 1 \le x^2\big/9 + y^2\big/4 \le 4;$$
$$y \ge 0, \ \ y \le x\big/2;$$
$$\mu = 8\,y\big/x^3.$$

9.6.
$$D: \ x^2\big/9 + y^2 \le 1;$$
$$x \ge 0;$$
$$\mu = 7xy^6.$$

9.7.
$$D: \ x^2\big/4 + y^2 \le 1;$$
$$\mu = 4y^4.$$

9.8.
$$D: \ 1 \le x^2\big/4 + y^2\big/9 \le 4;$$
$$x \ge 0, \ \ y \le 3x\big/2;$$
$$\mu = x\big/y.$$

9.9.
$$D: \ 1 \le x^2\big/16 + y^2\big/4 \le 4;$$
$$x \ge 0, \ \ y \le x\big/2;$$
$$\mu = x\big/y.$$

9.10.
$$D: \ x^2\big/4 + y^2\big/9 \le 1;$$
$$x \ge 0, \ \ y \ge 0;$$
$$\mu = x^3 y.$$

9.11.
$$D: \ x^2\big/4 + y^2 \le 1;$$
$$x \ge 0, \ \ y \ge 0;$$
$$\mu = 6x^3 y^3.$$

9.12.
$$D: \ 1 \le x^2\big/4 + y^2 \le 25;$$
$$x \ge 0, \ \ y \le x\big/2;$$
$$\mu = x\big/y^3.$$

9.13.
$$D: \ x^2\big/9 + y^2\big/4 \le 1;$$
$$\mu = x^2 y^2.$$

9.14.
$$D: \ x^2\big/16 + y^2 \le 1;$$
$$x \ge 0, \ \ y \ge 0;$$
$$\mu = 5xy^7.$$

9.15.
$$D: \ x^2\big/4 + y^2 \le 1;$$
$$x \ge 0, \ \ y \ge 0;$$
$$\mu = 30x^3 y^7.$$

9.16.
$$D: \ 1 \le x^2\big/9 + y^2\big/4 \le 3;$$
$$y \ge 0, \ \ y \le \frac{2}{3}x;$$
$$\mu = y\big/x.$$

9.17.
$$D: x^2 + y^2/25 \leq 1;$$
$$y \geq 0;$$
$$\mu = 7x^4 y.$$

9.18.
$$D: x^2 + y^2/9 \leq 1;$$
$$y \geq 0;$$
$$\mu = 35x^4 y^3.$$

9.19.
$$D: x^2/4 + y^2/9 \leq 1;$$
$$\mu = x^2.$$

9.20.
$$D: 1 \leq x^2 + y^2/16 \leq 9;$$
$$y \geq 0, \ y \leq 4x;$$
$$\mu = y/x^3.$$

9.21.
$$D: x^2/9 + y^2 \leq 1;$$
$$x \geq 0;$$
$$\mu = 11xy^8.$$

9.22.
$$D: 1 \leq x^2/4 + y^2/16 \leq 5;$$
$$x \geq 0, \ y \leq 2x;$$
$$\mu = x/y.$$

9.23.
$$D: 1 \leq x^2/9 + y^2/4 \leq 5;$$
$$x \geq 0, \ y \leq 2x/3;$$
$$\mu = x/y.$$

9.24.
$$D: x^2/4 + y^2/9 \leq 1;$$
$$x \geq 0, \ y \geq 0;$$
$$\mu = x^5 y.$$

9.25.
$$D: x^2/4 + y^2/25 \leq 1;$$
$$\mu = x^4.$$

9.26.
$$D: x^2 + y^2/4 \leq 1;$$
$$x \geq 0, \ y \geq 0;$$
$$\mu = 15x^5 y^3.$$

9.27.
$$D: 1 \leq x^2/4 + y^2/9 \leq 36;$$
$$x \geq 0, \ y \geq \frac{3}{2}x;$$
$$\mu = 9x/y^3.$$

9.28.
$$D: x^2/100 + y^2 \leq 1;$$
$$x \geq 0, \ y \geq 0;$$
$$\mu = 6xy^9.$$

9.29.
$$D: x^2/16 + y^2 \leq 1;$$
$$x \geq 0, \ y \geq 0;$$
$$\mu = 105x^3 y^9.$$

9.30.
$$D: 1 \leq x^2/9 + y^2/16 \leq 2;$$
$$y \geq 0, \ y \leq \frac{4}{3}x;$$
$$\mu = 27y/x^5.$$

9.31.
$$D: 1 \leq x^2/16 + y^2 \leq 3;$$
$$x \geq 0, \ y \geq x/4;$$
$$\mu = x/y^5.$$

Задача 10. Найти объем тела, заданного ограничивающими его поверхностями.

