TOMSK POLYTECHNIC UNIVERSITY

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CONTROL THEORY

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Main approaches of investigations, analysis and design of linear, nonlinear and pulse control system are considered. The tasks for course work and necessary data and examples of such problems solving are applied. Project includes mathematical description of control objects and systems; linear system analysis, including stability analysis, step response plot and frequency characteristics plot; linear systems synthesis, including controllers design; nonlinear control systems analysis.

Study aid manual is intended for training students majoring in the specialties 220400 «Control in technical systems».

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CONTENT

INTRODUCTION

Active development of the automatic control theory has begun with electromachine systems and radio automatics systems. Later it has appeared that methods of the automatic control theory allow to explain work of the various physical nature objects: in the mechanic, power, radio and the electrical engineer, that is everywhere where is feedback.

In the book the sections of the the automatic control theory, necessary for term paper performance are considered. Questions of the mathematical description of linear, nonlinear and pulse systems; algebraic and frequency criteria for an estimation of stability of systems of automatic control; indicators of quality of their process of regulation are considered. The concrete numerical examples facilitating development of a material are resulted.

The primary goals of a term paper are:

- Drawing up on a function chart circuit diagram.
- Drawing up of mathematical model in the form of the block diagram.
- System research on stability.
- Construction of system transient process for regulation quality estimation.
- An estimation of regulation process accuracy.

For term paper performance it is necessary to choose a circuit diagram of system and numerical values of parameters of its elements (the Appendix 1). Also it is possible to use additional information from books [1-10] in order to carry out task.

Linear continuous ACS.

- To give the short description of automatic control system (ACS).
- To describe a principle of ACS regulation.
- Using linear models of ACS elements (the Appendix 1) to make on the base of system circuit diagram functional and structural schemes.
- To get openloop transfer function of system.
- To find transfer functions of the closed-loop system on setting influence and to the desturbance factor.
- To write down differential equation of ACS.
- To check upACS on stability on the of roots of the characteristic equation of system.To check up ACS on stability, using criterion
- of Mikhailov stability.To check up ACS on stability, using criterion of Nyquist stability.
- To define margins of system stability on amplitude and a phase.

- To define by Gurvits stability criterion critical gain of open-loop system.
- Under zero conditions, to construct the transitive characteristic of system and to define its quality indicators.
- To define the full established error of system.

Nonlinear ACS.

- To accept that the amplyfing unit in system is a nonlinear element and to make the unit diagram of nonlinear ACS.
- To reduse the block diagram of nonlinear ACS to typical and to get transfer function of a linear part of system.
- To receive the differential equation of harmonoiusly linearized nonlinear system.
- To estimate stability of harmoniously linearized nonlinear system by Goldfarb method.
- Using Popova V. M. absolute stability criterion to investigate stability of system balance position in general.

Linear pulse ACS.

- To generate the scheme of pulse system.
- To get transfer function of a continuous part of pulse system
- To define, using the Kotelnikov theorem, the period of quantization.
- To find open-loop and closed-loop transfer functions of system.
- To define stability of system on the base of roots of the characteristic equation.
- To define stability of system, using Mikhailov stability criterion analog.
- Under conditions zero, to construct the discrete transitive characteristic of system and to define its quality indicators.
- To define a regulation error on setting influence.

1 LINEAR AUTOMATIC CONTROL SYSTEM

1.1 Automatic Control System Functional Diagram Design According to its Circuit Schematic

Any automatic control system (ACS) functional diagram includes ***plant*** with controllable output value ***x(t)*** and disturbance ***f***, control unit (***CU***), that provides output value stabilization with prescribed accuracy ***x*** that is $x(t) = x_0 = const$; setting device (***SD***), which provides required x_0 value; feedback; comparing summarizing unit (***CSU***) (fig. 1.1).

CU consists of amplifying element, execution unit and subsequent or parallel correction.

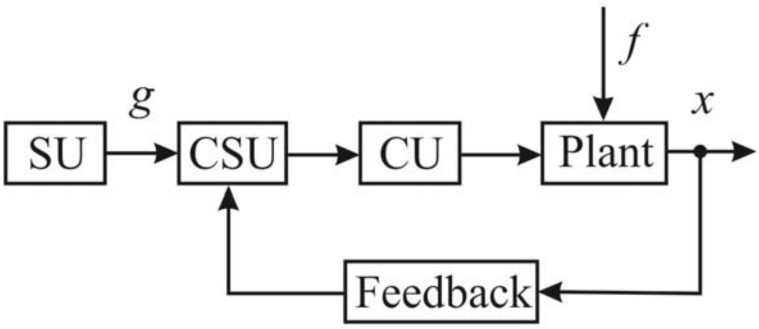


Fig. 1.1. ACS functional diagram

Besides, the control system could be realized additional disturbance *f* control or reference signal *g* control, or simultaneously disturbance and reference signal control (combined control).

CSU could be implemented on operational, magnetic or rotating amplifier, or on measurement device.

Various sensors which transform output controlled value x_0 of the plant into electrical signal present primary feedback.

Initial ACS circuit schematic divided into separate devices and nodes with taking into account the functions performed. Also ***SD*** and ***plant*** are identified in circuit. In the following systems ***plant*** is a DC motor with the reduction gear, and controlled value is rotation angle. It is necessary to remember that in the control system functional diagram, in the forward path of reference signal *g* passing the first place takes ***SD***, and ***plant*** takes the last. (Fig. 1.1).

Example 1.1. Make the functional diagram according to the circuit schematic of DC motor rotating frequency ω control system, represented in fig. 1.2.

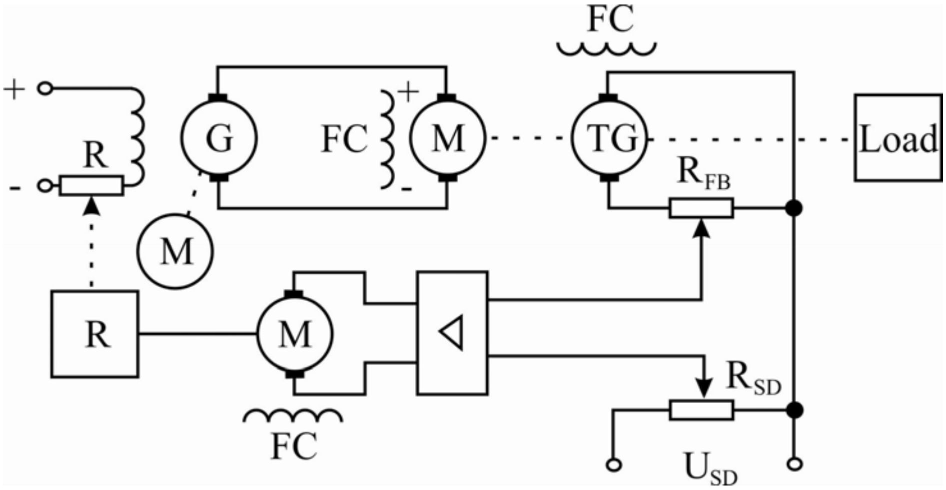


Fig. 1.2. Circuit schematic of DC motor rotating frequency control system

Setting device for this system is potentiometer ***R***. It is placed on the first place in the functional scheme. (fig. 1.2). According to the ACS tittle the ***plant*** is a ***Direct Current Motor (DCM)***, and its controlled value is a rotating frequency ω. Therefore ***DC motor*** $\boldsymbol{U_{in}}$ is placed on the last place in the forward path. $\boldsymbol{U_{in}}$ voltage is compared with $\boldsymbol{U_{FB}}$ voltage and forwards the signal by turns goes through electronic amplifier ***EA***, servomotor ***SM***, reducer ***RD***, direct current generator ***DCG*** and comes on ***DCM.*** Tacho-generator ***TG*** is a sensor, which transforms frequency ω into voltage $\boldsymbol{U_{FB}}$, measured on potentiometer $\boldsymbol{R_{FB}}$. Disturbance ***f*** in the given system is resistance (load) moment $\boldsymbol{M_L}$ (fig. 1.3).

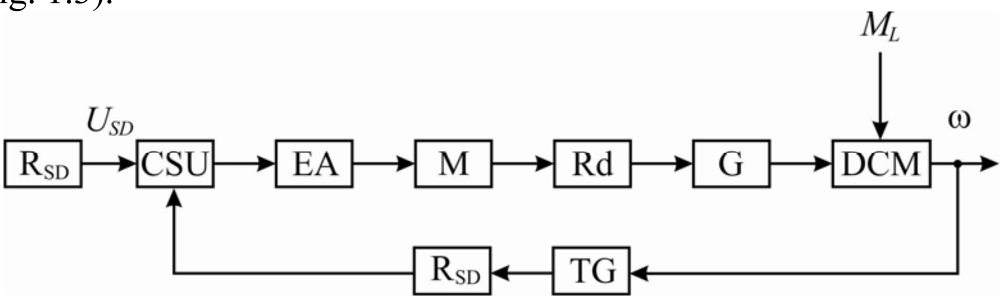


Fig. 1.3. System

Automated control or stabilization system task provides the required signal x_0 in the plant output. Deviation system control principle is in reference signal changing which acts on plant depending on difference between the set value and real output value.

Let's consider deviation control principle on the DCM rotation frequency system functional scheme as the following (example 1.1).

If the load on the DCM shaft increases, the disturbance M_L increases consequently too. This leads to ω_0 and the decreased $\boldsymbol{U_{FB}}$. Therefore, positive difference $U_{in} - U_{FB} = +\Delta U$ appears at the ***EA*** input that in its turn leads to the signal value magnitude, fed on servomotor, increasing, that means current in the DC motor circuit coil also increases. Rotating frequency will increase proportionally from $+\Delta U$ to ω_0.

Thus, any deviation of the output controlled value $x(t)$ from the required value x_0 leads to the error: $x(t) - x_0 = \pm\Delta x$.

This error $\pm\Delta x$ is reduced to zero with the given accuracy by the system during control process.

1.2 Automated Control System Unit Diagram Design

For the block diagram construction one should make the transfer functions of the control system devices (appendix 1) and equipment on the base of their differential equations. Herewith, differential equation disturbance *f* component (Mc, I etc.) needs to be taken into account only for the **plant**. That's why the **plant** will have two transfer functions: reference signal $W^{g}_{Plant}(s)$ and disturbance $W^{f}_{Plant}(s)$. ***CSU*** also have some kinds of transfer functions and their quantity determined by the quantity of inputs.

For the definition of transfer function expression according to the specific influence superposition principle is used.

Transfer function – relation between output and input signal in the Laplace transform, with zero initial conditions.

Example 1.2. Obtain transfer function for the direct current generator $W_{DCG}(s)$.

Solution:

Direct current generator differential equation (look appendix 1) has a view:

$$(T_G \cdot p + 1) \cdot \Delta U_G(t) = K_{G1} \cdot \Delta U_{FC}(t). \tag{1.1}$$

Applying Laplace transform to the equation (2.1) get

$$(T_G \cdot s + 1) \cdot \Delta U_G(s) = K_{G1} \cdot \Delta U_{FC}(s)$$

Then, according to the transfer function definition write

$$W_{DCG}(s) = \frac{\Delta U_G(s)}{\Delta U_{FC}(s)} = \frac{K_{G1}}{T_G \cdot s + 1}.$$

Example 1.3. Obtain plant transfer functions of automated control system represented in fig. 1.3.

Solution

Lets write direct current generator differential equation:

$$(T_E \cdot T_{EM} \cdot p^2 + T_{EM} \cdot p + 1)\Delta\omega(t) = K_{M1} \cdot \Delta U_{AV}(t) - K_{M2} \cdot (T_E \cdot p + 1) \cdot \Delta M_L. \tag{1.2}$$

Using superposition principle obtain direct currenct generator voltage anchor chain transfer function – $W_{DCG}^{U_{AV}}(s)$. For this, let's equate $M_L = 0$. Then equation (1.2) takes the view:

$$(T_E T_{EM} p^2 + T_{EM} \cdot p + 1)\Delta\omega(t) = K_{M1} \cdot \Delta U_{AV}(t)$$

Let's get direct current generator anchor chain transfer function:

$$W_{DCG}^{U_{AV}}(s) = \frac{\Delta\omega(s)}{\Delta U_{AV}(s)} = \frac{K_{M1}}{(T_E T_{EM} s^2 + T_{EM} \cdot s + 1)}.$$

Similarly get direct current generator resisting moment transfer function $W_{DCG}^{M_L}$(s), for this reason let's equate $\Delta U_{AV}(t) = 0$.

$$W_{DCG}^{M_L}(s) = \frac{\Delta\omega(s)}{\Delta M_L(s)} = \frac{-K_{M2}(T_E \cdot s + 1) \cdot}{(T_E \cdot T_M \cdot s^2 + T_M \cdot s + 1)}.$$

Let's define the concept of the unit diagram.

Unit diagram – a graphical representation of the device differential equation, when the transfer function expression is written inside the rectangle, input signal and output signal are represented by arrows.

ACS unit diagram composed according to it's functional diagram taking into account obtained transfer functions of devices and equipment included in this diagram. Unit diagram represented on fig. 1.4 corresponds to the functional diagram represented on fig. 1.1.

Example 1.4. Make unit diagram according to the functional scheme of DCG rotation frequency ADS, depicted on fig. 1.3.

Solution.

Let's get ADS devices and equipment transfer functions:

- Tachogenerator **TG** $W_{TG}(s) = \dfrac{\Delta\omega(s)}{\Delta U_{TG}(s)} = K_{TG}$;
- Resistence $\mathbf{R_{FB}}$ – $W(s) = \dfrac{\Delta U_{FB}(s)}{\Delta U_{TG}(s)} = K_{TG}$;
- Electronic amplifier **EA** – $W_{EA}(s) = \dfrac{\Delta U_{EA}(s)}{\Delta U} = K_{EA}$;
- Servomotor **SM** – $W_{SM}(s) = \dfrac{U_{SM}(s)}{U_{EA}(s)} = \dfrac{K_{SM}}{(T_{SM} s + 1)}$;
- Reducer **RD** – $W_{RD}(s) = K_{RD}$.

Unit diagram for this scheme is represented on fig. 1.4.

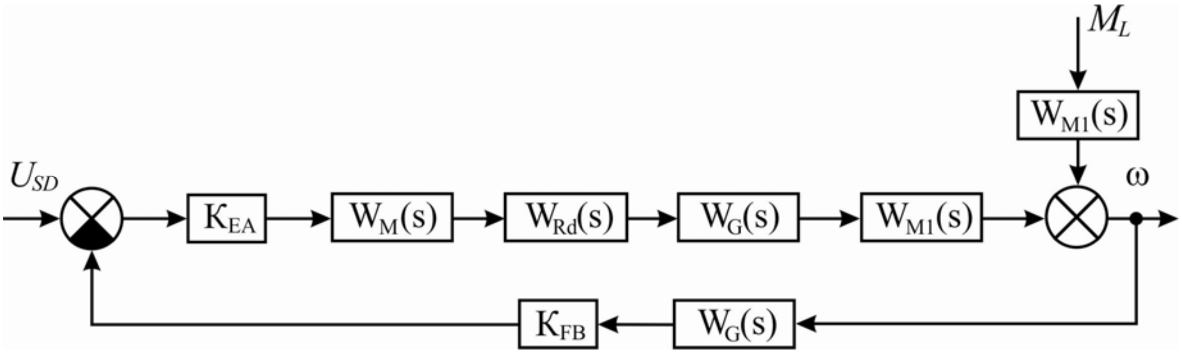


Fig. 1.4. Rotation frequency ADS unit diagram

1.3 ACS Transfer Functions

Using structural transformation rules, bring obtained ADS unit diagram to the form (fig. 1.5).

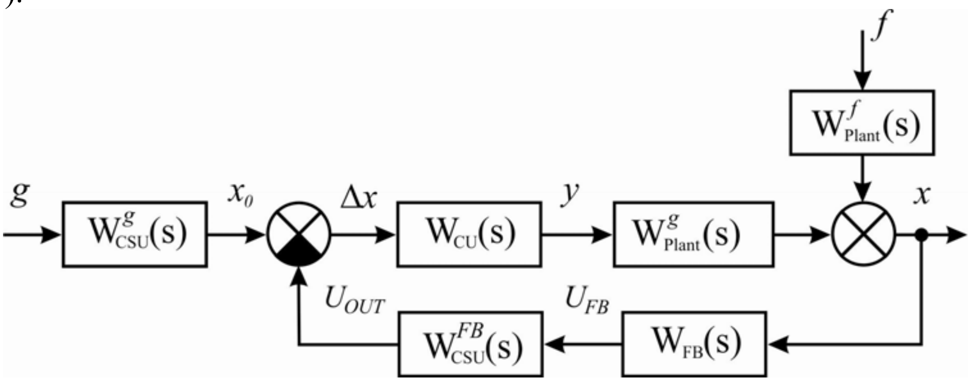


Fig. 1.5. ADS Unit diagram

Let's consider some ACS transfer functions formation:

- open-loop system transfer function $W_{OLS}(s)$;
- reference signal closed-loop system transfer function $W^g_{CLS}(s)$;
- disturbance closed-loop system transfer function $W^f_{CLS}(s)$;
- control error closed-loop system transfer function $W^\varepsilon_{CLS}(s)$.

For the formation of the open-loop system $W_{ST}(s)$ transfer function construct open-loop ACS unit diagram (see fig. 1.6.):

- all the impacts and blocks, which are not the parts of the main control loop not taking into account;
- the primary feedback is broken, and it's circuit is considered as an extension of the forward path of reference signal g passing (see fig. 1.6.)

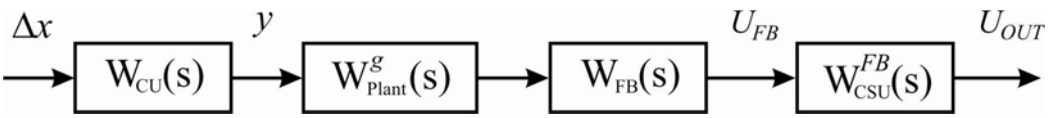


Fig. 1.6. Open-loop ACS unit diagram

Then we can write an equation for the open-loop system transfer function

$$W_{OLS}(s) = W_{CU}(s) \cdot W_{EA}(s) \cdot W_{FB}(s) \cdot W_{CSU}^{FB}(s). \quad \mathbf{(1.3)}$$

Let's use superposition principle for any influence closed-loop ACS transfer function obtaining. Unit diagram for reference-signal control closed-loop system obtaining is represented on fig. 1.7.

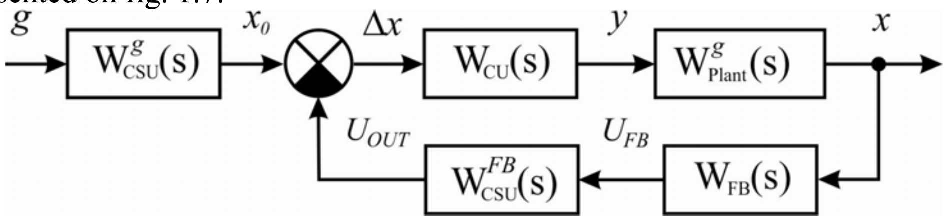


Fig. 1.7. Reference signal control ACS unit diagram

System transfer function has form

$$W_{CLS}^{g}(s) = \frac{W_{CU}(s) \cdot W_{EA}(s) \cdot W_{CSU}^{g}(s)}{1 + W_{CU}(s) \cdot W_{EA}(s) W_{FB}(s) \cdot W_{CSU}^{FB}(s)}. \quad (1.4)$$

Analyzing equation obtaining (1.4), one can note, that transfer function numerator is a transferfunction $W_{ST}^{g}(s)$ is a part of system between the system input and output point. Therefore expression (1.4) could be represented in the form:

$$W_{CLS}^{g}(s) = \frac{W_{ST}^{g}(s)}{1 + W_{OLS}(s)}. \quad (1.5)$$

Unit diagram for closed-loop system transfer function in the disturbance is represented in fig. 1.8.

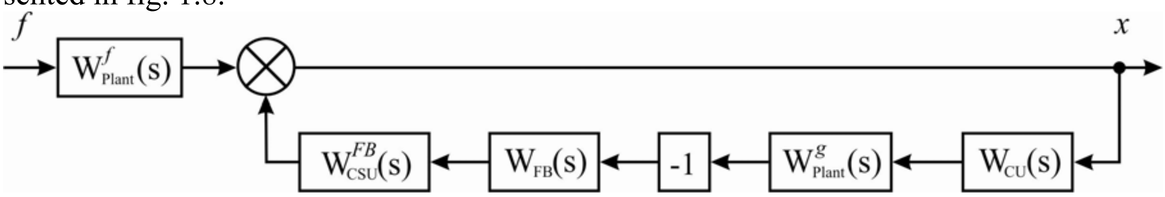


Fig. 1.8. Unit diagram of disturbance ACS

Then transfer function expression has form:

$$W_{CLS}^{f}(s) = \frac{W_{Plant}^{f}(s)}{1 - W_{CU}(s) \cdot W_{EA}^{y}(s) \cdot (-1) \cdot W_{FB}(s) \cdot W_{CSU}^{FB}(s)}.$$

Or

$$W_{3C}^{g}(s) = \frac{W_{\Pi P}^{f}(s)}{1 + W_{PC}(s)}, \quad (1.6)$$

where $W_{ST}^{f}(s)$ – transfer function of the disturbance signal straight passing.

Unit diagram for the closed-loop system transfer function $W_{CLS}^{\varepsilon}(s)$ for the control error is represented in fig. 1.9.

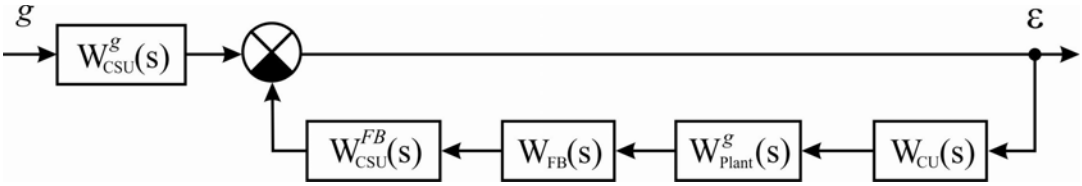


Fig. 1.9. ACS unit diagram for the control error

$$W^{\varepsilon}_{CLS}(s) = \frac{W^{g}_{CSU}(s)}{1 + W_{CU}(s) \cdot W_{EA}(s) W_{FB}(s) \cdot W^{FB}_{CSU}(s)} = \frac{W^{\varepsilon}_{ST}(s)}{1 + W_{OLS}(s)}, \quad (1.7)$$

Analyzing equations (1.5)-(1.7), could be made the conclusion that closed-loop transfer function for any influence z equals to

$$W_{CLS}(s) = \frac{W^{F}_{ST}(s)}{1 + W_{OLS}(s)}, \quad (1.8)$$

Where $W^{Z}_{ST}(s)$ is transfer function between the error signal f input and output of system.

Obtaining system transfer functions expressions, it is necessary to reduce them to a simple fraction.

Example 1.5. Get all the direct current motor rotation frequency ACS transfer functions, make unit diagram (fig. 1.3)

Solution

Let's make open-loop ACS unit diagram, breaking feedbacks, loping off reference signal U_{in} and disturbance M_L

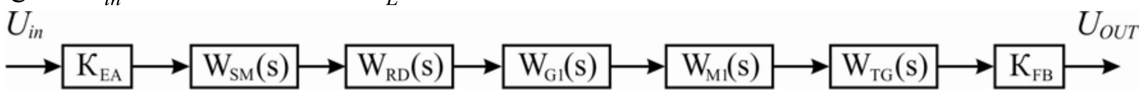


Fig. 1.10. Open-loop ACS unit diagram

$$W_{OLS}(s) = \frac{K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1} \cdot K_{TG} \cdot K_{FB}}{(T_{SM}s+1)\cdot(T_{G}s+1)\cdot(T_{E}T_{EM}s^{2}+T_{EM}s+1)}.$$

Let's use superposition principle, equal $M_L = 0$ and make closed-loop ACS unit diagram for reference signal U_{in}.

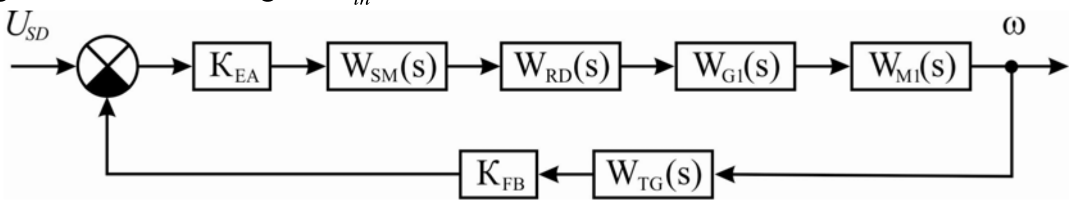


Fig. 1.11. ACS unit diagram for reference signal

Let's get closed-loop ACS transfer function for reference signal U_{in}:

$$W_{CLS}^{U_{in}}(s)=\frac{W_{ST}^{U_{in}}(s)}{1+W_{OLS}(s)}=\frac{\dfrac{K_{EA}K_{SM}K_{RD}K_{G1}K_{M1}}{(T_{SM}s+1)\cdot(T_Gs+1)\cdot(T_ET_{EM}s^2+T_{EM}s+1)}}{1+\dfrac{K_{EA}K_{EA}K_{RD}K_{G1}K_{M1}K_{TG}K_{FB}}{(T_{SM}s+1)\cdot(T_Gs+1)\cdot(T_ET_{EM}s^2+T_{EM}s+1)}}=$$

$$=\frac{K_{EA}K_{SM}K_{RD}K_{G1}K_{M1}}{(T_{SM}s+1)\cdot(T_Gs+1)\cdot(T_ET_{EM}s^2+T_{EM}s+1)+K_{EA}K_{SM}K_{RD}K_{G1}K_{M1}K_{TG}K_{FB}}.$$

Let’s equal $U_{in}=0$ and make closed-loop ACS unit diagram for disturbance M_L

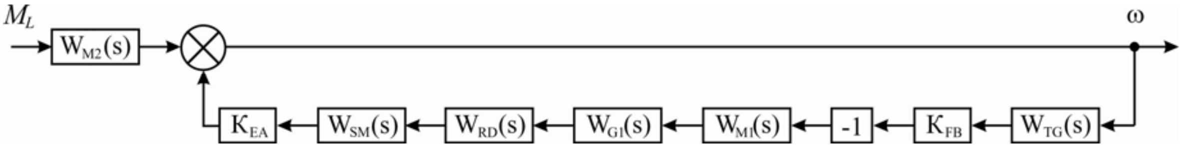


Fig. 1.12. ACS unit diagram for disturbance *f*

Now represent closed-loop ACS transfer function for disturbance as:

$$W_{CLS}^{M_L}(s)=\frac{W_{ST}^{M_L}(s)}{1+W_{OLS}(s)}=\frac{\dfrac{-K_{M2}}{(T_ET_{EM}s^2+T_{EM}s+1)}}{1+\dfrac{K_{EA}K_{SM}K_{RD}K_{G1}K_{M1}K_{TG}K_{FB}}{(T_{SM}s+1)\cdot(T_Gs+1)\cdot(T_ET_{EM}s^2+T_{EM}s+1)}}=$$

$$=\frac{-K_{M2}(T_{SM}s+1)\cdot(T_Gs+1)\cdot}{(T_{SM}s+1)\cdot(T_Gs+1)\cdot(T_ET_{EM}s^2+T_{EM}s+1)+K_{EA}K_{SM}K_{RD}K_{G1}K_{M1}K_{TG}K_{FB}}.$$

1.4 Differential Equation of ACS

Having obtained the closed-loop system transfer function for the reference signal $W_{CLS}^{g}(s)$ and disturbance $W_{CLS}^{f}(s)$, ACS unit diagram depicted in fig. 1.5 can be represented in form (fig. 1.13):

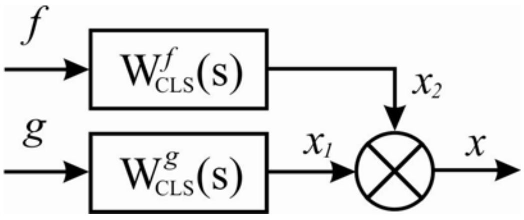


Fig. 1.13. ACS unit diagram

Let’s write the ACS output signal equation in image S

$$X(s)=X_1(s)+X_2(s)=W_{CLS}^{g}(s)\cdot G(s)+W_{CLS}^{f}(s)\cdot F(s),\qquad(1.9)$$

where $G(s),F(s)$ are the images of reference signal $g(t)$ and disturbance $f(t)$.

Let’s introduce the notation $W_{CLS}^{g}(s)=\dfrac{B(s)}{A(s)}$; $W_{CLS}^{f}(s)=\dfrac{C(s)}{A(s)}$ and write (1.9):

$$A(s)\cdot X(s)=B(s)\cdot G(s)+C(s)\cdot F(s),\qquad(1.10)$$

where $A(s), B(s), C(s)$ is polynomial of image S:

$$A(s) = (a_O \cdot s^n + a_1 \cdot s^{n-1} + a_2 \cdot s^{n-2} + \cdots + a_n);$$
$$B(s) = (b_0 \cdot s^m + b_1 \cdot s^{m-1} + \cdots + b_m);$$
$$C(s) = (c_0 \cdot s^l + c_1 \cdot s^{l-1} + \cdots + c_l).$$

Then (1.10) has a form:

$$(a_0 \cdot s^n + a_1 \cdot s^{n-1} + a_2 \cdot s^{n-2} + \cdots + a_n) \cdot X(s) = (b_0 \cdot s^m + b_1 \cdot s^{m-1} + \cdots + b_m) \times$$
$$\times G(s) + (c_0 \cdot s^l + c_1 \cdot s^{l-1} + \cdots + c_l) \cdot F(s).$$

If the transfer function denominator $A(s)$ equals to zero, we obtain the characteristic equation:

$$A(s) = a_0 \cdot s^n + a_1 \cdot s^{n-1} + a_2 \cdot s^{n-2} + \cdots + a_{n-1} \cdot s + a_n = 0. \qquad (1.11)$$

Solving this equation, characteristic equation roots $s_1, s_2, \cdots s_{n-1}, s_n$ are defined. Switching from signal images to their originals and replacing $s \to p \to \frac{d}{dt}$, we get ACS differential equation:

$$a_0 \frac{d^n X(t)}{dt^n} + a_1 \frac{d^{n-1} X(t)}{dt^{n-1}} + a_2 \frac{d^{n-2} X(t)}{dt^{n-2}} s^{n-2} + \cdots + a_n X(t) =$$
$$= b_0 \frac{d^m g(t)}{dt^m} + b_1 \frac{d^{m-1} g(t)}{dt^{m-1}} + \cdots + b_m + c_0 \frac{d^l f(t)}{dt^l} + c_1 \frac{d^{l-1} f(t)}{dt^{l-1}} + \cdots + c_l f(t). \qquad (1.12)$$

Example 1.6. Get DCM rotation frequency ACS differential equation, its unit diagram is represented on fig. 1.5.

Solution.

Let's write the output signal equation ω in image s, using the following equation (1.9).

$$\omega(s) = W_{CLS}^{U_{in}}(s) \cdot U_{in}(s) + W_{CLS}^{M_L}(s) \cdot M_L(s) =$$
$$= \frac{K_{EA} K_{SM} K_{RD} K_{G1} K_{M1}}{(T_{SM} s + 1)(T_G s + 1)(T_E T_{EM} s^2 + T_{EM} s + 1) + K_{EA} K_{SM} K_{RD} K_{G1} K_{M1} K_{TG} K_{FB}} \cdot U_{in}(\mathrm{s}) +$$
$$+ \frac{K_{M2}(T_{SM} s + 1) \cdot (T_G s + 1)}{(T_{SM} s + 1)(T_G s + 1)(T_E T_{EM} s^2 + T_{EM} s + 1) + K_{EA} K_{SM} K_{RD} K_{G1} K_{M1} K_{TG} K_{FB}} \cdot M_L(\mathrm{s})$$

Switching from signal images to their originals, and replacing $s \to p$, we obtain DCM rotation frequency ACS differential equation:

$$((T_{SM} p + 1)(T_G p + 1)(T_E T_{EM} p^2 + T_{EM} p + 1) + K_{EA} K_{SM} K_{RD} K_{G1} K_{M1} K_{TG} K_{FB}) \cdot \omega(t) =$$
$$= K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1} \cdot U_{in}(t) - K_{M2}(T_{SM} p + 1) \cdot (T_G p + 1) \cdot M_L(t).$$

Using the numerical values of system parameters and replacing $p \to \frac{d}{dt}$, the obtained equation could be written in form (1.12).

1.5 ACS Stability Estimation According to Characteristic Equation Roots

Solution of differential equation (1.12) for the known $g(t)$, $f(t)$ is the variation law of output control variable $X(t)$. It's necessary to implement inverse Laplace transform to equation (1.9) for ACS transient process finding:

$$X(\mathrm{t}) = L^{-1}\left[X_1(s) + X_2(s)\right] = L^{-1}\left[W_{CLS}^{g}(s)\cdot G(s) + W_{CLS}^{f}(s)\cdot F(s)\right] =$$
$$= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} W_{CLS}^{g}(s)\cdot G(s)\cdot e^{st}d\mathrm{s} + \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} W_{CLS}^{f}(s)\cdot F(s)\cdot e^{st}d\mathrm{s}. \tag{1.13}$$

If integrals (2.13) are "unsolvable", the Heaviside formula for transient process definition is used:

$$X(t) = U_{in}\left[\frac{B(0)}{A(0)} + \sum_{i=1}^{n}\frac{B(s_i)}{s_i\cdot A'(s_i)}e^{s_i t}\right], \tag{1.14}$$

where U_{in} is the input signal amplitude; $A'(\mathrm{s}_i)$ is the derivative value of numerator transfer function for value s_i; *n* is the roots number of system characteristic equation.

