Control System Robust Controller Parametric Synthesis Based on Coefficient Estimation of Stability and Oscillation Indices

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A lot of linear system controller design approaches allowing realization a reasonable structure and parameters choice to provide some specified requirements to system performance are developed. Among all the known methods coefficient methods allow to obtain approximate but rather simple correlations, which link quality indices of unconditioned order control system with desired controller parameters. Maximum system stability degree with restriction on oscillation is one of the most widespread criteria when designing an automatic control system When designing the system, one of the most extensively used criterions is maximal degree of stability. It is known, that the systems are designed according to this criterion, other conditions being equal, have higher performance, smaller overshoot and higher degree of stability. On the practice in real systems the plant parameters, usually, are known not precisely or can change according to unforeknowable rules in specified limits. Such parameters are accepted to call interval parameters, and control systems – interval control systems. To guarantee robust stability in interval systems, supposing its operability save by any interval parameters variations is very important and sometimes time-consuming task. The problem of providing in system maximal robust stability represents particular interest. The approach offered to solve referred above controller design problems is based on coefficient estimations of stability indices of interval system and enables to maximize system robust degree of stability for account of unsophisticated algebraic correlations.

Keywords-component; interval system, stability degree, controller, coefficient method, robust stability, oscillation

I. Introduction

A lot of automatic control linear system synthesis principles allowing realizing a reasonable structure and parameters choice to provide some specified requirements to system performance have been developed. Quality indices can be divided into four groups: frequency, time, root and coefficient, defined by the transfer function coefficients set. System synthesis principles analysis shows, that it is desirable to have simple analytical or graphical relations that provide an easy link between automatic control system quality indices and required controller parameters. Such relations are difficult to obtain for frequency or time quality indices except for simple cases. It's much easier to solve the controller parametric synthesis problem on the basis of root approach or using coefficients methods.

Process step response character is defined by characteristic polynomial roots decomposition and type (closed-loop system transfer function poles). Therefore requirements to stability margin and system performance, without step response curves consideration, can be stated by imposing constraints on characteristic polynomial roots. Hereby we impose constrains in time domain on step response type and on maximal values of control value, overshoot value, response time etc.

It is convenient to have linear system transfer function coefficients or some of their combinations as efficiency indices of the system because these indices are directly and simply connected with the system physical parameters, chosen when designing the system. This circumstance is one of the reasons to be interested in dynamic systems stability and quality estimation coefficient methods [1].

Coefficient methods allow obtaining approximate but rather simple relations, which link (usually by some inequalities) random order automatic control system quality indices and required controller parameters.

II. Problem Formulation

Maximum degree of system stability is one of the most widespread criteria when designing an automatic control system. It is known that systems designed according to this criterion have a higher performance, a smaller overshoot and a higher degree of stability [1].

The problem of linear controller synthesis providing a maximum degree of stability in stationary automatic control systems has attracted a considerable attention of researchers [2–8]. Offered in [2] method of typical normalized characteristic equations provides any preassignated system stability degree obtaining involving information about system state vector. However, increasing system stability degree dramatically increases of gain coefficient at low-order derivatives that complicate technical realization of such systems. Approach considered in [3, 4] provides controller synthesis by plant incomplete state vector, that allows to avoid redundant number of controller tunings. However offered

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approach doesn't take into account requirements to system overshoot and accuracy. To obtain the maximum degree of stability in automatic control systems the method of nonlinear programming [5, 6] is of a great interest. However the solution of nonlinear equations given in [5, 6] system is connected with difficulties of initial conditions choice (there is no formalized choice procedure) and as a result it's not always possible to find a desirable solution.

Synthesis of robust controller for a plant which parameters change in a priori certain or uncertain perturbations, which belong to some restricted range or set, is considered in [7]. Robust control problems are also widely solved with the help of H_2 and H_∞ theories. They are based on algorithms of direct synthesis with utilization of LMI instrument and have both pros and cons: solution dependence on initial conditions choice, dependence on plant parametric robustness and large quantity of iterations. Also stated methods are mathematically saturated, require intensive computations and serious mathematical background.

In connection with the stated above facts robust stability degree maximization problem solution of random order system based on coefficient method by the appropriate controller tunings choice is of a great interest. Note, that failure to take account of oscillation when controller synthesizing could leads to undesirable constituents appearance in system responses. Therefore system synthesis problem based on coefficient method by maximal stability degree criterion accounting oscillation restriction is actual.