10.1. $y = 16\sqrt{2x}, \ y = \sqrt{2x},$
$z = 0, \ x + z = 2.$

10.2. $y = 5\sqrt{x}, \ y = 5x/3,$
$z = 0, \ z = 5 + 5\sqrt{x}/3.$

10.3. $x^2 + y^2 = 2, \ y = \sqrt{x}, \ y = 0,$
$z = 0, \ z = 15x.$

10.4. $x + y = 2, \ y = \sqrt{x},$
$z = 12y, \ z = 0.$

10.5. $x = 20\sqrt{2y}, \ x = 5\sqrt{2y},$
$z = 0, \ z + y = 1/2.$

10.6. $x = 5\sqrt{y}/2, \ x = 5y/6,$
$z = 0, \ z = \dfrac{5}{6}\left(3 + \sqrt{y}\right).$

10.7. $x^2 + y^2 = 2, \ x = \sqrt{y}, \ x = 0,$
$z = 0, \ z = 30y.$

10.8. $x + y = 2, \ x = \sqrt{y},$
$z = 12x/5, \ z = 0.$

10.9. $y = 17\sqrt{2x}, \ y = 2\sqrt{2x},$
$z = 0, \ x + z = 1/2.$

10.10. $y = 5\sqrt{x}/3, \ y = 5x/9,$
$z = 0, \ z = 5\left(3 + \sqrt{x}\right)/9.$

10.11. $x^2 + y^2 = 8, \ y = \sqrt{2x}, \ y = 0,$
$z = 0, \ z = 15x/11.$

10.12. $x + y = 4, \ y = \sqrt{2x},$
$z = 3y, \ z = 0.$

10.13. $x = \dfrac{5}{6}\sqrt{y}, \ x = \dfrac{5}{18}y,$
$z = 0, \ z = \dfrac{5}{18}\left(3 + \sqrt{y}\right).$

10.14. $x = 19\sqrt{2y}, \ x = 4\sqrt{2y},$
$z = 0, \ z + y = 2.$

10.15. $x^2 + y^2 = 8, \ x = \sqrt{2y}, \ x = 0,$
$z = 30y/11, \ z = 0.$

10.16. $x + y = 4, \ x = \sqrt{2y},$
$z = 3x/5, \ z = 0.$

10.17. $y = 6\sqrt{3x}, \ y = \sqrt{3x},$
$z = 0, \ x + z = 3.$

10.18. $y = \dfrac{5}{6}\sqrt{x}, \ y = \dfrac{5}{18}x,$
$z = 0, \ z = \dfrac{5}{18}\left(3 + \sqrt{x}\right).$

10.19. $x^2 + y^2 = 18$, $y = \sqrt{3x}$, $y = 0$, $z = 0$, $z = 5x/11$.

10.20. $x + y = 6$, $y = \sqrt{3x}$, $z = 4y$, $z = 0$.

10.21. $x = 7\sqrt{3y}$, $x = 2\sqrt{3y}$, $z = 0$, $z + y = 3$.

10.22. $x = 5\sqrt{y}/3$, $x = 5y/9$, $z = 0$, $z = 5\left(3 + \sqrt{y}\right)/9$.

10.23. $x^2 + y^2 = 18$, $x = \sqrt{3y}$, $x = 0$, $z = 0$, $z = 10y/11$.

10.24. $x + y = 6$, $x = \sqrt{3y}$, $z = 4x/5$, $z = 0$.

10.25. $y = \sqrt{15x}$, $y = \sqrt{15x}$, $z = 0$, $z = \sqrt{15}\left(1 + \sqrt{x}\right)$.

10.26. $x^2 + y^2 = 50$, $y = \sqrt{5x}$, $y = 0$, $z = 0$, $z = 3x/11$.

10.27. $x + y = 8$, $y = \sqrt{4x}$, $z = 3y$, $z = 0$.

10.28. $x = 16\sqrt{2y}$, $x = \sqrt{2y}$, $z + y = 2$, $z = 0$.

10.29. $x = \sqrt{y}$, $x = 15y$, $z = 0$, $z = 15\left(1 + \sqrt{y}\right)$.

10.30. $x^2 + y^2 = 50$, $x = \sqrt{5y}$, $x = 0$, $z = 0$, $z = 6y/11$.

10.31. $x = 17\sqrt{2y}$, $x = 2\sqrt{2y}$, $z = 0$, $z + y = 1/2$.

Задача 11. Найти объем тела, заданного ограничивающими его поверхностями.

11.1. $x^2 + y^2 = 2y,$
$z = 5/4 - x^2, \ z = 0.$

11.2. $x^2 + y^2 = y, \ x^2 + y^2 = 4y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.3. $x^2 + y^2 = 8\sqrt{2}x,$
$z = x^2 + y^2 - 64,$
$z = 0 \ \left(z \geq 0 \right).$

11.4. $x^2 + y^2 + 4x = 0,$
$z = 8 - y^2, \ z = 0.$

11.5. $x^2 + y^2 = 6x, \ x^2 + y^2 = 9x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ \left(y \leq 0 \right)$

11.6. $x^2 + y^2 = 6\sqrt{2}y,$
$z = x^2 + y^2 - 36,$
$z = 0 \ \left(z \geq 0 \right).$

11.7. $x^2 + y^2 = 2y,$
$z = 9/4 - x^2, \ z = 0.$

11.8. $x^2 + y^2 = 2y, \ x^2 + y^2 = 5y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.9. $x^2 + y^2 + 2\sqrt{2}y = 0,$
$z = x^2 + y^2 - 4,$
$z = 0 \ \left(z \geq 0 \right).$