System characteristic equation roots (fig. 1.14) can be real (root s_1), complex-conjugative (S_2, S_3, S_7, S_8) and imaginary (S_5, S_6). Furthermore, roots can be located: in the loft half plane, in the right half plane or on the ordinate axis and respectively will be left, right or neutral.

The system will be stable, if the transient process for $t \to \infty$ tend to the steady-state value $X(\infty) = X_{SS}$. This means that exponent index of equation (1.14) must be negative, i.e. all the system characteristic equation roots must be located in the left half plane (fig. 1.14).

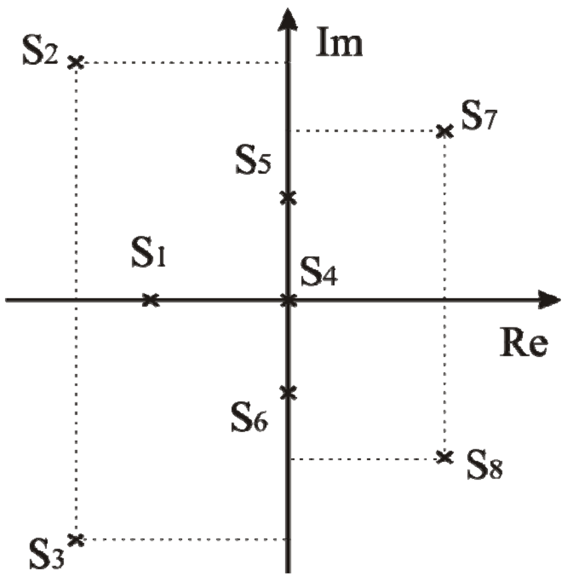


Fig. 1.14. Variation of characteristic equation roots location

Root stability criterion:

The necessary and sufficient condition for the system to be stable is that all the system characteristic equation roots were in the left half plane (have a negative real part).

If among the system characteristic equation roots even one is from the right half plane and the rest are from the left , it means that ACS is unstable.

If among the system characteristic equation roots even one is neutral, and the rest are from the left half plane, it means that ACS is neutral, that is situated on the stability boundary.

Example 1.7. Estimate the stability according to DCM rotation frequency ACS characteristic equation roots.

Solution.

Let's use the system characteristic equation

$$A(s) = (T_{SM}s+1)\cdot(T_G s+1)\cdot(T_E T_{EM} s^2 + T_{EM}s+1)+$$
$$+K_{EA}\cdot K_{SM}\cdot K_{RD}\cdot K_{G1}\cdot K_{M1}\cdot K_{TG}\cdot K_{FB}.$$

Take equation to the form:

$$T_{SM}T_G T_E T_{EM}s^4 + \left[(T_{SM}T_G T_{EM}) + (T_{SM}+T_G)\cdot T_E T_{EM}\right]s^3 +$$
$$+\left[(T_{SM}+T_G)T_{EM} + T_E T_{EM}\right]s^2 + \left[T_{SM}+T_G+T_{EM}\right]s +$$
$$+K_{EA}K_{SM}K_{RD}K_{G1}K_{M1}K_{TG}K_{FB}+1=0.$$

Set system parameters: $T_E = 0{,}02$ sec; $T_M = 0{,}5$ sec; $T_{SM} = 0{,}1$ sec; $T_G = 0{,}7$ sec;

$K_{EA} = 15; K_{SM} = 15; K_{RD} = 0.2; K_{G1} = 8; K_{M1} = 8.5; K_{TG} = 0.15; K_{FB} = 0.5$

Let's calculate the system characteristic equation coefficients:

$$a_0 = 0.1\cdot0.7\cdot0.02\cdot0.5 = 0.0007;$$
$$a_1 = 0.1\cdot0.7\cdot0.5 + (0.1+0.7)\cdot0.02\cdot0.5 = 0.043;$$
$$a_2 = (0.1+0.7)\cdot0.5 + 0.02\cdot0.5 = 0.41;$$
$$a_3 = 0.1+0.7+0.5 = 1.3;$$
$$a_4 = 15\cdot0.6\cdot0.2\cdot8\cdot8.5\cdot0.15\cdot0.5+1 = 13.24.$$

Using MatLab, we obtain roots values of system characteristic equation

```
>> W=tf([12.24],[0.0007 0.043 0.41 1.3 13.24])
 Transfer function:
 12.24
-------------------------------------------------
0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 13.24
>> pole(W)
ans =
–50.3881
 –11.3604
 0.1600 + 5.7460i
 0.1600 – 5.7460i
```

Conclusion: roots s_2, s_3 are locating in the right half plane, therefore, DCM rotation frequency ACS is unstable for the given parameters.

1.6 ACS Stability Estimation According to the Mikhailov Stability Criterion

It is necessary to get Mikhailov curve equation for the ACS stability estimation. Let's use closed-loop characteristic equation (1.11) for these purposes

$$A(s) = a_0 \cdot s^n + a_1 \cdot s^{n-1} + a_2 \cdot s^{n-2} + \cdots + a_{n-1} \cdot s + a_n = 0.$$

To get the Mikhailov curve equation it is necessary to go to the frequency domain, substitute $s \to j\omega$, separating real and imaginary components

$$D(j\omega) = a_0 \cdot (j\omega)^n + a_1 \cdot (j\omega)^{n-1} + a_2 \cdot (j\omega)^{n-2} + \cdots + a_{n-1} \cdot (j\omega) + \\ + a_n = U(\omega) + jV(\omega). \quad (1.15)$$

Where $U(\omega)$, $V(\omega)$ are real and imaginary components of Mikhailov curve equation.

According to the equation (1.15), when the ω is changing, one can draw the Mikhailov curve (fig. 1.15).

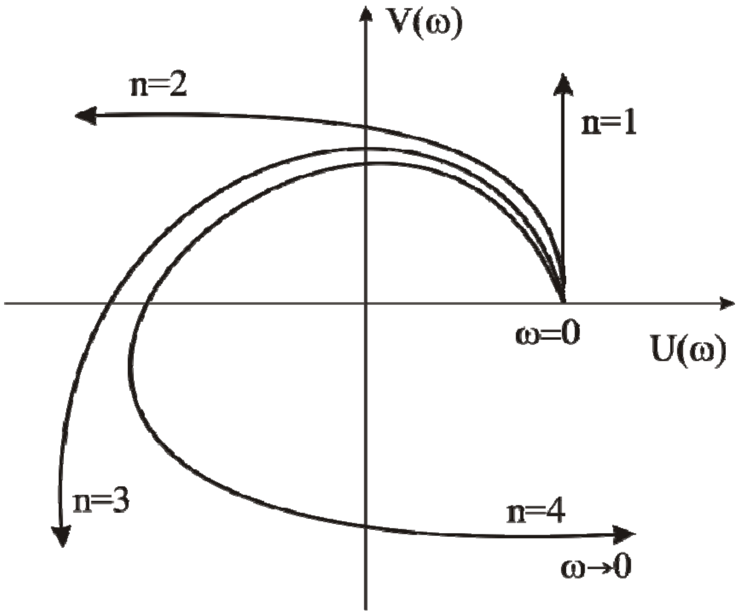


Fig. 1.15. Mikhailov curves for stable systems with $n = 1,\ n = 2;\ n = 3;\ n = 4$

For ACS stability necessary and sufficient conditions should hold:

- when $\omega = 0$ Mikhailov curve locus should begin in the positive part of the real axis;
- when $0 \le \omega \le +\infty$ is changing, Mikhailov curve locus should: sequentially, without vanish, in the positive (counterclockwise) derection pass n quadrants.

If the Mikhailov curve locus for the concrete frequency that does not equal zero pass through the coordinate origin, the system is neutral.

If any of these conditions are not fulfilled, the system is unstable.

Example 1.8. Estimate DCM rotation frequency ACS stability using Mikhailov criterion.

Solution.

Let’s use characteristic equation and system parameters from the example 1.7.

$A(s) = 0.0007s^4 + 0.43s^3 + 0.41s^2 + 1.3s + 13.24$.

For the Mikhailov curve equation obtaining substitute in A(s) $s \to j\omega$ and separate real and imaginary components.

$$D(j\omega) = 0.0007 \cdot (j\omega)^4 + 0.043 \cdot (j\omega)^3 + 0.41 \cdot (j\omega)^2 + 1.3 \cdot (j\omega) + 13.24 =$$
$$= 0.0007 \cdot \omega^4 - j \cdot 0.043 \cdot \omega^3 - 0.41 \cdot \omega^2 + j \cdot 1.3 \cdot \omega + 13.24 =$$
$$= \left(0.0007 \cdot \omega^4 - 0.41 \cdot \omega^2 + 13.24\right) - j \cdot \left(0.358 \cdot \omega^3 - 1.3 \cdot \omega\right).$$

Varying ω from 0 to 6.5, one can draw the Mikhailov curve (fig. 1.16).

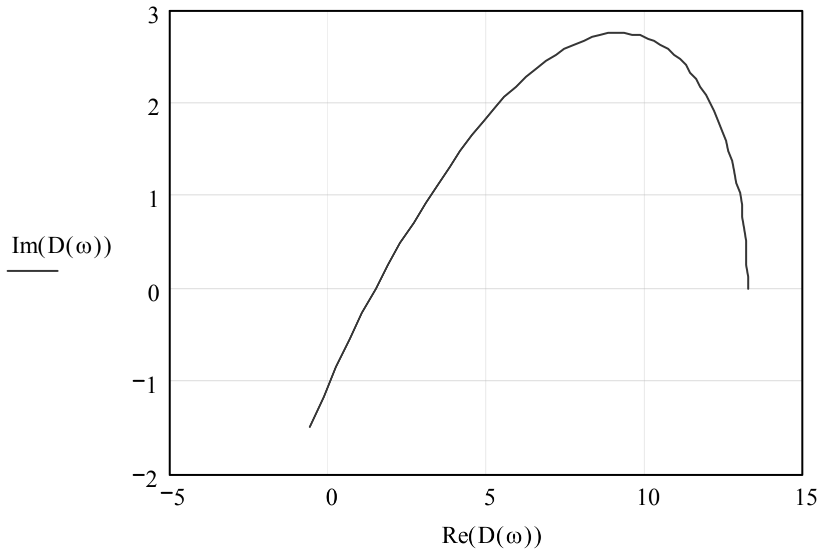


Fig. 1.16. Mikhailov curve for ω from 0 to 6.

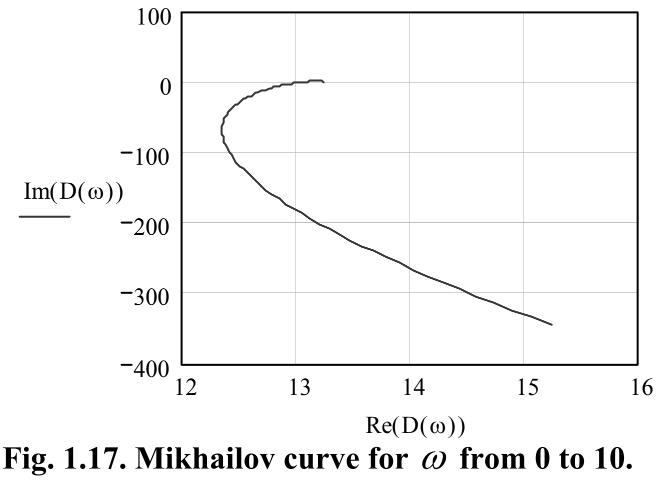


Fig. 1.17. Mikhailov curve for ω from 0 to 10.

Conclusion: not all the Mikhailov stability requirements are fulfilled:
The order of quadrant pass is broken.
Consequently, DCM rotation frequency ACS with the given parameters is unstable.

1.7 ACS Stability Estimation According to the Nyquist Stability Criterion

For the ACS stability estimation it is required to use open-loop system transfer function replacing $s \to j\omega$, and draw up the locus. The special feature of this criterion is closed-loop ACS stability estimation on the base of open-loop form of graph.

Open-loop ACS could be stable, unstable or neutral.Thus there are two approaches to the system stability estimation.

Open-loop system is stable.

If the open-loop system is stable, the closed-loop system is unstable for any encirclement of the point $(-1; j0)$.

If the locus happens to pass through the point $(-1; j0)$, then the closed-loop system is neutral, that means that it is boundary stable.

On the fig. 1.18 three ACS graphs are represented. Graph 1 corresponds to the stable closed-loop ACS, 2 is neutral, 3 is unstable.

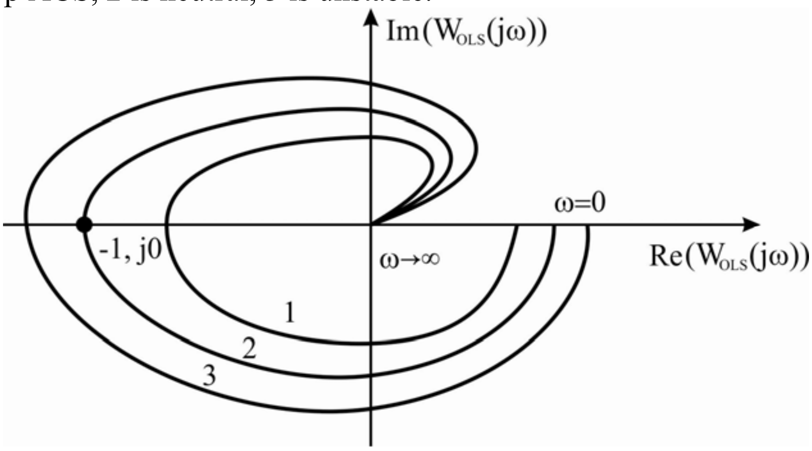


Fig. 1.18. Open-loop system locuses

Closed-loop system is unstable or neutral.

In this case, if among the left half plane roots even one is from the right half plane or located in the coordinates origin.

If the open-loop system is unstable or neutral, then it is necessary and sufficient that open-loop system locus encircled point $(-1; j0)$ in the positive direction $\frac{K}{2}$ times, for the closed-loop system stability, where K is number of right half plane roots of the open-loop system.

Unstable open-loop system locus, which has one right half plane root is represented on fig.1.19.

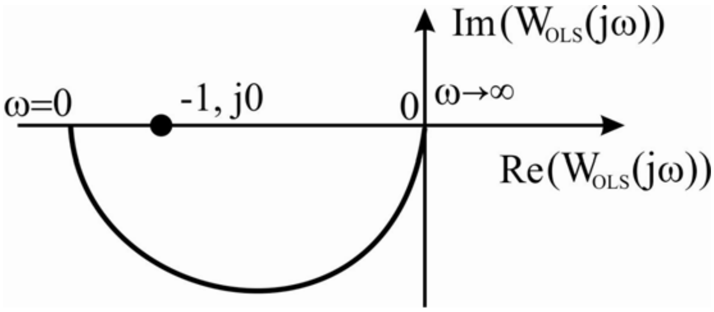


Fig. 1.19. Open-loop system locus for K=1

Locus encircles the point $(-1; j0)$ 0.5 times in the positive direction, consequently the closed-loop system is stable too.

Example 1.9. Estimate DCM rotation frequency ACS stability using Nyquist stability criterion

Solution.

Let's use DCM rotation frequency ACS transfer function $W_{OLS}(s) = \dfrac{K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1} \cdot K_{TG} \cdot K_{FB}}{(T_{SM}s+1) \cdot (T_G s+1) \cdot (T_E T_{EM} s^2 + T_{EM} s + 1)}$.

Let's set system parameters:

$$T_E = 0.02 \sec; T_{EM} = 0.5 \sec; T_{SM} = 0.1 \sec;$$

$$T_G = 0.7 \sec; K_{EA} = 15; K_{SM} = 0.6; T_{RD} = 0.2;$$

$$K_{G1} = 10; K_{M1} = 8.5; K_{TG1} = 0.16; K_{FB} = 0.5.$$

$$W_{OLS}(s) = \frac{K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1} \cdot K_{TG} \cdot K_{FB}}{(T_{SM}s+1) \cdot (T_G s+1) \cdot (T_E T_{EM} s^2 + T_{EM} s + 1)} =$$

$$= \frac{12.24}{0.0007s^4 + 0.043s^3 + 0.41s^2 + 1.3s + 1}.$$

Let's define open-loop system characteristic equation roots, using Matlab.

```
 W=tf([12.24], [0.0007 0.043 0.41 1.3 1])
 Transfer function:
 12.24
---------------------------------------------
0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 1
>> pole(W)
ans =
 -50.5593
 -4.8755 + 1.2244i
 -4.8755 - 1.2244i
 -1.1181
```

Then since all the roots are located in the left half plane, let's use the first system stability estimation approach.

Let's draw up the stable open-loop system locus, using Mathcad. (fig. 1.20 and fig. 1.21).

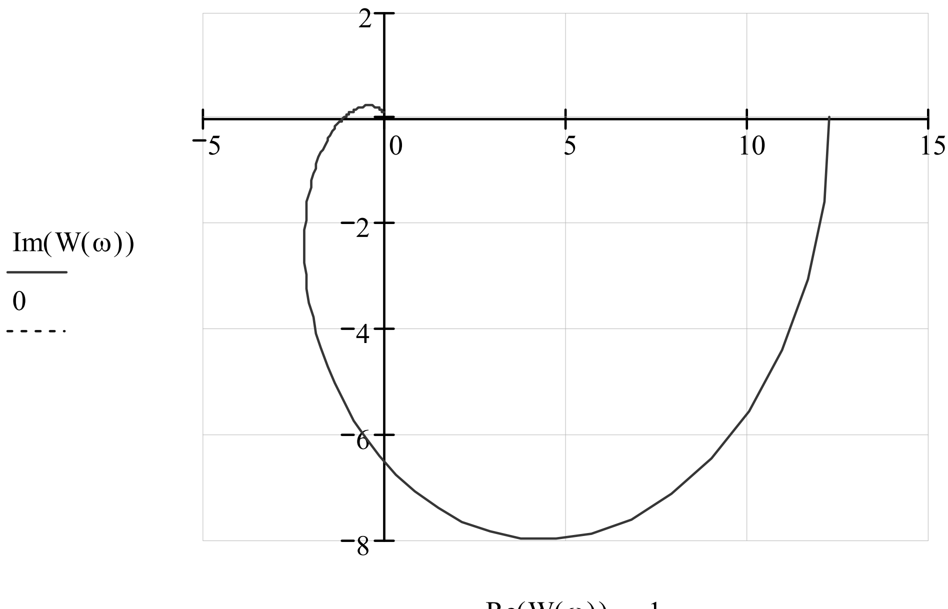


Fig. 1.20. Nyquist locus for ω from 0 to 15

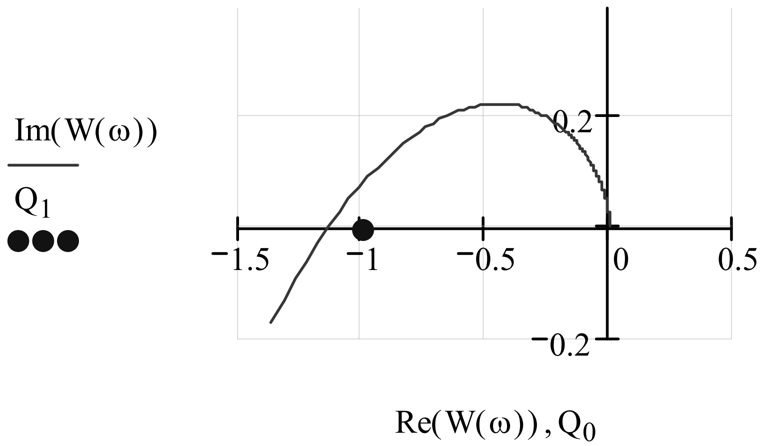


Fig. 1.21. Nyquist locus for ω from 5 to 45

Conclusion. Nyquist locus, according to fig 1.21, encircles the point $(-1; j0)$, consequently, the closed-loop system is stable.

1.8 Hurwitz Stability Criterion. ACS Critical Gain

Critical gain K_{CR} of ACS is the value of the open-loop system coefficient K_{OLS},when the closed-loop system is neutral. It's possible to use any stability criterion for the system critical gain K_{CR} value definition.

Let's consider the Hurwitz criterion to define K_{CR}.

This requires closed-loop system characteristic equation:

$$A(s) = a_0 \cdot s^n + a_1 \cdot s^{n-1} + a_2 \cdot s^{n-2} + \cdots + a_{n-1} \cdot s + a_n = 0.$$

The principal Hurwitz determinant is formed from equation coefficients on the basis of following rules:

- characteristic equation coefficients from a_1 are situated along the main diagonal of the Hurwitz determinant;
- determinant columns are filled out with coefficients regarding to principal diagonal: upwards with increasing indexes, downwards with decreasing indexes;
- zeros are set instead of default coefficients.

The rest of the Hurwitz determinant are formed from principal determinant by means of separating of rows (columns) number, which are equaled to sequence number of determinant

$$\Delta_n = \begin{bmatrix} a_1 & a_3 & a_5 & \ldots & 0 \\ a_0 & a_2 & a_4 & \ldots & 0 \\ 0 & a_1 & a_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & a_n \end{bmatrix}.$$

Criterion:

It is necessary, for a stable system, that all the coefficients of the characteristic equation be positive:

$a_n > 0, a_{n-1} > 0, \ldots, a_1 > 0, a_0 > 0$

If even one determinant is equaled to zero, the system is neutral.

To define critical gain K_{CR} it' is enough to take only penultimate determinant and equate it to zero.

Example 1.10. Define DCM rotation frequency ACS critical gain K_{CR} value, using Hurwitz criterion

Solution.

Before the solution it is necessary to define which coefficients make open-loop system coefficient K_{OLS}. For that, let's use DCM rotation frequency ACS transfer function from example 1.9. and find it's limit.

$$W_{OLS}(s) = \frac{K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1} \cdot K_{TG} \cdot K_{FB}}{(T_{SM}s+1)\cdot(T_G s+1)\cdot(T_E T_{EM} s^2 + T_{EM}s+1)}.$$

$$\lim_{s\to 0} W_{OLS}(s) = K_{OLS} = K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1} \cdot K_{TG} \cdot K_{FB}.$$

Analyzing system characteristic equation from example 1.7 one can mention that K_{OLS} is a part only of a_4.

$$A(s) = 0.0007 \cdot s^4 + 0.043 \cdot s^3 + 0.41 \cdot s^2 + 1.3 \cdot s + K_{OLS} + 1 = 0$$

Let's use the closed-loop system characteristic equation:

$$A(s) = 0.0007 \cdot s^4 + 0.043 \cdot s^3 + 0.41 \cdot s^2 + 1.3 \cdot s + K_{OLS} + 1$$

Let's form the fourth order determinant

$$\Delta_4(K_{OLS}) = \begin{bmatrix} 0,043 & 1,3 & 0 & 0 \\ 0,0007 & 0,41 & 1+K_{OLS} & 0 \\ 0 & 0,043 & 1,3 & 0 \\ 0 & 0,007 & 0,41 & 1+K_{OLS} \end{bmatrix}.$$

Let's use third order determinant:

$$\Delta_3(K_{OLS}) = \begin{bmatrix} 0,043 & 1,3 & 0 \\ 0,0007 & 0,41 & 1+K_{OLS} \\ 0 & 0,043 & 1,3 \end{bmatrix} = 0.$$

Expanding this determinant, we get $K_{CR} = 10.7555$.

1.9 Stability Plane Plotting in System Parameter Plane

Using the stability criteria doesn't give an answer to the question: "To what extent one can vary the system parameters saving it's stability". This problem was solved by Neimark and then being a part of control theory became known as « D-partition method» or «Stability plane plotting in system parameter plane». This method is graph-analytic and allows to define the varying range of one or two system parameters.

The method is as follows. Varying the system parameters in certain sequence, one can value the parameters combinations, when the system characteristic equation roots are neutral (located on the ordinate axis). In the fig 1.22 represented the situation when points $1,2,3\ldots m$ are combination of parameters C and D, when even one of the system (1.11) characteristic equation roots is imaginary. Connecting these points, we get D-partition curve.

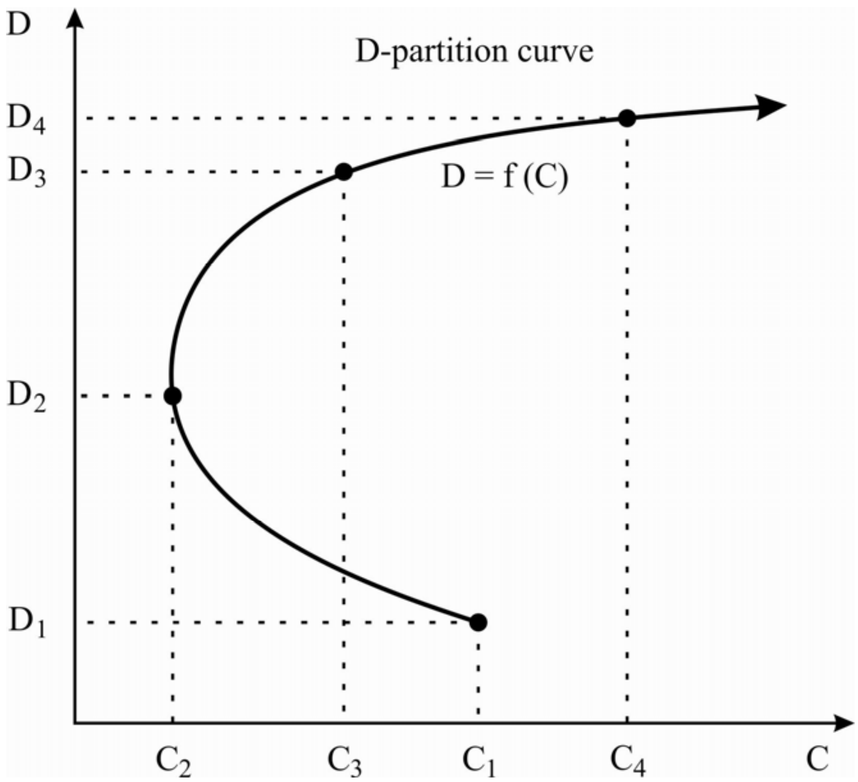


Fig. 1.22. D **-partition curve in parameters plane** C **and** D

D-partition curve divides parameters plane C and D into areas with different content of the right and the left roots. Plane area where all the system characteristic equation roots are left claims to be stable. For the stability area identification D-partition curve shading is used. Closed-loop system characteristic equation, where the varying parameters C and D are contained, is the initial equation for stability region plotting.

The stability area plotting algorithm in a single parameter plane C:

Varying parameter C is detected in closed-loop system characteristic equation (1.11).

The given equation is expressed with respect to the variable parameter C.

After passing to a frequency domain, replacing $s \to j\omega$ and separating real and imaginary components, *D*-partition curve equation is obtained $N(j\omega) = \mathrm{Re}(j\omega) + \mathrm{Im}(j\omega)$. Let's set a frequency ω from 0 to ∞, and plot one branch of *D*-partition curve and for ω from $-\infty$ to 0 – another branch.

Causing a hatch on the branch of the *D*-partition, select the region of stability.

Choose parameter C variation limits from the stability region.

For the chosen value C, using any stability criterion, make found region checking.

Example 1.11. Plot stability region in plane of the parameter $C = K_{CR}$. Define variation limits of K_{CR} and critical gain K_{CR} value of DCM rotation frequency ACS.

Solution.

Let's use closed-loop system characteristic equation from example 1.10:

$$A(s) = 0.0007 \cdot s^4 + 0.043 \cdot s^3 + 0.41 \cdot s^2 + 1.3 \cdot s + K_{OLS} + 1 = 0$$

Express K_{OLS} from the given equation:

$$K_{OLS} = -0.0007 \cdot s^4 - 0.043 \cdot s^3 - 0.41 \cdot s^2 - 1.3 \cdot s - 1.$$

Let's go to frequency domain and plot *D*-partition curve in the varying parameter K_{OLS} plane.

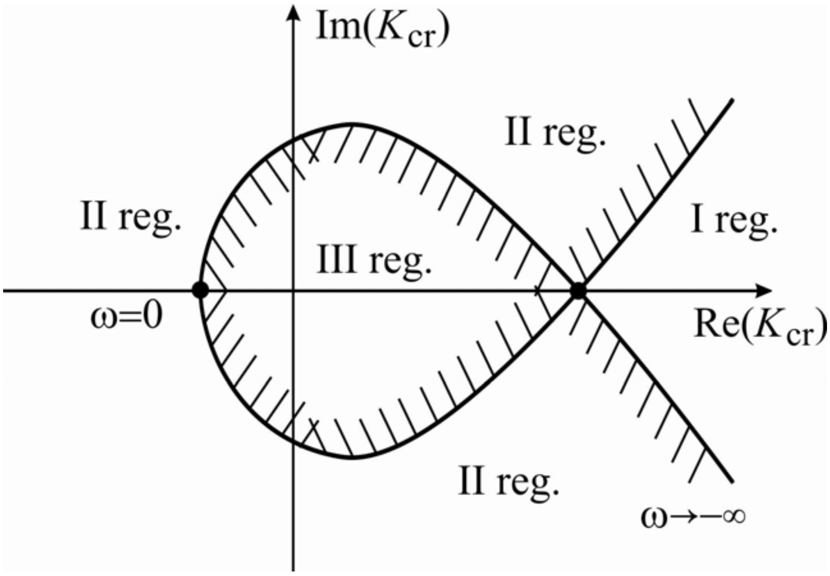


Fig. 1.23. *D*-partition curve in the parameter K_{OLS} plane

In the fig. 1.23 one can see that stability region is the III-rdregion. Variation limits $K_{OLS} = (0 \ldots 10.7)$ are chosen from this region. Therefore, critical gain value $K_{CR} = 10.7$, which coincides with the value is in the example 1.10.

1.10 Step Response of the System and Quality Indexes of the Control Process.

Performance quality of any control system is characterized by quantitative and qualitative indexes, which are defined by the step response curve or other dynamic system characteristics. System step response is the system reaction on the external influence, which, in general, could be the complex time function. Usually system performance is considered in terms of following standard influence: unit step function $1(t)$, impulse function $\delta(t)$ and harmonic function. Often direct quality indexes (transient character, control time – t_S, and overshoot – $\sigma, \%$) are obtained from the step response $h(t)$, for unit step input signal $1(t)$.

Both numerator and denominator influence on step response character. If the closed-loop system transfer function $W_{CLS}(s)$ has no zeros, i.e. has the form:

$$W_{CLS}(s) = \frac{K}{a_0 s^n + a_1 s^{n-1} + \ldots + a_n} = \frac{K}{A(s)}, \tag{1.16}$$

the character of the step response is completely determined by the closed-loop characteristic equation roots:

$$a_0 s^n + a_1 s^{n-1} + \ldots + a_n = 0. \tag{1.17}$$

If characteristic equation (1.17) roots are real $S_i = \alpha_i$, the character of the step response is monotonous, fig. 1.24

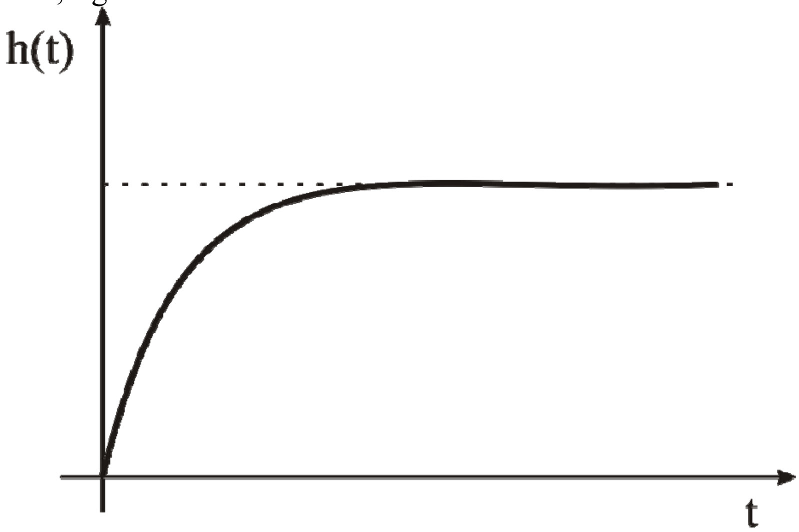


Fig. 1.24. Aperiodic step responce

If the roots are real $S_i = \alpha_i$ and complex conjugate $S_{l,k} = \alpha_k \pm j\beta_k$ and α_k complex roots much more than α_i real, the character of the step response is oscillating (periodical), fig 1.24.

If the pair of roots located on the ordinate axes and others in the left half plane, that means that the step response is oscillating with constant amplitude and frequency. The system is situated on stability boundary.

If the closed-loop system characteristic polynomial roots are situated in the left half plane, such system is stable. If even one of roots is situated in the right half plane, and the others are in left, this system is claimed to be unstable.

