1.2 MAGNETIC FIELD

At each point of space the magnetic field is characterized in terms of a magnetic flux density vector [T]

$$B = \lim_{l \to 0} \frac{f}{i \cdot l}.$$

The mutual direction of vectors is clear from Fig. 1.3, where the thin straight wire carrying current i is shown in the uniform magnetic field. It is underlined by *Lorentz Force Law* (*Ampere's Law*) for a conductor element

$$\vec{df} = i[\vec{dl}\,\vec{B}],$$

where the direction of $\vec{dl}$ is in the direction of i.

All materials may be divided into *ferromagnetic* materials and *non-magnetic* materials by their magnetic properties taken into account in terms of a *vector of a magnetic field intensity (strength)* [A/m]:

$$\vec{H} = \frac{\vec{B}}{\mu_0},$$

where $\mu_a = \mu_0 \cdot \mu_r$ is the absolute magnetic permeability [G/m],

μ_r is the relative magnetic permeability of the material,

$\mu_0 = 4\pi \cdot 10^{-7}$ [G/m] is the permeability of a vacuum.

For the non-magnetic materials $\mu_r = 1$, for the ferromagnetic materials $\mu_r >> 1$ and it also depends on the intensity of a field (an effect of a saturation). Furthermore, an effect of hysteresis occurs in the ferromagnetic materials.

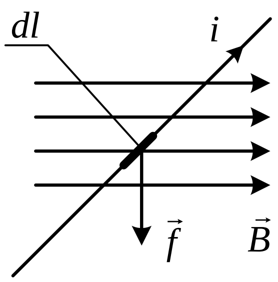

Fig. 1.3

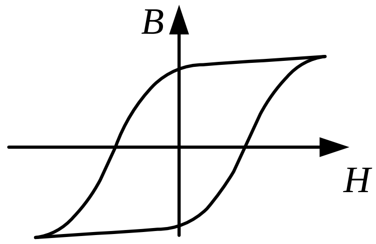

Fig. 1.4

The *magnetic flux* [Wb] through a surface S is

$$\Phi = \int_S (\vec{B}\,\vec{ds}).$$

Principle of the continuity of magnetic flux. Magnetic flux through any closed surface S is equal to zero

$$\oint_S (\vec{B}\,\vec{ds}) = 0.$$

By analogy to an electric field the concept of magnetic voltage (difference of scalar magnetic potentials) between points A and B is defined as

$$u_{MAB} = \varphi_{MA} - \varphi_{MB} = \int_A^B (\vec{H}\, d\vec{l}).$$

Ampere's Law (*The total current law). The circulation of a vector of a magnetic field intensity along a closed loop is equal to the total current passing through this loop:*

$$\oint_l (\vec{H}\, d\vec{l}) = \Sigma i.$$

The direction of tracing around loop and a current direction are associated by the right-hand screw rule (corkscrew rule). It is depicted in Fig. 1.5.

The *magnetic flux linkage* [Wb] of coil is equal to the number of turns *w* multiplied by a magnetic flux *Φ*, passing them:

$$\Psi = w \cdot \Phi .$$

Faraday's law of an electromagnetic induction. The magnetic flux through a surface, bounded by any loop, induces E.M.F., which is proportional to the rate of change of magnetic flux linkage in this loop.

$$e = -\frac{d\Psi}{dt}.$$

If the EMF may produce a current around the loop, it will be in such a direction as to oppose the change of magnetic flux. The «minus» sign shows this in the formula (Lenz's rule).

Example 1.3

The winding carrying current i with the number of turns w is placed on a toroidal ferromagnetic core of a cross-section area S and centerline length l (Fig. 1.6).

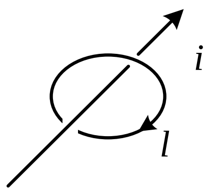

Fig. 1.5

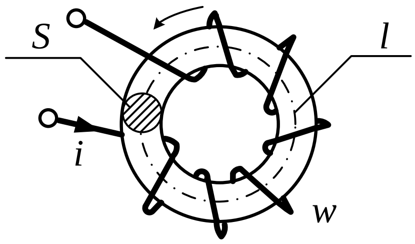

Fig. 1.6

The flux is distributed uniformly throughout the core, so that

$$\Phi = \int_S \vec{B}\, d\vec{S} = B \cdot S = \mu_a \cdot H \cdot S \quad \text{and} \quad iw = \oint_l (\vec{H} d\vec{l}) = H \cdot l.$$

From here, $\Psi = \Phi \cdot w = w^2 \cdot \mu_a \cdot S \cdot \frac{i}{l} = Li$,

where $L = \frac{\Psi}{i} = w^2 \cdot \mu_a \cdot \frac{S}{l}$ is the *inductance* of a coil [G].

The energy [J] stored in the coil is equal to

$$W_M = \Psi \cdot \frac{i}{2} = L \cdot \frac{i^2}{2}.$$

Under alternating current the voltage across coil terminals, compensates a self-induction E.M.F. induced into it

$$u_L = -e = \frac{d\Psi}{dt} = L \cdot \frac{di}{dt}.$$

There are two approaches to a research of electromagnetic processes they are *field theory* and *theory of circuits*.

The field theory operates with differential quantities characterizing these processes at each point of space at each instant:

$$E, \quad D, \quad \frac{dq}{dV}, \quad \frac{dq}{ds}, \quad B, \quad H, \quad \delta.$$

The theory of circuits uses with integral quantities defining a condition of electric devices and their elements:

$$q, \ \Phi, \ \Psi, \ \Psi_E, \ \Psi_D, \ u, \ u_M, \ i.$$

The most part of our course is just devoted to the theory of circuits.