$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40}\ \mu F^{-1}$$

$$= 0.1 + 0.05 + 0.025 = 0.175\ \mu F^{-1}$$

hence

$$C_S = \frac{1}{0.175} = 5.714\ \mu F \text{ (Ans.)}$$

Note
The value of C_S is **less than** the smallest capacitance (10 μF) in the circuit.

6.16 CAPACITORS IN PARALLEL

When capacitors are in parallel with one another (Figure 6.8(a)), they have the same voltage across them. The charge stored by the capacitors in the

fig 6.8 *(a) capacitors in parallel and (b) their electrical equivalent capacitance*

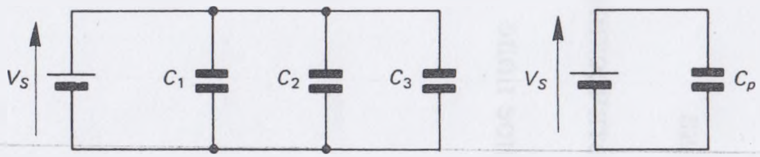


figure is, therefore, as follows

$$Q_1 = C_1 V_S$$

$$Q_2 = C_2 V_S$$

$$Q_3 = C_3 V_S$$

The total charge stored by the circuit is

$$Q_1 + Q_2 + Q_3 = C_1 V_S + C_2 V_S + C_3 V_S = V_S(C_1 + C_2 + C_3)$$

The parallel bank of capacitors in diagram (a) can be replaced by the single equivalent capacitor C_P in diagram (b). Since the supply voltage V_S is applied to C_P, the charge Q_P stored by the equivalent capacitance is

$$Q_P = V_S C_P$$

For the capacitor in diagram (b) to be equivalent to the parallel combination in diagram (a), both circuits must store the same charge when connected to V_S. That is

$$Q_P = Q_1 + Q_2 + Q_3$$

or

$$V_S C_P = V_S(C_1 + C_2 + C_3)$$

when V_S is cancelled on both sides of the equation above, the expression for the capacitance C_P is

$$C_P = C_1 + C_2 + C_3$$

The equivalent capacitance of a parallel connected bank of capacitors is equal to the sum of the capacitances of the individual capacitors. It is of interest to note that *the equivalent capacitance of a parallel connected bank of capacitors is greater than the largest value of capacitance in the parallel circuit.*

Example
Calculate the equivalent capacitance of three parallel-connected capacitors of capacitance 10, 20 and 40 microfarads, respectively.

Solution

$$C_1 = 10\ \mu\text{F};\ C_2 = 20\ \mu\text{F};\ C_3 = 40\ \mu\text{F}$$

equivalent capacitance, $C_P = C_1 + C_2 + C_3 = 10 + 20 + 40$

$$= 70\ \mu\text{F (Ans.)}$$

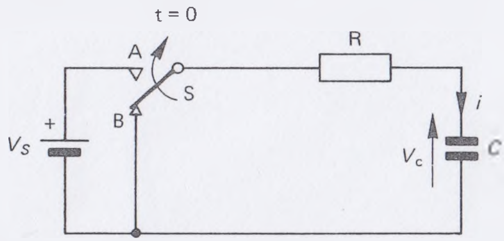
Note
The value of C_P is **greater** than the largest capacitance (40 μF) in the circuit.

6.17 CAPACITOR CHARGING CURRENT

In this section of the book we will investigate what happens to the current in a capacitor which is being charged and what happens to the voltage across the capacitor.

The basic circuit is shown in Figure 6.9. Initially, the blade of switch S is connected to contact B, so that the capacitor is discharged; that is the voltage V_C across the capacitor is zero.

You will observe that we now use the *lower-case* letter v rather than the upper case letter V to describe the voltage across the capacitor. The reason fig 6.9 *charging a capacitor. The blade of switch S is changed from position B to position A at time t = 0*



is as follows. Capital letters are used to describe either d.c. values or 'effective a.c.' values (see chapter 10 for details of the meaning of the latter phrase) in a circuit. Lower case (small) letters are used to describe **instantaneous values**, that is, values which may change with time. In this case the capacitor is initially discharged so that at 'zero' time, that is, $t = 0$, we can say that $v_C = 0$. As you will see below, a little time after switch S is closed the capacitor will be, say, half fully-charged (that is $v_C = \frac{V_S}{2}$). As time progresses, the voltage across C rises further. Thus, v_C changes with time and has a different value at each instant of time. Similarly, we will see that the charging current, i, also varies in value with time. We will now return to the description of the operation of the circuit.