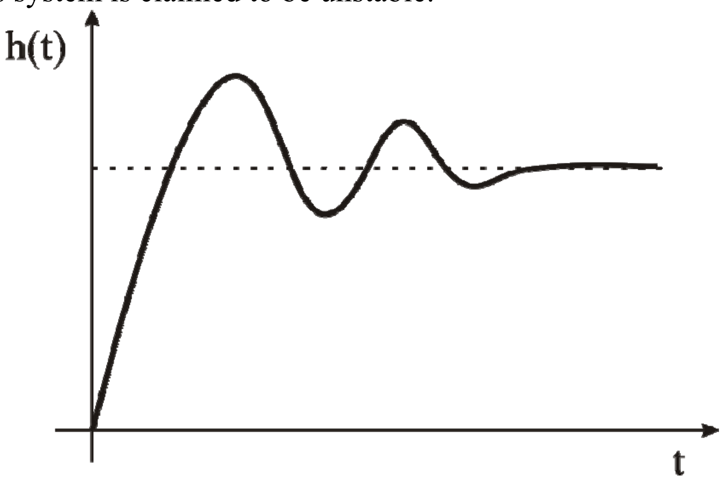


Fig. 1.25. Oscillating step response

The system tendency to oscillation is characterized by a maximum value of a control variable $h_{\max}$ (fig. 1. 25) or by an overshoot value – $\sigma, \%$.

$$\sigma = \frac{h_{\max} - h_{\infty}}{h_{\infty}} \cdot 100\%,$$

where h_{∞} is steady-state value of control variable after the completion of the step response.

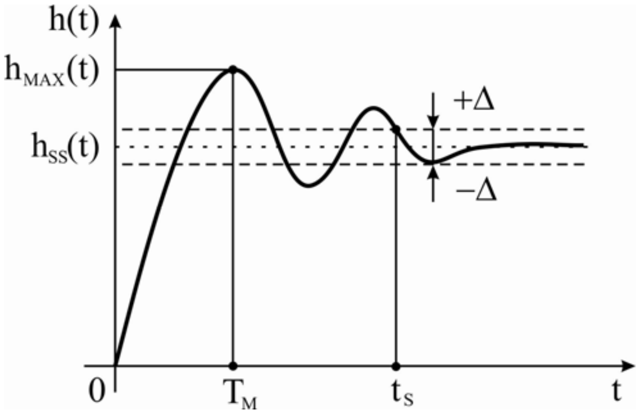


Fig. 1.26. Qualitative indexes of step response

System performance settling time is characterized by the duration of step response t_S. Settling time t_S (step response duration) is defined as a period of time from application of influence at the system input to the moment, when the following inequality is held: $\left|h(t) - h_{\infty}\right| \le \Delta h$, where Δh is a small constant value, representing the specified accuracy. In the control theory it is $\Delta = 0.05$.

Degree of stability η represents an absolute value of the shortest distance from real axes to the nearest root (or complex conjugated roots). Oscillating μ is $tg(\phi)$ (fig. 1.26). Settling time t_S and $\sigma, \%$ are connected with degree of stability η and oscillating μ by following correlations:

$$t_S = \frac{1}{\eta} \cdot \ln\frac{1}{\Delta} \approx \frac{3}{\eta}, \ \sigma\ \% = e^{-\pi/\mu} \cdot 100\%.$$

For a more accurate estimation t_S and σ according to the correlations, it is necessary for the system not to have zeros and all the system characteristic equation roots were located inside or on the boundary of trapezium in the roots plane fig. 1.27.

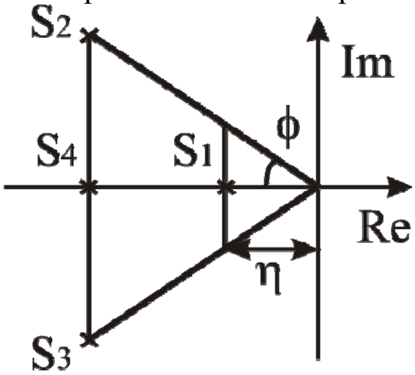


Fig. 1.27. Roots qualitative indexes

Example 1.12. Plot DCM rotation frequency ACS step response. Define qualitative indexes.

Solution.

Let's use the closed-loop system transfer function expression for the reference signal from example 1.5

$$W_{CLS}^{U_{in}}(s) = \frac{K_{EA} \cdot K_{SM} \cdot K_{RD} \cdot K_{G1} \cdot K_{M1}}{(T_{SM}s+1) \cdot (T_G s+1) \cdot (T_E T_{EM} s^2 + T_{EM} s + 1) + K_{EA} K_{SM} K_{RD} K_{G1} K_{M1} K_{TG} K_{FB}}.$$

Let's set the system parameters:

$T_E = 0.02$ sec; $T_{EM} = 0.5$ sec; $T_{SM} = 0.1$ sec; $T_G = 0.7$ sec;

$K_{EA} = 10; K_{SM} = 0.6; K_{RD} = 0.2; K_{G1} = 8; K_{M1} = 8.5; K_{TG} = 0.15; K_{FB} = 0.5.$

Then

$$W_{CLS}^{U_{in}}(s) = \frac{12.24}{0.0007s^4 + 0.043s^3 + 0.41s^2 + 1.3s + 7.12}.$$

Let's use Matlab to plot the step response. The results are shown on fig. 1.27.

```
> W=tf([12.24],[0.0007 0.043 0.41 1.3 7.12])
 Transfer function:
 12.24
-----------------------------------------------
0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 7.12
>> pole(W)
ans =
 -50.4742
 -9.7133
 -0.6205 + 4.5124i
 -0.6205 - 4.5124i
>> step(W)
```

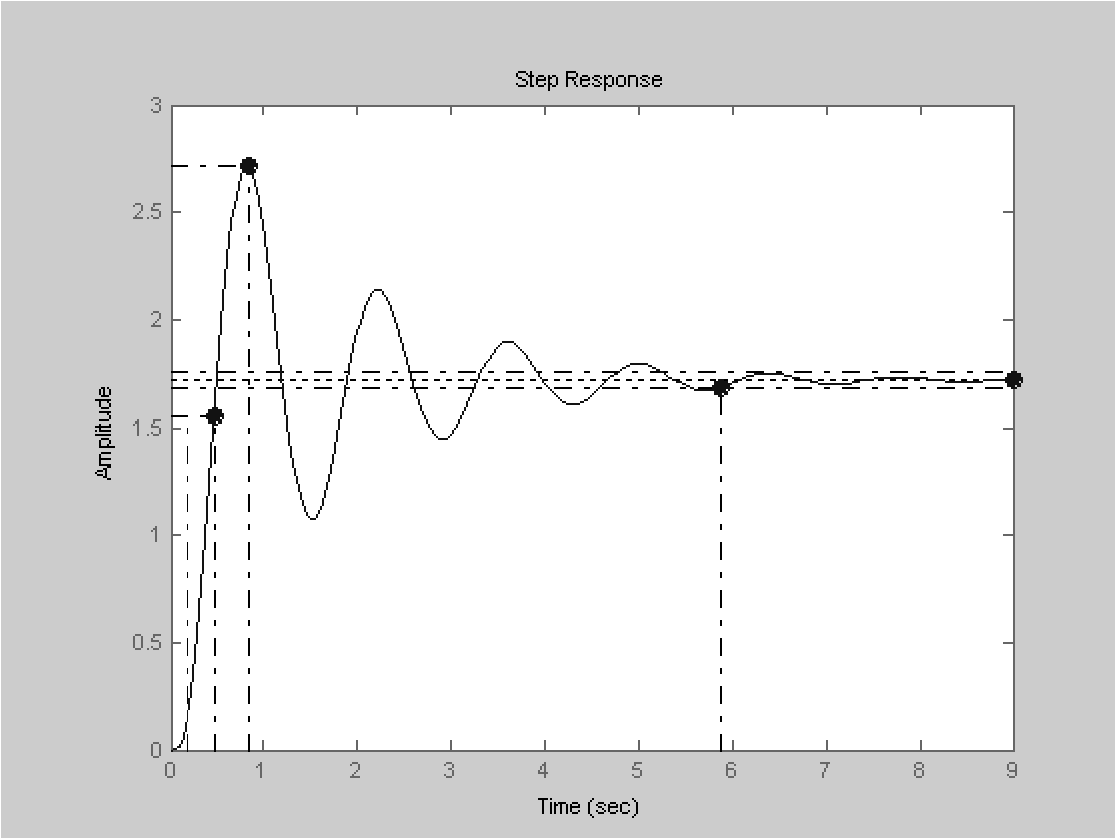


Fig. 1.28. Step response of DCM rotation frequency ACS

Let's give all the system indexes:

$h_{\max} = 2.71\ rad/\sec$;

$h_{\infty} = K_{CLS} = 1.72$;

$\sigma = 57.9\ \%$;

$T_M = 0.278\ \sec$;

$t_S = 5.87\ \sec$;

$T_{FR} = 1.3\ \sec$;

$\omega_{FR} = 2\pi/T_{FR} = 4.83\ \sec^{-1}$;

$\eta = 0.6205$;

$$\mu = tg(\varphi) = \frac{4.5123}{0.6205} = 7.27\ .$$

1.11 Automated Control System Control Process Accuracy Estimation

Control accuracy research of ACS is conducted by means of the system steady-state process analysis, i.e. the accuracy of control system is estimated by the steady-state errors, which is defined by the system structure (transfer functions) and influences (reference signals and disturbances).

1.11.1 Control error in stabilization systems

Estimating the stabilization control system accuracy the reference signal is assumed to be constant, i.e. $g(t) = g_0 \cdot 1(t)$. Total control error $\varepsilon_F(t)$ of linear system, which functional scheme is depicted on fig. 1.29, could be represented as $\varepsilon_F(t) = g(t) - x(t)$, where $g(t)$ is the reference signal; $x(t)$ is the output signal.

In the image domain s the equation can be written as

$$E_F(s) = G(s) - X(s) \tag{1.18}$$

Connection between reference signal $g(t)$, disturbance $f(t)$ and output signal $x(t)$ in the image domain s is established by means of transfer functions

$$X(s) = W_{CLS}^{g}(s) \cdot G(s) + W_{CLS}^{f} \cdot F(s) \tag{1.19}$$

where $W_{CLS}^{g}(s)$ is the closed-loop system transfer function for the reference signal $g(t)$; while $W_{CLS}^{f}(s)$ is closed-loop system transfer function for disturbance $f(t)$.

For the given control system (fig. 1.**Ошибка! Источник ссылки не найден.**29) transfer functions have form:

$$W_{CLS}^{g}(s) = \frac{W_{OLS}(s)}{1 + W_{CLS}(s)}; \quad W_{CLS}^{Z}(s) = \frac{-W_{Plant}(s)}{1 + W_{OLS}(s)}, \tag{1.20}$$

Where $W_{OLS}(s) = W_{CU}(s) \cdot W_{Plant}(s)$ is the transfer function of the open-loop system; $W_{CU}(s)$ is the controller transfer function; $W_{Plant}(s)$ is the object transfer function.

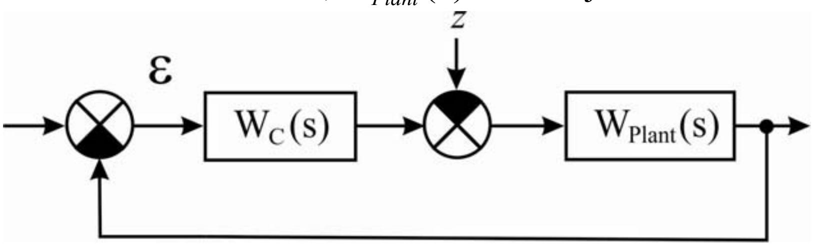


Fig. 1.29. Standard ACS unit diagram

Substituting(1.20), (1.19) into (1.18), we get

$$E_F(s) = \left[1 - \frac{W_{OLS}(s)}{1 + W_{OLS}}\right] \cdot G(s) + \frac{-W_{Plant}(s)}{1 + W_{OLS}} \cdot F(s). \tag{1.21}$$

Where $\left[1 - \frac{W_{OLS}(s)}{1 + W_{OLS}(s)}\right] = \frac{1}{1 + W_{OLS}} = W_{CLS}^{\varepsilon}(s)$ is the closed-loop system transfer function for the control error.

Therefore, total control error $E_F(s)$ consists of 2 components

$$E_F(s) = E_g(s) + E_f(s), \tag{1.22}$$

where $E_g(s)$ – control error, caused by reference signal $g(t)$; $E_f(t)$ – control error, caused by disturbance $f(t)$.

Using expressions (1.21), (1.22) and limiting value theorem

$\lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to\infty} s \cdot W(s) \cdot F(s)$, for standard signals $g(t) = g_0 \cdot 1(t)$, $f(t) = f_0 \cdot 1(t)$ systems steady-state errors can be defined according to the following expressions:

$$\varepsilon_F^{ss} = \varepsilon_g^{ss} + \varepsilon_f^{ss}, \qquad (1.23)$$

$$\varepsilon_g^{ss} = W_{g\varepsilon}(0) \cdot g_0, \qquad (1.24)$$

$$\varepsilon_f^{ss} = W_{fx}(0) \cdot f_0. \qquad (1.25)$$

where ε_F^{ss} – steady-state value of the total error; ε_g^{ss} – steady-state value of the error, caused by reference signal; ε_f^{ss} – steady-state value of error, caused by disturbance.

Equations (1.23)-(1.25) are static equations, which in static stationary mode ($t = \infty$, $s = 0$) connect steady-state control error values with transfer function values, defined for $s = 0$.

The first total control error component in stabilization systems ($g(t) = const$) ε_F^{ss} can be reduced to zero by scaling. Then control system accuracy will be fully characterized by steady-state error δ_{SS}:

$$\delta_{SS} = \frac{\varepsilon_f^{ss}}{g_0} \cdot 100\% = \frac{W_{CLS}^Z(0) \cdot f_0}{g_0} \cdot 100\%.$$

1.11.2 Control error in servo systems

In servo control systems and servo drive, used in aircrafts, reference signal is changed with constant speed υ_0.

$$g(t) = v_0 \cdot t, \; v_0 = const, \qquad (1.26)$$

or with constant acceleration

$$g(t) = \frac{a \cdot t^2}{2}, \; a = const. \qquad (1.27)$$

Control process accuracy estimated with the help of number of errors.

$$\varepsilon_{SS}(t) = c_0 \cdot g(t) + c_1 \cdot g'(t) + \frac{c_2 \cdot g''(t)}{2!} + \ldots + \frac{c_n}{n!} \cdot \frac{d^n g(t)}{dt^n}. \qquad (1.28)$$

where $\varepsilon_{SS}(t)$ steady-state error; $c_0, c_1, \ldots, \frac{c_n}{n!}$ – number of errors coefficients; $g'(t), \; g''(t), \ldots, \frac{d^n g(t)}{dt^n}$ – the first, the second, …, n – derivative of reference signal.

Coefficients $c_0, c_1, \ldots, \frac{c_n}{n!}$ of number of errors (1.28) expressed in terms of transfer function W_{CLS}^{ε} for control error:

$$c_0 = W^{\varepsilon}_{CLS}(s)\Big|_{s=0}; \qquad c_1 = \frac{\partial W^{\varepsilon}_{CLS}(s)}{\partial s}\Bigg|_{s=0};$$
$$\frac{c_2}{2!} = \frac{\partial^2 W^{\varepsilon}_{CLS}(s)}{\partial s^2}\Bigg|_{s=0}; \qquad \frac{c_n}{n!} = \frac{\partial^n W^{\varepsilon}_{CLS}(s)}{\partial s^n}\Bigg|_{s=0}. \tag{1.29}$$

Number of errors (1.29) is restricted, both left and right. Restriction from the right depends on equality to zero of some derivative from the reference signal $g(t)$. For example, for the standard signal $g(t) = g_0 \cdot 1(t)$ steady-state error is defined according to the expression

$$\varepsilon_{ss} = c_0 \cdot g_0 \tag{1.30}$$

In this case number of errors coefficient c_0 characterizes steady-state error.

If the reference signal is changed with the constant speed (1.26), steady-state error expressed as

$$\varepsilon_{ss}(t) = c_0 \cdot v_0 \cdot t + c_1 \cdot v_0, \tag{1.31}$$

where coefficient c_1 characterizes speed error.

Steady-state error for the reference signal (1.27) expressed as

$$\varepsilon_{ss}(t) = c_0 \cdot \frac{a \cdot t^2}{2} + c_1 \cdot a \cdot t + \frac{c_2}{2!} \cdot a. \tag{1.32}$$

Coefficient $\frac{c_2}{2!}$ characterizes acceleration error.

From the expressions (1.30) - (1.32) follows, that for the static, speed and acceleration errors elimination it is necessary equality to zero of coefficients $c_0, c_1, \frac{c_2}{2!}$, respectively. For this purpose, it is necessary to provide appropriate astatism order for the system.

Under the astatism order meant degree v of the image S^v, which is situated in the open-loop system transfer function denominator. For example for $W_{CLS}(s) = \frac{B(s)}{s^2 \cdot A(s)}$ astatism order equals to 2.

For the 1st order astatic systems coefficient c_0 equals to zero, for the 2nd order astatic systems – c_0, c_1 equals to zero, for the 3rd order astatic systems – $c_0, c_1, \frac{c_2}{2!}$ equals to zero. Thereby 1st order astatic systems reproduce constant reference signals $g(t) = g_0 \cdot 1(t)$ without error, systems with the 2nd order of astaticism reproduce reference signals, which change with the constant speed $g(t) = v_0 \cdot t, \;\; v_0 = const$ without errors etc.

2 Nonlinear ACS

ACS is considered to be nonlinear, if even one element of the system described by the nonlinear differential equation. Practically all the ACS are nonlinear. If after substitution of nonlinear system characteristic by the linear the ACS behavior doesn't change, such system called linearized. Nonlinearities can be:

- accompanying, if nonlinearity is a part of the composition of ACS invariable part;
- not accompanying, if nonlinearity is a part of synthesized ACS part;
- essential;
- inessential nonlinearity;
- single-valued nonlinearity;
- mixed nonlinearity.

Nonlinearity is consider to be **inessential**, if the nonlinear component substitution by the linear unit doesn't change fundamental system features and processes, occurring in the linearzed ACS, have no qualitative difference from the real system processes.

In the unit diagrams the nonlinear element is represented by means of the rectangle with static characteristic or functional dependence of the output signal Y from the input signal X, written inside. For a single-valued nonlinearity is $y = F(x)$. For mixed nonlinearity y depends not only on output signal value x, but also on direction (i.e. derivative) $y = F(x, px)$.

Nonlinear ACS transformations have their own features. They are specified by the fact that superposition principle and commutativity rule are not held for them, i.e. $y_{out} \neq y_{in1} + y_{in2}$.

Also not all the structural transformation rules are held for nonlinear ACS, for example:

- it's not allowed to transfer the summer through a nonlinear unit;
- it's not allowed to rearrange linear and nonlinear units, etc.

Nonlinear ACS transformation consists in linear units transformation, standing from the different sides of nonlinear element.

2.1 ACS Differential Equation in Implicit Form

There is no notation for the closed-loop nonlinear ACS. Therefore differential equation obtaining approach for such type of systems is different from the obtaining of linear ACS equation approach. Let's obtain close-loop ACS differential equation, which unit diagram represented on fig. 2.1.

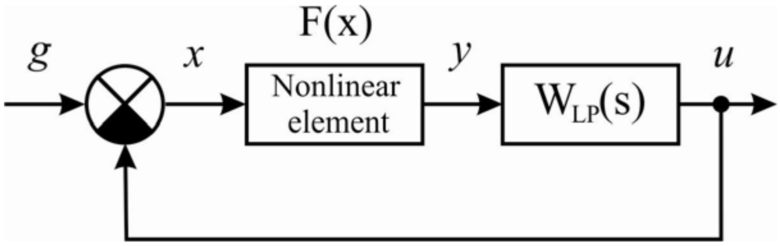


Fig. 2.1. Typical nonlinear ACS bock diagram

Let's designate the linear part transfer function of nonlinear ACS $W_{LP}(s)$ as $W_{LP}(s)=\frac{B(s)}{A(s)}$, then its differential equation has a form

$$A(s)\cdot Y(t)=B(s)\cdot U(t). \tag{2.1}$$

Nonlinear equation element in the implicit form

$$Y(t)=F(x,px). \tag{2.2}$$

Let's write the equation for x(t)

$$x(t)=g(t)-U(t). \tag{2.3}$$

Let's put (2.3), (2.2) in (2.1) and get the closed-loop nonlinear ACS differential equation relative to $U(t)$ in implicit form.

$$A(s)\cdot U(t)=F\Big[\big(g(t)-u(t)\big),s\big(g(t)-u(t)\big)\Big]\cdot B(s).$$

Practically this equation is not used, therefore we get differential equation regarding $X(t)$. For this purpose, let's evaluate $U(t)$ from (2.3) and put it in (2.1), then we obtain the differential equation regarding $X(t)$ in the explicit form

$$A(p)\cdot x(t)+B(s)\cdot F(x,sx)=A(s)\cdot g(t). \tag{2.4}$$

If reference signal $g(t)=0$, then free motion differential equation of nonlinear ACS in implicit form will be obtained from (2.4).

$$A(s)\cdot x(t)+B(s)\cdot F(x,sx)=0. \tag{2.5}$$

Due to the fact that the nonlinear ACS does not have a differential equation in explicit form, for analysis and synthesis of such type of systems following approaches are used:

1st approach.

Accepting hypothesis of linearity of nonlinear element static characteristic, ACS linearization is conducted.

Then, in terms of the harmonic linearization method, the V.M. Popov or N.I. Tsipkin stability criterion, the nonlinear ACS stability is estimated.

2nd approach.

Mathematical model for every segment of the nonlinear element static characteristic is formed.

In terms of the system state space and taking into account the obtained mathematical models, nonlinear ACS description in form of 1^{st} order differential equations is performed. Analyzing 1^{st} order differential equation systems solutions for each segment of nonlinear static characteristic, nonlinear ACS stability is estimated.

2.2 Harmonic Linearization Method Application for Nonlinear ACS

It's convenient to use harmonic linearization (harmonic balance) method for nonlinear ACS study. This method is based on frequency characteristics usage, applied in linear control theory. It requires taking into account some assumptions:

- Unit diagram should be typical (fig. 2.1).
- Nonlinear element characteristic should be symmetric in relation to the coordinate origin.
- The system should have self-oscillation with constant amplitude a_n and frequency ω_n.
- System should be autonomous, i.e. $g(t)=0$.

If a closed-loop autonomous (without external influences) nonlinear system can be represented as the compounds of the nonlinear element and a stable linear part with transfer function $W_{LP}(s)$ (fig. 2.1), then under a certain conditions could be applied harmonic linearization method to it. The main idea of this method is that the possible stable oscillations on linear part of nonlinear system output approximately considered to be harmonic (sinusoidal).

Let's assume, at the nonlinear element output sinusoidal signal $x(t)=a\cdot\sin(\omega\cdot t)$ is feed. Therefore, nonlinear element output signal $y(t)$ is also periodical and could be expanded in the Fourier series. This series contains components with frequencies multiple to frequency $\omega, 2\omega, \ldots$ of output signal $x(t)$. Supposing, that this signal, passing through the linear part is filtered to the extent that higher harmonics can be neglected, we write the harmonic linearization equation of nonlinear element:

$$y(t)=F(x,sx)=F\left(a\cdot\sin\psi, a\omega\cdot\cos\psi\right)=q(a)\cdot x(t)+\frac{g'(a)}{\omega}\cdot x(t), \quad (2.6)$$

where $\psi=\omega\cdot t$, $q(a)$, $q'(a)$ are the harmonic linearization coefficients of nonlinear element:

$$q(a)=\frac{1}{\pi\cdot a}\int_0^{2\cdot\pi}F\left(a\cdot\sin\psi,\ a\cdot\omega\cdot\cos\psi\right)\cdot\sin\psi\ d\psi\ ;$$

$$q'(a) = \frac{1}{\pi \cdot a} \int_0^{2\cdot\pi} F\left(a \cdot \sin\psi,\ a \cdot \omega \cdot \cos\psi\right) \cdot \cos\psi \ d\psi .$$

Equation (2.6) is a harmonic linearization equation up to highest harmonics from the case, when nonlinear element has the ambiguous characteristic. For the case, when nonlinear element has the single-valued characteristic

$$y(t) = q(a) \cdot x(t). \tag{2.7}$$

Expressions for the harmonic linearization coefficients $q(a), q'(a)$, definition represented in.

2.3 Differential and Characteristic Equations of ACS Harmonic Linearization

Harmonic linearization method application allows to obtain nonlinear ACS differential equations in the implicit form.

For this purpose expression (2.6) or (2.7) is put into equation (2.4). As a result nonlinear ACS harmonic linearization differential equation with ambiguous or single-valued characteristics is obtained:

$$A(s) \cdot X(t) + \cdot B(s) \cdot \left(q(a) \cdot x(t) + \frac{g'(a)}{\omega} \cdot s \cdot x(t) \right) = A(s) \cdot g(t), \tag{2.8}$$

$$A(s) \cdot X(t) + \cdot B(s) \cdot \left(q(a) \cdot x(t) \right) = A(s) \cdot g(t) \tag{2.9}$$

For the autonomous ACS expressions have the following form:

$$A(s) \cdot X(t) + \cdot B(s) \cdot \left(q(a) \cdot x(t) + \frac{g^{'}(a)}{\omega} \cdot s \cdot x(t) \right) = 0, \tag{2.10}$$

$$A(s) \cdot X(t) + \cdot B(s) \cdot \left(q(a) \cdot x(t) \right) = 0 \tag{2.11}$$

For the expressions (2.8)-(2.11) nonlinear ACS harmonic linearization differential equation are

$$A(s) + \cdot B(s) \cdot \left(q(a) + \frac{g^{'}(a)}{\omega} \cdot s \right) = 0, \tag{2.12}$$

$$A(s) + B(s) \cdot \left(q(a) \right) = 0. \tag{2.13}$$

2.4 Obtaining of Typical Unit Diagram of Nonlinear ACS

To reduce nonlinear ACS unit diagram to a standard form (fig. 2.1), use the following rules:

- Since the system should be autonomous, it's necessary to discard the reference signal and the disturbance with the surrounding chains.
- Due to the fact that the nonlinear element should be in a typical scheme, right after the main summer, it is necessary to add one more summer at the output of the nonlinear element, in initial schemes.
- If nonlinear element has time lag (thyristor transducer), then gain is realized in its static characteristic, and time lag remains a separate unit.

- Standard scheme should be drawn beginning with the included summer.
- After nonlinear element all the other units of initial scheme, forwards the reference signal to the introduced summer are drawn.
- If the initial scheme has the local feedbacks or additional control channels, they also should be drawn.

Example 2.13. Cast DCM rotation frequency ACS unit diagram with nonlinear DCG characteristic to typical. Obtain differential and characteristic equations of harmonic linearized system. Nonlinear DCG characteristics are shown on fig. 2.2.

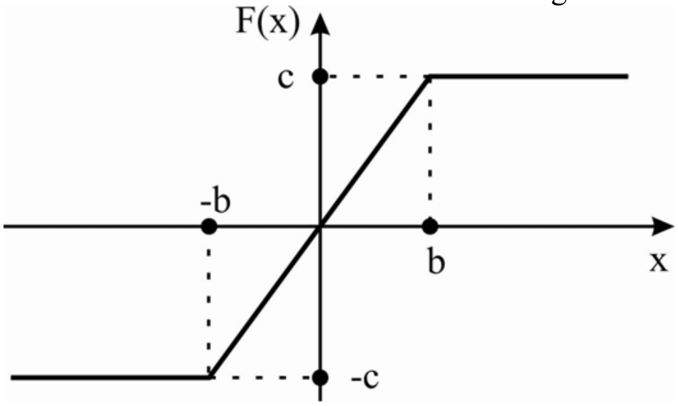


Fig. 2.2. Nonlinear DCG characteristics «saturation»

Coefficient of linearization for such nonlinearity has a form

$$q(a)=\frac{2\cdot k}{\pi}\cdot\left(\arcsin\frac{b}{a}+\frac{b}{a}\cdot\sqrt{1-\frac{b^2}{a^2}}\right);\ q'(a)=0. \qquad (2.14)$$

Solution.

Let's use ACS unit diagram, represented on (fig. 1.4). Discard all the signals and represent DCG in form of two units (nonlinear element and inertial unit with transfer function $W_{DCG}(s)=\dfrac{1}{T_G\cdot s+1}$). At the nonlinear element the input supplementary summer is added (fig 2.3).

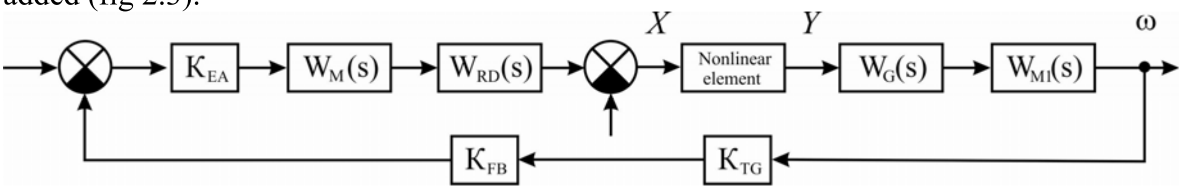


Fig. 2.3. Unit diagram of DCM rotation frequency nonlinear ACS

Utilizing standard unit diagram organization rules, obtain the system (fig. 2.4).

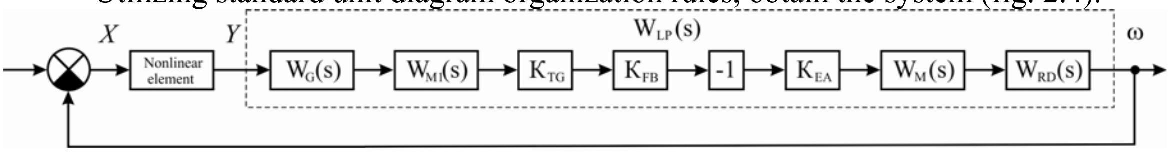


Fig. 2.4.Reduction of the nonlinear ACS unit diagram to the standard

Let's obtain the transfer function of linear part of nonlinear system

$$W_{LP}(s)=\frac{K_{EA}\cdot K_M\cdot K_{RD}\cdot K_{M1}\cdot K_{TG}\cdot K_{FB}}{(T_M s+1)\cdot(T_G s+1)\cdot(T_E T_M s^2+T_M s+1)}=\frac{B(s)}{A(s)}.$$

Using expressions (2.11) and (2.14), write differential and characteristic equations of the harmonic linearized system

$$A(s)x(t)+B(s)\big(q(a)x(t)\big)=(T_M s+1)(T_G s+1)(T_E T_M s^2+T_M s+1)x(t)+$$
$$+K_{EA}K_M K_{RD}K_{M1}K_{TG}K_{FB}\cdot\frac{2\cdot k}{\pi}\cdot\left(\arcsin\frac{b}{a}+\frac{b}{a}\cdot\sqrt{1-\frac{b^2}{a^2}}\right)\cdot x(t). \quad (2.15)$$

$$A(s)+B(s)\cdot\big(q(a)\big)=(T_M s+1)\cdot(T_G s+1)\cdot(T_E T_M s^2+T_M s+1)+$$
$$+K_{EA}\cdot K_M\cdot K_{RD}\cdot K_{M1}\cdot K_{TG}\cdot K_{FB}\cdot\frac{2\cdot k}{\pi}\cdot\left(\arcsin\frac{b}{a}+\frac{b}{a}\cdot\sqrt{1-\frac{b^2}{a^2}}\right). \quad (2.16)$$

2.5 Goldfarb Method for Nonlinear ACS Stability Estimation Application

The stability analysis of harmonic linearized nonlinear ACS conducted in 2 stages. On the first stage we take a hypothesis, that system has the self-oscillations and define amplitude a_n and frequency ω_n of these oscillations. On the second stage stability of the found periodical solution and the nonlinear ACS is estimated. For these purposes the Mikhailov criterion or the Goldfarb approach can be applied.

The main equation harmonic balance (linearization) approach has the form

$$1+W_{NP}(a)\cdot W_{LP}(j\omega)=0, \quad (2.17)$$

where $W_{LP}(j\omega)$ is the linear part transfer function of nonlinear ACS; $W_{NP}(a)$ is the complex gain of harmonic linearized nonlinear element.

On the basis of equations (2.6), (2.7) one can write

$$W_{NP}(a)=q(a)+j\cdot q'(a); \quad (2.18)$$

$$W_{NP}(a)=q(a). \quad (2.19)$$

Solving equation (2.17) regarding ω and a, self-oscillation parameters can be defined. Goldfarb suggested to solve this problem in a graphic way, representing this equation as

$$W_{LP}(j\cdot\omega)=-G_{NP}(a), \quad (2.20)$$

where $G_{NP}(a)=1/W_{NP}(a)$ are the nonlinear reverse characteristics.

Linear part $W_{LP}(j\omega)$ locus (fig 3.3) and nonlinear element negative characteristic $-G_{NP}(a)$ are plot on the complex plane. Nodes of these characteristics give us the equation (2.52) solution. The oscillation amplitude a_n defined according to characteristic $-G_{NP}(a)$, and the frequency ω_n defined according to $W_{LP}(j\omega)$.

Fig. 3.5 shows the case when system has 2 periodic solutions: diagram nodes 2 (a_{n1}, ω_{n1}) and 5 (a_{n2}, ω_{n2}).

For the positive increment of amplitude $a_n + \Delta a$, locus $W_{LP}(j\omega)$ encircles point 4 and doesn't encircles point 11, and for negative $a_n - \Delta a$ – encircles point 3 and doesn't encircles point 6.