11.10. $x^2 + y^2 = 4x,$
$z = 10 - y^2, \ z = 0.$

11.11. $x^2 + y^2 = 7x, \ x^2 + y^2 = 9x,$
$z = \sqrt{x^2 + y^2}, \ z = 0,$
$y = 0 \ \left(y \leq 0 \right)$

11.12. $x^2 + y^2 = 8\sqrt{2}y,$
$z = x^2 + y^2 - 64,$
$z = 0 \ \left(z \geq 0 \right).$

11.13. $x^2 + y^2 = 2y,$
$z = 13/4 - x^2, \ z = 0.$

11.14. $x^2 + y^2 = 3y, \ x^2 + y^2 = 6y,$
$z = \sqrt{x^2 + y^2}, \ z = 0.$

11.15. $x^2 + y^2 = 6\sqrt{2}x,$
$z = x^2 + y^2 - 36,$
$z = 0 \ \left(z \geq 0 \right).$

11.16. $x^2 + y^2 = 2\sqrt{2}y,$
$z = x^2 + y^2 - 4,$
$z = 0 \ \left(z \geq 0 \right).$

11.17. $\begin{aligned} & x^2 + y^2 = 4x, \\ & z = 12 - y^2, \ z = 0. \end{aligned}$

11.18. $\begin{aligned} & x^2 + y^2 = 8x, \ x^2 + y^2 = 11x, \\ & z = \sqrt{x^2 + y^2}, \ z = 0, \\ & y = 0 \ (y \le 0) \end{aligned}$

11.19. $\begin{aligned} & x^2 + y^2 = 4\sqrt{2}x, \\ & z = x^2 + y^2 - 16, \\ & z = 0 \ (z \ge 0). \end{aligned}$

11.20. $\begin{aligned} & x^2 + y^2 = 4y, \\ & z = 4 - x^2, \ z = 0. \end{aligned}$

11.21. $\begin{aligned} & x^2 + y^2 = 4y, \ x^2 + y^2 = 7y, \\ & z = \sqrt{x^2 + y^2}, \ z = 0. \end{aligned}$

11.22. $\begin{aligned} & x^2 + y^2 = 4\sqrt{2}y, \\ & z = x^2 + y^2 - 16, \\ & z = 0 \ (z \ge 0). \end{aligned}$

11.23. $\begin{aligned} & x^2 + y^2 + 2x = 0, \\ & z = 17/4 - y^2, \ z = 0. \end{aligned}$

11.24. $\begin{aligned} & x^2 + y^2 = 9x, \ x^2 + y^2 = 12x, \\ & z = \sqrt{x^2 + y^2}, \ z = 0, \\ & y = 0 \ (y \ge 0) \end{aligned}$

11.25. $\begin{aligned} & x^2 + y^2 + 2\sqrt{2}x = 0, \\ & z = x^2 + y^2 - 4, \\ & z = 0 \ (z \ge 0). \end{aligned}$

11.26. $\begin{aligned} & x^2 + y^2 = 4y, \\ & z = 6 - x^2, \ z = 0. \end{aligned}$

11.27. $\begin{aligned} & x^2 + y^2 = 10x, \ x^2 + y^2 = 13x, \\ & z = \sqrt{x^2 + y^2}, \ z = 0, \\ & y = 0 \ (y \ge 0) \end{aligned}$

11.28. $\begin{aligned} & x^2 + y^2 = 2\sqrt{2}x, \\ & z = x^2 + y^2 - 4, \\ & z = 0 \ (z \ge 0). \end{aligned}$

11.29. $\begin{aligned} & x^2 + y^2 = 2x, \\ & z = 21/4 - y^2, \ z = 0. \end{aligned}$

11.30. $\begin{aligned} & x^2 + y^2 = 5y, \ x^2 + y^2 = 8y, \\ & z = \sqrt{x^2 + y^2}, \ z = 0. \end{aligned}$

11.31. $\begin{aligned} & x^2 + y^2 + 2x = 0, \\ & z = 25/4 - y^2, \ z = 0. \end{aligned}$

Задача 12. Найти объем тела, заданного ограничивающими его поверхностями.

