Problems and Questions

Module 1

1. When the concept of the trajectory of a particle is valid?

2. Evaluate the linear dimensions of atoms and nuclei if the energy of an atomic electron is about 10 eV and the energy of a nucleon is about 10 MeV.

3. Evaluate the width of an excited energy level of atomic electron if the lifetime of an excited state is $\tau \approx 10^{-8}$ c?

4. What conditions are needed to form the discrete energy spectrum of a particle?

Module 2

1. What is the force between the neighbor atoms of cesium and chlorine in a cesium chloride molecule? (See Fig.2.9).
$1{,}8 \cdot 10^{-4}$ dyne.

2.While melting the ice, the volume of the water produced is smaller than the initial ice volume and decreases when the water is heated up to 4°C. Explain that phenomenon using the concept of molecule bonds.

3. How many atoms are there in an elementary crystalline cubic cell: a) simple, b) volume centered, and c) side centered?

3. Give definitions of fermions and bosons.

5. Describe the process of phase space quantization. What quantity is called the density of states?

6. What is the difference between a degenerated and non-degenerated ensemble of particles? At what condition, the quantum distribution transforms into the classic one?

7. Give definition of the Fermi energy. What factors do affect its position at zero temperature?

8. How does depend the Fermi level on temperature? Which temperature is called the degeneracy temperature?

9. What is the number of atoms per an elementary cell in crystals with a cubic simple, volume- centered and side-centered one?
Answer: 1; 2; 4.

10. How many atoms are there per an elementary cell of crystals with a simple and tight packed hexagon structure?
Answer: 1; 2.

Module 3, Module 4

1.Prove that for an ideal hexagon structure with the tight packing, the quantity *c/a* = 1,633.

2. Evaluate the volume of an elementary cell of a hexagon crystal with parameters *a* and *c*.
Answer: $\frac{\sqrt{3}}{2}a^2c$.

3. Prove that [*hkl*] direction in a cubic cell is perpendicular to the plane (*hkl*).

4. Which planes in the structure of a side-centered and volume-centered cube are of more atom density packing? What are directions in those planes where the atomic linear density is maximal?

5.Find the parameter of an aluminum crystalline lattice (side-centered cube).
Answer: 4.04 Å.

6. The hexagon structure of a cadmium crystal is tightly packed (a = 2.97 Å, c = 5,61 Å). Find the cadmium density.
Answer: 8,65 $g \cdot cm^{-3}$.

7. The crystalline lattice of many metals can be found in two modifications: cubic volume-centered and cubic side-centered. Transition from one structure to the other one is accompanied by very small changing of the volume. Assuming that there is no volume change at all, find the ratio D_1/D_2. The quantity D_1 and D_2 is the inter-atomic distance in the side-centered and volume-centered lattice.

Module 5

1. Find the minimal Debye wavelength in titan if the characteristic temperature is 5°C, and the sonic speed $6 \cdot 10^3 m \cdot c^{-1}$
Answer: 10,2 Å.

2. Find the maximum photon energy in lead crystals if the characteristic temperature is 94K. Answer: $8,2 \cdot 10^{-2}$ eV.

3. Find the specific heat f zinc at 100° C.
Answer: 0,092 $cal \cdot g^{-1} \cdot dergee^{-1}$.

Модуль 6

1. The specific heat of aluminum at 20° C is 840 $J \cdot кg^{-1} \cdot K^{-1}$. Can you trust the Dulong and Petit law in that situation?

2. The energy 8300J is needed to heat a metal (mass 100g) from 20C to 50C. That temperature interval is higher than the characteristic temperature. How does that metal is called?
Answer: beryllium.

Module 7

1. In accordance with $k_B \theta_D = h\nu_D$, the quantity θ_D is proportional to ν_D (and hence to ν). Taking into account $\nu = \frac{1}{2\pi}\sqrt{\frac{\alpha}{m}}$ we can state that the quantity θ_D is greater for light (hard) metals and smaller for heavy metals with small elasticity modules. Is that statement always true? Give examples.

2. Find the specific heat of diamond at the temperature 30K.
Answer: $1,3 \cdot 10^{-4}$ $cal \cdot g^{-1} \cdot degree^{-1}$.