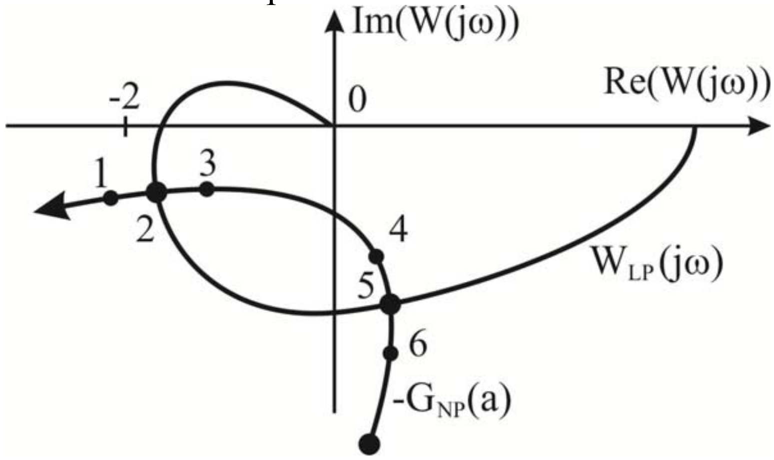


Fig. 2.5. Graphic representation of the Goldfarb approach

If the locus $W_{LP}(j\omega)$ doesn't encircle point with positive increment of amplitude $a_n + \Delta a$ (point 1), and encircles point with negative increment $a_n - \Delta a$, then obtained solution will be stable (point 2), in this case the system is stable in general. If not, found solution is unstable (point 5), and system is stable in small.

Example 2.14. Using Goldfarb approach, estimate DCM rotation frequency ACS stability with nonlinear DCM characteristic. Nonlinear DCG characteristic represented on fig. 3.2.

Solution.

Let's use transfer function of linear part and harmonic linearization coefficients from example 2.13.

$$W_{LP}(s) = \frac{K_{EA} \cdot K_M \cdot K_{RD} \cdot K_{M1} \cdot K_{TG} \cdot K_{FB}}{(T_M s + 1) \cdot (T_G s + 1) \cdot (T_E T_M s^2 + T_M s + 1)}.$$

$$q(a) = \frac{2 \cdot k}{\pi} \cdot \left(\arcsin \frac{b}{a} + \frac{b}{a} \cdot \sqrt{1 - \frac{b^2}{a^2}} \right); \; q'(a) = 0.$$

Let's set the system parameters:

$T_E = 0.02$ sec; $T_M = 0.5$ sec; $T_M = 0.1$ sec; $T_G = 0.7$ sec; $K_{EA} = 10$;

$K_M = 0.6$; $K_{RD} = 0.2$;

$K_{G1} = 8$; $K_{M1} = 8.5$; $K_{TG} = 0.15$; $K_{FB} = 0.5$; $k = K_{G1}$; $b = 2$.

Then

$$W_{LP}(s) = \frac{6.12}{0.0007s^4+0.043s^3+0.41s^2+1.3s+1}.$$

$$W_{NP}(a) = q(a) = \frac{2 \cdot 8}{3.14} \cdot \left(\arcsin\frac{2}{a} + \frac{2}{a} \cdot \sqrt{1 - \frac{2^2}{a^2}} \right) = 5.096 \cdot \left(\arcsin\frac{2}{a} + \frac{2}{a} \cdot \sqrt{1 - \frac{2^2}{a^2}} \right).$$

$$G_{NP}(a) = 1/W_{NP}(a) = 0.196 \cdot \frac{1}{\left(\arcsin\frac{2}{a} + \frac{2}{a} \cdot \sqrt{1 - \frac{2^2}{a^2}} \right)}.$$

$W_{LP}(j\omega)$ and $-G_{NP}(a)$ locuses are plot on the complex plane. The results represented in fig. 3.4.

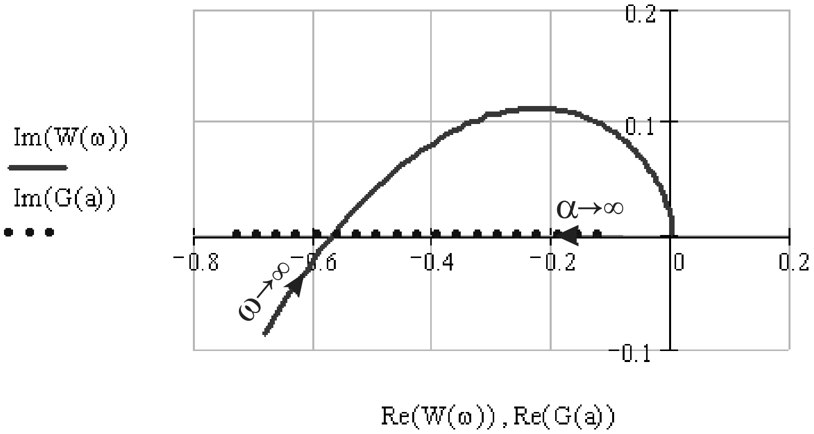


Fig. 2.6. $W_{LP}(j\omega)$ **and** $-G_{LP}(a)$ **locuses**

Conclusion. Locuses cut across, therefore, there is general solution of equation (2.52). Obtained solution is stable and ACS is stable in general.

2.6 Application of Popov Stability Criterion for Nonlinear ACS Stability Analysis

The frequency criterion research of the nonlinear ACS equilibrium position absolute stability was introduced by V.M. Popov in 1959. To use this criterion is necessary to take into account the following restrictions and assumptions:

- unit diagram should be typical (fig. 2.1);
- nonlinear element characteristic should be single-valued;
- linear part of nonlinear ACS should be stable;
- nonlinear characteristic should belong to sector $[0,\ k]$ (fig. 2.7), i.e. the condition $0 \le f(x) \le kx$ should hold.

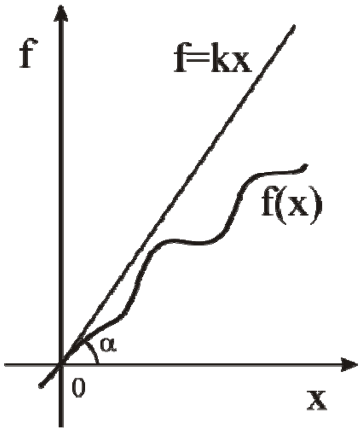


Fig. 2.7.Nonlinear element characteristic

For the equilibrium position of nonlinear ACS is stable, the following inequality should hold

$$\mathrm{Re}\left[(1+j\omega\cdot\alpha)\cdot W_{LP}(j\omega)\right]+1/k_1>0, \qquad (2.21)$$

for all $\omega\geq 0$, where α is unconditioned real number.

In other words, if the final real number α can be chosen in such a way that inequality (2.53) held, the equilibrium position of closed-loop ACS is absolutely stable.

As it follows from the criterion statement, he gives just necessary, but not sufficient condition of stability, i.e. system could be stable when this criterion is not held.

This inequality (2.21) is called Popov inequality, its graphic solution is used on practice. The transformed frequency characteristic of $W_{LP}(j\omega)$ linear part is introduced into consideration for convenience.

$$W_{LP}(j\omega)=U^*(\omega)+jV^*(\omega);$$
$$U^*(\omega)=\mathrm{Re}(W_{LP}(j\omega)); \qquad (2.22)$$
$$V^*(\omega)=\omega\cdot\mathrm{Im}(W_{LP}(j\omega)).$$

Let's extract real component from the square bracket in inequality (2.21) :

$$\mathrm{Re}\left[(1+j\omega\alpha)\cdot W_{LP}(j\omega)\right]=\mathrm{Re}\left[(1+j\omega\alpha)\cdot\mathrm{Re}(W_{LP}(j\omega))+\mathrm{Im}(W_{LP}(j\omega))\right]=$$
$$=\mathrm{Re}(W_{LP}(j\omega))-\alpha\omega\cdot\mathrm{Im}(W_{LP}(j\omega)).$$

Taking into account the equations (2.22) write inequality (2.21) in form

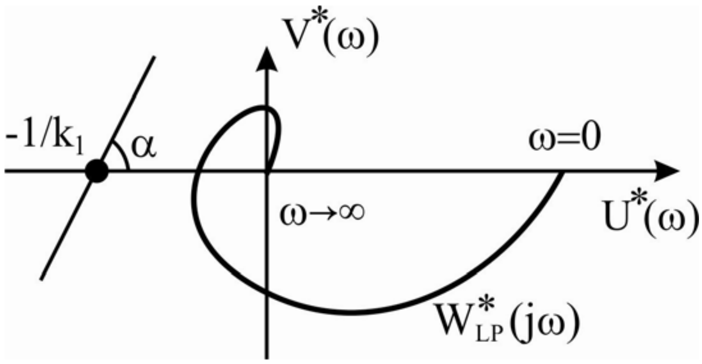
$$U^*(\omega)-\alpha V^*(\omega)+1/k>0. \qquad (2.23)$$

Solution of equation (2.54) reduced to following (fig. 3.5):

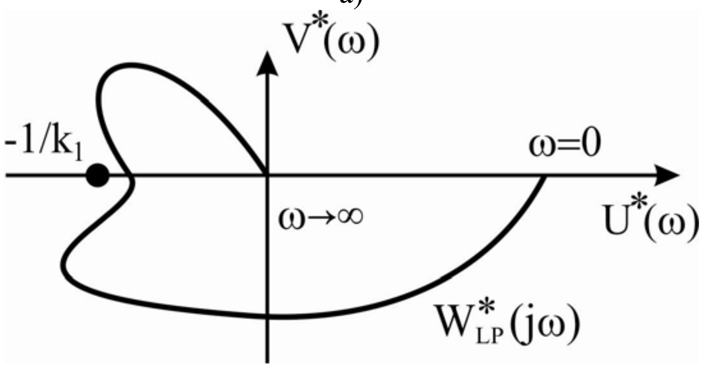
When varying frequency ω from 0 to ∞, the transformed frequency characteristic of linear part $W_{LP}^*(j\omega)$ is plot on the complex plane, and a straight line under any angle α is drawn though the point $(-1/k_1; j0)$ (fig. 2.8a).

Popov criterion.

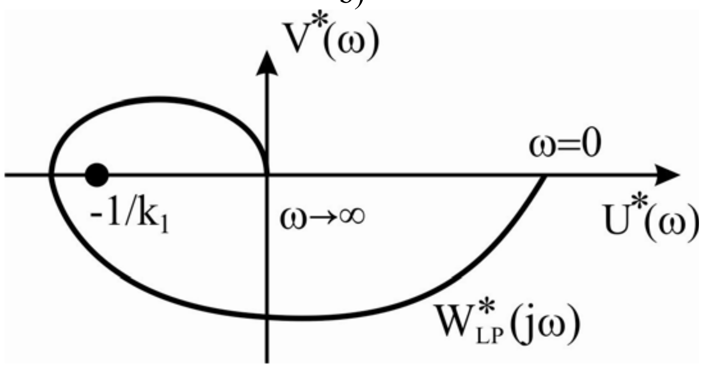
For the nonlinear ACS equilibrium position was stable, all transformed frequency characteristic linear part $W^*_{LP}(j\omega)$ locus is necessary to be located on the right side from the straight line, drawn under any angle α, through the point $(-1/k_1; j0)$. Where k_1 is the straight line slope ratio, restricting sector $(0,\ k_1)$.



a)



b)



c)

Fig. 2.8. Popov inequality solution

According to fig. 2.8, for the case a) ACS equilibrium position is absolutely stable; for b) and c) Popov criterion is not held, but the system can be stable.

Example 2.15. It is necessary to estimate DCM rotation frequency ACS stability with nonlinear characteristic of DCG, using Popov criterion.

Nonlinear characteristic parameters of DCG: $K_{G1} = 8;\ b = 4;\ m = 0.1$.

Solution.

Let's plot the nonlinear characteristic of DCG taking into account its parameters (fig. 2.9).

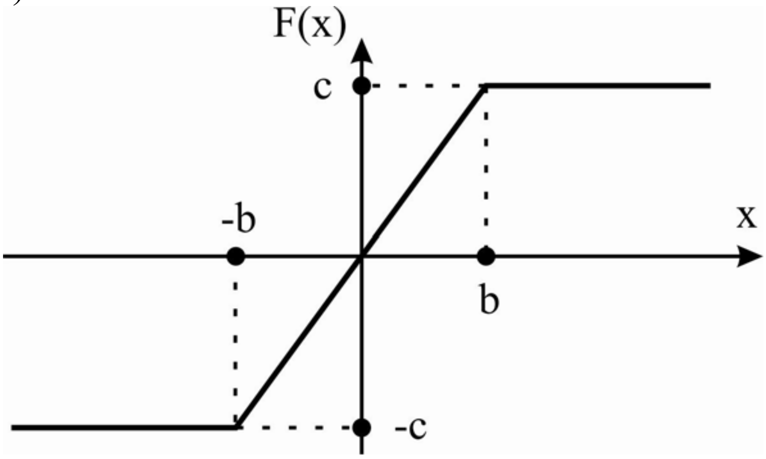


Fig. 2.9. Nonlinear characteristic of DCG

Let's use linear part transfer function and system parameters from the example 2.14

$$W_{LP}(s) = \frac{6.12}{0.0007s^4 + 0.043s^3 + 0.41s^2 + 1.3s + 1}.$$

Bode plot of transformed frequency characteristic linear part $W^*_{LP}(j\omega)$ and point $(-0.139;\ j0)$.

Plot results represented in fig. 2.10.

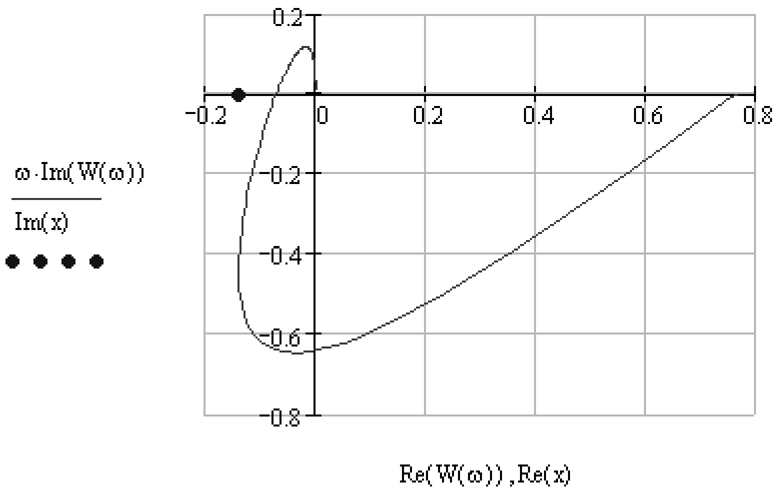


Fig. 2.10. Popov criterion application for the system stability estimation

Conclusion. The Popov criterion is held, because it is possible to draw the straight line under any angle through the found point, in such a way that all the Bode plot located on the right side of the straight line.

2.7 V.M. Popov Stability Criterion Application for the Case of Neutral or Unstable Linear Part

In case when the linear part is neutral or unstable, the Popov criterion is inapplicable. For the Popov criterion generalization for the given case, the unit diagram transformation is made in such a way that the linear part was stable. For this, in the unit diagram in parallel with the nonlinear element proportional link with the transfer ration – r is introduced, and the linear part is covered by a negative feedback with the transfer ratio r (fig. 2.11).

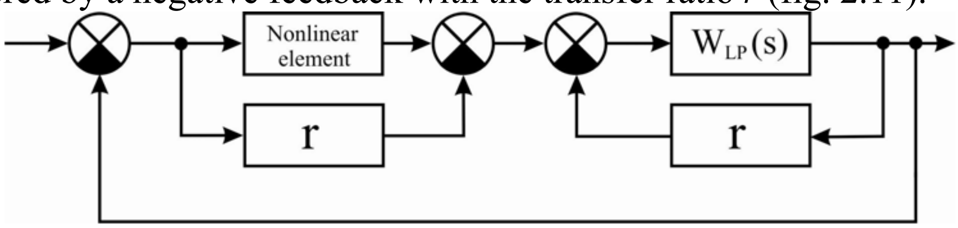


Fig. 2.11. Unit diagram transformation

Let's write the transfer function of transformed linear part of nonlinear ACS

$$W_{LP1}(s) = \frac{W_{LP}(s)}{1 + W_{LP}(s) \cdot r}.$$

Value r is chosen in such way, that the transformed linear part of nonlinear ACS becomes stable.

According to the Popov criterion statement: the system equilibrium position is absolutely stable, if the following inequality is held:

$$\mathrm{Re}[(1 + j\alpha\omega) \cdot W_{LP1}(\omega)] + \frac{1}{k_1} > 0,$$

and the nonlinear element $f_1(x)$ characteristic should be located in sector $[0, k_1]$, i.e.

$$0 \le f_1(x)/x \le k_1; \;\; f_1(x) = f(x) - rx.$$

Both expressions could be reduced to initial:

$$\mathrm{Re}[(1 + j\alpha\omega) \cdot \frac{W_{LP}(\omega)}{1 + W_{LP}(\omega)}] + \frac{1}{K_1} > 0,$$

$$r \le \frac{f(x)}{x} \le K_1 + r.$$

Nonlinear characteristic should be located in sector $\left[r,\, k_1 + r\right]$. If linear part of nonlinear ACS is neutral, then r should be extremely small value.

3 LINEAR PULSE ACS

Depending on signal transmission and transformation methods ACS can be divided into:

- continuous ACS;
- discrete ACS.

In the continuous systems signals during the transformation process are not interrupted. There are elements or units, which transform continuous signals into the pulse sequence, or quantized signals series, or the digital code in discrete systems. In many modern ACS the discrete devices or digital processors are used.

Discrete method of signals transmission and transformation supports their amplitude quantization or time quantization, or amplitude and time quantization. There are 3 types of quantization and 3 groups of discrete ACS:

1. Amplitude quantization. In this case the signal is fixed in some discrete levels. For the amplitude quantization the multiposition relay element is used, represented in fig 3.1, its static characteristic represented in fig. 3.2:

Fig. 3.1. Multiposition relay element

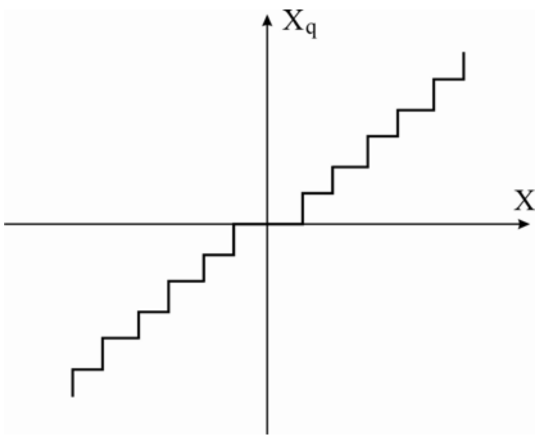


Fig. 3.2. Multiposition relay element characteristic

Results of amplitude quantization are represented in fig.3.3, where X_p – quantized signal.

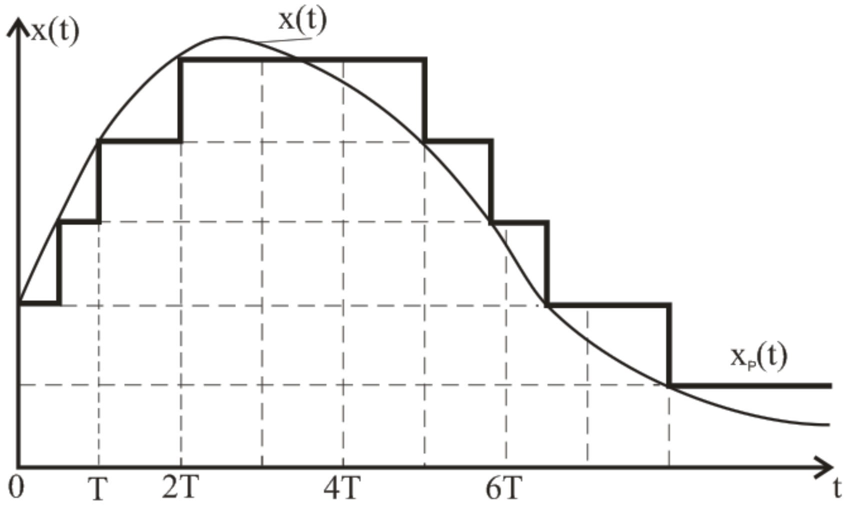


Fig. 3.3. Level quantization

Since the relay element is used as a continuous signal $X(t)$ quantizer, then the discrete ACS also called relay ACS. Such type of discrete systems refers to nonlinear ACS type, and for their analysis and synthesis the nonlinear system theory is used.

2. Time quantization. In this case continuous signal is fixed in discrete moments of time: $0, T, 2T, 3T$ etc. Continuous signal quantization can be obtained by passing continuous signal through the switch (fig. 3.4), which periodically, with the quantization cycle T, closed on time h. In the discrete ACS this element is called the pulse element. Quantization result is represented in fig. 3.5.

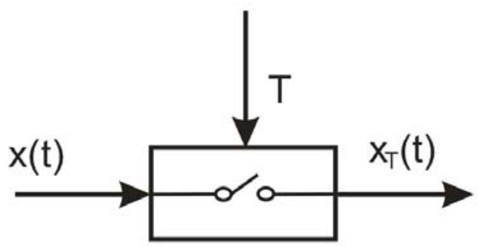


Fig. 3.4.Pulse element

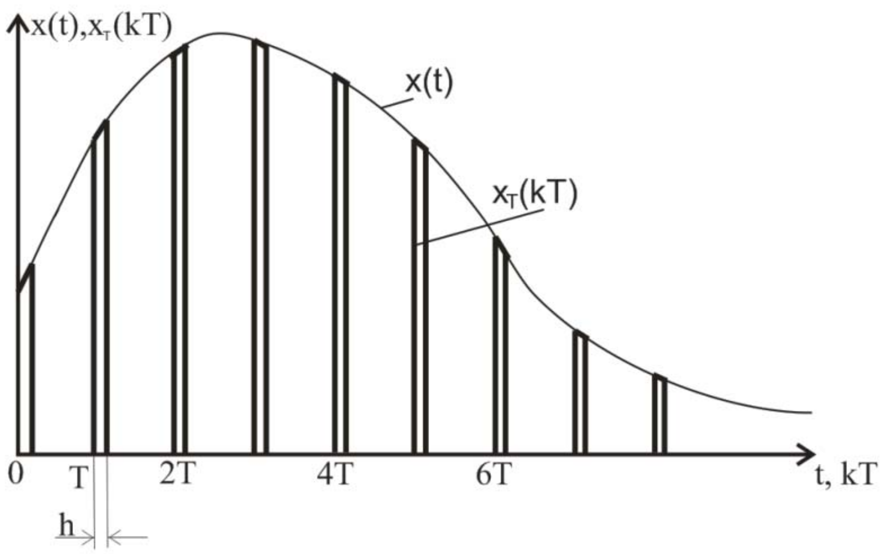


Fig. 3.5.Time quantization

If pulse h duration essentially smaller then quantization cycle T, and after the switch situated linear unit with time constant *T*>>*h*, then pulse sequence $X_T(t)$ can be considered as series of instantenuous pulses of δ-functions, which amplitudes equale to input signal $X(t)$ values in quantization time (fig. 3.6).

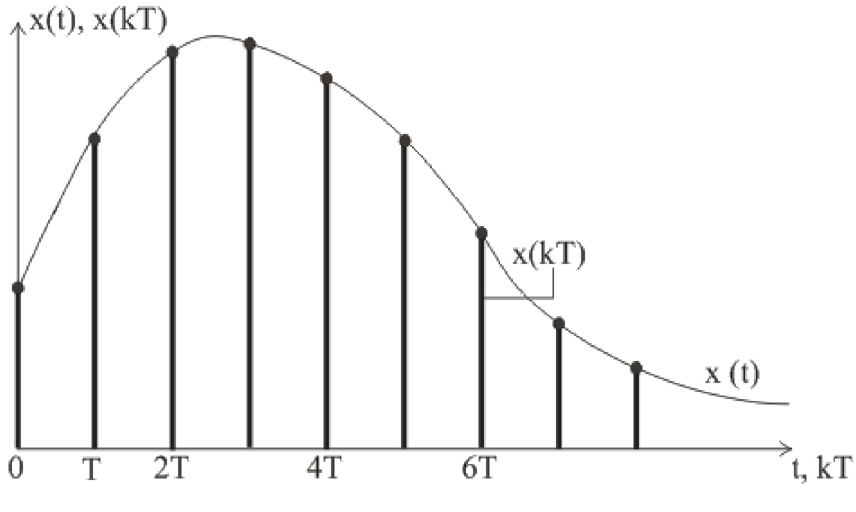


Fig. 3.6. Signal quantization for the case when *T>>h*

The information between the quantization periods is lost. The discrete signal can be represented as following:

$$\begin{cases} X_T(t) = X_T(nT), \quad for\ t = nT, \\ X_T(t) = 0, \quad for\ nT < t < (n+1)T,\ n = 0,1,2,... \end{cases}$$

Since the pulse element is used as a continuous signal quantizer, discrete systems are called the pulse ACS.

3. Amplitude and time quantization. In this case, in the discrete moments of time: $0, T, 2T, 3T$ etc., the continuous function $X(t)$ values are chosen and fixed on the nearest specified level. The results of the amplitude and time quantization are represented in fig. 3.7.

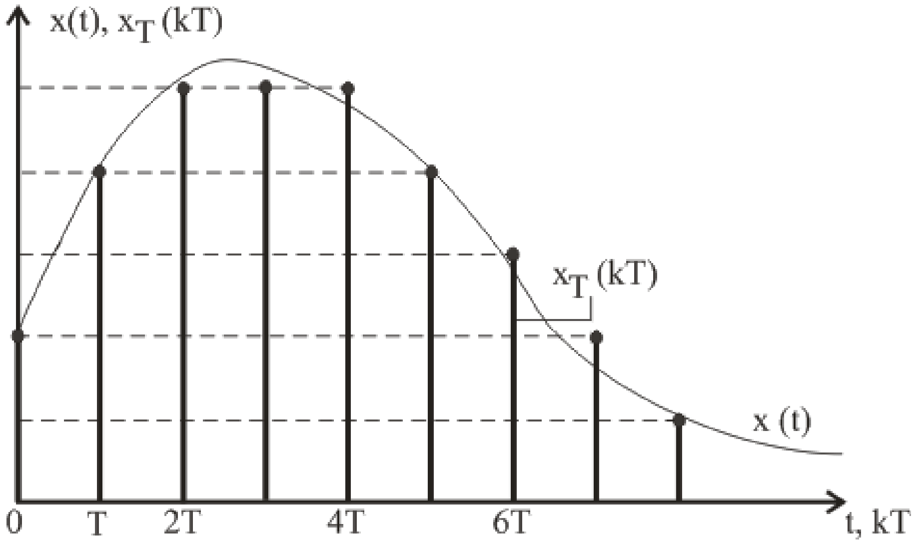


Fig. 3.7.Amplitude and time quantization

Quantization is implemented by the code pulse modulator or the analog-to-digital converter (ADC) embedded into the computer. Therefore the discrete ACS of such type called digital.

The amplitude quantization introduce nonlinearity in the digital system, but for ADC capacity 32 and higher, differences between the signals at the nearby lying levels are not significant. Therefore, amplitude quantization can be neglected. Moreover, the pulse ACS and the digital are united by one feature – time quantization is realized by the pulse element. Hereby for analysis and synthesis of the digital systems pulse ACS theory can be applied.

The continuous signal transformation to pulse sequence process, which parameters depend on this signal value in discrete moments of time, called the pulse modulation. Continuous signal called system input signal of pulse element or modulator, and output – pulse modulated sequence.

Depending on which pulse parameter (amplitude, duration, phase) is modulated by continuous signal, there are: the pulse-amplitude modulation (PAM), the pulse-duration modulation (PDM), the pulse-phase modulation (PPM). Also possible modulation, when amplitude, duration and phase are constant, and the pulse period or pulse frequency at the modulator output is the continuous signal function at the modulator input. Such type of modulation called the pulse-frequency modulation (PFM).

If the modulated parameter of the pulse sequence is defined by the input signal values in the fixed equidistant moments of time and remains constant on all the period of pulse existence, then such a modulation called the pulse modulation of the first type. There may be instances when the modulated parameter of the pulse sequence on all the period of pulse existence changes according to the current input signal value. Such modulation called the modulation of the second type.

First type pulse-amplitude modulation ACS refers to the category of linear system, therefore only the linear pulse ACS analysis and the design theory will be considered.

Linear pulse system is an automated control system, which besides units, described by the linear differential equations, has the pulse element, which transforms input signal into pulse sequence.

3.1 General Pulse System Unit Diagram

Single-circle pulse ACS can be represented as the interacting pulse and the continuous ACS parts (fig. 3.8).

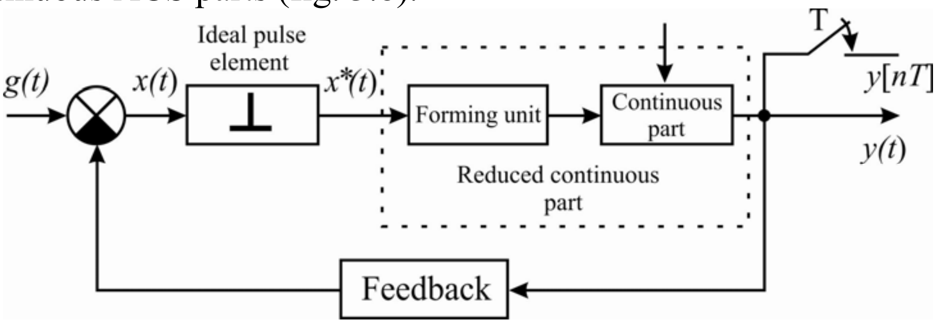


Fig. 3.8. Functional scheme of pulse system.

Plant, amplifying element and execution unitare usually a part of continuous part of the system. The pulse part is usually a control unit and consists of functional elements, which participate in the pulse signal transformation. This part can be realized by switches, modulators, pulsecontrollers, digital computing devices with the analog-digital and the digital-analog converters etc.

Functionally a pulse part can be considered as some continuous signal transformer into the pulse reference signal of any type. In linear pulse-amplitude systems output signal of pulse part is a pulse sequence, which amplitudes are proportional to the continuous signal values, in the equidistant quantization moments T. In the simplest case the pulse part is a real pulse element or a pulse modulator.

When studying the pulse systems their real pulse elements are usually substituted by successive connection of the ideal pulse element (IPE) and the forming unit (FU) (fig. 3.8).

The ideal pulse element under the influence of continuous input signal $x(t)$ (fig. 3.9) form ideal instantenuous pulses $x^*(t)$ of δ-function type, which «amplitude areas» are equal to the input signal values in quantization moments. Usually the gain of pulse element refers to the continuous system part, considering the ideal pulse element transmission gain equaled one.

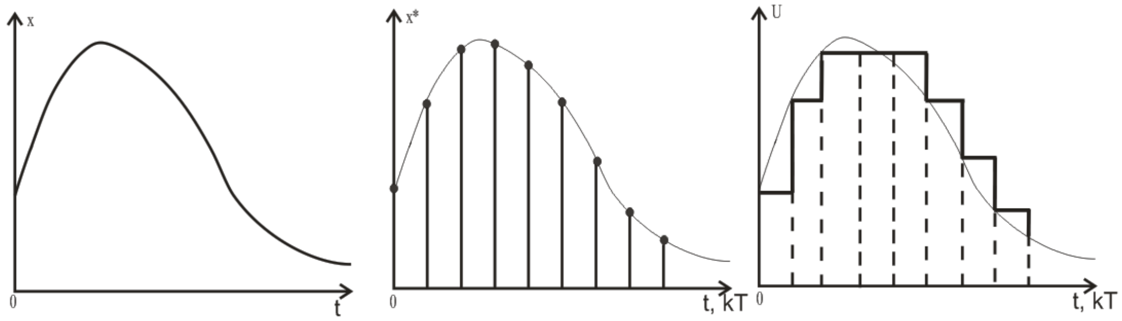


Fig. 3.9.Signal forming by real pulse element

The forming unit transform these pulses into signals $u(t)$ of the required form. Forming unit is pulse-amplitude modulator. Forming element response to instantenuous pulse of sequence $x^*(t)$ coincide with the real pulse sequence $u(t)$ at the real pulse element output. In practice, most often the data-hold device of zero order with the transfer function (2.56) is used as a forming unit

$$W_{FU}(s) = \frac{1-e^{-T\cdot s}}{s}. \qquad (3.1)$$

For the convenience of system analysis the forming unit is combined with the continuous part. In this case independently of real pulses form, pulse systems with amplitude modulation can be represented as the combination of ideal pulse element and reduced continuous part (fig. 3.10). Output signal of pulse system reduced continuous part is continuous signal, described by time function $y(t)$. To apply the discrete Laplace transform it is accepted to consider this signal in discrete moments of time, coinciding with moments of ideal pulse element shorting at input. This is equivalent to fictitious ideal pulse element switching on (fig. 3.10) at the systems output, operating synchronously and in-phase with the main pulse element. Reduced continuous part (RCP) reaction on δ-functions is a sum of pulse (weighting) step response $w(t)$. The transfer function of the reduced continuous part is equaled to

$$W_{RCP}(s) = W_{FU}(s)W_{CP}(s) \tag{3.2}$$

Unit diagram of pulse ACS depicted in fig. 3.10.