$y = 5x^2 + 2, \ y = 7,$

12.1. $z = 3y^2 - 7x^2 - 2,$

$z = 3y^2 - 7x^2 - 5.$

$y = 5x^2 - 2, \ y = -4x^2 + 7,$

12.2. $z = 4 + 9x^2 + 5y^2,$

$z = -1 + 9x^2 + 5y^2.$

$x = -5y^2 + 2, \ x = -3,$

12.3. $z = 3x^2 + y^2 + 1,$

$z = 3x^2 + y^2 - 5.$

$x = 2y^2 - 3, \ x = -7y^2 + 6,$

12.4. $z = 1 + \sqrt{x^2 + 16y^2},$

$z = -3 + \sqrt{x^2 + 16y^2}.$

$y = -6x^2 + 8, \ y = 2,$

12.5. $z = x - x^2 - y^2 - 1,$

$z = x - x^2 - y^2 - 5.$

$y = 5x^2 - 1, \ y = -3x^2 + 1,$

12.6. $z = -2 + \sqrt{3x^2 + y^2},$

$z = -5 + \sqrt{3x^2 + y^2}.$

$x = 5y^2 - 9, \ x = -4,$

12.7. $z = x^2 + 4x - y^2 - 4,$

$z = x^2 + 4x - y^2 + 2.$

$y = 6x^2 - 1, \ y = 5,$

12.8. $z = 2x^2 + x - y^2,$

$z = 2x^2 + x - y^2 + 4.$

$x = 5y^2 - 1, \ x = -3y^2 + 1,$

12.9. $z = 2 - \sqrt{x^2 + 6y^2},$

$z = -1 - \sqrt{x^2 + 6y^2}.$

$x = -3y^2 + 7, \ x = 4,$

12.10. $z = 2 + \sqrt{6x^2 + y^2},$

$z = 3 + \sqrt{6x^2 + y^2}.$

$y = -5x^2 + 3, \ y = -2,$

12.11. $z = 2x^2 - 3y - 6y^2 - 1,$

$z = 2x^2 - 3y - 6y^2 + 2.$

$y = x^2 - 5, \ y = -x^2 + 3,$

12.12. $z = 4 + \sqrt{5x^2 + 8y^2},$

$z = 1 + \sqrt{5x^2 + 8y^2}.$

$x = 3y^2 - 5, \ x = -2,$

12.13. $z = 2 - \sqrt{x^2 + 16y^2},$

$z = 8 - \sqrt{x^2 + 16y^2}.$

$x = y^2 - 2, \ x = -4y^2 + 3,$

12.14. $z = \sqrt{16 - x^2 - y^2} + 2,$

$z = \sqrt{16 - x^2 - y^2} - 1.$

$y = 2x^2 - 1, \ y = 1,$

12.15. $z = x^2 - 5y^2 - 3,$

$z = x^2 - 5y^2 - 6.$

$y = x^2 - 2, \ y = -4x^2 + 3,$

12.16. $z = 2 + \sqrt{x^2 + y^2},$

$z = -1 + \sqrt{x^2 + y^2}.$

$$x = -4y^2 + 1, \quad x = -3,$$

12.17. $z = x^2 - 7y^2 - 1,$

$\quad\quad z = x^2 - 7y^2 + 2.$

$$x = 7y^2 - 6, \quad x = -2y^2 + 3,$$

12.18. $z = 3 - 12y^2 + 5x^2,$

$\quad\quad z = -2 - 12y^2 + 5x^2.$

$$y = 1 - 2x^2, \quad y = -1,$$

12.19. $z = x^2 + 2y + y^2 - 2,$

$\quad\quad z = x^2 + 2y + y^2 + 1.$

$$y = x^2 - 7, \quad y = -8x^2 + 2,$$

12.20. $z = 3 - 12y^2 + 5x^2,$

$\quad\quad z = -2 - 12y^2 + 5x^2.$

$$x = 2y^2 + 3, \quad x = 5,$$

12.21. $z = 1 + \sqrt{9x^2 + 4y^2},$

$\quad\quad z = 4 + \sqrt{9x^2 + 4y^2}.$

$$y = 3x^2 + 4, \quad y = 7,$$

12.22. $z = 5 - \sqrt{2x^2 + 3y^2},$

$\quad\quad z = 1 - \sqrt{2x^2 + 3y^2}.$

$$x = 5y^2 - 2, \quad x = -4y^2 + 7,$$

12.23. $z = 4 - \sqrt{2x^2 + 3y^2},$

$\quad\quad z = -1 - \sqrt{2x^2 + 3y^2}.$

$$x = -2y^2 + 5, \quad x = 3,$$

12.24. $z = 5 - \sqrt{x^2 + 25y^2},$

$\quad\quad z = 2 - \sqrt{x^2 + 25y^2}.$

$$y = -3x^2 + 5, \quad y = 2,$$

12.25. $z = 3 + \sqrt{5x^2 + y^2},$

$\quad\quad z = -1 + \sqrt{5x^2 + y^2}.$

$$y = 3x^2 - 5, \quad y = -6x^2 + 4,$$

12.26. $z = 2 + 10x^2 - y^2,$

$\quad\quad z = -2 + 10y^2 - y^2.$

$$x = 4y^2 + 2, \quad x = 6,$$

12.27. $z = x^2 + 4y^2 + y + 1,$

$\quad\quad z = x^2 + 4y^2 + y + 4.$

$$x = 3y^2 - 2, \quad x = -4y^2 + 5,$$

12.28. $z = 4 - 7x^2 - 9y^2,$

$\quad\quad z = 1 - 7x^2 - 9y^2.$

$$y = 2x^2 - 5, \quad y = -3,$$

12.29. $z = 2 + \sqrt{x^2 + 4y^2},$

$\quad\quad z = -1 + \sqrt{x^2 + 4y^2}.$

$$y = 2x^2 - 3, \quad y = -7x^2 + 6,$$

12.30. $z = 1 - 5x^2 - 6y^2,$

$\quad\quad z = -3 - 5x^2 - 6y^2.$

$$y = -2x^2 + 7, \quad y = 5,$$

12.31. $z = 1 - 2x^2 + 3y^2,$

$\quad\quad z = 4 - 2x^2 + 3y^2.$

Задача 13. Найти объем тела, заданного ограничивающими его поверхностями.

13.1. $z = \sqrt{9 - x^2 - y^2}$,
$9z/2 = x^2 + y^2$.