3. Accordingly to Einstein, the crystalline body can be considered as the set of N independent quantum oscillators of the same frequency. Deuce the distribution law. Find the average energy and specific heat of the system at law and high frequency.

4. The specific heat of a certain modification of carbon depends on temperature as T^2 (not as T^3 that is usual for most solids). What can you tell about the structure of that carbon modification?

Module 8

1. Wright the basic relations of the Drude-Lorenz theory.

2. How does the relaxation time depend on the free path time (in accordance with that theory)?

3. What object is called the hole in the electric conductivity theory of crystals?

4. How does the free path depend on temperature of an electron gas? Evaluate the free path.

5. Find the Fermi energy at zero temperature for aluminum. Assume that there are three free electrons per an aluminum atom.
Answer:12eV.

6. Find the difference of energy (in units of k_BT) of an electron at the Fermi level and at the levels with the population probability 0.20 and 0.80.
Answer: -1,38 k_BT and +1,38 k_BT.

7. Find the electron population probability of an energy level, which is at 0.01eV lower than the Fermi level at temperature +18° C.
Answer: 0.6.

Module 9

1. Find the free electron concentration in a crystal if the degeneracy temperature is 0°C. Answer: $1,86 \cdot 10^{25}$ m^3.

2. How would change the population probability of an electron energy level in metal if the temperature decreases from 1000K up 300K? The level is at 0.1eV higher than the Fermi level.
Answer: decreases by the factor of 11.4.

3.Find the free electron kinetic energy density in cesium at zero temperature.
Answer: 1280 J/cm^3.

4. Transform the energy distribution function of conduction electrons in metal into the velocity distribution function at an arbitrary and zero temperature. Draw the graphs.

5. What fraction of conduction electrons in metal at zero temperature has a kinetic energy greater than 0,5 E_F
Answer: 0.65.

6. Formulate the dispersion law of electrons in a metal. Give geometrical interpretation.

7. The quantity *e/m* of an electron was measured by the method of accelerating a metallic body. Explain the basic idea of the method.

Module 10

1. Explain the mechanism of generation of the allowed electron energy zones in crystals.

2. Find the de-Broglie wavelength of an electron at upper or down edge of a free zone.

3. Explain the basic idea of adiabatic and a single-electron approximation when solving the Schrödinger equation of an electron in a crystal. Draw the wave functions.

4. Formulate the rule of building the Brillouin zones.

5. Explain the basic ideas of quasi-free and quasi-bound electron approximation. What deduction can you make? Do you know some other methods?

Module 11

1. What is the difference of zone schemes of semiconductors, dielectrics, and metals?

2. List the different types of electron local energy levels in the forbidden zone.

3. Explain the concept of an ideal crystalline lattice. How does the lattice affect the motion of electrons through the crystal?

4. Explain the concept of the effective mass of an electron in a crystal.

5. How does the effective mass of an electron depend on the curve of the energy surface, speed, and quasi-momentum?

Module 12

1. Give definition of semiconductors, metals, and dielectrics in accordance with the zone theory.

2. Give definition of the hydrogen-like model of impurity states.

3. Find the specific conductivity of a silicon crystal, if the Hall coefficient
$R_x = 7 \cdot 10^{-4} m^3 \cdot K^{-1}$.
Answer: 390 om^{-1}·m^{-1}.

4. Phosphorus (25,7μкg) and gallium (38,2 μкg) are uniformly distributed in a silicon crystal (120g). Assuming that all the impurity atoms are ionized, find the specific resistance of the crystal.
Answer: 7,4·10^{-3} om^{-1}·m^{-1}.

5. Explain how to find the electron concentration in the conduction zone of an intrinsic semiconductor or dielectric.

6. How does the electron concentration depend on temperature?

7. How does the temperature affect the free electron concentration in an impurity semiconductor?

Module 13

1.Find the electron mobility in an *n*-type germanium. The specific resistance ρ = 1,6·10^{-2} om·m, the Hall coefficient 7·10^{-3} m^3·K$^{-1.}$
Answer: 0.37 m^2/(V·c).