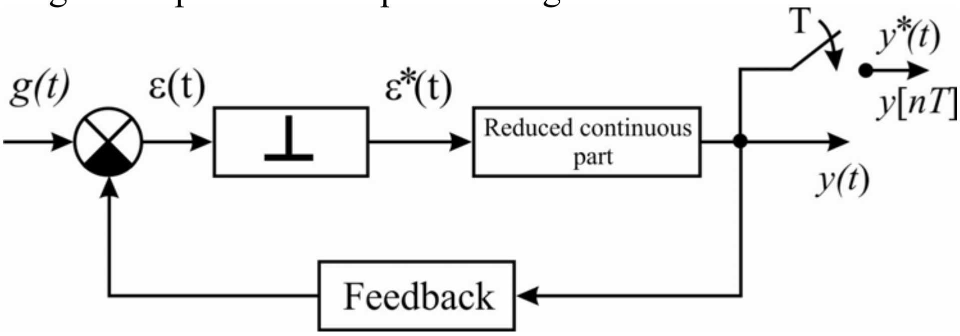


Fig. 3.10. Unit diagram of pulse ACS

Example 3.1. Form unit diagram of DCM rotation frequency pulse ACS.

Solution.

Let’s place the simplest pulse element and the pulse former after the summer in the unit diagram of DCM rotation frequency ACS, represented in fig. 1.4. On the basis of fig. 3.10 the pulse system unit diagram can be formed (fig. 3.11).

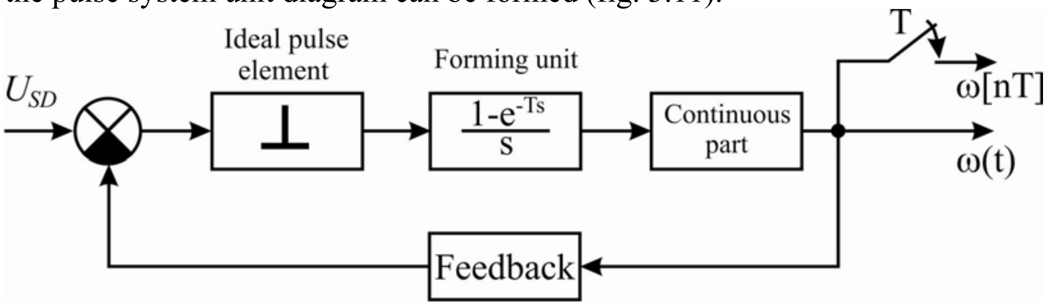


Fig. 3.11. DCM rotation frequency pulse ACS unit diagram

where $W_{FB}(s) = K_{FB} \cdot K_{TG}$;

$$W_{CP}(s) = \frac{K_{EA} \cdot K_M \cdot K_{RD} \cdot K_{G1} \cdot K_{M1}}{(T_M s+1) \cdot (T_G s+1) \cdot (T_E T_M s^2 + T_M s+1)}.$$

3.2 Mathematical Tools of Pulse Systems

3.2.1 Lattice function and differential equation

Reduced continuous part response only to discrete values of the continuous signal in quantization moments *nT*. Therefore, continuous function *x(t)*, defining continuous signal, can be substituted by appropriate lattice function

$x(nT) = x(t) \quad for \quad t = nT$;

$x(nT) = 0 \quad for \quad nT < t < (n+1)$, where $n = 0,1,2,...$

Thereby, for the lattice function obtaining according to specified continuous function *x(t)*, it is necessary to substitute in continuous function *t* by nT (fig. 3.12).

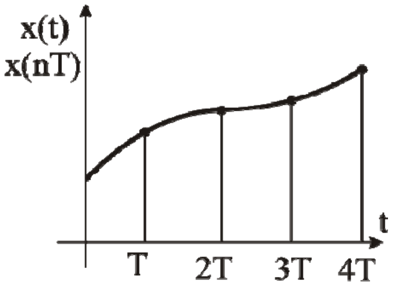


Fig. 3.12. Function $x(t)$ and its lattice function $x(nT)$

Lattice functions describe their "generative" continuous functions only in discrete moments of time, coincident with quantization moments. In intervals between the quantization moments the information about continuous functions changes is absent. If the quantization interval *T* is set, then the lattice function $x(nT)$ of function $x(t)$ is uniquely determined. The converse proposition is false.

For the identification of the continuous function behavior between quantization moments, the intermediate fixed time Δt=σ is introduced. In this case continuous function *x(t)* can be substituted by shifted lattice function

$x(nT,\sigma T)=x(t)\ \ for\ \ t=nT+\sigma T$.

Varying σT from 0 to T, collection of lattice functions $x(nT,\sigma T)$, $n=1,2,3,...$ can be obtained, which defines the function $x(t)$ for all t values.

When analyzing the continuous systems the differential equations are used, defining relationship between the continuous function $x(t)$ and its derivatives $\frac{d^k x(t)}{dt^k}$. Similarly, the correlation between the lattice function $x(n)$ and its difference $\Delta^k x(n)$ defines finite-difference equation or differential equation. If this correlation is linear, so the differential equation is called linear.

Linear differential equation with constant coefficients can be represented in the following form

$$a_k\Delta^k x(n)+a_{k-1}\Delta^{k-1}x(n)+\ldots+a_0x(n)=f(n), \tag{3.3}$$

or

$$b_kx(n+k)+b_{k-1}x(n+k-1)+\ldots+b_0x(n)=f(x), \tag{3.4}$$

where $f(n)$ is the certain lattice function, $x(n)$ is the desired lattice function, represents the solution of the difference equation.

This differential equation, which contains $x(n)$ and $x(n+k)$ called the differential equation of *k*-order. Classical approaches to differential equations solution in many respects analogous to the classical methods differential equations solution.

Differential equation solution gives the output signal values only in discrete moments of time $t = nT$. In many cases it's rather enough for the system behavior estimation. If appears the necessity in output signal information obtaining, the offset sequence is used.

In case, when $f(n) \equiv 0$, equations (3.3) and (3.4) called homogenous.

3.2.2 Z-transform application

For sequences *f(n)* discrete Laplace transform concept, defined by the expression (2.60), can be introduced

$$F^*(s) = D\{f(n)\} = \sum_{i=0}^{\infty} f(n) e^{-snT}. \tag{3.5}$$

In expression, like in case of the continuous Laplace transform, complex value $s = c + j\omega$, where c is the abscissa of absolute convergence. If $c < \infty$, then series, defined by the expression (3.5), convergences, and some expression corresponds to the original $f(n)$.

Z-transform is widespread for the pulse systems research, which is connected with the discrete Laplace transform.

Under Z-transform one understand sequence of images, defined by the expression

$$F(z) = \sum_{i=0}^{\infty} f(n) z^{-n}. \tag{3.6}$$

Here new notation is introduced $z = e^{ST}$.

Principle rules and theorems in respect to Z-transform are also true for discrete Laplace transform.

If image $F(z)$ is represented in the simplest table form, then transition to original doesn't make any difficulty. Complex fractionally rational form can be represented in form of the first order sum, then z-transform table can be used for original obtaining from every simple fraction.

Moreover, if $F(z)$ is a ratio of two polynomials $F(z) = \dfrac{B(z)}{A(z)}$, then analogue of the Heaviside decomposition expression, used for continuous systems, can be applied.

$$f(n) = \frac{B(1)}{A(1)} - \sum_{i=1}^{l} \frac{B(z_i)}{(1 - z_i) A'(z_i)} z_i^n,$$

where $A'(z)$ is the derivative $A(z)$ for z, and z_i are the denominator roots

($i = 1, 2, \ldots, l$).

Depending on the numerator polynomial order $F(z)$ and on roots, the decomposition formula can be changed.

Moreover, $F(z)$ can be expanded in a Laurent series (series in decreasing orders of z)

$$F(z) = C_0 + C_1 z^{-1} + \ldots + C_k z^{-k} + \ldots,$$

where $C_0 = f(0)$, $C_1 = f(1)$, $C_2 = f(2), \ldots, C_k = f(k)$ etc.

Series expansion can be performed in any manner, because such expansion is unique. Most suitable approach for the fractionally rational functions is dividing numerator and denominator.

Applying the Laurent series expansion, the original values of $f(n)$ or $f(n,\varepsilon)$ can be calculated in the discrete points without definition of images $F(z)$ poles.

$$f(n =)C_0 + C_1 \cdot \delta(t - T) + C_2 \cdot \delta(t - 2 \cdot T) + ... + C_k \cdot \delta(t - k \cdot T) + ... \quad (3.7)$$

3.3 Sampling Theorem

If the continuous dependence, in the result of quantization substituted by the lattice function, loss of data is taking place. Such data loss occurs also as the result of pulse modulator work. In limit, for the infinite quantization frequency, the continuous signal comes out. However, the low quantization frequency limit is of the most interest. If the frequency is too low, the continuous signal can considerably change in one interval. Therefore, initial signal restoration may appear to be impossible, with the help of its lattice function.

Let's define the condition, which fulfillment allows restoring initial signal completely.

Let's assume, continuous part of pulse system has amplitude-frequency characteristic, represented in fig. 3.13, with the bandwidth from 0 to ω_c.

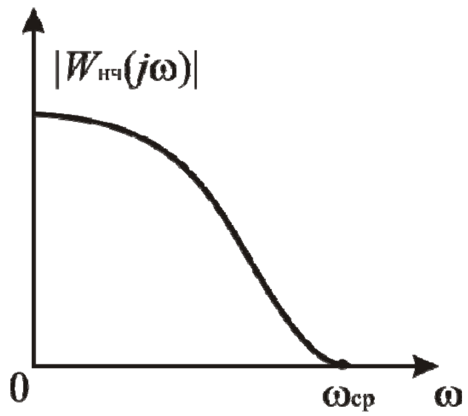


Fig. 3.13. Low frequency bandwidth of pulse ACS

The sampling theorem was stated and proved by V.A. Kotelnikov in 1933. According to this theorem, if the signal doesn't contain the frequencies higher then ω_c, it is entirely described by its values, measured in discrete moments of time with the interval $T = \pi / \omega_c$.

Thereby, quantization period must be

$$T \le \pi / \omega_c, \tag{3.8}$$

Example 3.2. Applying sampling theorem define quantization period of DCM rotation frequency pulse ACS.

Solution.

Let's use the system parameters from example 2.9 and continuous part transfer function from example 2.16:

$T_E = 0.02$ sec; $T_M = 0.5$ sec; $T_M = 0.1$ sec; $T_G = 0.7$; $K_{EA} = 15$;

$K_{SM} = 0.6$; $K_{RD} = 0.2$; $K_{G1} = 10$; $K_{M1} = 8.5$; $K_{TG} = 0.16$; $K_{FB} = 0.5$

$$W_{CP}(s) = \frac{K_{EA} \cdot K_M \cdot K_{RD} \cdot K_{G1} \cdot K_{M1}}{(T_M s + 1) \cdot (T_G s + 1) \cdot (T_E T_M s^2 + T_M s + 1)} =$$

$$= \frac{153}{0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 1}.$$

To plot the continuous part amplitude-frequency characteristic Mathcad is used. The continuous part bandwidth ω_{CP} is restricted to 10% from $H_{CP\,\max}(\omega)$.

Results are represented in fig. 3.14. Let's chose $\omega_{CP} = 3.25\ rad/\sec$ from the plot and, using expression (3.8), define pulse system quantization period $T \le \pi/\omega_C \le 3.14/3.25 \le 0.97$ sec.

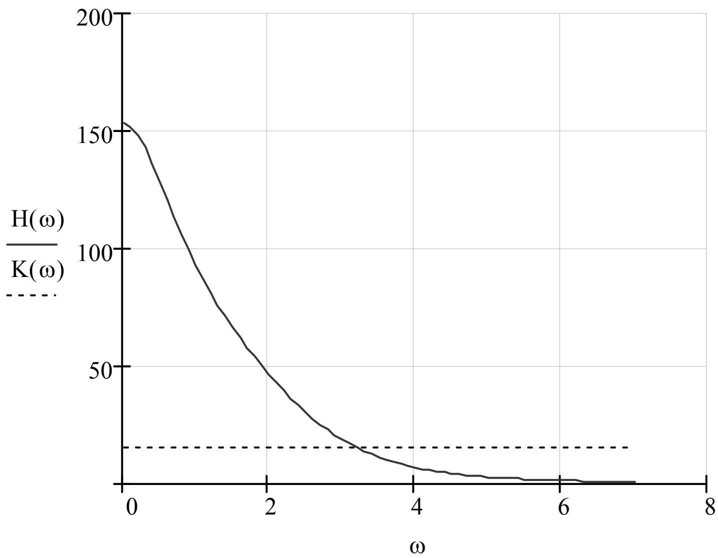


Fig. 3.14. Continuous part amplitude-frequency characteristic, $\omega_{CP} = 3.25\ rad/\sec$.

3.4 Pulse Transfer Function of Open-Loop Pulse System

Let's consider unit diagram depicted in fig. 3.15, pulse transfer function of the open-loop ACS for the case, when $W_{FB}(s)=1$.

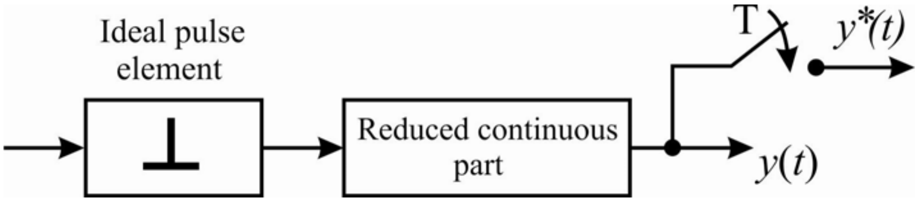


Fig. 3.15. Closed-loop pulse ACS unit diagram

The direct Laplace transform expression (L-transform) of continuous function $x(t)$ has the form $X(s)=\int_0^\infty x(t)e^{-st}dt$.

For the pulse system study discrete analogue of this transform is used – so-called the direct discrete Laplace transform (LD-transform).

$$X^*(s)=\sum_{n=0}^{\infty} x(nT)e^{-nsT}.$$

The difference of these transforms is, that integral in L-transform substituted by the sum, and instead of continuous function $x(t)$ corresponding lattice function $x(nT)$ is introduced.

Let's define LD-transform for output signal $y^*(t)$ of pulse system

$$Y^*(s)=L_D\{y^*(t)\}=\sum_{n=0}^{\infty} y(nT)e^{-nsT}. \tag{3.9}$$

Since the reduced continuous part response on δ-function represents pulse step response *w(t)*, so the signal value $y(t)$ at the output of the reduced continuous part is defined from the expression:

$$y(t)=\sum_{i=0}^{\infty} w(n-iT)x(iT),$$

Therefore, output signal value in moments of time $t=nT$ equals

$$y(nT)=\sum_{i=0}^{\infty} w(nT-iT)x(iT). \tag{3.10}$$

Substituting (3.10) in (3.9), obtain

$$Y^*(s)=\sum_{n=0}^{\infty}\sum_{i=0}^{\infty} w(nT-iT)x(iT)e^{-nsT}. \tag{3.11}$$

By means of substitution $m=n-i$ and $n=i+m$ expression (3.11) is reduced to

$$Y^*(s) = \sum_{i=0}^{\infty} x(iT). \sum_{m=-i}^{\infty} w(mT) e^{-isT} e^{-msT}.$$

Taking into account, that $w(mT) \equiv 0$ for $m < 0$, finally get

$$Y^*(s) = \sum_{i=0}^{\infty} x(iT) \cdot e^{-isT} \sum_{m=0}^{\infty} w(mT) \cdot e^{-msT}. \tag{3.12}$$

Proceeding from the definition of LD-transform, the expression (3.12) can be reduced to the form

$$Y^*(s) = X^*(s) W^*(s), \tag{3.13}$$

Then

$$W^*(s) = \frac{Y^*(s)}{X^*(s)} = \sum_{m=0}^{\infty} w(mT) e^{-msT} = L_D\left\{w(mT)\right\}, \tag{3.14}$$

where $W^*(s)$ is the open-loop pulse transfer function in S-image (so-called pulse transfer function with an asterisk).

Thereby, the open-loop pulse transfer function in S-form is the ratio of discrete Laplace transforms output to input at zero initial conditions.

Substituting $z = e^{sT}$ In (3.12) Z-transform equation can be obtained, i.e. $Y(z) = X(z) \cdot W_{OLS}(z)$,

$$W_{OLS}(z) = \frac{Y(z)}{X(z)} = \sum_{m=0}^{\infty} w(mT) z^{-m}. \tag{3.15}$$

Where $W_{OLS}(z)$ is the open-loop system pulse transfer function in z-transform. Therefore, pulse transfer function of open-loop pulse system can be defined as ration of z-image pulse output to image pulse input at zero initial conditions. Expression (3.15) shows that the pulse transfer function is Z-transform of the pulse transition function of system reduced continuous part, i.e. $W(z) = Z\{w(t)\} = Z\{w(nT)\}$.

Thereby, to define system pulse transfer function with forming unit of unconditioned type, it is necessary:

- to define the reduced continuous part transfer function: $W_{RCP}(s) = W_{FU}(s) W(s)$;
- to define the pulse transition function of reduced continuous part with the help of inverse Laplace transform: $w(t) = L^{-1}\left\{W_{RCP}(s)\right\}$;
- to define the system weighting sequence (lattice weighting function): $w(nT) = w(t)\big|_{t=nT}$;
- to find series sum in right part of the expression: $W(z) = \sum_{n=0}^{\infty} w(nT) z^{-n}$.

Since the δ-function image equals to one, and pulse transition function equals to $w(t) = L^{-1}\{W(s)\}$, then pulse transfer function in z-form can be defined as $W(z) = Z\{W(s)\}$, i.e., knowing the transfer function expression $W(s)$, and applying z-transform table, $W(z)$ can be obtained.

For the given case, when $W_{FB}(s) = 1$, the pulse transfer function in z-transform of the reduced continuous part $W_{RCP}(z)$ equals to the transfer function of the open-loop system $W_{OLS}(z)$.

Based on the proposed approach and the unit diagram (fig. 3.16) expression of the open-loop pulse transfer function $W_{OLS}(z)$ in z-transform can be written for any case

$$W_{OLS}(z) = Z\{W_{RCP}(s) \cdot W_{FB}(s)\}, \tag{3.16}$$

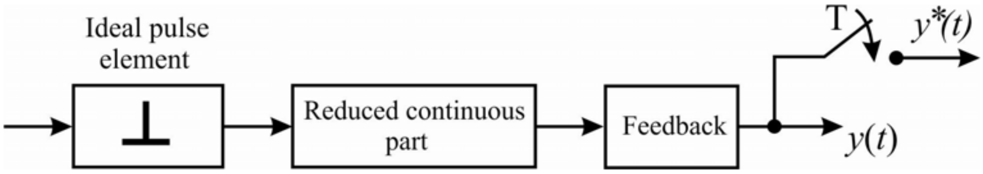


Fig. 3.16. Open-loop pulse ACS unit diagram

Applying the equations (3.1), (3.2), equation (3.16) can be represented as the following

$$W_{OLS}(z) = Z\left\{\frac{1-e^{-sT}}{s} \cdot W_{CP}(s) \cdot W_{FB}(s)\right\}.$$

Taking into account, that $e^{-sT} = z^{-1}$, finally write

$$W_{OLS}(z) = \frac{z-1}{z} \cdot Z\left\{\frac{1}{s} \cdot W_{CP}(s) \cdot W_{FB}(s)\right\}. \tag{3.17}$$

In the absence of the pulse former in ACS scheme, the expression $W_{OLS}(s)$ can be written as $W_{OLS}(z) = Z\{W_{CP}(s) \cdot W_{FB}(s)\}$.

Z-transform table (appendix 2) allows the obtaining expressions for the partial fraction only. Therefore, the complex fraction should be decomposed into partial fractions and then the table can be applied.

Example 3.3. Obtain pulse transfer functions of continuous and open-loop DCM rotation frequency ACS.

Solution.

Let's use the system parameters from the example 2.9 and continuous part transfer function expression from the example 2.16:

$$W_{CP}(s) = \frac{K_{EA} \cdot K_M \cdot K_{RD} \cdot K_{G1} \cdot K_{M1}}{(T_M s + 1) \cdot (T_G s + 1) \cdot (T_E T_{EM} s^2 + T_{EM} s + 1)}.$$

In this example to simplify the solution, the system order reduced to the second order, for $T_G = 0;\ T_E = 0$.

$$W_{CP}(s) = \frac{K_{EA} \cdot K_M \cdot K_{RD} \cdot K_{G1} \cdot K_{M1}}{(T_M s+1)\cdot(T_{EM} s+1)} = \frac{153}{(0.1s+1)\cdot(0.5s+1)}.$$

Let's use expression (2.71)

$$W_{CP}(z) = \frac{z-1}{z} \cdot Z\left\{\frac{1}{s} \cdot W_{CP}(s) \cdot W_{FB}(s)\right\} = \frac{z-1}{z} \cdot Z\left\{\frac{1}{s} \cdot \frac{153}{(0.1s+1)(0.5s+1)}\right\}.$$

Denominator roots are: $s_1 = 0;\ s_2 = -10;\ s_3 = 2$.

Applying Viete theorem, let's decompose expression in braces on partial fractions:

$$\left\{\frac{1}{s} \cdot \frac{153}{(0.1s+1)(0.5s+1)}\right\} = \left\{\frac{A}{s} + \frac{B}{(s+10)} + \frac{C}{(s+2)}\right\} = \tag{3.18}$$
$$= \frac{A(s+10)(s+2) + B\cdot s(s+2) + C\cdot s(s+10)}{s\cdot(s+10)(s+2)}$$

Left side of equation (3.18) will be equaled to right side, if their numerators are equal:

$$153 = A(s+10)(s+2) + B\cdot s(s+2) + C\cdot s(s+10) =$$
$$= \left(A+B+C\right)s_2 + \left(12A+2B+10C\right)s + 20A$$

Let's form three equations system, choosing expression at $s_2,\ s_1,\ s_0$

$\left(A+B+C\right) = 0$;

$\left(12A+2B+10C\right) = 0$;

$20A = 153$.

Solving this system, obtain coefficients values

$A = 7.65;\ B = 1.9125;\ C = -9.5625$.

Let's use Z-transormation table (look appendix 2),

For $T = 0.9$ sec. (look example 2.17) obtain

$$W_{CP}(z) = \frac{z-1}{z} \cdot \left[\frac{7.65z}{z-1} + \frac{1.9125z}{z-e^{-10T}} - \frac{9.5625z}{z-e^{-2T}}\right] =$$
$$= \frac{z-1}{z} \cdot \left[\frac{7.65z}{z-1} + \frac{1.9125z}{z-0.0001187} - \frac{9.5625z}{z-0.164}\right]$$

$$W_{CP}(z) = \frac{6.081z^2 - 5.7693z - 0.3127}{z^2 - 0.164z + 0.00001947}.$$

Open-loop ACS transfer function

$$W_{OLS}(z) = K_{FB} \cdot K_{TG} \cdot W_{CP}\left(z\right) = \frac{0.4865z^2 - 0.4615z - 0.025}{z^2 - 0.164z + 0.00001947}.$$

3.5 Closed-Loop Pulse System Transfer Function

In the pulse closed-loop system unit diagram (fig. 3.17) the pulse element can be located in any place, but there is a single approach for transfer function and output equation obtaining.

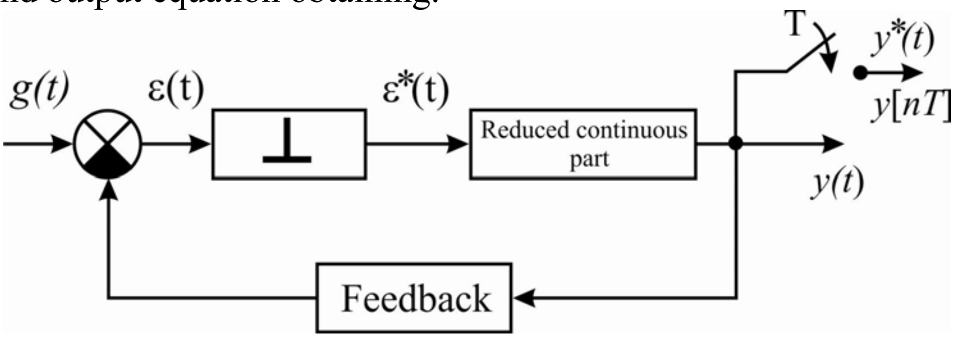


Fig. 3.17. Unit diagram of the pulse closed-loop system

The output function equation of the obtaining pulse system realized in the following form:

- It's considered, that the pulse element is a switch and the pulse ACS described for the case, when the switch is open-ended.
- It's considered, that discrete signal at the output of an open-ended switch exists and is written in Z-transform.
- Output ACS signal equation is written in Z-transform.
- When excluding intervening variables in equations, the output system equation is written, when it's possible its transfer function is also written.

Let's consider the introduced approach for some variants of the unit diagram.

The first case. Pulse element located after summer (fig. 3.17)

Let's write signal in the pulse element output in Z-transform:

$$\varepsilon^*(z) = Z\{g(s)\} - \varepsilon^*(z) \cdot Z\{W_{RCP}(s) \cdot W_{FB}(s)\}. \tag{3.19}$$

System output equation is written in Z-transform:

$$y^*(z) = \varepsilon^*(z) \cdot Z\{W_{RCP}(s)\}. \tag{3.20}$$

Let's evaluate $\varepsilon^*(z)$ from (3.19):

$$\varepsilon^*(z) + \varepsilon^*(z) \cdot Z\{W_{RCP}(s) \cdot W_{FB}(s)\} = g^*(z).$$

$$\varepsilon^*(z) = \frac{g^*(z)}{1 + Z\{W_{RCP}(s) \cdot W_{FB}(s)\}} \tag{3.21}$$

Substituting (3.21) in (3.20), obtain:

$$y^*(z) = \frac{Z\{W_{RCP}(s)\}}{1 + Z\{W_{RCP}(s) \cdot W_{FB}(s)\}} \cdot g^*(z);$$

Let's write system differential equation:

$$\left[1+Z\left\{W_{RCP}(s)\cdot W_{FB}(s)\right\}\right]y^{*}(z)=Z\left\{W_{RCP}(s)\right\}g^{*}(z) \qquad (3.22)$$

Divide in (3.22) $y^{*}(z)$ by $g^{*}(z)$ obtain the closed-loop system pulse transfer function

$$W_{CLS}(z)=\frac{W_{RCP}(z)}{1+W_{OLS}(z)}. \qquad (3.23)$$

The second case (fig. 3.18).

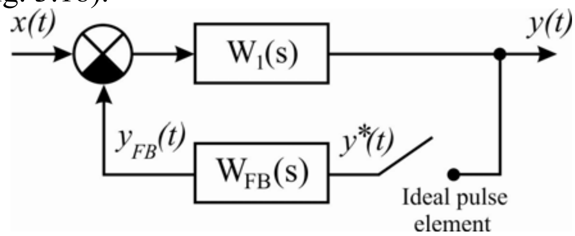


Fig. 3.18. System with pulse element in feedback loop

Let's write the signal equation $y^{*}(z)$ at the pulse element output when feedback loop is broken

$$y^{*}(z)=Z\left\{x^{*}(s)\cdot W_{1}(s)\right\}-z\left\{W_{FB}(s)\cdot W_{1}(s)\right\}\cdot y^{*}(z). \qquad (3.24)$$

Evaluating $y^{*}(z)$ from the equation (3.24), obtain the system differential equation: $\left[1+z\left\{W_{FB}(s)\cdot W_{1}(s)\right\}\right]\cdot y^{*}(z)=Z\left\{x^{*}(s)\cdot W_{1}(s)\right\}$

The third case (fig. 3.19).

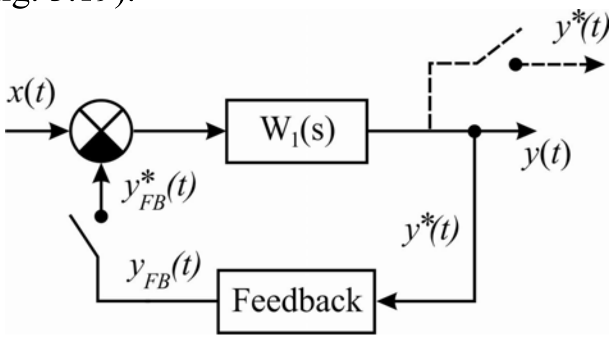


Fig. 3.19. System with pulse element in the feedback loop

Let's write signal equation coming to pulse element, when the feedback loop is broken:

$$y^{*}{}_{FB}(z)=Z\left\{x^{*}(z)\cdot W_{1}(s)\right\}-Z\left\{W_{1}(s)\cdot W_{OC}(s)\right\}\cdot y^{*}{}_{OC}(z). \qquad (3.25)$$

Let's evaluate $y^{*}{}_{FB}(z)$ from this equation

$$y^{*}{}_{FB}(z)=\frac{Z\left\{x^{*}(z)\cdot W_{1}(s)\right\}}{1+Z\left\{W_{1}(s)\cdot W_{oc}(s)\right\}},$$

where

$$W_1 \cdot W_{FB} \cdot x^*(z) = z \cdot \{W_1(s) \cdot W_{OC}(s) \cdot x(s)\},$$
$$W_1 \cdot W_{FB}(z) = z \cdot \{W_1(s) \cdot W_{FB}(s)\}.$$

Output signal system in Z-image:

$$y^*(z) = W_1 \cdot x^*(z) - W_1(z) \cdot y^*_{FB}(z). \qquad (3.26)$$

Substituting equation (3.25) into equation (3.26), obtain:

$$y^*(z) = \frac{W_1 \cdot x^*(z) + W_1 \cdot W_{FB}(z) \cdot W_1 \cdot x^*(z) - W_1 \cdot W_{FB} \cdot x^*(z) \cdot W_1(z)}{1 + W_1 \cdot W_{FB}(z)}.$$

Example 3.4. Obtain pulse transfer function of DCM rotation frequency closed-loop ACS.

Solution.

Let's use expression (3.23) and pulse transfer functions of continuous part and DCM rotation frequency open-loop ACS from the example 3.3:

$$W_{CLS}(z) = \frac{W_{RCP}(z)}{1 + W_{OLS}(z)}; \; W_{CP}(z) = \frac{6.081z^2 - 5.7693z - 0.3127}{z^2 - 0.164z + 0.00001947}.$$

$$W_{OLS}(z) = K_{FB} \cdot K_{TG} \cdot W_{CP}(z) = \frac{0.4865z^2 - 0.4615z - 0.025}{z^2 - 0.164z + 0.00001947}$$

$$W_{CLS}(z) = \frac{\dfrac{6.081z^2 - 5.7693z - 0.3127}{z^2 - 0.164z + 0.00001947}}{1 + \dfrac{0.4865z^2 - 0.4615z - 0.025}{z^2 - 0.164z + 0.00001947}} = \frac{6.081z^2 - 5.7693z - 0.3127}{1.4865z^2 - 0.6255z - 0.02498}.$$

3.6 Stability Analysis of Pulse Closed-Loop Systems

3.6.1 Pulse ACS stability estimation based on system characteristic equation roots

Pulse closed-loop ACS transfer function has a form

$$W_{CLS}(z) = \frac{B(z)}{A(z)} = \frac{b_0 z^e + b_{e-1} z^{e-1} + \ldots + b_e}{a_0 z^m + a_1 z^{m-1} + \ldots + a_m},$$

and its characteristic equation $A(z) = a_0 z^m + a_1 z^{m-1} + \ldots + a_m = 0$.

On the basis of correlation between *s* and *z*-planes system stability condition can be stated, having characteristic equation roots.

Statement: For the pulse closed-loop system to be stable, it's necessary and sufficient, that system characteristic equation roots were modulo smaller than one, i.e. $|z_i| < 1$, if $|z_i| = 1$ – system is on the stability boundary, and if $|z_i| > 1$ – system is unstable.

Example 3.4. Estimate DCM rotation frequency pulse ACS stability, using root method.

Solution.

Let's use transfer function of DCM rotation frequency closed-loop ACS from the example 3.3.

$$W_{CLS}(z) = \frac{6.081z^2 - 5.7693z - 0.3127}{1.4865z^2 - 0.6255z - 0.02498} = \frac{B(z)}{A(z)}.$$

Using Matlab , obtain

```
> W=tf([6.081 -5.7693 -0.3127],[1.4865 -0.6255 -0.02498])
Transfer function:
 6.081 s^2 - 5.769 s - 0.3127
------------------------------
1.486 s^2 - 0.6255 s - 0.02498
>> pole(W)
ans = 0.4575; -0.0367
```

Conclusion. Since characteristic equation modulo $|z_1|$, $|z_2|$ smaller then 1, DCM rotation frequency closed-loop ACS is stable.

3.6.2 Mihailov criterion analogue application for pulse system stability estimation

The physical sense of pulse and continuous systems frequency characteristics is similar. Feature of these characteristics for the pulse systems is correlation between harmonic sequences (harmonic lattice function) between output and input signals of pulse filter with transfer function $W^*(s)$ or $W(z)$. Envelopes of lattice functions change according to harmonic law.

If at linear pulse filter input harmonic sequence $x(nT) = A \cdot x \cdot \sin\omega \cdot nT$ is fed, then after step response finishing harmonic sequence $y(nT) = A \cdot y \cdot \sin(\omega \cdot nT + \varphi)$ will be at the system otput.

If the initial system information represented as a pulse transfer function $W^*(s)$ or $W(z)$, so for transition to frequency characteristics the argument substitution of $s = j\omega$ or $z = e^{j\omega T}$ are used.