III. Conditions of Interval System Stability Degree Maximization

Consider linear stationary system with characteristic polynomial

$$A(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0, \quad a_n > 0. \qquad (1)$$

Stability indices λ_i could be formed by quadruple of nearby coefficients of polynomial (1)

$$\lambda_i = \frac{a_{i-1} a_{i+2}}{(a_i a_{i+1})}, \quad i = \overline{1, n-2}. \qquad (2)$$

In [1] linear systems sufficient stability conditions on the basis of coefficient stability indices (2) are obtained

$$\lambda_i < \lambda^* \approx 0,465, \ \forall\, i = \overline{1, n-2}. \qquad (3)$$

Conditions (3) can be implemented for the controller parameters choice, which provide system stability. Their simplicity allows forming well algorithmized synthesis procedures and some redundancy facilitates to obtain stability with a margin, which is always necessary when designing real systems.

It is obvious, that when designing control system it is important to get not only stable system but provide specific system performance quality as well. From this point of view sufficient conditions of specified stability degree η offered in [3] could be useful.

$$\begin{cases} \dfrac{a_{i-1} a_{i+2}}{\left[a_i - a_{i+1}(n-i-1)\eta\right]\left[a_{i+1} - a_{i+2}(n-i-2)\eta\right]} < \lambda^*, \\ \qquad i = \overline{1, n-2}; \\ a_l - a_{l+1}(n-l-1)\eta \ge 0, \quad l = \overline{1, n-1}; \\ a_0 - a_1\eta + \dfrac{2a_2\eta^2}{3} \ge 0. \end{cases} \qquad (4)$$

Fulfillment of conditions (4) guarantees characteristic polynomial (1) roots location on the left side of the vertical line going through the point $(-\eta, j0)$.

It is obvious, that increasing η in specified conditions allows finding its maximum value, which will be considered as a lower estimate of system maximum stability degree. Designate it as η^*. In such case a synthesis problem is to choose controller parameters $\bar{k}^*$, which provide maximum. Designate this maximum as $\eta^*_{\max}$. Consider $\eta^*_{\max} = \max\limits_{\bar{k}} \eta^*$, where $\eta^*_{\max}$ – lower estimate of maximum stability degree. It is called a system quazimaximum stability degree.

Let's introduce the following designations

$$\begin{cases} \lambda_i(\bar{k}, \eta) = \dfrac{a_{i-1}(\bar{k}) a_{i+2}(\bar{k})}{\left[a_i(\bar{k}) - a_{i+1}(\bar{k})(n-i-1)\eta\right] \times} \\ \qquad \overline{\times \left[a_{i+1}(\bar{k}) - a_{i+2}(\bar{k})(n-i-2)\eta\right]}, \quad i = \overline{1, n-2}; \\ f_l(\bar{k}, \eta) = a_l(\bar{k}) - a_{l+1}(\bar{k})(n-l-1)\eta, \quad l = \overline{1, n-1}; \\ g(\bar{k}, \eta) = a_0(\bar{k}) - a_1(\bar{k})\eta + \dfrac{2a_2(\bar{k})\eta^2}{3}. \end{cases} \qquad (5)$$

Increasing of η in each expression $\lambda_i(\eta)$ from (5) by controller tunings change is possible up to the value, when $\lambda_i(\bar{k}, \eta) = \lambda^*$, $i = \overline{1, n-2}$. Thereby, for quazimaximum stability degree and appropriate controller tunings $\bar{k}^*$ determination, it is necessary to solve the following system of equations (n-2) times

$$\begin{cases} \lambda_i(\bar{k}, \eta) = \lambda^*, \quad i = \overline{1, n-2}; \\ \lambda_j(\bar{k}, \eta) < \lambda^*, \ j = \overline{1, n-2}, \ j \ne i; \\ f_l(\bar{k}, \eta) \ge 0, \quad l = \overline{1, n-1}; \\ g(\bar{k}, \eta) \ge 0, \end{cases} \qquad (6)$$

defining at each step the maximum value of η^*, and after to choose the maximum among them.