When the contact of switch S is changed to position A at time $t = 0$, current begins to flow into the capacitor. Since the voltage across the capacitor is zero at this point in time, the **initial value** of the **charging current** is

$$i = \frac{\text{supply voltage} - \text{voltage across the capacitor}}{\text{circuit resistance}}$$

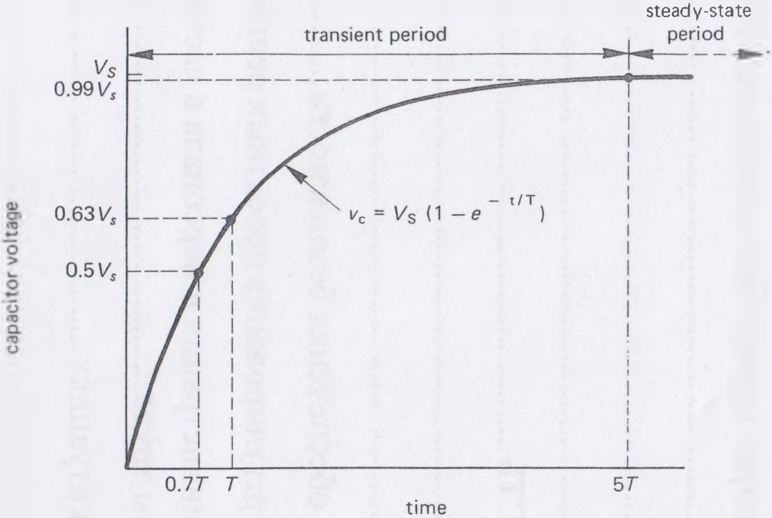
$$= \frac{(V_S - 0)}{R} = \frac{V_S}{R}$$

Let us call this value i_0 since it is the current at 'zero time'. As the current flows into the capacitor, it begins to acquire electric charge and the voltage across it builds up in the manner shown in Figure 6.10(a).

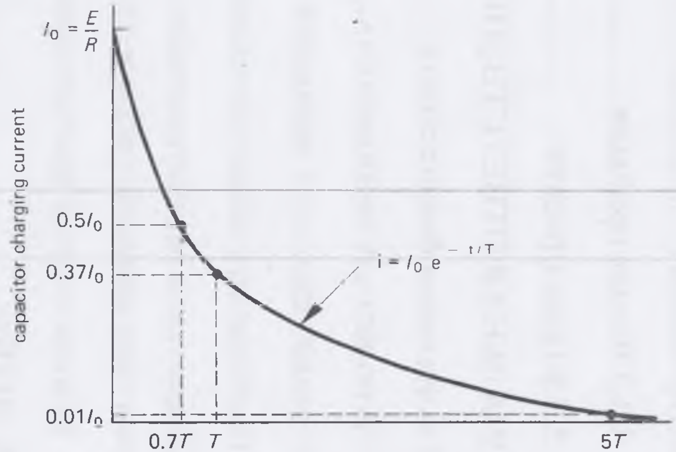
Just after the switch is closed and for a time less than $5T$ (see Figure 6.10), the current through the circuit and the voltage across each element in the circuit change. This period of time is known as the **transient period** of operation of the circuit. During the transient period of time, the voltage v_C across the capacitor is given by the mathematical expression

$$v_C = V_S\,(1 - e^{-t/T}) \text{ volts}$$

fig 6.10 *capacitor charging curves for (a) capacitor voltage, (b) capacitor current*



(a)



(b)

where v_C is the voltage across the capacitor at time t seconds after the switch has been closed, V_S is the supply voltage, T is the *time constant* of the circuit (see section 6.18 for details), and e is the number 2.71828 which is the base of the natural logarithmic series.

For example, if the time constant of an RC circuit is 8 seconds, the voltage across the capacitor 10 seconds after the supply of 10 V has been connected is calculated as follows

$$v_C = 10(1 - e^{-10/8}) = 10(1 - e^{-1.125})$$
$$= 10(1 - 0.325) = 6.75 \text{ V}$$

The curve in Figure 6.10(a) is described as an **exponentially rising curve**.