As the result of such substitution amplitude-phase-frequncy characteristic (complex gain) of pulse system is obtained.

$$W^*(j\omega) = W(e^{j\omega T}). \tag{3.27}$$

Let's consider the transfer function of the following form

$$W(z) = \frac{b_m z^m + b_{m-1} z^{m-1} + ... + b_0}{a_n z^n + a_{n-1} z^{n-1} + ... + a_0} = \frac{B(z)}{A(z)}.$$

Let's make a substitution $z = e^{j\omega T}$, obtain amplitude-phase-frequency characteristic.

$$W(e^{j\omega T}) = \frac{b_m e^{jm\omega T} + b_{m-1} e^{j(m-1)\omega T} + \ldots + b_0}{a_n e^{jn\omega T} + a_{n-1} e^{j(n-1)\omega T} + \ldots + a_0}. \qquad (3.28)$$

Complex expression can be represented in form

$$W^*(j\omega) = P^*(\omega) + jQ^*(\omega) = R^*(\omega) e^{j\phi^*(\omega)},$$

where $P^*(\omega)$, $Q^*(\omega)$, $R^*(\omega)$, $\varphi^*(\omega)$ accordingly is real, imaginary, amplitude and phase characteristics of pulse system. Apparently, $R^*(\omega) = \sqrt{P^{*2}(\omega) + Q^{*2}(\omega)}$, $\phi^*(\omega) = \text{arctg}\frac{Q^*(\omega)}{P^*(\omega)} + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$,

$P^*(\omega) = R^*(\omega)\cos\phi^*(\omega)$, $Q^*(\omega) = R^*(\omega)\sin\phi^*(\omega)$.

For the fixed value of ω amplitude-phase-frequency characteristic represented as a vector on the plane $\left(P^*, jQ^*\right)$. When changing ω the end of vector $W^*(j\omega)$ plot some curve, which is called the amplitude-phase-frequency characteristic locus.

Let's mention the main frequency characteristic features of pulse systems, which result from the pulse transfer function properties.

1. For frequency characteristics plotting, it's sufficient to limit oneself to ω changing in the range from 0 to $\frac{\pi}{T}$.

2. Amplitude-phase-frequency characteristics of pulse system finish on real axis, because for $\omega = \frac{\pi}{T}$ complex gain (3.27) is always a real number. Among frequency criterions for pulse systems analysis the Nyquist and Mikhailov criterions analogues are used. Let's consider the Mikhailov criterion.

For the stability estimation of pulse ACS characteristic equation of closed-loop system is used. Making substitution $z = e^{j\omega T}$, obtain Mikhailov curve equation

$$D^*(j\omega) = a_0 + a_1 e^{j\varpi T} + a_2 e^{(j\omega T)^2} + \ldots + a_n e^{(j\omega T)m} = U^*(\omega) + jV^*(\omega), \qquad (3.29)$$

Applying Euler formula $e^{j\omega T} = \cos T\omega + j\sin T\omega$, write (3.29)

$$D(j\omega) = a_0 + a_1(\cos T\omega + j\sin T\omega) + a_2(\cos 2T\omega + j\sin 2T\omega) +$$
$$+ \ldots + a_m(\cos mT\omega + j\sin mT\omega).$$

When changing frequency ω from 0 to $\frac{\pi}{T_0}$, the Mikhailov curve points on the complex plain $U(\omega), jV(\omega)$ (fig. 3.20) are defined.

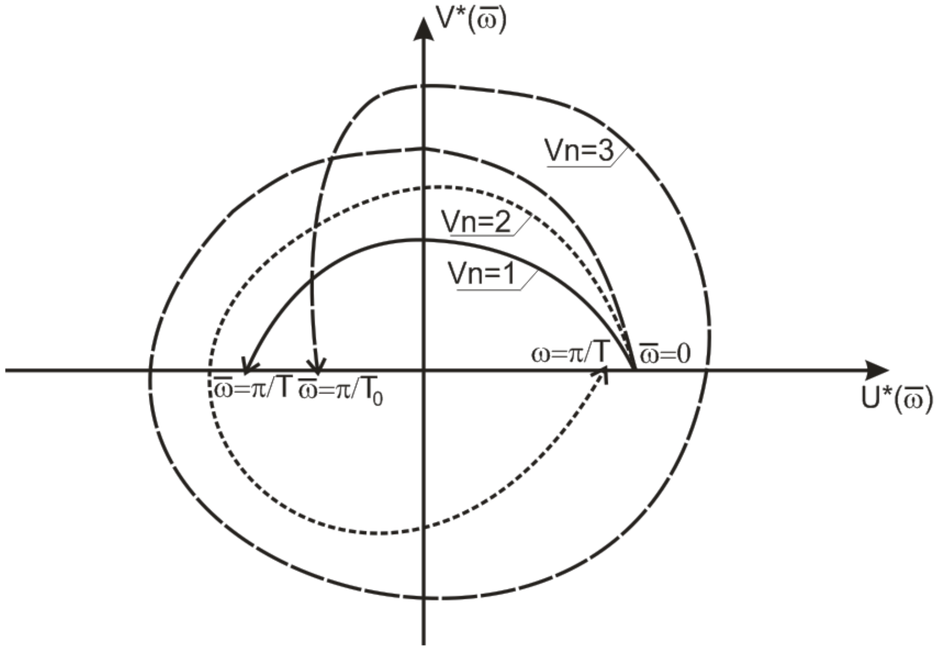


Fig. 3.20. Mikhailov curve locuses for stable system of 1st, 2nd , 3rd orders.

For the close-loop pulse ACS to be stable, it's necessary and sufficient that for ω=0 Mikhailov curve takes the beginning in the positive part of the real axis and by increasing the frequency from 0 to $\frac{\pi}{T_0}$ characteristic curve $D^*(j\omega)$ sequentially, without vanish, in the positive (counterclockwise) direction pass $2m$ quadrants, where m is the system order.

Example 3.5. Estimate DCM rotation frequency pulse ACS stability, using the Mikhailov criterion analogue.

Solution.

Let's use transfer function and characteristic equation of DCM rotation frequency closed-loop ACS from the example 3.4.

$$W_{CLS}(z) = \frac{6.081z^2 - 5.7693z - 0.3127}{1.4865z^2 - 0.6255z - 0.02498} = \frac{B(z)}{A(z)}.$$

Using Mathcad, obtain the Mikhailov locus (fig. 3.21)

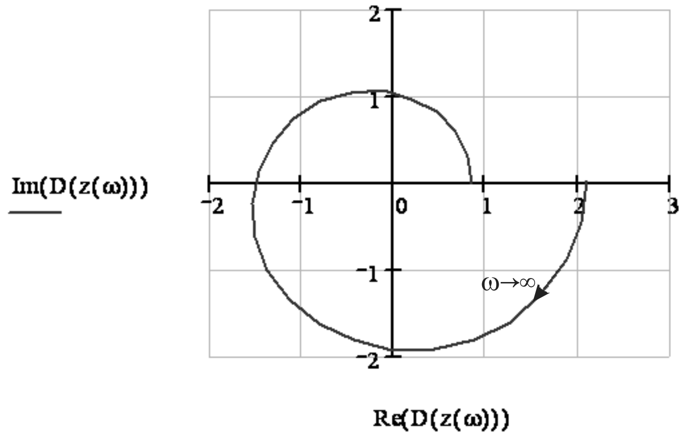


Fig. 3.21. Mikhailov locus

The Mikhailov curve for $\omega = 0$ takes the beginning on the positive real axis (0,836) and finish on the real axis (2,087). Pass sequentially, without vanish, $2m = 4$ quadrants. Therefore, DCM rotation frequency pulse ACS is stable.

3.7 Control Process Quality Estimation of Pulse ACS

For the pulse ACS quality indexes estimation applied the same approach as in the linear systems, but it has its own specific. Pulse system output signal $y(t)$ is continuous, but, as far as, for the system analysis discrete Laplace transform and fictitious quantizer are used, it can be assumed that output signal $y^*(t)$ is discrete or $y[nT]$. Having the discrete signal and making its approximation, obtain the continuous output signal. Applying the pulse transfer function of closed-loop ACS it can be written: $Y(z) = W_{CLS}(z) \cdot G(z)$. To obtain $y[nT]$ the Heaviside equation or Laurent series can be applied. The easier way for discrete signal obtaining is using program Control System Toolbox Matlab. Let's consider this approach on the example.

Example 3.6. Obtain transfer function and discrete signal of DCM rotation frequency closed-loop ACS. Define system quality indexes.

Solution.

Let's use system parameters and continuous part transfer function expression

$T_E = 0.02$ sec; $T_{EM} = 0.5$ sec; $T_M = 0.1$ sec; $T_G = 0.7$ sec; $K_{EA} = 15$; $K_{RD} = 0.2$; $K_{G1} = 10$; $K_{M1} = 8.5$; $K_{TG} = 0.16$; $K_{FB} = 0.5$.

$$W_{CP}(s)=\frac{K_{EA}\cdot K_M\cdot K_{RD}\cdot K_{G1}\cdot K_{M1}}{(T_M s+1)\cdot(T_G s+1)\cdot(T_E T_{EM} s^2+T_{EM} s+1)}=$$
$$=\frac{12.24}{0.0007s^4+0.043s^3+0.41s^2+1.3s+1}.$$

Discrete signal of DCM rotation frequency pulse ACS represented in fig. 3.22 and its quality indexes in fig. 3.23.

```
>> Wn=tf([12.24],[0.0007 0.043 0.41 1.3 1])
 Transfer function:
 12.24
--------------------------------------------
0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 1
```

>> *Wnd*=c2d(Wn,0.9) – conversion $W_{CP}(s)$ into pulse $W_{CP}(z)$ with sampling period $T=0.9$ sec.

```
Transfer function:
 5.031 z^3 + 2.607 z^2 + 0.04084 z + 1.89e-007
-------------------------------------------------------
z^4 - 0.3768 z^3 + 0.00426 z^2 - 5.644e-005 z + 1.602e-022
 Sampling time: 0.9
>> Woc=tf([0.08],1)
 Transfer function:
0.08
```

>> *Wz*=feedback(*Wnd*, *Woc*) – closed-loop pulse ACS transfer function obtaining $W_{3C}(z)$

```
 Transfer function:
 5.031 z^3 + 2.607 z^2 + 0.04084 z + 1.89e-007
-----------------------------------------------------
z^4 + 0.02571 z^3 + 0.2129 z^2 + 0.003211 z + 1.512e-008
 Sampling time: 0.9
>> pole(Wz)
ans =
 -0.0053 + 0.4612i
 -0.0053 - 0.4612i
 -0.0151
-0.0000
>> step(Wz)
```

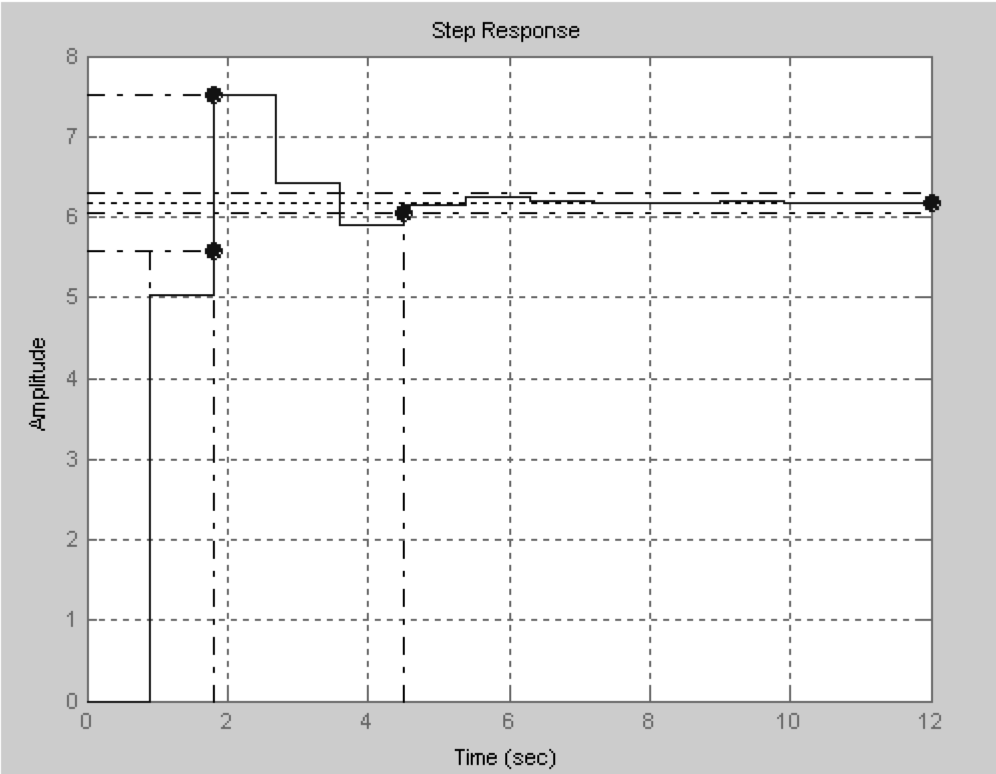


Fig. 3.22. Pulse ACS discrete signal

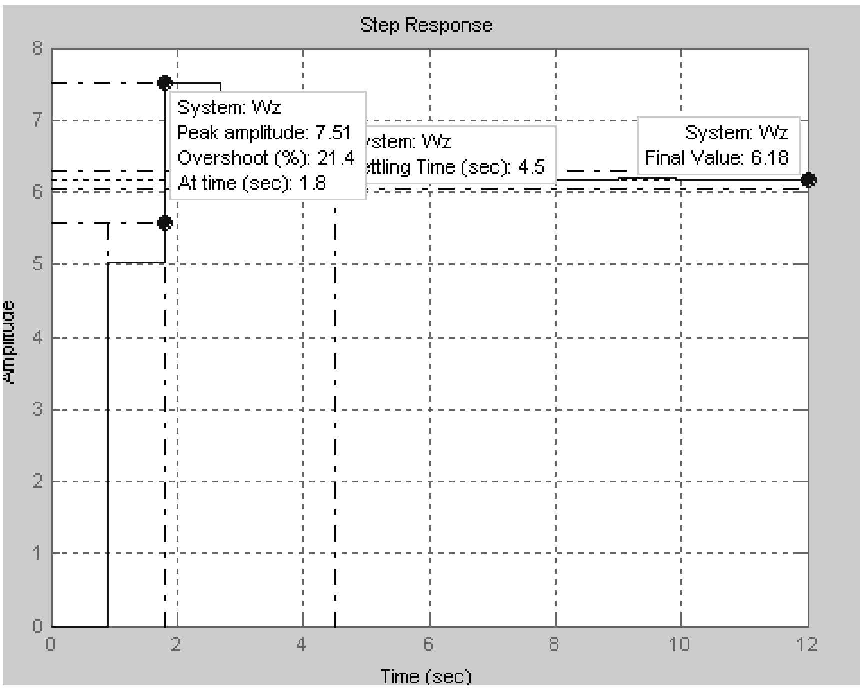


Fig. 3.23. Pulse ACS quality indexes

For the accuracy estimation of the pulse control systems in the steady-state condition the value of steady-state error for different reference signals is used.

In closed-loop pulse system (fig. 3.10) error *e*, reference signal *g* and the disturbance *f* related to each other by the following equation regarding *z*-image $\varepsilon(z) = W_{CLS}^{\varepsilon}(z)G(z) + W_{CLS}^{f}(z)F(z)$.

This expression contains two components of error, the first $E_g(z)$ specified by reference signal, and the second $E_f(z)$ by disturbance.

Steady-state error of pulse system can be calculated by the expression, which defines the finite value of original, i.e.

$$\underset{n\to\infty}{e(nT)} = \lim_{z\to 1}\frac{z-1}{z}E_g(z) + \lim_{z\to 1}\frac{z-1}{z}E_f(z). \tag{3.30}$$

Let's define steady-state error for the reference signal, assuming $f(t)\equiv 0$.

$$\underset{n\to\infty}{e(nT)} = \underset{n\to\infty}{e_g(nT)} = \lim_{z\to 1}\left[\frac{z-1}{z}\frac{1}{1+W_{OLS}(z)}G(z)\right]. \tag{3.31}$$

If the constant signal $g(t) = g_0 \cdot 1(t)$ fed at the system input, which z-image $G(z) = \frac{g_0 z}{z-1}$, then according to (3.30) position steady-state system error

$$\underset{n\to\infty}{e(nT)} = \lim_{z\to 1}\frac{g_0}{1+W_{OLS}(z)}. \tag{3.32}$$

For the reference signal $g(t) = g_1 \cdot t$, linearly dependent on time, Z-image $G(z) = \frac{g_1 T z}{(z-1)^2}$, and steady-state error, according to (3.30), defined by the following expression

$$\underset{n\to\infty}{e(nT)} = \lim_{z\to 1}\frac{g_1 T}{(z-1)(1+W_{OLS}(z))}. \tag{3.33}$$

And called speed system error.

If the input signal changes with the constant acceleration, i.e. $g(t) = g_2 t^2/2$, then Z-image has the form $G(z) = \frac{T^2 g_2 z(z+1)}{2(z-1)^3}$.

Steady-state error

$$\underset{n\to\infty}{e(nT)} = \lim_{z\to 1}\frac{g_2 T^2}{2(z-1)^2(1+W_{OLS}(z))}, \tag{3.34}$$

and that is called the acceleration system error.

For the given errors definition, one can use the error series

$$e(nT) = C_0 g(nT) + C_1 g'(nT) + \frac{C_2}{2!}g''(nT) + \ldots + \frac{C_m}{m!}g^{(m)}(nT) + \ldots \tag{3.35}$$

where $g', g'', \ldots, g^{(m)}$ are derivatives of $g(t)$ in the time moments T;

$$C_0 = W^{\varepsilon}_{CLS}(z) \, / \, z = 1; \; C_1 = \frac{\partial^1 W^{\varepsilon}_{CLS}(z)}{\partial z} \, / \, z = 1; \; \frac{C_m}{m!} = \frac{\partial^m W^{\varepsilon}_{CLS}(z)}{\partial z^m} \, / \, z = 1;$$

$$W^{\varepsilon}_{CLS}(z) = \frac{1}{1 + W_{OLS}(z)}.$$

Example 3.7. Define the control error of DCM rotation frequency pulse ACS for input $U_{in} = U_0 \cdot 1(t)$, $U_0 = 5\ V$.

Solution.

Let's use the expression (3.32) $\lim\limits_{n \to \infty} e(nT) = \lim\limits_{z \to 1} \frac{g_0}{1 + W_{OLS}(z)}$ and, using Control System Toolbox Matlab obtain $W_{OLS}(z)$.

```
Wn=tf([12.24],[0.0007 0.043 0.41 1.3 1])
 Transfer function:
 12.24
--------------------------------------------
0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 1
>> Woc=tf([0.08],1)
 Transfer function:
0.08
>> Wpc=Wn*Woc
 Transfer function:
 0.9792
--------------------------------------------
0.0007 s^4 + 0.043 s^3 + 0.41 s^2 + 1.3 s + 1
>> Wpcd=c2d(Wpc,0.9)
 Transfer function:
 0.4025 z^3 + 0.2086 z^2 + 0.003267 z + 1.512e-008
---------------------------------------------------------
z^4 - 0.3768 z^3 + 0.00426 z^2 - 5.644e-005 z + 1.602e-022
Sampling time: 0.9
```

$$\lim_{n \to \infty} e(nT) = \lim_{z \to 1} \frac{5(z^4 - 0.3768\, z^3 + 0.00426\, z^2 - 5.644 \cdot 10^{-5}\, z + 1.602 \cdot 10^{-22})}{z^4 - 0.0257\, z^3 + 0.21286\, z^2 - 0.0033\, z} =$$

$$= \frac{3.137}{1.1839} = 2.65.$$

4 CONTROL TASKS AND STUDY GUIDE

4.1 General Study Guide

In the course paper (part 1) linear stationary ACS is a subject of study. In the project (part 2) nonlinear pulse ACS is investigated.

Initial data for the ACS study is given as a system circuit schematic, its parameters numerical values table and list of questions are to be considered.

When linear ACS stability region plotting, take gain constant of amplifying element as varying parameter A.

Matlab or Mathcad programs are acceptable to use during the project carrying out.

Nonlinear static characteristic types of nonlinear element, electronic amplifier, magnetic amplifier and thyristor converter.

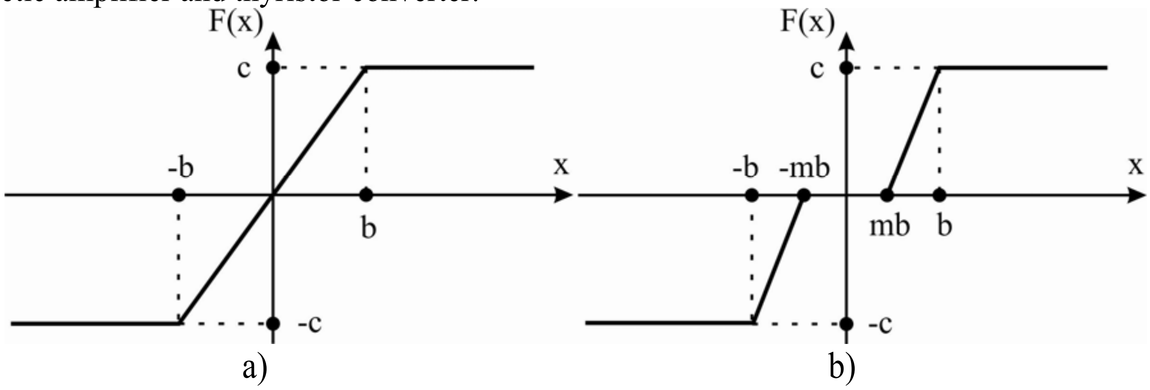


Fig. 4.1. Nonlinear element static characteristics: a) EA, b) MA and TT.

Static characteristic parameter values assume equaled «*b*»=4 for thyristor converter, «*b*»=0.5 – for magnetic amplifier; «*b*»=1 – for electronic amplifier; parameter «*m*»=0.1; value «*c*» defined from gain constant of the given amplification element.

If there will be no periodical solutions, when carrying out paragraph 4, then it is necessary to substitute amplification element coefficient or nonlinear element parameters.

To form pulse system scheme is necessary:

- use the unit diagram of closed-loop system for the reference signal;
- in the given scheme, place the ideal pulse element with the pulse former after the summer;

4.2 Guide Lines for Project Text Document Content

The project must be drawn on the format sheets A4 and contain:

- cover page;
- project content;
- project task;
- ACS unit diagram;
- open-loop and closed-loop ACS transfer functions;
- estimation results of system stability in Matlab or Matcad;
- step response calculation results according to system unit diagram in

Matlab;

- conclusion about system analysis quality;
- references.

4.3 Guide Lines for Project Graphic Material Appearance

Requirements for project graphic material:

- functional and structural schemes must be drawn according to the required format;
- signal motion direction and their title must be drawn in structural schemes;
- diagrams must be represented with dimensional axis and obligatory with grid lines.

Conclusion

In the book authors made an attempt to represent all the basic approaches to analysis and design of linear, nonlinear and pulse systems. In order to make an explanation more clear, a lot of different examples were attached. Theoretical part contains methods that allow solving main problems of control theory as identification procedure, system stability analysis and controller design.

Actually, book contains three parts, the first one is devoted to linear systems questions study; the second part is more complicated and consequently more interesting; in the third part approaches to pulse systems research are considered.

The main particularity of this book is to acquire practical knowledge in control theory, so it is focused on course work realization. In order to understand the biggest part of control system design procedure and to make clear corresponding between control theory methods and real systems the tasks for course paper begin with principle scheme of systems.

The results of control theory application are everywere around us, it makes this course important and very interesting.

Scheme №1. SYNCHRONOUS GENERATOR VOLTAGE ACS

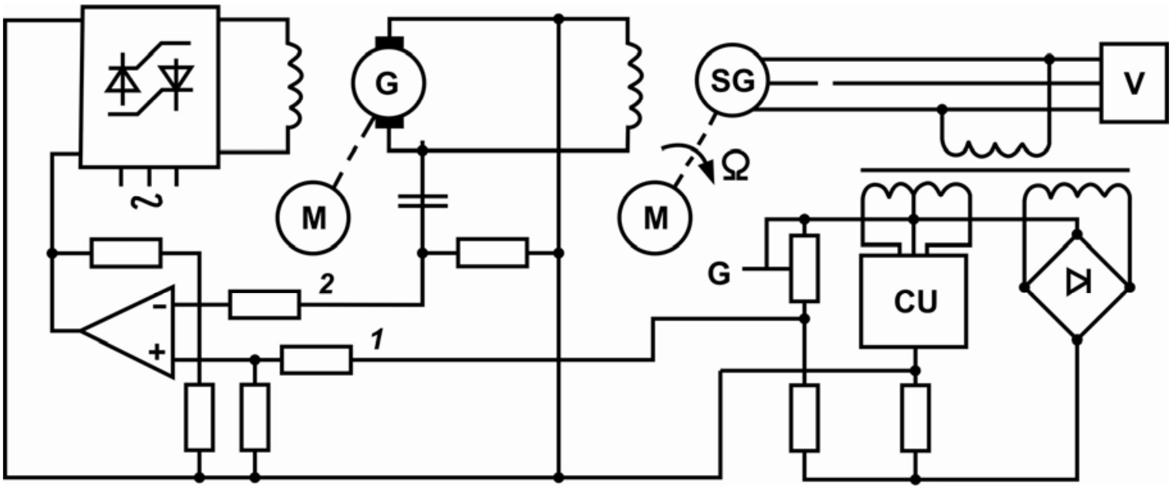


Fig. 0.1. Circuit schematic

Table 1

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{I_1}	V/rad	20	8	10	7,2	12,5	6,5	9,8	5,6	6,4	12
K_{I_2}		0,2	0,08	0,1	0,07	0,12	0,1	0,13	0,1	0,15	0,08
T_I	s	0,025	0,018	0,014	0,028	0,018	0,022	0,02	0,016	0,03	0,021
K_{A_1}		4,6	8,25	12,3	11,3	8,7	18,9	12,2	20	9,1	23,7
K_{A_2}		1	1	1	1	1	1	1	1	1	1
K_{TC}		11,8	12,8	8,1	8,8	9,08	14,2	11,5	8,3	9	4,6
T_{TC}	s	0	0	0	0	0	0	0	0	0	0
K_{G_1}		1,05	1,09	1,2	1,12	1,15	1,07	1,11	1,08	1,18	1,1
K_{G_2}	V/A	0	0	0	0	0	0	0	0	0	0
T_G	s	0,0425	0,127	0,087	0,079	0,12	0,07	0,78	0,066	0,1	0,042
K_{SG_1}		2,2	3,6	3,5	3,45	3,4	2,1	2,05	2,12	2,08	3,3
K_{SG_2}	V/A	16	23	22	15	18	20	17	14	13	24
T_{SG}	s	0,55	0,27	0,42	0,37	0,34	0,45	0,3	0,28	0,385	0,6
T	s	0,1	0,085	0,079	0,112	0,089	0,071	0,085	0,076	0,126	0,13
I_V	A	0,5	1	1,5	0,75	0,8	1,75	2	2,5	2,25	2,75

Scheme №2. ARTERIAL PREASSURE ACS BY THE EXTRA CORPOREAL CIRCULATION

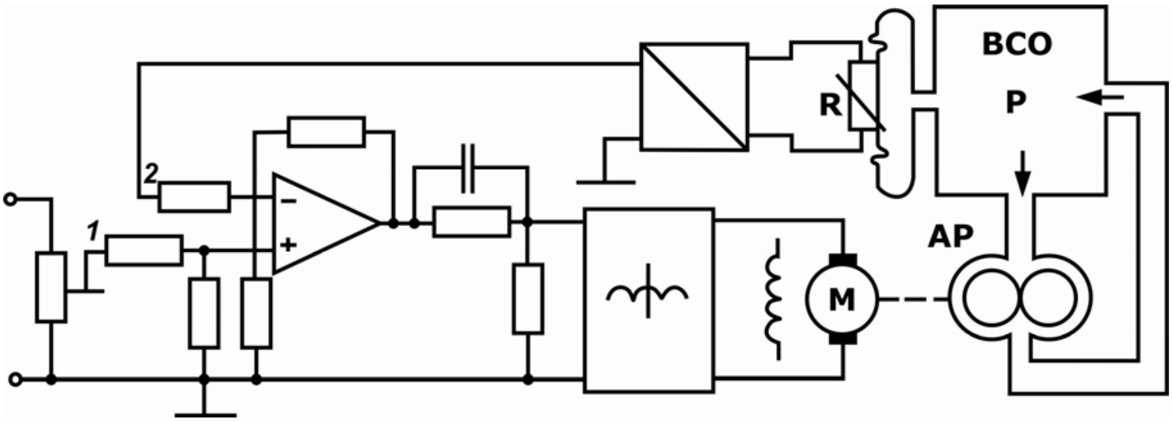


Fig. 0.2.Circuit schematic

Table 2

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{A_1}		35,7	36,3	19,8	24,7	24,3	14,8	10	27,7	14,7	21,1
K_{A_2}		35,7	36,3	19,8	24,7	24,3	14,8	10,	27,7	14,7	21,1
T	s	0,25	0,28	0,36	0,27	0,29	0,42	0,32	0,45	0,33	0,4
δ		0,15	0,11	0,147	0,16	0,103	0,133	0,172	0,1	0,08	0,105
K_{MA}		24	28	22	19,8	27,6	18,2	15	16,2	15,2	14,8
T_{MA}	s	0	0	0	0	0	0	0	0	0	0
K_{M_1}	$rad/V \cdot s$	17,2	14,6	16,8	21	17,5	24	18,8	15,6	20	18
K_{M_2}	$rad/N \cdot m \cdot s$	0	0	0	0	0	0	0	0	0	0
T_E		0	0	0	0	0	0	0	0	0	0
T_{EM}		0,5	0,63	0,56	0,48	0,59	0,91	0,52	0,83	0,76	0,7
K_a	$mm\,Hg/V$	0,65	0,5	0,56	0,54	0,4	0,6	0,5	0,4	0,7	0,7
K_b		0,2	0,35	0,36	0,25	0,6	0,48	0,42	0,51	0,4	0,25
T_a	s	8,3	14	15	5,8	8,9	14	7,4	17,8	12	11
T_b	s	25	35	44	23	50	40	19	34	32	42
K_{PS}	$V/mm\,Hg$	0,4	0,35	0,42	0,36	0,25	0,32	0,45	0,28	0,33	0,3
K		115	100	91	80	72,5	63	46	40	33	31
f_L		10	15	20	22	18	25	24	12	14	17

Scheme № 3. ELECTRONIC FURNACE TEMPERATURE ACS

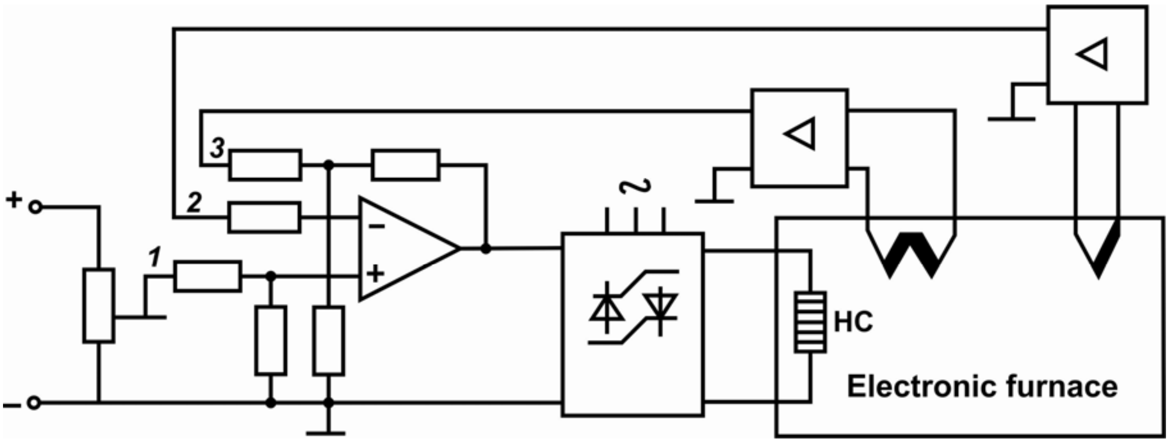


Fig. 0.3.Circuit schematic

Table3

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{A_1}		4,3	4	5	4	2	1	6,2	5,2	4	2
K_{A_2}		4,3	6,5	8,8	2,4	2	2,6	4	8,6	2,2	2
K_{A_3}		4,3	6,5	4,4	2,4	2	2,6	4	4,3	2,2	2,16
K_{TC}		6,5	8	14,2	9,6	5,1	6,4	8	4,2	7,5	6
T_{TC}	s	0	0	0	0	0	0	0	0	0	0
K_H	$\deg/V$	5	4,8	6,4	5,6	4,4	3,8	6,4	2,4	6	4
T_H	s	250	140	220	180	120	160	170	275	320	87
K_P		0,9	0,8	0,94	0,88	0,96	0,7	0,85	0,92	0,76	0,65
T_P	s	790	400	690	660	420	580	440	760	910	600
K_{TS_1}	V/rad	0,5	0,2	0,1	0,8	1,2	0,75	0,4	0,5	1,05	0,8
T_{TS_1}	s	2,35	2,15	2,3	3,6	2,2	5,6	2,3	5,9	3,4	3,8
K_{TS_2}	V/rad	0,5	0,4	1	3,2	2	0,75	1	1,8	2,4	2
T_{TS_2}	s	28,1	12,2	8,3	7,2	14	21,5	7,7	28,6	16,	10
K		63	40	75	90	52	33	70	40	80	25
f_L	deg	18	20	25	26	28	30	31	24	29	19