2. Find the minimal energy needed to produce the electron-hole pair in GaAs crystal. The electric conductivity changes ten folds within the temperature interval (+20 до –3)°C.
Ansver:1.4eV.

3. The electric resistance of a PbS crystal at temperature 20°C is 10^4 Ohm. Find its resistance at the temperature +80° C.
Answer:1350 Ohm.

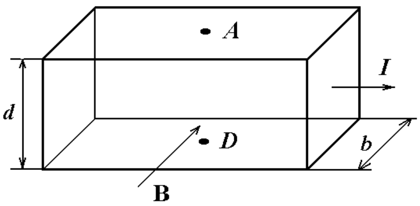
4. Explain the basic peculiarities while populating by electrons the local levels in a forbidden zone.

5. The conductivity of an intrinsic semiconductor at T = 273 K is 0.01 symens. The optical measurements have proved that the valence zone is at 0.1eV lower than the conduction zone. Find the electric conductivity of the semiconductor at T = 500K.

Module 14

1. At what condition, in a semiconductor with the free charge carriers, there is no the Hall effects?

Отв.: $\frac{p}{n} = \left(\frac{u_n}{u_p} \right)^2$



Problem 2, module 14 Through the sample in the form of a parallelepiped, electric current I runs. The magnetic field **B** is perpendicular to the current and sides of the sample

2. Find the sign, concentration, and mobility of the free charge carriers in an impurity semiconductor. The electric resistance is 338 Ohm. In a magnetic field 0.1 Tesla, the current 50mA runs through the semiconductor. The Hall voltage produced is 200 mV. The size of the sample (see Figure) b = 0,1 мм, d = 5 мм.

Answ.: $n = 1{,}8 \cdot 10^{21}$ m-3, $u_n = 2\ \mathrm{m}^2 \cdot \mathbf{V}^{-1} \cdot \mathrm{c}^{-1}$.

3. Write the Boltzmann kinetic equation and explain the basic ideas of its applying.

4. While investigating kinetic phenomena, the approximation of the relaxation time is used. Explain the main idea of that approximation.

5. In germanium here is no the Hall effects. What fraction of a current do electrons transport? The electron mobility is 3500 $\mathrm{cm}^2/(\mathrm{V}\cdot\mathrm{c})$, the mobility of holes is 1400 $\mathrm{cm}^2/(\mathrm{V}\cdot\mathrm{c})$?

Module 15

1. Find the thermionic current density (at the zero electric field) in tungsten at the temperature 2500 K.
Answer: 12.3A/m^2.

2. Using the Fig.7.2, find the work of exit of tungsten.
Answer: 4.43 eV.

3. The distance between plate electrodes is 0.01 meter. The voltage across the electrodes is 1000 V. Find the electric field at the cathode. Find the position of the maximum of potential energy $x_0 = \left(\dfrac{e}{16\pi\varepsilon_0 E}\right)^{1/2}$ (in angstroms). What is the decrement of the exit work caused by the Schottky defect? Find the ratio of the current density (at a given field) to that one (when there is no field) at temperature 1700 K. Assume that there are no space charges.
Answer: 10^5 V/m; $6.0\cdot10^{-8}$ m; 0.012 V; 1.085.

Module 16

1. Draw the zone scheme of a metal-n-type semiconductor contact if the thermodynamic exit work from the semiconductor is greater than that one from the metal.

2. Why the energy zones near the surface of a semiconductor are curved?

3. What quantity is called the screening (Debye) length?

4. On what does depend the contact voltage between a metal and semiconductor?

5. Explain why at a metal-semiconductor interface, the contact field is located mainly in the semiconductor and practically does not penetrate into the metal.