During the transient period, the mathematical expression for the transient current, i, in the circuit is

$$i = I_O e^{-t/T}$$

where I_O is the initial value of the current and has the value

$$I_O = \frac{V_S}{R} \text{ A}$$

The curve in Figure 6.10(b) is known as an **exponentially falling curve**.

After a time equal to $5T$ seconds ($5T = 5 \times 8 = 40$ seconds in the above example) the transients in the circuit 'settle down', and the current and the voltages across the elements in the circuit reach a steady value. The time period beyond the transient time is known as the **steady-state period**.

As mentioned above, T is the *time constant* of the circuit, and it can be shown that after a length of time equal to one time constant, the voltage across the capacitor has risen to 63 per cent of the supply voltage, that is $v_C = 0.63\ V_S$. The charging current at this instant of time is

$$i = \frac{(V_S - \text{voltage across the capacitor})}{R}$$
$$= \frac{(V_S - 0.63\ V_S)}{R} = \frac{0.37\ V_S}{R} = 0.37 I_O$$

This is illustrated in Figure 6.10. That is, as the capacitor is charged, the voltage across it rises and the charging current falls in value.

On completion of the transient period, the voltage across the capacitor has risen practically to V_S, that is the capacitor is 'fully charged' to voltage V_S. At this point in time the current in the circuit has fallen to

$$i = \frac{(V_S - \text{voltage across } C)}{R} \simeq \frac{(V_S - V_S)}{R} = 0$$

Thus, when the capacitor is fully charged, **it no longer draws current from the supply**.

6.18 THE TIME CONSTANT OF AN *RC* CIRCUIT

For a circuit containing a resistor R and a capacitor C, the **time constant**, T, is calculated from

$$T = RC \text{ seconds}$$

where R is in ohms and C is in farads. For example, if $R = 2000\ \Omega$ and $C = 10\ \mu\text{F}$, then

$$T = RC = 2000 \times (10 \times 10^{-6}) = 0.02 \text{ s or } 20 \text{ ms}$$

If the supply voltage is 10 V, it takes 0.02 s for the capacitor to charge to $0.63\ VS = 0.63 \times 10 = 6.3$ V.

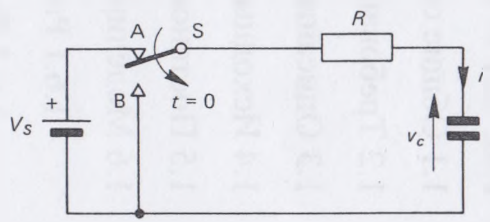
The time taken for the transients in the circuit to vanish and for the circuit to settle to its steady-state condition (see Figure 6.10, is about $5T$ seconds. This length of time is referred to as the **settling time**. In the above case, the settling time is $5 \times 0.02 = 0.1$ s.

6.19 CAPACITOR DISCHARGE

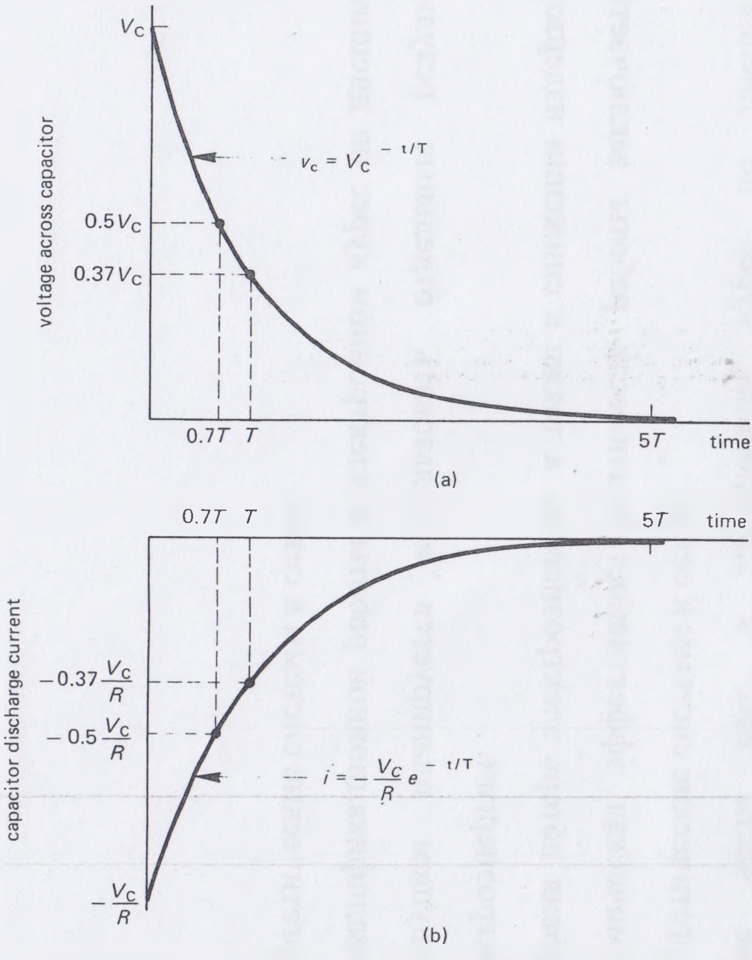
While the contact of switch S in Figure 6.11 is in position A, the capacitor is charged by the cell. When the contact S is changed from A to B, the capacitor is discharged via resistor R.