In case of interval uncertainty of system parameters system characteristic polynomial (1) is reduced to the form

$$A(s) = [a_n]s^n + [a_{n-1}]s^{n-1} + \ldots + [a_0],$$

$$\underline{a_i(\bar{k})} \le a_i(\bar{k}) \le \overline{a_i(\bar{k})},\; i=\overline{0,\,n}.$$

Let's write sufficient conditions of robust system stability [1] on the basis of (2), (3) taking into account interval coefficients of polynomial (1)

$$\frac{\overline{a_{i-1}}\,\overline{a_{i+2}}}{\underline{a_i}\,\underline{a_{i+1}}} < 0.465,\; i=\overline{1,n-2}.$$

It is obvious that existence of interval parameters in system is of a great interest for designer not so much providing system robust stability, as obtaining a specified robust quality. Therefore interval analysis is applicable to conditions (6) for the robust controller synthesis, providing quazimaximum stability degree of interval system. It is obvious, that the given conditions should held at any values of system interval parameters from the certain intervals. That is why it is necessary to set such values of interval coefficients in $\lambda_i\left(\bar{k},\eta\right)$, when $\lambda_i\left(\bar{k},\eta\right)$ possesses maximal values.

Note that it is necessary to substitute such values of interval coefficients, which provide minimum of expressions $f_l\left(\bar{k},\eta\right)$ and $g\left(\bar{k},\eta\right)$. This way conditions (6) assume the form

$$\begin{cases} \dfrac{\overline{a_{i-1}(\bar{k})}\,\overline{a_{i+2}(\bar{k})}}{\left[\underline{a_i(\bar{k})}-a_{i+1}(\bar{k})\left(n-i-1\right)\eta\right]\times\left[a_{i+1}(\bar{k})-\overline{a_{i+2}(\bar{k})}\left(n-i-2\right)\eta\right]}=\lambda^*,\; i=\overline{1,\,n-2}; \\ \dfrac{\overline{a_{j-1}(\bar{k})}\,\overline{a_{j+2}(\bar{k})}}{\left[\underline{a_j(\bar{k})}-a_{j+1}(\bar{k})\left(n-j-1\right)\eta\right]\times\left[a_{j+1}(\bar{k})-\overline{a_{j+2}(\bar{k})}\left(n-j-2\right)\eta\right]}<\lambda^*,\; j=\overline{1,\,n-2},\, j\ne i; \\ \underline{a_l(\bar{k})}-\overline{a_{l+1}(\bar{k})}\left(n-l-1\right)\eta\ge 0,\quad l=\overline{1,\,n-1}; \\ \underline{a_0(\bar{k})}-\overline{a_1(\bar{k})}\eta+\dfrac{2a_2(\bar{k})\eta^2}{3}\ge 0. \end{cases} \tag{7}$$

It is defined, that for requirements (7) to functions $\lambda_i\left(\bar{k},\eta\right)$ and $f_l\left(\bar{k},\eta\right)$ fulfillment coefficients $a_{i+1}(\bar{k})$ and $a_{j+1}(\bar{k})$ can take both minimal and maximal values. Therefore the system of inequalities (7) should be solved for both limits of the given coefficients.

Therefore, for the verification of conditions $\lambda_i(\bar{k},\eta)=\lambda^*$, $\lambda_j(\bar{k},\eta)<\lambda^*$ and $f_l(\bar{k},\eta)$ from (6) fulfillment is necessary to consider the following polynomial pairs

$$D_1\left(s\right)=\overline{a_0}+\underline{a_1}s+\underline{a_2}s^2+\overline{a_3}s^3+\overline{a_4}s^4+\underline{a_5}s^5+\underline{a_6}s^6+\ldots,$$
$$D_2\left(s\right)=\overline{a_0}+\underline{a_1}s+\overline{a_2}s^2+\overline{a_3}s^3+\underline{a_4}s^4+\overline{a_5}s^5+\overline{a_6}s^6+\ldots,$$
$$D_3\left(s\right)=\overline{a_0}+\overline{a_1}s+\underline{a_2}s^2+\underline{a_3}s^3+\overline{a_4}s^4+\overline{a_5}s^5+\underline{a_6}s^6+\ldots,$$
$$D_4\left(s\right)=\overline{a_0}+\overline{a_1}s+\underline{a_2}s^2+\overline{a_3}s^3+\overline{a_4}s^4+\underline{a_5}s^5+\overline{a_6}s^6+\ldots,$$
$$D_5\left(s\right)=\underline{a_0}+\overline{a_1}s+\overline{a_2}s^2+\underline{a_3}s^3+\underline{a_4}s^4+\overline{a_5}s^5+\overline{a_6}s^6+\ldots,$$
$$D_6\left(s\right)=\underline{a_0}+\overline{a_1}s+\overline{a_2}s^2+\underline{a_3}s^3+\overline{a_4}s^4+\overline{a_5}s^5+\underline{a_6}s^6+\ldots,$$
$$D_7\left(s\right)=\underline{a_0}+\underline{a_1}s+\overline{a_2}s^2+\overline{a_3}s^3+\underline{a_4}s^4+\underline{a_5}s^5+\overline{a_6}s^6+\ldots.$$