6. What is called the blackout layer? What is its thickness?

7. Why does a contact metal-semiconductor rectify currents?

Module 17

1. Using the zone scheme explain how a *p-n-p* transistor acts.

2. Draw the zone scheme of the surface region of an *n*-type semiconductor for poured, enriched, and inverse layers.

3. Draw the scheme of a *p-n* transition.

4. Draw the ampere-voltage characteristic of a *p-n* trasition. перехода.

5. Describe an active resistance (ohm) contact. What types of contacts do you know? .

6. Heterotransitions

Module 18

1.To find the relaxation time τ of polar dielectric (at a fixed temperature), the quantity ε''(in certain frequency interval) is measured. It appeared that the relaxation time corresponds to a frequency considerably greater then the theoretical one.
Prove, that if the following relation is true $\varepsilon''=\left(L-Mf^2\right)f$ (*f* –frequency), then $\tau^2 = M/4\pi^2 L.$
If the electric conductivity of dielectric were appreciable, it would be convenient to use the logarithm scale to build the dependence of ε'' on the frequency. Why?

2. In a vacuum chamber there is a certain quantity of diatomic molecules. An electric field is applied across the chamber.
Prove that the polarization of the gas depends on orientation of molecules relative the field. For polar molecules, the orientation effect is more noticeable (especially at low temperature). The refraction indices for polarized light depend on orientation of polarization plane relative the field (perpendicular and parallel).
Get the expression for the difference of the refraction index for these situations. Assume that the ‘length’ of a molecule is *l* and the electron polarizability of both atoms of the molecule is identical.

Module 19

1. An alternating voltage is applied through a capacitor. The capacitor dielectric is of polar kind. The relaxation time τ is known. Get the expression for the heat loss as a function of frequency. Prove that the minimal value of ε'' corresponds to the frequency

$\omega_{рел}$ ($\omega_{рел} = 1/\tau$ - the relaxation frequency) when the energy loss reaches the half of its maximum. You can think that this method is very convenient to find the relaxation frequency $\omega_{рел}$, but it appears that in practice it is not very good. Why?

2. Let the quantity *n* be the free electron concentration at low pressure. Prove that the frequency dependence of the dielectric permittivity can be given by: $\varepsilon_r = 1 - \frac{ne^2}{4\pi f^2 m\varepsilon_0}$,

f –is the frequency of the electromagnetic field ($\omega = 2\pi f$)
How does the electromagnetic field of different frequencies propagate through that plasma? Apply the result to a metallic sodium and compare it with the experimental data concerning the critical magnitude of the frequency when ε_r becomes zero.

Module 20

1. What crystalline defects do you know? How are they generated?

2. What factors do affect the speed of defect translation in crystals?

3. Describe the dependence of the charge defect current on the electric field strength.

4. a) To build a vacancy in aluminum, the energy of 0.75 eV is needed. How many vacancies are there at the room temperature, at 550° C?
b) To build a penetration defect in aluminum, the energy 3eV is needed. Find the ratio n_i/n_v at room temperature and at 550° C.

Module 21

1. The impurity atoms uniformly diffuse through a solid of unique cross-section along the x-axis. Prove that the concentration rate can be given as follows:
$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}.$$

2. A substance distributed on the surface of a body evaporates inside it.
a) Prove that the concentration depends on a distance and time as follows:
$$C = \frac{A}{\sqrt{t}} e^{X^2/4Dt}.$$
b) Build the graph of that equation.
c) Find the distance at which the curvature of the function is zero.
d) Find the relative quantity of atoms, which have diffused at the least at the distance equal to the double mean square one.

Module 22

1. Compare the features of crystals, polymers, liquid crystals, and amorphous matters.

2. Explain what the expression ‘the jump electric conductivity’ means.

Control Tasks

Section 1

1. Explain the physics concept of the quantities $|\psi(x)|^2$ and $|\psi(x)|^2 dx$. Write the normalization condition.

2. Prove that at $r = r_B$ (r_B – the Bohr radius), the density probability for an hydrogen electron in the state (n = 1, l = 0) is maximum.

3. How can you describe the wave function of a state, which can be built in different ways? Give examples. [As for me, I don't understand. May be you mean the superposition principle?]

4. A body (mass 1 g) is located at x=0 with possible deviation (of quantum character) 10^{-10} m. How long would the body displace through the distance 1 cm, if we assume that the uncertainty principle is true?

