1. PHYSICAL FUNDAMENTALS of ELECTRICAL ENGINEERING

In a research of the unified electromagnetic field existing objectively it is convenient to think of it as being into two components. One of them is an *electric* field detected by force action on fixed charged bodies, and another is a *magnetic* field, that can be detected by force action on uncharged current-carrying conductors.

1.1 Electric Field

The main term describing it, is the vector of electric field intensity:

$$\vec{E} = \lim_{q \to 0} \left(\frac{\vec{f}}{q} \right).$$

Unit of intensity is [V/m], charge is in [C], and force is in [N]. The direction of the electric field intensity vector is defined as the direction of the force acting on a point positive charge.

By electric properties materials are divided into *conductors* and *dielectrics* (*semiconductors* take intermediate position). Practically, the dielectrics do not contain free charges, but they can be polarized. A measure of this ability of dielectric is the *vector of the electric flux density* $\left[\mathrm{C/m^2} \right]$:

$$\vec{D} = \varepsilon_a \vec{E},$$

where $\varepsilon_a = \varepsilon_0 \varepsilon_r$ is the absolute permittivity [F/m],

ε_r is the relative permittivity (for vacuum and air its value is $\varepsilon_r = 1$),

$\varepsilon_0 = (4\pi \cdot 9 \cdot 10^9)^{-1}$ [F/m] is the electric constant.

The field of fixed charges is called electrostatic.

Gauss's theorem: The flux of an electrostatic field intensity vector through a closed surface in the homogeneous isotropic dielectric is equal to a ratio of the free charge inside the volume, enclosed within this surface to the absolute permittivity of a material:

$$\oint_S (\vec{E}\, d\vec{s}) = \frac{q_{\text{СВ}}}{\varepsilon_a}$$

In the case of an arbitrary dielectric **Maxwell's postulate** generalizes this theorem:

$$\oint_S (\vec{D}\, d\vec{s}) = q_{\text{СВ}}.$$

The electric voltage (electric potential difference) between points M and N is defined as

$$u_{MN} = \varphi_M - \varphi_N = \int_M^N (\vec{E}\, d\vec{l}).$$

It is connected with a work being done by the electric field forces to move a positive point charge from point M to point N:

$$A = \int_M^N (\vec{f}\, d\vec{l}) = q\int_M^N (\vec{E}\, d\vec{l}) = q \cdot u_{MN} = q(\varphi_M - \varphi_N).$$

The potential of point M is determined accurate to the constant from the condition $\varphi_N = 0$. Usually for the single bodies a potential of indefinitely far points is taken as zero and in actual problems a potential is equal to zero on the Earth surface). A corollary from the principle of energy conservation is the equality

$$\oint_l (\vec{E}\, d\vec{l}) = 0$$

(in the absence of external sources along the integration contour).

Unit of voltage and potential is [V].

Example 1.1

In the flat capacitor with the displacement d between the plates (of area S_0 and charge q) the field is uniform (Fig. 1.1).

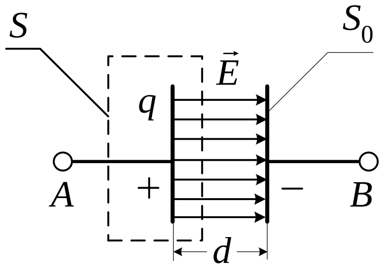

Fig. 1.1

By Gauss's theorem we have $\oint_S (\vec{E}\, d\vec{s}) = \int_{S_0} Eds = ES_0 = \dfrac{q}{\varepsilon_a}$.

The voltage between the plates is

$$u = \int_A^B (\vec{E}\, d\vec{l}) = \int_A^B Edl = Ed.$$

From here $q = \varepsilon_a S_0 \cdot E = \varepsilon_a S_0 \dfrac{u}{d} = C \cdot u$, where $C = \varepsilon_a \cdot \dfrac{S_0}{d}$ is the capacitance [F].

The energy stored in the capacitor is $W_Э = \dfrac{q \cdot u}{2} = \dfrac{C \cdot u^2}{2}$ [J].

If the voltage varies in time, then *a displacement current* [A] appears between the plates due to a reorientation of dipoles in the dielectric

$$i = \frac{dq}{dt} = C\frac{du}{dt}.$$

Under fixed voltage no current flows through a capacitor, though the capacitor has been charged.

The conductors contain free charges (electrons and ions) which under the influence of an external electrical field can move, producing a *conduction current*. Ohm's law in differential form determines its density [A/m^2]:

$$\vec{\delta} = \gamma \vec{E} \;,$$

where γ is the conductivity of a material [S/m = 1/(Ω·m)].

The current through a cross-section of a conductor of area S

$$i = \int_S (\vec{\delta}\, \vec{ds})$$

produces along a circuit path of length l the voltage drop

$$u = \int_l (\vec{E}\, \vec{dl}).$$

As well as in an electrostatic field when going around a closed path, in which the external sources are absent, it is

$$\oint_l \vec{E}\, \vec{dl} = 0.$$

Principle of the continuity of electric current. The current through a closed surface is equal to zero.

$$\oint_S (\vec{\delta}\, \vec{ds}) = 0.$$

Example 1.2

A cylindrical conductor of length l and cross-sectional area S (Fig. 1.2) carries current i.

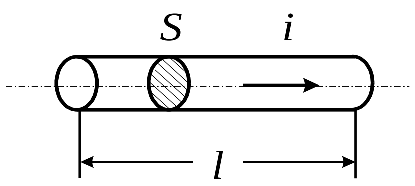

Fig. 1.2

At low frequencies a current density is practically the same all over the volume, therefore

$$i = \int_S \left(\vec{\delta}\, \vec{ds} \right) = \int_S \delta ds = \delta S = \gamma E S$$

then $$u = \int_l (\vec{E}\, \vec{dl}) = E \cdot l = i \cdot \frac{l}{\gamma S}.$$

In fact this expression is *Ohm's law:*

$$u = R \cdot i \,,$$

where $R = \dfrac{l}{\gamma \cdot S}$ is the *resistance* [Ω].

$G = \dfrac{1}{R}$ is the *conductance* [S},

$p = u \cdot i = i^2 \cdot R$ is the *power* [W].