Scheme № 4. DIRECT CURRENT MOTOR ROTATION FREQUENCY ACS

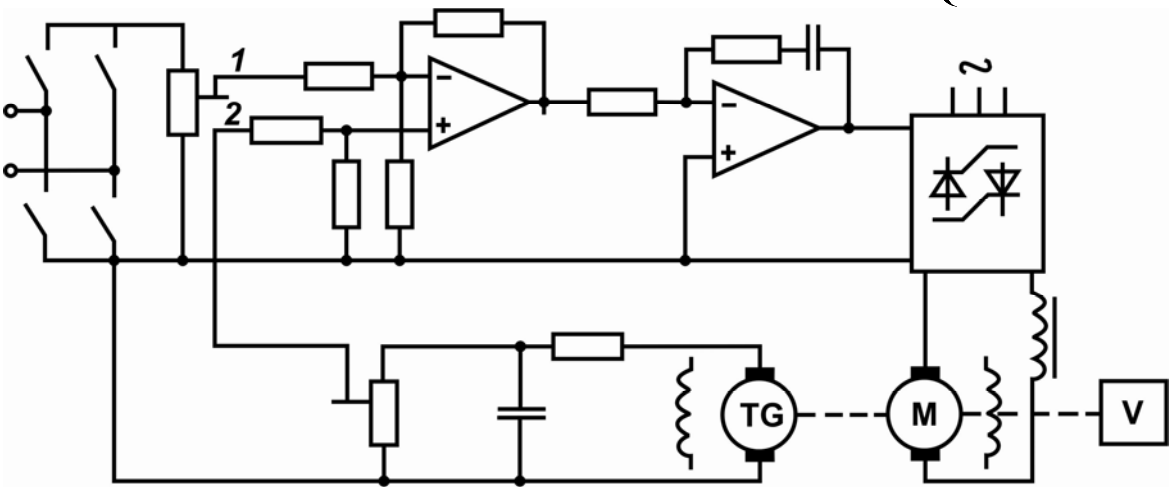


Fig.0.4.Circuit schematic

Table 4

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{A_1}		10	9,8	6,5	5	5,6	12,5	7,8	10,6	6,9	5,6
K_{A_2}		10	9,8	6,5	5	5,6	12,5	7,8	10,6	6,9	5,6
T_1	s	T_{TC}	T_{TC}	T_{TC}	T_{TC}	T_{TC}	T_{TC}	T_{TC}	T_{TC}	T_{TC}	T_{TC}
T_2	s	0,126	0,044	0,063	0,109	0,085	0,08	0,071	0,068	0,095	0,056
T_3	s	0,016	0,0063	0,01	0,02	0,015	0,008	0,0085	0,012	0,01	0,0085
T_4	s	0,126	0,044	0,063	0,109	0,085	0,08	0,071	0,68	0,095	0,056
K_{TC}		13,8	13,8	12,7	11,5	13,8	13,2	12,5	13,8	12,7	13,8
T_{TC}		T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1	T_1
K_{M_1}	$rad/V \cdot s$	2,85	0,95	1,43	1,9	2,4	0,96	1,43	2,85	1,9	2,4
K_{M_2}	$rad/N \cdot m \cdot s$	4,6	8,4	6,4	2,8	3,6	4,2	2	5,6	3,2	4
T_E	s	0,021	0,009	0,013	0,012	0,011	0,013	0,011	0,009	0,013	0,01
T_{EM}	s	0,522	0,233	0,264	0,448	0,391	0,368	0,327	0,456	0,413	0,366
K_P		0,2	0,4	0,35	0,25	0,6	0,4	0,45	0,2	0,34	0,25
K_{TG}	$V \cdot s/rad$	0,13	0,2	0,2	0,4	0,1	0,2	0,2	0,22	0,2	0,3
K		81,16	233,6	131,2	100,2	130,9	158, 4	176,7	270	119,2	248,4
M_L	$N \cdot m$	46	84	64	28	36	42	20	56	32	40

Scheme №5. SERVO SYSTEM

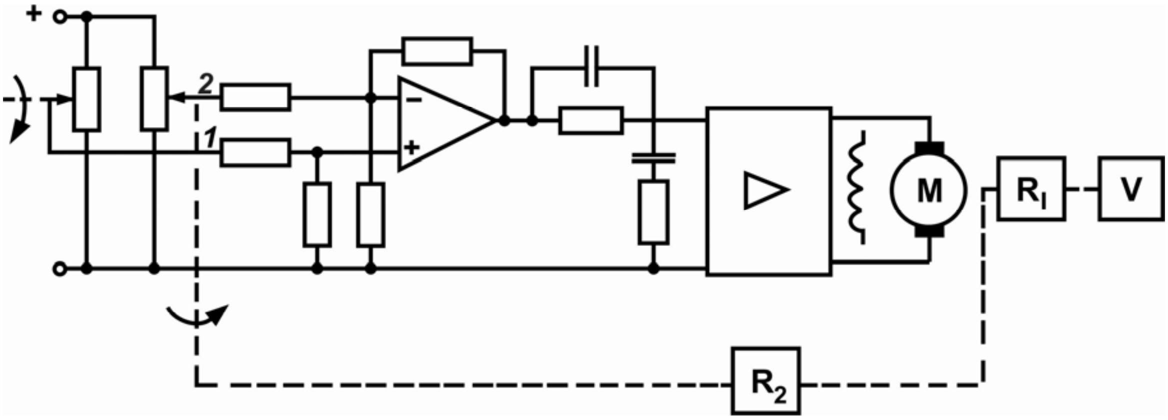


Fig. 0.5.Circuit schematic

Table 5

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{AMS}	V/rad	30	25	27,6	32	18	15	20	29,6	28,6	33
K_{A_1}		25	20	19	22	50	22,5	23	15	14	12,5
K_{A_2}		25	20	19	22	50	22,5	23	15	14	12,5
T_1	s	0,28	0,174	0,166	0,126	0,063	0,112	0,056	0,19	0,2	0,158
T_2	s	0,08	0,105	0,112	0,091	0,051	0,083	0,038	0,05	0,1	0,102
T_3	s	0,8	0,7	0,477	0,546	0,268	0,387	0,164	0,594	0,87	0,403
T_4	s	0,028	0,026	0,039	0,021	0,012	0,024	0,013	0,016	0,023	0,04
K_{EA}		20	21	18	24	18,5	30	16	27	22	17
K_{M_1}	$rad/V \cdot s$	0,95	1,43	1,9	1,5	0,98	1,44	1,95	1,9	1,45	0,95
K_{M_2}	$rad/N \cdot m \cdot s$	5,2	0,65	15	8,6	36	0,8	24	17,2	7,8	40
T_E	s	0	0	0	0	0	0	0	0	0	0
T_{EM}	s	0,25	0,33	0,398	0,295	0,166	0,224	0,107	0,135	0,141	0,27
K_{R_1}		-	-	-	-	-	-	-	-	-	-
K_{R_2}		0,01	0,008	0,008	0,009	0,007	0,011	0,009	0,0088	0,01	0,012
M_L	$N \cdot m$	5	0,5	15	8	4	0,8	2	3	4	5

Scheme №6. HEAT EXCHANGER TEMPERATURE ACS

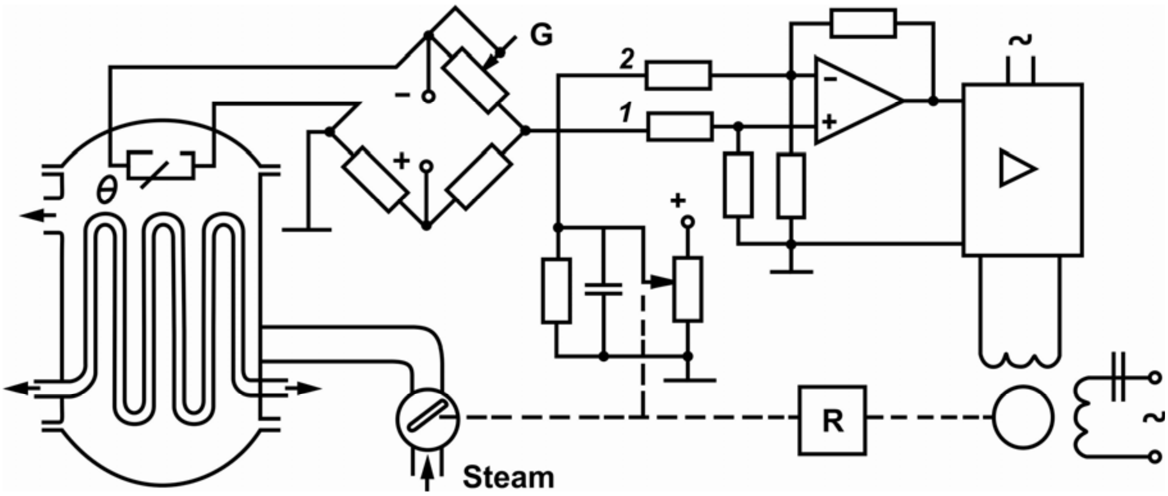


Fig. 0.6. Circuit schematic

Table 6

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{I_1}	V/rad	12	15	12,5	14	12	10	15	10	12,5	14
K_{I_2}	$V/\deg$	2,1	1,8	1,7	1,9	2,2	2,1	2,4	2,	2,3	2,6
T_I	s	0	0	0	0	0	0	0	0	0	0
K_{A_1}		1	1	1	1	1	1	1	1	1	1
K_{A_2}		1	1	1	1	1	1	1	1	1	1
K_{EA}		10,2	18,7	8,5	17,5	10	12,5	12	12	11,1	10,9
K_{M_1}	$rad/V\cdot s$	1,4	1,1	1,24	0,95	1,4	1,15	1	0,94	1,05	0,9
K_{M_2}	$rad/N\cdot m\cdot s$	0	0	0	0	0	0	0	0	0	0
T_{EA}	s	0	0	0	0	0	0	0	0	0	0
T_{EM}	s	0	0	0	0	0	0	0	0	0	0
K_R		0,011	0,01	0,013	0,008	0,012	0,01	0,011	0,014	0,015	0125
K_P	V/rad	8	12	15	8,5	9	7,5	8,2	11	9,1	10
T_{FB}	s	4,5	1,78	2,2	2,6	4	4,8	4,26	3,9	3,23	5,4
K_{HE_1}	$\deg/rad$	127	183	172	156	95,6	171	118	178	153	150
K_{HE_2}	$rad/\deg$	1	1	1	1	1	1	1	1	1	1
T_{HE_1}	s	65	30	24	55	50	125	100	90	85	110
T_{HE_2}	s	1,25	0,5	0,63	0,91	1,07	1,6	1,4	1,17	1	1,38
f_L	deg	20	25	24	28	30	25	26	31	32	23

Scheme №7. STEAM TEMPERATURE ACS

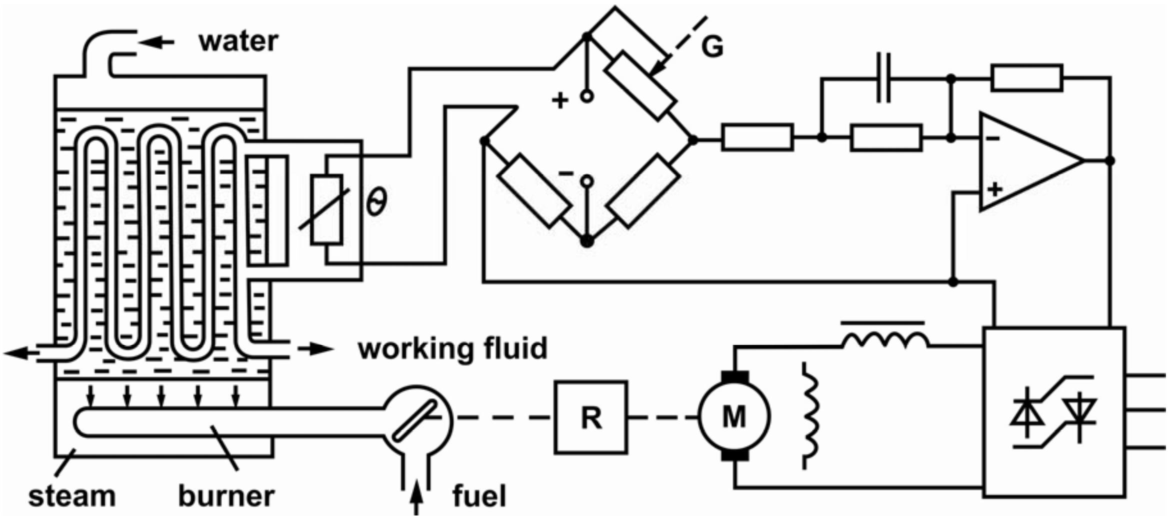


Fig. 0.7. Circuit schematic

Table7

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{I_1}	V/rad	14	12	10	15	12	20	14	15	10	11
K_{I_2}	V/deg	0,5	0,2	0,4	0,24	0,21	0,15	0,12	0,25	0,4	0,3
T_I	s	0	1	1,6	0	1,4	1	0	1,25	1,7	0
K_1		0,4	0,25	0,2	0,18	0,1	0,2	0,15	0,12	0,16	0,2
T_1	s	2,2	2,3	2,3	2,85	2	1,52	0,63	1,66	1,84	2
$T_2 = T_E = T_F$	s	0	0	0	0	0	0	0	0	0	0
K_{TC}		6,4	10,2	13,7	9,8	12	14,5	18	16,6	14	10
K_{M_1}	$rad/V \cdot s$	11	5	4	3,2	4,8	9,6	9,6	5,6	4,8	3,2
K_{M_2}	$rad/N \cdot m \cdot s$	0	0	0	0	0	0	0	0	0	0
T_{EM}	s	0,5	0	0,42	0,55	0	0,32	0,35	0	0,32	0,25
K_R	V/rad	0,028	0,07	0,05	0,09	0,06	0,04	0,12	0,08	0,075	0,04
K_G	deg/rad	40	85	120	52	180	152	100	90	65	230
T_B	s	1,3	0,35	0	1,8	0,4	0	1,2	1	0	1,5
K_P		0,8	0,9	0,95	1	0,6	0,75	0,62	0,5	0,55	0,4
T_P	s	630	950	2500	1150	1260	790	690	2000	1380	660
f_L	deg	30	22	34	31	35	40	42	45	50	55

Scheme№8.DIRECT CURRENT MOTOR ROTATION FREQUENCE ACS

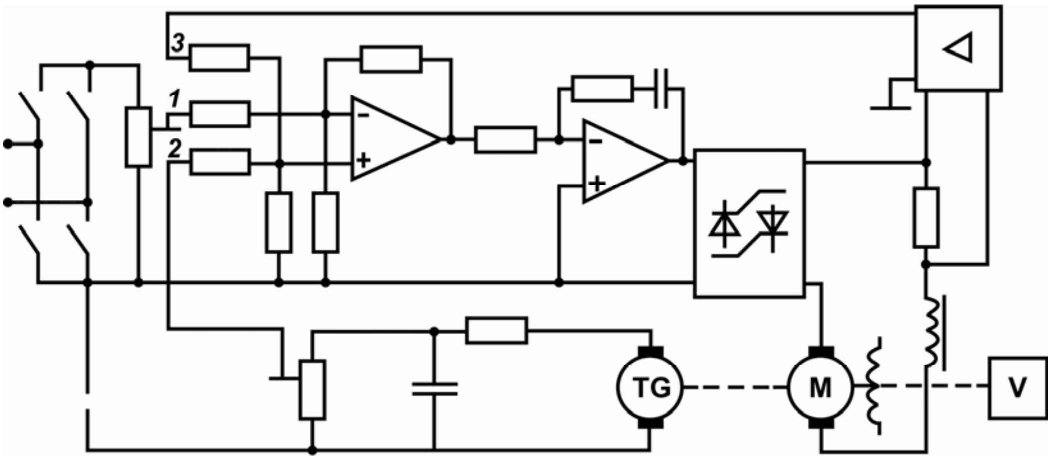


Fig. 0.8. Circuit schematic

Table8

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{A_1}		4	4,2	3,5	10	4,2	1	8,5	6	3	1
K_{A_2}		6	16,3	22,3	41,7	39,5	4,5	28,8	17,5	10,65	2,84
K_{A_3}		0,035	0,27	0,026	0,36	0,092	0,11	1	1	0,101	0,1
T_1	s	0,083	0,063	0,093	0,054	0,112	0,08	0,068	0,072	0,1	0,04
T_2	s	0,05	0,044	0,04	0,064	0,1	0,012	0,08	0,12	0,06	0,01
K_{TC}		8, 6	9	5	6,4	10,2	7,5	12,5	9,6	8,2	6,6
T_{TC}	s	0	0	0	0	0	0	0	0	0	0
K_{M_1}	$rad/V \cdot s$	1,4	2,4	1,9	1,43	0,96	1,8	2,4	0,95	2,85	1,9
K_{M_2}	$rad/N \cdot m \cdot s$	6,4	26	2,8	24	8	10	3,6	5,6	21	36
T_E	s	0,012	0,018	0,016	0,01	0,015	0,018	0,05	0,022	0,035	0,011
T_{EM}	s	0,297	0,497	0,382	0,482	0,42	0,247	0,155	0,575	0,58	0,247
R_{FB}	Ω	0,8	1,65	0,3	1	1,2	2,4	2,1	1,9	1,5	1,2
K_{VS}	$V \cdot s/rad$	0,1	0,08	0,12	0,2	0,08	0,13	0,2	0,16	0,15	0,1
T_{VS}	s	0,022	0,01	0,027	0,015	0,025	0,01	0,011	0,012	0,014	0,013
K_P		0,4	0,25	0,1	0,2	0,2	0,15	0,016	0,3	0,12	0,25
K_{CS}	Ω	0,12	0,1	0,05	0,08	0,17	0,11	0,25	0,11	0,2	0,085
M_L	$N \cdot m$	6	10	3	12	8	10	4	5	2	4

Scheme №9. SERVOSYSTEM WITH COMBINED CONTROL

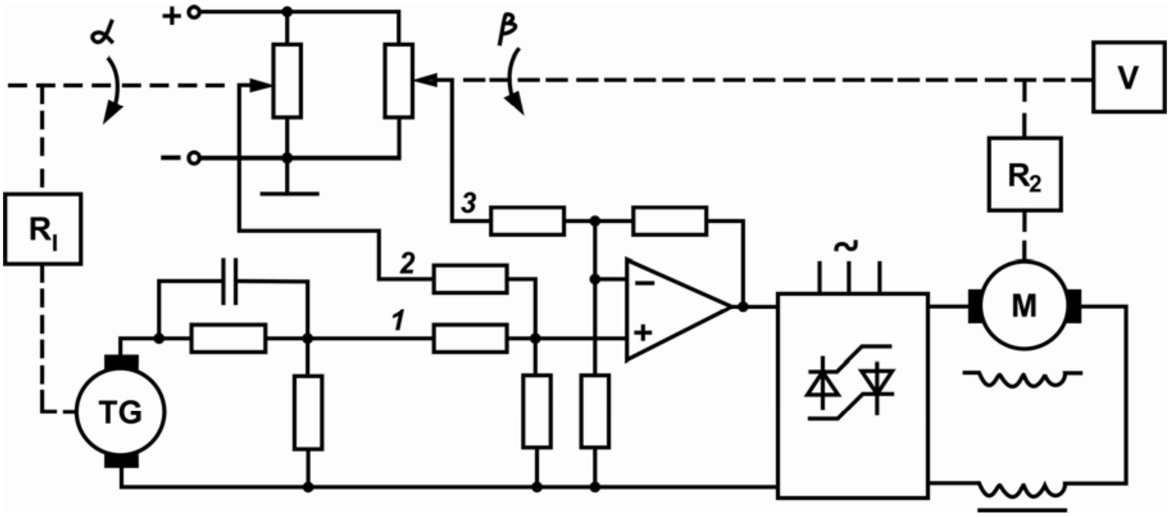


Fig. 0.9. Circuit schematic

Table9

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{AMS}	V/rad	16	18	20	28	21	15	24	20	30	25,2
K_{AMS}	V/rad	16	18	20	28	21	15	24	20	30	25,2
K_{A_1}		0,216	0,202	0,192	0,03	0,154	0,04	0,3	0,12	0, 2	0,2
K_{A_2}		0,65	0,88	0,78	2,85	1,34	3,42	0,68	1,22	0,93	1,09
K_{A_3}		0,65	0,88	0,78	2,85	1,34	3,42	0,68	1,22	0,93	1,09
K_{TC}		12,8	11,2	18,6	14,8	12,8	20	13,6	16,2	10	15,2
T_{TC}		0,02	0,03	0,01	0,04	0,02	0,008	0,012	0,015	0,01	0,03
K_{M_1}	$rad/V \cdot s$	14,1	8,2	6,2	2,2	8,7	2,6	15,6	5,6	12,3	4,8
K_{M_2}	$rad/N \cdot m \cdot s$	2,6	1,5	8,7	7,8	2,4	10	12	7,2	4,5	6,5
T_E	s	0,03	0,02	0,03	0,01	0,01	0,015	0,008	0,02	0,018	0,01
T_{EM}	s	0,15	0,12	0,09	0,2	0,24	0,18	0,14	0,21	0,12	0,15
K_{R_1}		20	10	12	14	10	15	13,55	11,8	17,6	20,4
K_{R_2}		0,008	0,011	0,01	0,006	0,008	0,012	0,01	0,009	0,007	0,01
K_{TG}	$V \cdot s/rad$	0,5	0,1	0,2	0,08	0,11	0,2	0,18	0,15	0,11	0,1
δ		0,15	0,2	0,4	0,172	0,19	0,19	0,345	0,26	0,45	0,56
T	s	0,03	0,0375	0,067	0,05	0,061	0,044	0,08	0,079	0,106	0,23
M_L	$N \cdot m$	2	2	6	5	2	8	6	4	3	2

Scheme№10. HERMETIC CHAMBER TEMPERATURE ACS

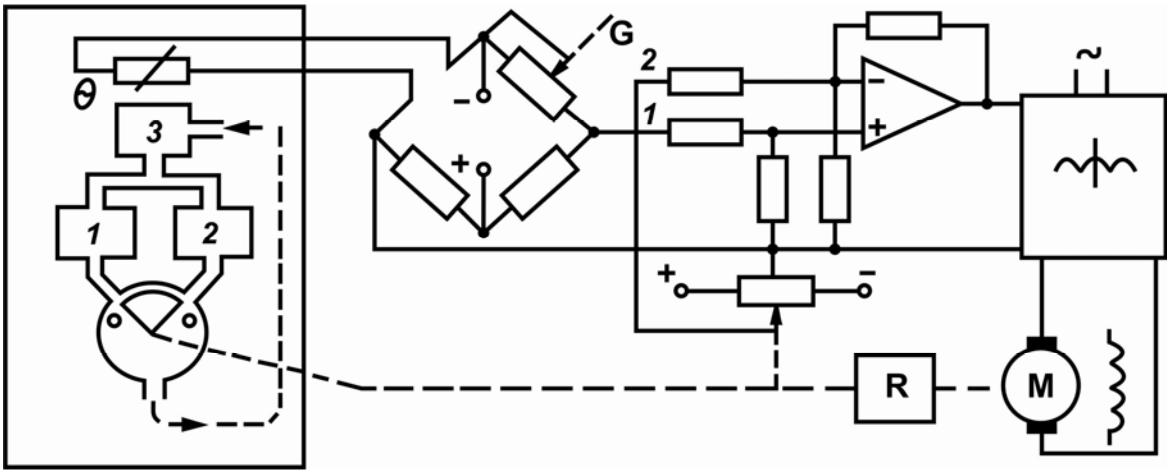


Fig. 0.10. Circuit schematic

Table 10

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{I_1}	$V/\deg$	21	15	8	16	12	10	12	15	20	18
K_{I_2}	$V/\deg$	0,4	0,16	0,25	0,1	0,2	0,12	0,15	0,1	0,22	0,24
T_I	s	0	1,6	0	6,3	0	2,5	0	8	0	4
K_{A_1}		2	0,64	4	1,82	3	1,35	3,5	0,47	2	4,5
K_{A_2}		1	2	0,5	0,4	1	0,25	0,8	0,4	1,2	1
K_{MA}		2,4	4,5	3	2	87	3,5	6,4	2,2	5,6	1,8
$T_{MA} = T_E$	s	0	0	0	0	0	0	0	0	0	0
K_{M_1}	$rad/V \cdot s$	1,3	1,15	1,2	1,04	1,12	1	0,93	0,95	1,05	0,83
K_{M_2}	$rad/N \cdot m \cdot s$	0	0	0	0	0	0	0	0	0	0
T_{EM}	s	0,54	0	0,255	0	0,23	0	0,38	0	0,2	0
K_R		0,02	0,081	0,074	0,002	0,03	0,003	0,006	0,01	0,038	0,002
K_{CE}	$\deg/rad$	25	15	27,5	18	35	16	18	22	30	26
K_C		0,744	0,853	0,667	0,886	0,338	0,789	0,65	0,717	0,782	0,823
T_C	s	60	15	90	30	60	25	40	20	20	100
K_{CE}	s	22	63	40	90	35	60	100	115	80	25
K_{AMS}	V/rad	3,6	2,5	5	2,4	2	2	4,2	0,5	2,8	4
f_L	deg	8	6	12	15	9	10	14	18	21	16

Scheme №11. SERVO SYSTEM

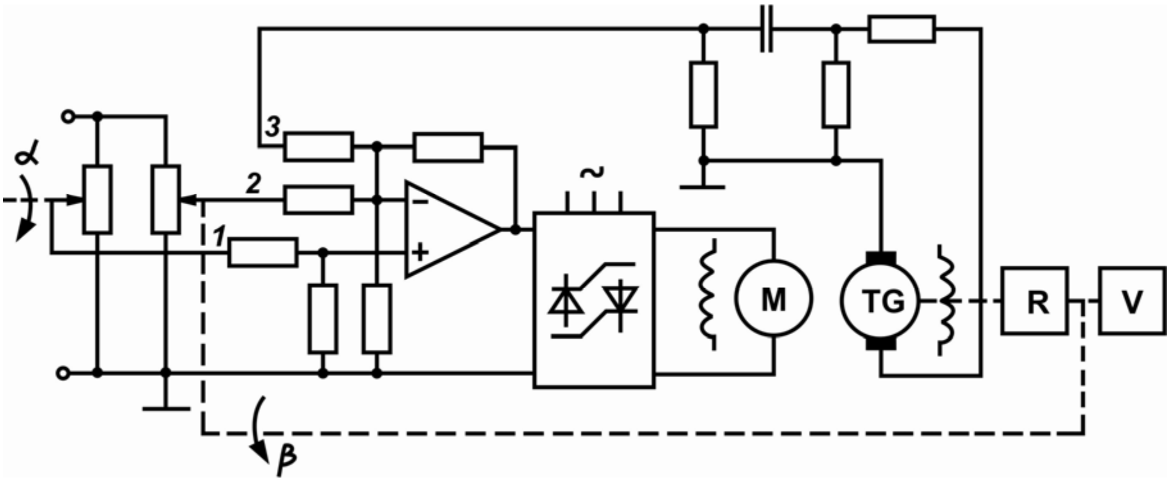


Fig. 0.11. Circuit schematic

Table 11

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{AMS}	V/rad	12	15	16	12,5	10	13,5	14	9	15	14,5
K_{AMS}	V/rad	12	15	16	12,5	10	13,5	14	9	15	14,5
K_{A_1}		22,4	17,3	25	20,2	18	17,5	15,3	11,5	27,2	16
K_{A_2}		22,4	17,3	25	20,2	18	17,5	15,3	11,5	27,2	16
K_{A_3}		1	1	1	1	1	1	1	1	1	1
K_{TC}		18,2	15	17	16,8	13,7	16,6	16	18,7	14,	10,4
T_{TC}		0,01	0	0,008	0	0,012	0	0,006	0	0,011	0
K_{M_1}	$rad/V \cdot s$	1,43	2,1	1,84	2	2,85	1,43	1,95	1,5	1	1,95
K_{M_2}	$rad/N \cdot m \cdot s$	21	36,5	40	32	20	27	18	24	26	42
T_E	s	0	0,015	0	0,02	0	0,016	0	0,022	0	0,018
T_{EM}	s	0,162	0,307	0,13	0,272	0,191	0,355	0,158	0,189	0,2	0,256
T_1	s	0,04	0,05	0,03	0,055	0,075	0,085	0,06	0,1	0,08	0,1
T_2	s	0,1	0,08	0,07	0,107	0,115	0,112	0,085	0,155	0,125	0,126
K_{TG}	$V \cdot s/rad$	0,25	0,16	0,2	0,27	0,13	0,15	0,18	0,12	0,15	0,2
K_R		0,01	0,011	0,008	0,01	0,009	0,008	0,012	0,011	0,007	0,012
M_L	$N \cdot m$	2	4	3	2	2	3	2	4	3	4

Scheme №12. SERVO SYSTEM

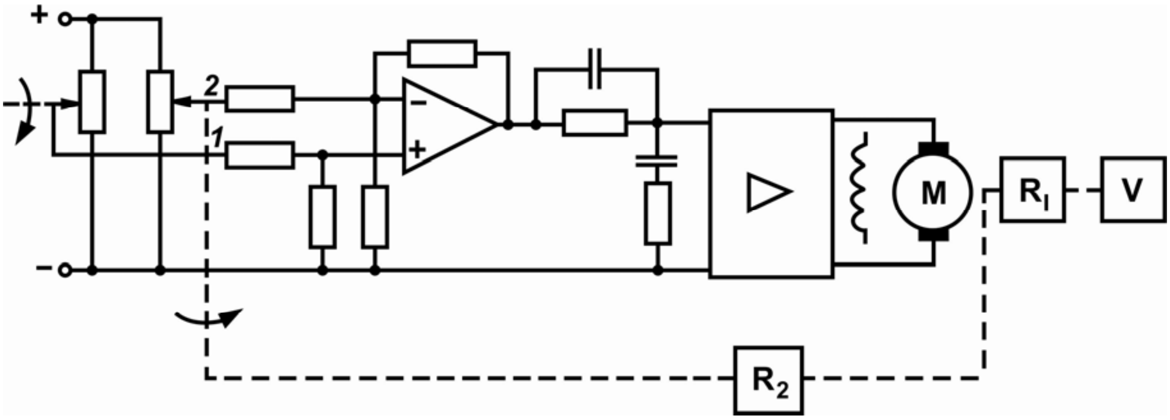


Fig. 0.12. Circuit schematic

Table 12

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{AMS}	V/rad	30	25	27,6	32	18	15	20	29,6	28,6	33
K_{A_1}		25	20	19	22	50	22,5	23	15	14	12,5
K_{A_2}		25	20	19	22	50	22,5	23	15	14	12,5
T_1	s	0,28	0,174	0,166	0,126	0,063	0,112	0,056	0,19	0,2	0,158
T_2	s	0,08	0,105	0,112	0,091	0,051	0,083	0,038	0,05	0,1	0,102
T_3	s	0,8	0,7	0,477	0,546	0,268	0,387	0,164	0,594	0,87	0,403
T_4	s	0,028	0,026	0,039	0,021	0,012	0,024	0,013	0,016	0,023	0,04
K_{EA}		20	21	18	24	18,5	30	16	27	22	17
K_{M_1}	$rad/V \cdot s$	0,95	1,43	1,9	1,5	0,98	1,44	1,95	1,9	1,45	0,95
K_{M_2}	$rad/N \cdot m \cdot s$	5,2	0,65	15	8,6	36	0,8	24	17,2	7,8	40
T_E	s	0	0	0	0	0	0	0	0	0	0
T_{EM}	s	0,25	0,33	0,398	0,295	0,166	0,224	0,107	0,135	0,141	0,27
K_{R_1}		-	-	-	-	-	-	-	-	-	-
K_{R_2}		0,01	0,008	0,008	0,009	0,007	0,011	0,009	0,0088	0,01	0,012
M_L	$N \cdot m$	5	0,5	15	8	4	0,8	2	3	4	5