13.2. $z = 15\sqrt{x^2 + y^2}\big/2$,
$z = 17/2 - x^2 - y^2$.

13.3. $z = \sqrt{4 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)\big/255}$.

13.4. $z = \sqrt{64 - x^2 - y^2}$, $z = 1$,
$x^2 + y^2 = 60$
(внутри цилиндра).

13.5. $z = \sqrt{\dfrac{16}{9} - x^2 - y^2}$,
$2z = x^2 + y^2$.

13.6. $z = 3\sqrt{x^2 + y^2}$,
$z = 10 - x^2 - y^2$.

13.7. $z = \sqrt{25 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)\big/99}$.

13.8. $z = \sqrt{100 - x^2 - y^2}$, $z = 6$,
$x^2 + y^2 = 51$
(внутри цилиндра).

13.9. $z = 21\sqrt{x^2 + y^2}\big/2$,
$z = 23/2 - x^2 - y^2$.

13.10. $z = \sqrt{16 - x^2 - y^2}$,
$6z = x^2 + y^2$.

13.11. $z = \sqrt{9 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)\big/80}$.

13.12. $z = \sqrt{81 - x^2 - y^2}$, $z = 5$,
$x^2 + y^2 = 45$
(внутри цилиндра).

13.13. $z = \sqrt{1 - x^2 - y^2}$,
$3z/2 = x^2 + y^2$.

13.14. $z = 6\sqrt{x^2 + y^2}$,
$z = 16 - x^2 - y^2$.

13.15. $z = \sqrt{36 - x^2 - y^2}$,
$z = \sqrt{(x^2 + y^2)\big/63}$.

13.16. $z = \sqrt{64 - x^2 - y^2}$, $z = 4$,
$x^2 + y^2 = 39$
(внутри цилиндра).

13.17. $z = \sqrt{144 - x^2 - y^2},$
$18z = x^2 + y^2.$

13.18. $z = 3\sqrt{x^2 + y^2}\big/2,$
$z = 5/2 - x^2 - y^2.$

13.19. $z = \sqrt{9 - x^2 - y^2},$
$z = \sqrt{\left(x^2 + y^2\right)\big/35}.$

13.20. $z = \sqrt{49 - x^2 - y^2}, \quad z = 3,$
$x^2 + y^2 = 33$
(внутри цилиндра).

13.21. $z = \sqrt{36 - x^2 - y^2},$
$9z = x^2 + y^2.$

13.22. $z = 9\sqrt{x^2 + y^2},$
$z = 22 - x^2 - y^2.$

13.23. $z = \sqrt{16 - x^2 - y^2},$
$z = \sqrt{\left(x^2 + y^2\right)\big/15}.$

13.24. $z = \sqrt{36 - x^2 - y^2}, \quad z = 2,$
$x^2 + y^2 = 27$
(внутри цилиндра).

13.25. $z = \sqrt{4/9 - x^2 - y^2},$
$z = x^2 + y^2.$

13.26. $z = 12\sqrt{x^2 + y^2},$
$z = 28 - x^2 - y^2.$

13.27. $z = \sqrt{9 - x^2 - y^2},$
$z = \sqrt{\left(x^2 + y^2\right)\big/8}.$

13.28. $z = \sqrt{25 - x^2 - y^2}, \quad z = 1,$
$x^2 + y^2 = 21$
(внутри цилиндра).

13.29. $z = \sqrt{64 - x^2 - y^2},$
$12z = x^2 + y^2.$

13.30. $z = 9\sqrt{x^2 + y^2}\big/2,$
$z = 11/2 - x^2 - y^2.$

13.31. $z = \sqrt{36 - x^2 - y^2},$
$z = \sqrt{\left(x^2 + y^2\right)\big/3}.$

Задача 14. Найти объем тела, заданного ограничивающими его поверхностями.