Whilst the capacitor discharges current through resistor R, energy is extracted from the capacitor so that the voltage v_C across the capacitor gradually decays towards zero value. When discharging energy, current flows out of the positive plate (the upper plate in Figure 6.11); that is, the current in Figure 6.11 flows in the reverse direction when compared with the charging condition (Figure 6.9).

fig 6.11 *capacitor discharge. The blade of switch S is changed from A to B at time t = 0*



The graph in Figure 6.12(a) shows how the capacitor voltage decays with time. The graph in diagram (b) shows how the discharge current rises to a maximum value of $\frac{-VS}{R}$ at the instant that the switch blade is moved fig 6.12 *capacitor discharge curves for (a) capacitor voltage, (b) capacitor current*



to position B (the negative sign implies that the direction of the current is reversed when compared with the charging condition); the current then decays to zero following an exponential curve.

The mathematical expression for the voltage v_C across the capacitor at time t *after* the switch blade in Figure 6.11 has been changed from A to B is

$$v_C = V_C e^{t/T} \text{ volts}$$

where V_C is the voltage to which the capacitor has been charged just before the instant that the switch blade is changed to position B. The time constant of the circuit is $T = RC$ (T in seconds, R in ohms, C in farads), and $e = 2.71828$. The expression for the discharge current is

$$i = -\frac{V_C}{R} e^{-t/T}$$

Once again, it takes approximately $5T$ seconds for the transient period of the discharge to decay, during which time the current in the circuit and the voltage across the circuit elements change. When the steady-stage period is reached, the current in the circuit and the voltage across R and C reach a steady value (zero in this case, since the capacitor has discharged its energy).

Theoretically, it takes an infinite time for the transient period to disappear but, in practice, it can be thought of as vanishing in a time of $5T$.

6.20 **TYPES OF CAPACITOR**

Capacitors are generally classified according to their dielectrics, for example, paper, polystyrene, mica, etc. The capacitance of all practical capacitors varies with age, operating temperature, etc, and the value quoted by the manufacturer usually only applies under specific operating conditions.

Air dielectric capacitors Fixed capacitors with air dielectrics are mainly used as laboratory standards of capacitance. Variable capacitance air capacitors have a set of fixed plates and a set of moveable plates, so that the capacitance of the capacitor is altered as the overlapping area of the plates is altered.

Paper dielectric capacitors In one form of paper capacitor, shown in Figure 6.13, the electrodes are metal foils interleaved with layers of paper which have been impregnated with oil or wax with a plastic (polymerisable) impregnant. In the form of construction shown, contact is made between the capacitor plates and the external circuit via pressure contacts.

In capacitors known as **metallised paper capacitors**, the paper is metallised so that gaps or voids between the plates and the dielectric are avoided. Important characteristics of this type when compared with other 'paper' types are their small size and their 'self-healing' action after electrical breakdown of the dielectric. In the event of the paper being punctured when a transient voltage 'spike' is applied to the terminals of the capacitor (this is a practical hazard for any capacitor), the metallising in the region of the puncture rapidly evaporates and prevents the capacitor from developing a short-circuit.

Plastic film dielectric capacitors These use plastic rather than paper dielectric and are widely used. The production techniques provide low