Scheme №13. DIRECT CURRENT MOTOR ROTATION FREQUENCY COMBINED ACS

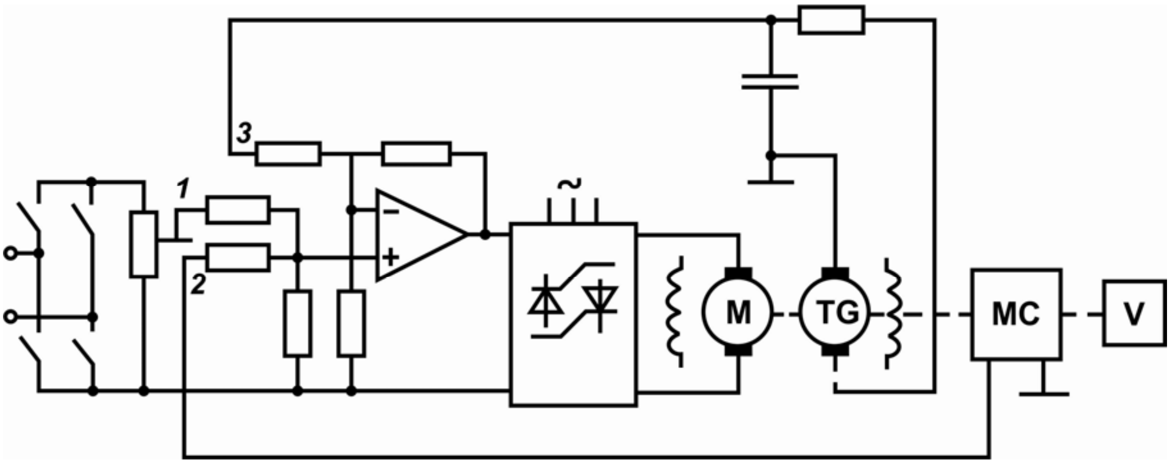


Fig. 0.13. Circuit schematic

Table 13

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{A_1}		5	7,5	6,4	10	8	7,2	5,6	6	5,5	8,8
K_{A_2}		23,33	15,25	5,6	6,25	20	20	12	15	8	12
K_{A_3}		20	8	15	6,25	25	20	15	15	20	20
K_{TC}		18,75	12,5	12,5	15	20	12,5	15	19	16	25
T_{TC}	s	0,06	0,02	0,03	0,02	0,01	0,01	0,015	0,005	0,006	0,008
K_{M_1}	$rad/V \cdot s$	1,6	2,8	2	3,2	2,4	3,2	2,4	3	2,5	1,8
K_{M_2}	$rad/N \cdot m \cdot s$	14	32	5,6	15	24	8	13,86	17,1	9,6	21,6
T_E	s	0,02	0,1	0,08	0,15	0,12	0,1	0,14	0,11	0,08	0,05
T_{EM}	s	0,3	0,4	0,5	0,35	0,4	0,32	0,45	0,3	0,4	0,36
K_{VS}	$V \cdot s/rad$	0,02	0,05	0,04	0,04	0,015	0,025	0,03	0,035	0,04	0,03
K_{VS}	s	0,01	0,005	0,006	0,005	0,008	0,003	0,01	0,002	0,003	0,002
K_{MC}	$V/N \cdot m$	0,02	0,06	0,04	0,05	0,025	0,01	0,035	0,02	0,03	0,04
M_L	$N \cdot m$	10	3	6	2	4	8	13	15	9	12

Scheme №14. TURBOJET ENGINE ROTATION FREQUENCY ACS

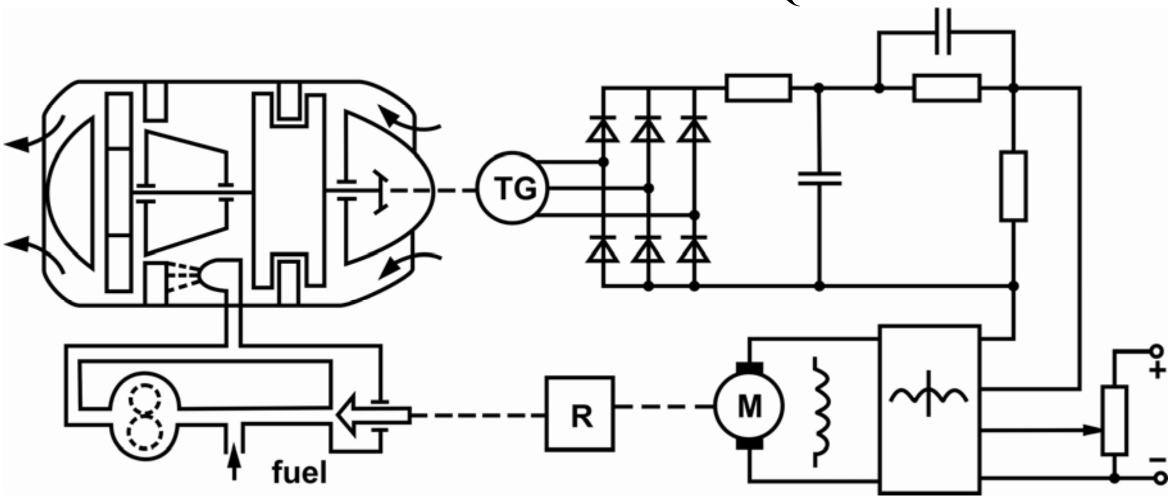


Fig. 0.14. Circuit schematic

Table 14

Parameters		ACS parameters values according to variants									
		1	2	3	4	5	6	7	8	9	0
K_{MA_1}		8	5,4	5,5	5	5	4	7,2	8,5	7,5	4,7
K_{MA_2}		8	5,4	5,5	5	5	4	7,2	8,5	7,5	4,7
T_{MA}	s	0	0	0	0	0	0	0	0	0	0
K_{M_1}	$rad/V \cdot s^2$	2,4	1,8	2,1	4,5	8,6	4,25	6,4	8,6	8,	4
K_{M_2}	$rad/N \cdot m \cdot s$	0	0	0	0	0	0	0	0	0	0
T_E	s	0	0	0	0	0	0	0	0	0	0
T_{EM}	s	0,11	0,07	0,126	0,06	0,055	0,085	0,05	0,046	0,058	0,09
K_R	mm/rad	4,8	5,2	3,9	4	3,2	5	3,6	4	4,4	3
K_{T_1}	$rad/mm \cdot s$	48	55	40	36	50	32	45	54	42	60
K_{T_2}		1	1	1	1	1	1	1	1	1	1
T_{TE}	s	6	4	5,5	3,5	6,3	4,3	1,75	2,9	3,2	5
K_{VS}	$V \cdot s/rad$	0,1	0,15	0,12	0,08	0,08	0,07	0,05	0,04	0,05	0,06
T_{VS}	s	0	0	0	0	0	0	0	0	0	0
δ		0,024	0,06	0,065	0,12	0,06	0,084	0,115	0,1	0,09	0,11
T	s	0,022	0,027	0,036	0,03	0,023	0,042	0,024	0,017	0,022	0,032
f_L	Rad/s	2	4	3	5	6	7	8	11	10	12

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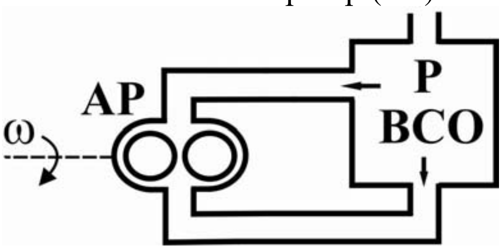
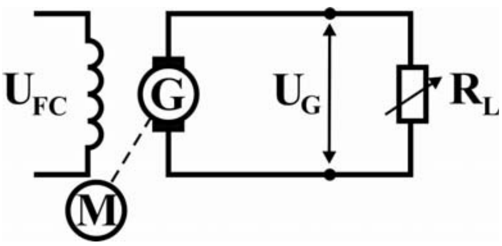
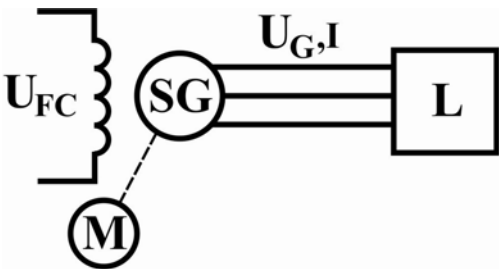
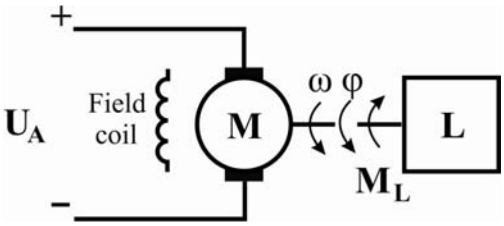
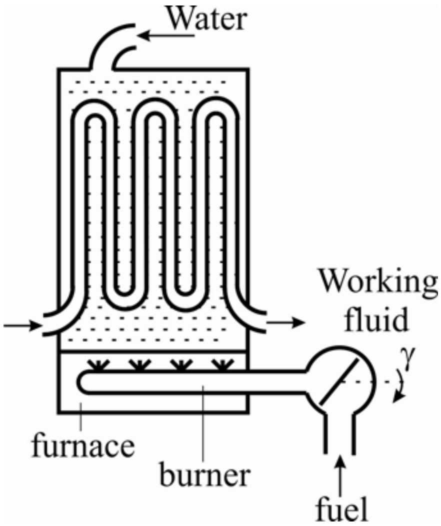
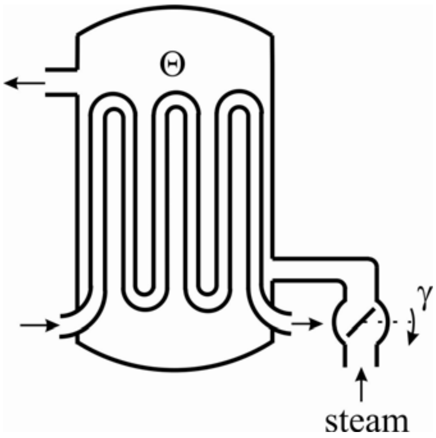
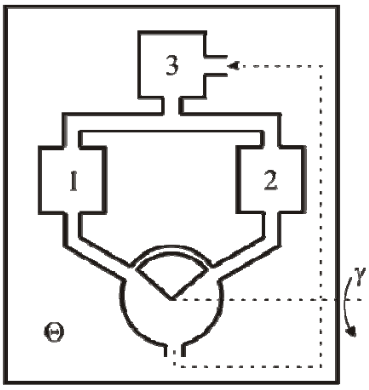
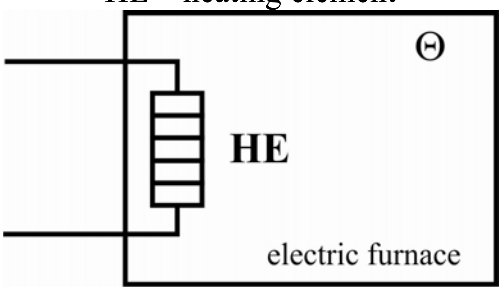
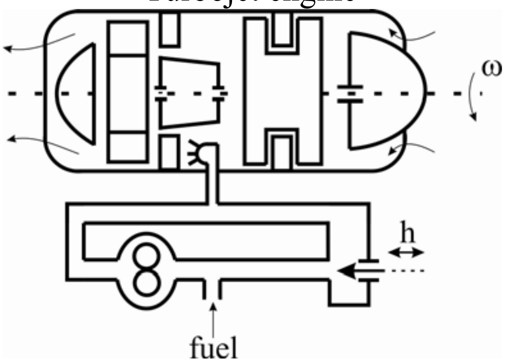
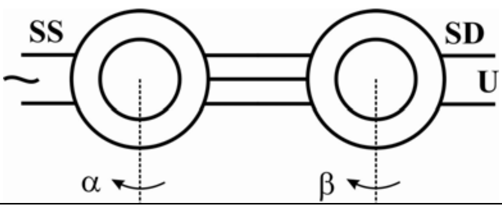
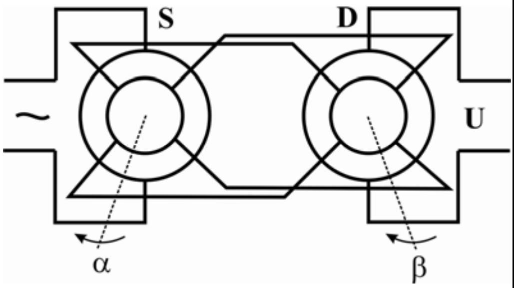
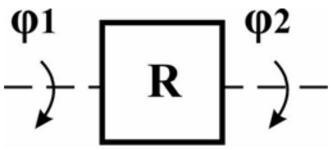
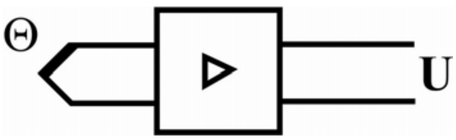
10. Andrei A. Agrachev, A. Stephen Morse, Eduardo D. Sontag, Hector J. Sussmann, Vadim I. Utkin. Nonlinear and Optimal Control Theory: Lectures given at the C.I.M.E. Summer School held in Cetraro. – Springer, 2004. – 360 p.

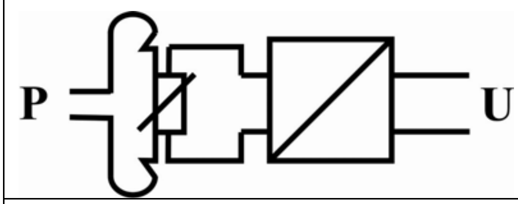
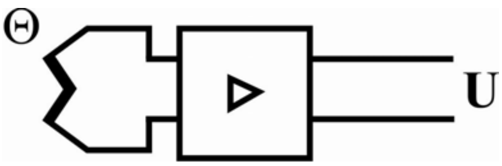
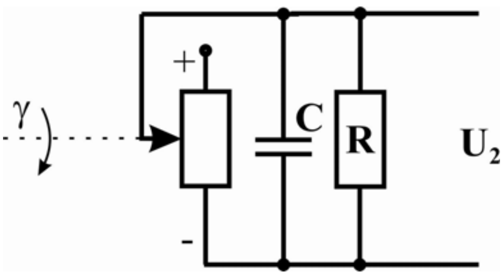
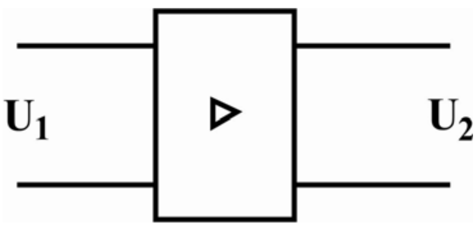
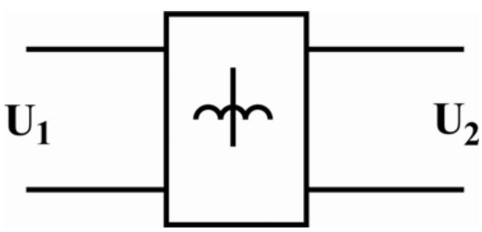
Appendix 1

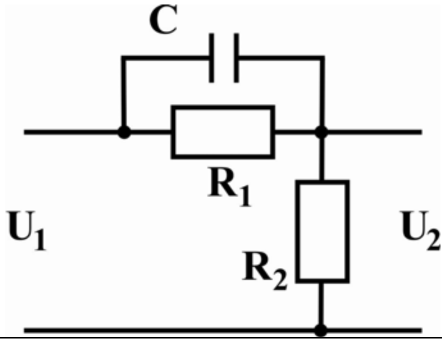
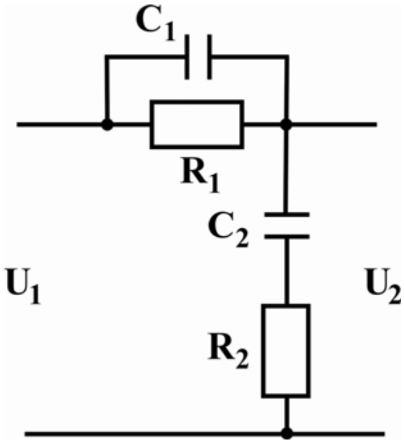
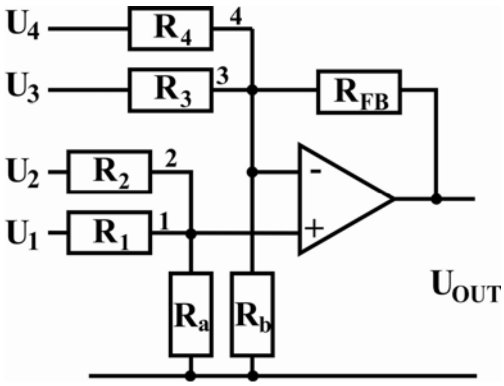
Device scheme	Device equation
Biological control object (BCO) with the arterial pump (AP) 	$(T_a p+1)(T_b p+1)\Delta P(t)=$ $=K_a K_b \Delta\omega(t)-K_b(T_a p+1)\Delta f(t),$ P – preasure in BCO; ω – AP rotation frequency; f – disturbance; K_a, K_b – gain; T_a, T_b – time constant.
Direct current generator with separate excitation (G) 	$(T_G p+1)\Delta U_G(t)=$ $=K_{G_1}\Delta U_{FC}(t)-K_{G_2}(T_G p+1)\Delta R_L(t),$ U_G – output voltage of G; U_{FC} – voltage on field coil; R_L – load resistance; K_{G_1}, K_{G_2} – gain of G; T_G – time constant of G.
Synchronous generator (SG) 	$(T_{SG} p+1)\Delta U_G(t)=$ $=K_{SG_1}\Delta U_B(t)-K_{SG_1}(T_{SG} p+1)\Delta I(t),$ U_Γ – output voltage of SG; U_{FC} – voltage on field coil; K_{SG_1}, K_{SG_2} – gain of SG; T_{SG} – time constant of SG; I – load current.

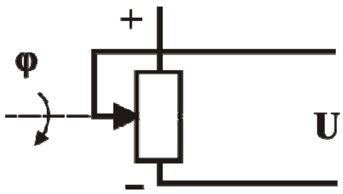
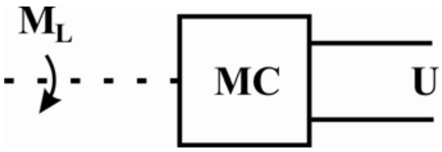
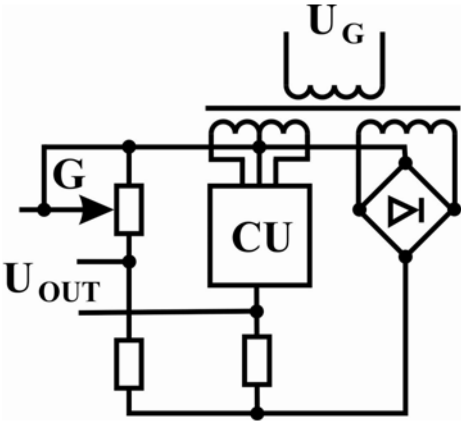
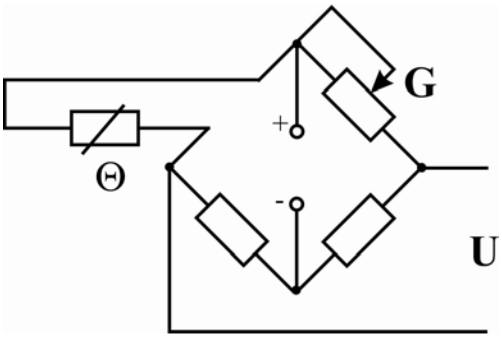
Direct current motor with separate excitation 	$(T_E T_{EM} p^2 + T_{EM} p + 1)\Delta\omega(t) =$ $= K_{M_1}\Delta U_{AV}(t) - K_{M_2}(T_E p + 1)\Delta\varphi_L(t)$ or $(T_E T_{EM} p^2 + T_{EM} p + 1) p\Delta\varphi(t) =$ $= K_{M_1}\Delta U_{AV}(t) - K_{M_2}(T_E p + 1)\Delta M_{RM}(t),$ ω – rotation frequency of output shaft; φ – rotation angle of output shaft; U_{AV} – armature voltage; M_L – resisting moment on shaft; K_{M_1}, K_{M_2} – voltage and moment shaft; T_E, T_{EM} – electromagnetic and electromechanical time conctants.
Furnace with burner 	$(T_B p + 1)(T_F p + 1)\Delta\theta(t) =$ $= K_B K_F \Delta\gamma(t) - K_F(T_B p + 1)\Delta f(t),$ θ – temperature in furnace; γ – rotation angle of furnace control element; f – disturbance; K_B, K_F – gain; T_B, T_F – time constants.

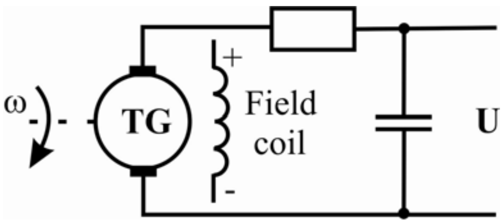
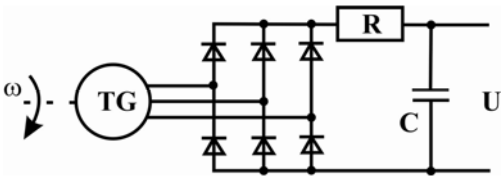
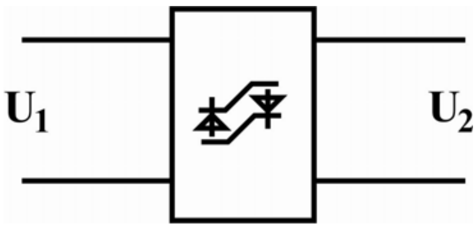
Heat exchanger 	$(T_{HE_1}p+1)(T_{HE_2}p+1)\Delta\theta(t)=$ $=K_{HE_1}\Delta\gamma(t)-K_{HE_1}K_{HE_2}\Delta f(t),$ θ – temperature in heat exchanger; γ – rotation angle of control element; f – disturbance; K_{T1}, K_{T2} – gain; T_{T1}, T_{T2} – time constants.
Hermetic cabin 	1-heater, 2-cooler, 3-pump $\left[(T_k p+1)(T_p p+1)-K_0\right]\Delta\Theta(t)=$ $=K_{po}(T_p p+1)\Delta\gamma(t)-(T_p p+1)\Delta f(t),$ Θ – temperature in cabin, γ – rotation angle of control element, K_{CE}, K_C – gain of control element and cabin, T_{CE}, T_C – time constants of control element and cabin.
Electric furnace (EF) HE – heating element 	$(T_I p+1)(T_P p+1)\Delta\theta(t)=$ $=K_I K_P\Delta U(t)-K_P\Delta f(t),$ θ – temperature in electric furnace; U – voltage on HE; f – disturbance; K_I, K_P – gains; T_I, T_P – time constants.

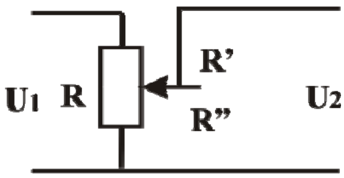
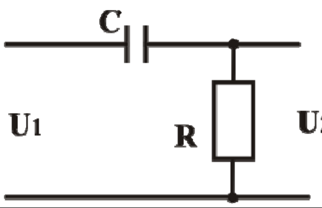
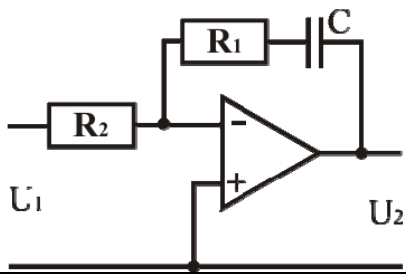
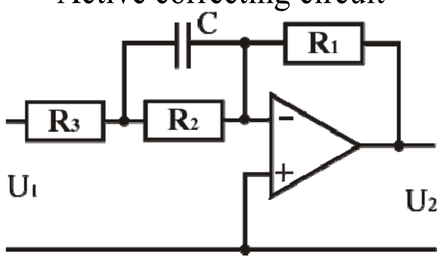
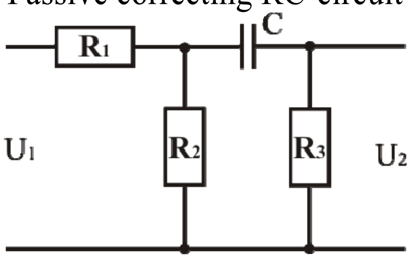
Turbojet engine 	$(T_{TE}p+1)\Delta\omega(t)=K_{TE_1}\Delta h(t)-K_{TE_2}\Delta f(t),$ ω – output shift rotation frequency, h –control element position, f –disturbance, K_{TE_1}, K_{TE_2} – gains, T_{TE} – time constants.
Error angle measurement unit on selsyns 	$\Delta U(t)=K_S(\alpha(t)-\beta(t));$ α – sensor rotation angle, β – detector rotation angle, U– voltage on measurement unit output, K_S – sensor gain.
Error angle measurement unit on rotary transformer 	$\Delta U(t)=K_S(\alpha(t)-\beta(t));$ α – sensor rotation angle, β – detector rotation angle, U– voltage on measurement unit output, K_S – sensor gain.
Reducer 	$\Delta\varphi_2(t)=K_R\Delta\varphi_1(t),$ φ_1 – input shift rotation angle; φ_2 – output shift rotation angle; K_p – reducer gain.
Temperatur sensor 	$(T_{T_1}p+1)\Delta U(t)=K_{T_1}\Delta\theta(t),$ θ – measurement object temperatur; U – sensor output voltage; K_{T1} – sensor gain; T_{T1} – sensor time constant.

Preassure tensometric sensor 	$\Delta U(t) = K_{PS} \Delta P(t)$, U – sensor outoput voltage; P – measurement object preassure; K_{PS} – sensor gain.
Elastic feedback element 	$\left(T_{TS_2} p + 1\right) \Delta U\left(t\right) = K_{TS_2} T_{TS_2} p \Delta \theta\left(t\right)$, θ – measurement object temparatur; U – element output voltage; K_{TS_2} – element gain; T_{TS_2} – element time constant.
Elastic feedback element 	$\left(T_{FB} p + 1\right) \Delta U\left(t\right) = K_P T_{FB} p \Delta \theta\left(t\right)$, $\gamma\left(t\right)$ - rotation angle of potentiometer motor, U – element output voltage; K_P – element gain; T_{FB} – element time constant.
Electronic amplifier (EA) 	$\Delta U_2\left(t\right) = K_{EA} \Delta U_1\left(t\right)$, U_2 – EA output voltage; U_1 – EA input voltage; K_{EA} –EA gain.
Magnetic amplifier (MA) 	$\left(T_{MA} p + 1\right) \Delta U_2\left(t\right) = K_{MA} \Delta U_1\left(t\right)$, U_2 – MA output voltage; U_1 – MA input voltage; K_{MA} – MA gain; T_{MA} – MA time constant.

Demultiplier 	$(T_P p+1)\Delta U_2(t)=(T_P+\delta)\Delta U_2(t)$, where $T=R_1R_2C/(R_1+R_2)$, $\delta=R_2/(R_1+R_2)$.
Correcting circuit 	$(T_3p+1)(T_4p+1)\Delta U_2(t)=(T_1+1)(T_2+1)\Delta U_1(t)$, where $T_1=R_1C$; $T_2=R_2C$; $T_3T_4=R_1R_2C_1C_2$; $T_3+T_4=R_1C_2+(R_2+R_1)$.
Comparing-summarizing unit (CSU) 	$\Delta U_{OUT}(t)=K_{A_1}\Delta U_1(t)+K_{A_2}\Delta U_2(t)-$ $-K_{A_3}\Delta U_3(t)-K_{A_4}\Delta U_4(t)$, where $K_{A_1}=R_{FB}/R_1$; $K_{A_2}=R_{FB}/R_2$; $K_{A_3}=R_{FB}/R_3$; $K_{A_4}=R_{FB}/R_4$;

Unipolar angular motion sensor 	$\Delta U = K_{AS}\Delta\varphi(t)$, φ – rotation angle of potentiometer motor (measured value); U – sensor output voltage; K_{AS} – sensor gain.
Moment clutch 	$\Delta U(t) = K_{MC}\Delta M_L(t)$, M_L – measured moment; U – voltage on MC output, K_{MC} – MC gain.
Measurement unit 	$(T_G p+1)\Delta U_{OUT} = K_{G_1}(T_G p+1)\Delta G(t) -$ $-K_{G_2}\Delta U_{OUT}(t)$, G – motor rheostat position (reference signal); U_G – measured voltage; U_{OUT} – output voltage; K_{G_1} – reference signal gain; K_{G_2} – measured signal gain; T_G – device time constant.
Measurement unit 	$(T_{И}p+1)\Delta U(t) = K_{И1}(T_{И}p+1)\cdot$ $\cdot\Delta G(t) - K_{И2}\Delta\theta(t)$, G – rheostat motor (reference signal); θ – measurement object temperatur; U – device output voltage; K_{MU_1} – reference signal gain; K_{MU_2} – measured value gain; T_{MU} – device time constant.

Velosity sensor  	$(T_{VS}\,p+1)\Delta U(t)=K_{VS}\Delta\omega(t)$, ω – VS shaft rotation frequency, U – VS output voltage, K_{VS} – VS gain, T_{VS} – VS time constant.
Velosity sensor  	$(T_{VS}\,p+1)\Delta U(t)=K_{VS}\Delta\omega(t)$, ω – VS shaft rotation frequency, U – VS output voltage, K_{VS} – VS gain, T_{VS} – VS time constant.
Thyristor transducer (TP)  	$(T_{TT}\,p+1)\Delta U_2(t)=K_{TT}\Delta U_1(t)$, U_2 – TT output voltage; U_1 – TT output voltage; K_{TT} – TT gain; T_{TT} – TT time constant.

Demultiplier  	$U_2(t) = K_P U_1(t)$, where $K_P = R''/R$.
Passive correcting RC - circuit  	$(T_P + 1)\Delta U_2(t) = T_P U_1(t)$, where $T = RC$.
Active correcting circuit  	$T_2 p \Delta U_2(t) = -(T_1 + 1)\Delta U_1(t)$ where $T_1 = R_1 C; T_2 = R_2 C;$
Active correcting circuit  	$(T_2 p + 1)\Delta U_2(t) = -(T_1 + K_1)\Delta U_1(t)$, where $K = \frac{R_1}{R_2 + R_3}$; $T_1 = \frac{R_1 R_2}{R_2 + R_3} C;\ T_2 = \frac{R_2 R_3}{R_2 + R_3} C.$
Passive correcting RC-circuit  	$(T_2 p + 1)\Delta U_2(t) = T_1 p \Delta U_1(t)$; where $T_1 = \frac{R_2 R_3}{R_1 + R_2} C$; $T_2 = \left(R_3 + \frac{R_1 R_2}{R_1 + R_2} \right) C;$

Appendix 2

Lattice functions images			
Original	Laplace transform	Unbiased lattice function	z-transform
$f(t)=\begin{cases}1, & for\ t=0\\ 0, & for\ t\neq 0\end{cases}$	-	$\delta_o[n]$	1
$1(t)-1(t-T)$	$\frac{1-e^{-sT}}{s}$	$\nabla 1[n]=\Delta 1[n-1]$	1
$1(t)$	$\frac{1}{s}$	$1[n]$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	nT	$\frac{Tz}{(z-1)^2}$
$\frac{t^2}{2!}$	$\frac{1}{s^3}$	$\frac{(nT)^2}{2!}$	$\frac{T^2 z(z+1)}{2!(z-1)^3}$
$\frac{t^3}{3!}$	$\frac{1}{s^4}$	$\frac{(nT)^3}{3!}$	$\frac{T^3 z(z^2+4z+1)}{3!(z-1)^4}$
$\frac{t^k}{k!}$	$\frac{1}{s^{k+1}}$	$\frac{(nT)^k}{k!}$	$\frac{T^k zR_z(z)}{k!(z-1)^{k+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$e^{-\alpha nT}=d^n$	$\frac{z}{z-e^{-\alpha T}}$
$1-e^{-\alpha t}$	$\frac{\alpha}{s(s+\alpha)}$	$1-e^{-\alpha nT}$	$\frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}$
$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$	$nTe^{-\alpha nT}$	$\frac{ze^{-\alpha T}}{(z-e^{-\alpha T})^2}$
$\frac{t^2}{2!}e^{-\alpha t}$	$\frac{1}{(s+\alpha)^3}$	$\frac{(nT)^2}{2!}e^{-\alpha nT}$	$\frac{z(z+e^{-\alpha T})e^{-2\alpha T}}{2!(z-e^{-\alpha T})^3}$
$\frac{t^k}{k!}e^{-\alpha t}$	$\frac{1}{(s+\alpha)^{k+1}}$	$\frac{(nT)^k}{k!}e^{-\alpha nT}$	$\frac{zR_k(ze^{\alpha T})e^{-k\alpha T}}{k!(z-e^{-\alpha T})^{k+1}}$
$\sin\pi\frac{t}{T}$	$\frac{\pi T^{-1}}{s^2+\pi^2 T^{-2}}$	$\sin\pi n=0$	0

$\cos\pi\frac{t}{T}$	$\frac{s}{s^2+\pi^2/T^2}$	$\cos\pi n=(-1)^n$	$\frac{z}{z+1}$
$\sin\frac{\pi}{2}\frac{t}{T}$	$\frac{0,5\pi/T}{s^2+0,25\pi^2/T^2}$	$\sin\frac{\pi}{2}n$	$\frac{z}{z^2+1}$
$\cos\frac{\pi}{2}\frac{t}{T}$	$\frac{s}{s^2+0,25\pi^2/T^2}$	EMBED	$\frac{z^2}{z^2+1}$
$\sin\beta t$	$\frac{\beta}{s^2+\beta^2}$	$\sin\beta nT$	$\frac{z\sin\beta T}{z^2-2z\cos\beta T+1}$
$\cos\beta t$	$\frac{s}{s^2+\beta^2}$	$\cos\beta nT$	$\frac{z^2-z\cos\beta T}{z^2-2z\cos\beta T+1}$
$e^{-\alpha t}\sin\beta t$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$	$e^{-\alpha nT}\sin\beta nT$	$\frac{ze^{-\alpha T}\sin\beta T}{z^2-2z\cos\beta T+e^{-2\alpha T}}$
$e^{-\alpha t}\cos\beta t$	$\frac{s+\beta}{(s+\alpha)^2+\beta^2}$	$e^{-\alpha nT}\cos\beta nT$	$\frac{z^2-ze^{-\alpha T}\cos\beta T}{z^2-2ze^{-\alpha T}\cos\beta T+e^{-2\alpha T}}$
-	-	$\delta(t-nT_0)$	z^{-n}

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