14.1. $z = 2 - 12(x^2 + y^2),$
$z = 24x + 2.$

14.2. $z = 10\left[(x-1)^2 + y^2\right] + 1,$
$z = 21 - 20x.$

14.3. $z = 8(x^2 + y^2) + 3,$
$z = 16x + 3.$

14.4. $z = 2 - 20\left[(x+1)^2 + y^2\right],$
$z = -40 - 38x.$

14.5. $z = 4 - 14(x^2 + y^2),$
$z = 4 - 28x.$

14.6. $z = 28\left[(x+1)^2 + y^2\right] + 3,$
$z = 56x + 59.$

14.7. $z = 32(x^2 + y^2) + 3,$
$z = 3 - 64x.$

14.8. $z = 4 - 6\left[(x-1)^2 + y^2\right],$
$z = 12x - 8.$

14.9. $z = 2 - 4(x^2 + y^2),$
$z = 8x + 2.$

14.10. $z = 22\left[(x-1)^2 + y^2\right] + 3,$
$z = 47 - 44x.$

14.11. $z = 24(x^2 + y^2) + 1,$
$z = 48x + 1.$

14.12. $z = 2 - 18\left[(x+1)^2 + y^2\right],$
$z = -36x - 34.$

14.13. $z = -16(x^2 + y^2) - 1,$
$z = -32x - 1.$

14.14. $z = 30\left[(x+1)^2 + y^2\right] + 1,$
$z = 60x + 61.$

14.15. $z = 26(x^2 + y^2) - 2,$
$z = -52x - 2.$

14.16. $z = -2\left[(x-1)^2 + y^2\right] - 1,$
$z = 4x - 5.$

14.17. $z = -2(x^2 + y^2) - 1,$
$z = 4y - 1.$

14.18. $z = 26\left[(x-1)^2 + y^2\right] - 2,$
$z = 50 - 52x.$

14.19. $z = 30(x^2 + y^2) + 1,$
$z = 60y + 1.$

14.20. $z = -16\left[(x+1)^2 + y^2\right] - 1,$
$z = -32x - 33.$

14.21. $z = 2 - 18(x^2 + y^2),$
$z = 2 - 36y.$

14.22. $z = 24[(x+1)^2 + y^2] + 1,$
$z = 48x + 49.$

14.23. $z = 22(x^2 + y^2) + 3,$
$z = 3 - 44y.$

14.24. $z = 2 - 4[(x-1)^2 + y^2],$
$z = 8x - 6.$

14.25. $z = 4 - 6(x^2 + y^2),$
$z = 12y + 4.$

14.26. $z = 32[(x-1)^2 + y^2] + 3,$
$z = 67 - 64x.$

14.27. $z = 28(x^2 + y^2) + 3,$
$z = 56y + 3.$

14.28. $z = 4 - 14[(x+1)^2 + y^2],$
$z = -28x - 24.$

14.29. $z = 2 - 20(x^2 + y^2),$
$z = 2 - 40y.$

14.30. $z = 8[(x+1)^2 + y^2] + 3,$
$z = 16x + 19.$

14.31. $z = 10(x^2 + y^2) + 1,$
$z = 1 - 20y.$

Задача 15. Найти объем тела, заданного неравенствами.

15.1.
$$1 \le x^2 + y^2 + z^2 \le 49,$$
$$-\sqrt{\frac{x^2 + y^2}{35}} \le z \le \sqrt{\frac{x^2 + y^2}{3}},$$
$$-x \le y \le 0.$$

15.2.
$$1 \le x^2 + y^2 + z^2 \le 64,$$
$$\sqrt{\frac{x^2 + y^2}{15}} \le z \le \sqrt{\frac{x^2 + y^2}{3}},$$
$$-\sqrt{3}x \le y \le 0.$$

15.3.
$$4 \le x^2 + y^2 + z^2 \le 64,$$
$$z \le \sqrt{\frac{x^2 + y^2}{3}}, \quad -\frac{x}{\sqrt{3}} \le y \le 0.$$

15.4.
$$4 \le x^2 + y^2 + z^2 \le 36,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{63}}, \quad 0 \le y \le -\frac{x}{\sqrt{3}}.$$

15.5.
$$1 \le x^2 + y^2 + z^2 \le 36,$$
$$z \ge \sqrt{\frac{x^2 + y^2}{99}}, \quad -\sqrt{3}x \le y \le \sqrt{3}x.$$

15.6.
$$25 \le x^2 + y^2 + z^2 \le 100,$$
$$z \le -\sqrt{\frac{x^2 + y^2}{99}}, \quad \sqrt{3}x \le y \le -\sqrt{3}x.$$

15.7.
$$1 \le x^2 + y^2 + z^2 \le 49,$$
$$0 \le z \le \sqrt{\frac{x^2 + y^2}{24}},$$
$$y \le -\frac{x}{\sqrt{3}}, \quad y \le -\sqrt{3}x.$$

15.8.
$$25 \le x^2 + y^2 + z^2 \le 49,$$
$$-\sqrt{\frac{x^2 + y^2}{24}} \le z \le 0,$$
$$y \ge -\frac{x}{\sqrt{3}}, \quad y \ge -\sqrt{3}x.$$

15.9.
$$4 \le x^2 + y^2 + z^2 \le 64,$$
$$-\sqrt{\frac{x^2 + y^2}{35}} \le z \le \sqrt{\frac{x^2 + y^2}{3}},$$
$$x \le y \le 0.$$

15.10.
$$16 \le x^2 + y^2 + z^2 \le 100,$$
$$\sqrt{\frac{x^2 + y^2}{15}} \le z \le \sqrt{\frac{x^2 + y^2}{3}},$$
$$\sqrt{3}x \le y \le 0.$$

15.11.
$$16 \le x^2 + y^2 + z^2 \le 100,$$
$$z \le \sqrt{\frac{x^2 + y^2}{3}}, \quad -\sqrt{3}x \le y \le -\frac{x}{\sqrt{3}}.$$

15.12.
$$16 \le x^2 + y^2 + z^2 \le 64,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{63}},$$
$$-\frac{x}{\sqrt{3}} \le y \le -\sqrt{3}x.$$

15.13.
$$4 \le x^2 + y^2 + z^2 \le 49,$$
$$z \ge \sqrt{\frac{x^2 + y^2}{99}}, \quad y \le 0, \quad y \le \sqrt{3}x.$$

15.14.
$$36 \le x^2 + y^2 + z^2 \le 121,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{99}}, \quad y \ge 0, \quad y \ge \sqrt{3}x.$$