Note, that $D_1\left(s\right)$, $D_3\left(s\right)$, $D_5\left(s\right)$, $D_7\left(s\right)$ are Kharitonov's polynomials. For the verification of condition $g\left(\bar{k},\eta\right)\ge 0$ is necessary to consider the additional polynomial

$$D_8\left(s\right)=\underline{a_0}+\overline{a_1}s+\underline{a_2}s^2+\overline{a_3}s^3+\underline{a_4}s^4+\overline{a_5}s^5+\underline{a_6}s^6+\ldots.$$

IV. Interval System Oscillation Restriction Conditions

System inclination to oscillations appears by existence of complex roots $-\alpha+j\beta$ in its characteristic equation solution. It is easy to define angle sector $\pm\varphi$ relating to oscillation within all the system roots are disposed, having some characteristic equation roots. However this characteristic is better to obtain directly by characteristic polynomial coefficients. According to [1] for the stationary system oscillation analysis on the basis of coefficient methods the following parameters are used

$$\delta_l=\frac{a_l(\bar{k})^2}{a_{l-1}(\bar{k})a_{l+1}(\bar{k})},\; l=\overline{1,n-2}.$$

This parameters are non-dimentional and called oscillation indices.

In [3] on the basis of δ_l sufficient conditions of specified oscillation is obtained: for the roots disposition in the given sector φ it is sufficient that the following conditions are held

$$\delta_l\ge\delta_P(n,\varphi),\;\forall\; l=\overline{1,n-1}, \tag{8}$$

where δ_P – permissible oscillation index, which is defined from fig. 1.

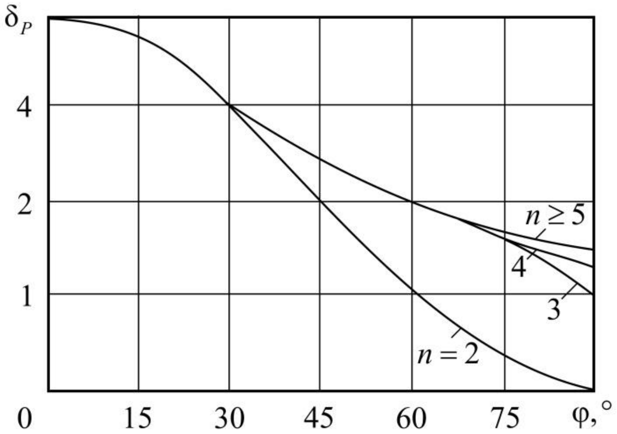

Figure 1. Permissible oscillation index definition

In case of interval parameters in system for oscillation analysis all interval coefficients range of interval characteristic polynomial should be defined and for each δ_l its minimal value should be found. However solution of the given problem can be simplified on the basis of interval analysis: it is offered to verify only inequalities, formed by the specified boundary values of interval coefficients

$$\frac{\underline{a_l(\bar{k})^2}}{\overline{a_{l-1}(\bar{k})}\,\overline{a_{l+1}(\bar{k})}} > \delta_P,\ l=\overline{1,n-1}. \qquad (9)$$

In this way, for the verification of conditions (9) is necessary to consider two additional polynomials

$$D_9(s)=\overline{a_0}+\underline{a_1}s+\overline{a_2}s^2+\underline{a_3}s^3+\overline{a_4}s^4+\underline{a_5}s^5+\overline{a_6}s^6+\ldots,$$

$$D_{10}(s)=\underline{a_0}+\overline{a_1}s+\underline{a_2}s^2+\overline{a_3}s^3+\underline{a_4}s^4+\overline{a_5}s^5+\underline{a_6}s^6+\ldots.$$

Note, that polynomials $D_8(s)$ and $D_{10}(s)$ coincide. This way, for the robust controller parametric synthesis providing system maximal stability degree at oscillation restriction it is necessary to fulfill conditions (7) and (9) for the stated above interval polynomials. For this purpose appropriate algorithm and programming tool set for robust controller synthesis are developed.

V. CONCLUSION

Developed systems synthesis approach with prescribed root quality indices is based on the system sufficient conditions of the specified stability degree and permissible oscillation indices. Robust extension of the given approach in case of interval systems provides verification of the given conditions in certain vertexes of the parametric polytope formed by the characteristic polynomial interval coefficients. These vertexes are determined on the basis of the interval analysis theory application to the interval inequalities system, which are formed on the basis of referred above sufficient conditions of specified stability degree and oscillation indices.

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