15.15.
$$4 \le x^2 + y^2 + z^2 \le 64,$$
$$0 \le z \le \sqrt{\frac{x^2 + y^2}{24}},$$
$$y \le \sqrt{3}x, \quad y \le \frac{x}{\sqrt{3}}.$$

15.16.
$$36 \le x^2 + y^2 + z^2 \le 144,$$
$$-\sqrt{\frac{x^2 + y^2}{24}} \le z \le 0,$$
$$y \ge \sqrt{3}x, \quad y \ge \frac{x}{\sqrt{3}}.$$

15.17.
$$9 \le x^2 + y^2 + z^2 \le 81,$$
$$-\sqrt{\frac{x^2 + y^2}{3}} \le z \le \sqrt{\frac{x^2 + y^2}{35}},$$
$$0 \le y \le -x.$$

15.18.
$$36 \le x^2 + y^2 + z^2 \le 144,$$
$$-\sqrt{\frac{x^2 + y^2}{3}} \le z \le \sqrt{\frac{x^2 + y^2}{35}},$$
$$0 \le y \le -\sqrt{3}x.$$

15.19.
$$36 \le x^2 + y^2 + z^2 \le 144,$$
$$z \le \sqrt{\frac{x^2 + y^2}{3}}, \quad \sqrt{3}x \le y \le \frac{x}{\sqrt{3}}.$$

15.20.
$$36 \le x^2 + y^2 + z^2 \le 100,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{63}},$$
$$\frac{x}{\sqrt{3}} \le y \le \sqrt{3}x.$$

15.21.
$$9 \le x^2 + y^2 + z^2 \le 64,$$
$$z \ge \sqrt{\frac{x^2 + y^2}{99}},$$
$$y \le \frac{x}{\sqrt{3}}, \quad y \le -\frac{x}{\sqrt{3}}.$$

15.22.
$$49 \le x^2 + y^2 + z^2 \le 144,$$
$$z \le -\sqrt{\frac{x^2 + y^2}{99}},$$
$$y \ge \frac{x}{\sqrt{3}}, \quad y \ge -\frac{x}{\sqrt{3}}.$$

15.23.
$$9 \le x^2 + y^2 + z^2 \le 81,$$
$$0 \le z \le \sqrt{\frac{x^2 + y^2}{24}},$$
$$y \le 0, \quad y \le \frac{x}{\sqrt{3}}.$$

15.24.
$$49 \le x^2 + y^2 + z^2 \le 81,$$
$$-\sqrt{\frac{x^2 + y^2}{24}} \le z \le 0,$$
$$y \ge 0, \quad y \ge \frac{x}{\sqrt{3}}.$$

15.25.
$$16 \le x^2 + y^2 + z^2 \le 100,$$
$$-\sqrt{\frac{x^2 + y^2}{3}} \le z \le \sqrt{\frac{x^2 + y^2}{35}},$$
$$0 \le y \le x.$$

15.26.
$$64 \le x^2 + y^2 + z^2 \le 196,$$
$$-\sqrt{\frac{x^2 + y^2}{3}} \le z \le -\sqrt{\frac{x^2 + y^2}{15}},$$
$$0 \le y \le \sqrt{3}x.$$

15.27.
$$64 \le x^2 + y^2 + z^2 \le 196,$$
$$z \le \sqrt{\frac{x^2 + y^2}{3}}, \quad \frac{x}{\sqrt{3}} \le y \le 0.$$

15.28.
$$64 \le x^2 + y^2 + z^2 \le 144,$$
$$z \ge -\sqrt{\frac{x^2 + y^2}{63}}, \quad 0 \le y \le \frac{x}{\sqrt{3}}.$$

15.29.
$$16 \le x^2 + y^2 + z^2 \le 81,$$
$$z \ge \sqrt{\frac{x^2 + y^2}{99}},$$
$$y \le 0, \quad y \le -\sqrt{3}x.$$

15.30.
$$64 \le x^2 + y^2 + z^2 \le 169,$$
$$z \le -\sqrt{\frac{x^2 + y^2}{99}},$$
$$y \ge 0, \quad y \ge -\sqrt{3}x.$$

15.31.
$$16 \le x^2 + y^2 + z^2 \le 100,$$
$$0 \le z \le \sqrt{\frac{x^2 + y^2}{24}},$$
$$y \le 0, \quad y \le \frac{x}{\sqrt{3}}.$$

Задача 16. Тело V задано ограничивающими его поверхностями, μ - плотность.

Найти массу тела.

$$64\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 4,$$
16.1. $y = 0, \quad z = 0 \quad \left(y \geq 0, \ z \geq 0\right),$

$\mu = 5\left(x^2 + y^2\right)\big/4.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 1,$$
16.2. $\left(x^2 + y^2 \leq 1\right), \quad x = 0 \quad \left(x \geq 0\right);$

$\mu = 4|z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 2z,$$
16.3. $x = 0, \ y = 0, \ z = 0 \quad \left(x \geq 0, \ y \geq 0\right);$

$\mu = 10x.$

$$x^2 + y^2 = \frac{16}{49}z^2, \quad x^2 + y^2 = \frac{4}{7}z,$$
16.4. $x = 0, \ y = 0, \quad \left(x \geq 0, \ y \geq 0\right);$

$\mu = 80\,yz.$

$$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = 4z^2,$$
16.5. $x = 0, \ y = 0, \quad \left(x \geq 0, \ y \geq 0, \ z \geq 0\right);$

$\mu = 20z.$

$$36\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 1,$$
16.6. $x = 0, \quad z = 0 \quad \left(x \geq 0, \ z \geq 0\right),$

$\mu = \dfrac{5}{6}\left(x^2 + y^2\right).$

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 4,$$
16.7. $\left(x^2 + y^2 \leq 4\right);$

$\mu = 2|z|.$

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 8z,$$

16.8. $x = 0, \ y = 0, \ z = 0 \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 5x.$

$$x^2 + y^2 = \frac{4}{25} z^2, \quad x^2 + y^2 = \frac{2}{5} z,$$

16.9. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 28xz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = z^2,$$

16.10. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0, \ z \geq 0 \right);$

$\mu = 6z.$

$$25\left(x^2 + y^2 \right) = z^2, \quad x^2 + y^2 = 4,$$

16.11.
$x = 0, \quad y = 0, \ z = 0$

$\left(x \geq 0, \ y \geq 0, \ z \geq 0 \right),$

$\mu = 2\left(x^2 + y^2 \right).$

$$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$$

16.12. $\left(x^2 + y^2 \leq 4 \right), \quad y = 0 \ \left(y \geq 0 \right);$

$\mu = |z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 6z,$$

16.13. $x = 0, \ y = 0, \ z = 0 \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 90y.$

$$x^2 + y^2 = \frac{1}{25} z^2, \quad x^2 + y^2 = \frac{1}{5} z,$$

16.14. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 14yz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 9z^2,$$

16.15. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0);$

$\mu = 10z.$

$$9(x^2 + y^2) = z^2, \quad x^2 + y^2 = 4,$$

$$x = 0, \quad y = 0, \quad z = 0$$

16.16. $(x \geq 0, \quad y \geq 0, \quad z \geq 0),$

$\mu = 5(x^2 + y^2)/3.$

$$x^2 + y^2 + z^2 = 4,$$

16.17. $x^2 + y^2 = 1, \quad (x^2 + y^2 \leq 1);$

$\mu = |z|.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = z,$$

$$x = 0, \quad y = 0, \quad z = 0,$$

16.18. $(x \geq 0, \quad y \geq 0);$

$\mu = 10y.$

$$x^2 + y^2 = \frac{1}{49}z^2, \quad x^2 + y^2 = \frac{1}{7}z,$$

16.19. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0);$

$\mu = 10xz.$

$$x^2 + y^2 + z^2 = 4, \quad x^2 + y^2 = 4z^2,$$

16.20. $x = 0, \quad y = 0, \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0);$

$\mu = 10z.$

$$16(x^2 + y^2) = z^2, \quad x^2 + y^2 = 1,$$

16.21. $x = 0, \quad y = 0, \quad z = 0 \quad (x \geq 0, \quad y \geq 0, \quad z \geq 0),$

$\mu = 5(x^2 + y^2).$

$$x^2 + y^2 + z^2 = 16,$$

16.22. $x^2 + y^2 = 4 \quad \left(x^2 + y^2 \leq 4 \right);$

$\mu = |z|.$

$$x^2 + y^2 = 4, \quad x^2 + y^2 = 4z,$$

16.23. $x = 0, \ y = 0, \ z = 0 \quad \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 5y.$

$$x^2 + y^2 = z^2, \quad x^2 + y^2 = z,$$

16.24. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0 \right);$

$\mu = 35yz.$

$$x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 = z^2,$$

16.25. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0, \ z \geq 0 \right);$

$\mu = 32z.$

$$x^2 + y^2 = z^2, \quad x^2 + y^2 = 4,$$

$$x = 0, \quad y = 0, \ z = 0$$

16.26. $\left(x \geq 0, \ y \geq 0, \ z \geq 0 \right),$

$\mu = 5\left(x^2 + y^2 \right) / 2.$

$$x^2 + y^2 + z^2 = 9, \quad x^2 + y^2 = 4,$$

16.27. $\left(x^2 + y^2 \leq 4 \right), \quad z = 0 \quad \left(z \geq 0 \right);$

$\mu = 2z.$

$$x^2 + y^2 = 1, \quad x^2 + y^2 = 3z,$$

$$x = 0, \ y = 0, \ z = 0$$

16.28. $\left(x \geq 0, \ y \geq 0 \right);$

$\mu = 15x.$

$$x^2 + y^2 = \frac{4}{49}z^2, \quad x^2 + y^2 = \frac{2}{7}z,$$

16.29. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0\right);$

$\mu = 20xz.$

$$x^2 + y^2 + z^2 = 16, \quad x^2 + y^2 = 9z^2,$$

16.30. $x = 0, \ y = 0, \ \left(x \geq 0, \ y \geq 0, \ z \geq 0\right);$

$\mu = 5z.$

$$4\left(x^2 + y^2\right) = z^2, \quad x^2 + y^2 = 1,$$

16.31. $y = 0, \ z = 0 \ \left(y \geq 0, \ z \geq 0\right),$

$\mu = 10\left(x^2 + y^2\right).$