Section 2

1. Prove that direction [*hkl*] in a cubic lattice is normal to the plane (*hkl*).

2. Prove that the distance between the neighbor *(hkl)* planes in a cubic lattice (the side a) can be written as follows: $d = \dfrac{a}{\left(h^2 + k^2 + l^2\right)^{1/2}}$

3. How many modifications of the planes of {*hkl*} and {*hk*0} type are there in cubic systems?

Section 3

1. In a solid with the length L, the maximum phonon wavelength is λ_{max} = 2L. The minimum wavelength λ_{min} is about c/ν /*y*. The frequency $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{\alpha}{m}}$; c is the sonic speed in the solid. Evaluate λ_{min} for some metals. Compare the found quantities with the inter-atomic distance.

2. In accordance with the classic theory, the specific heat is 6 cal/mole.
a) Evaluate the heat energy of a metal mole at 300° K.
б) The Debye temperature of gold is 170° K. Using the Debye theory of heat capacitance, calculate the heat energy of one mole of gold at 300° K. What is the error while evaluating in accordance with the classic theory?

Section4

1. Find the specific electric conductivity of a metal if the charge carrier concentration is n=5·10^{22}см$^{-3}$ and the relaxation time τ=2·10^{-13}с. Assume that the charge carrier mass is identical with that of an electron.

3. Evaluate the drift speed of electrons in a metal if the current density j = 1 A/cm^3, and the charge carrier concentration is n=5·10^{22}см$^{-3}$. What is the path of an electron (in terms of a lattice parameter) per second in the crystal (d = 6 Å)?

4. Find the time of free path of an electron. The charge carrier mobility is 0.4 SI units. The carrier mass is identical with the electron mass.

5.What properties of charge carriers does the function $f(E,T)$ describe? What types of functions $f_0(E,T)$ can describe the states of electrons?

Section 5

1. Draw the first four Brillouin zones of a simple two-dimensional rectangular lattice (the inter-atomic distance a and $b = 3a$).

2. Define the density state function of electrons and holes in **k**-space and in **p**-space.

3. What is the form of the density state function for the square dispersion law and scalar effective mass?

4. Give definition of the Fermi integral of the order of 1/2. Try to calculate the integral.

4. A particle is confided inside a three-dimensional potential box. Find the number of states with the energy less then E^*.

Advise: let k_x, k_y, and k_z be the coordinate axes in k-space. Build the sphere of radius E^* and find the number of possible quantities of wave vectors inside that sphere.

Section 6

1. Explain how to apply the temperature dependence of specific conductivity of an impurity semiconductor in order to find the width of forbidden zone and depth of impurity levels.

2. Draw the temperature dependence of specific conductivity of an intrinsic semiconductor, p-type semiconductor, and metal.

3. The intrinsic electric conductivity of semiconductor can be written as follows: $\sigma_i = en_i(\mu_n + \mu_p)$. Prove that the minimal conductivity is expressed by: $\sigma_{\min} = 2en_i(\mu_n\mu_p)^{1/2}$. The electron concentration $n = n_i(\mu_p/\mu_n)^{1/2}$, the concentration of holes: $p = p_i(\mu_n/\mu_p)^{1/2}$

3. A sample with size: l_x = 5 cm, l_y = 0.5 cm, and l_z = 0,5 cm is located in magnetic field B_z = 1000 gauss. While applying the voltage V_x = 3,5 mV, the current I = 25 mkA runs. The Hall voltage is 12 mkV. Find the electric conductivity, mobility, and concentration of charge carriers.

4. Describe the electric conductivity of a crystal using concepts of the Boltzman kinetic equation. Explain why in mostly cases, the electric conductivity is not an isotropic quantity.

Section 7

1. Find the dependence of the electric field voltage on coordinates for displaced in the blacking direction metal-semiconductor contact. Draw the graphs $E(x)$ and $\varphi(x)$.

2. Find the penetration depth of a contact field in germanium. The surface is covered by by a metallic film. The equilibrium electron concentration is $n_0 = 10^{14}$ см$^{-3}$, the difference of exit works at the metal-germanium interface is 0.3eV, the dielectric constant $\varepsilon = 16$.

3. The difference of exit works at the interface of two metals is 1.7 eV. When in contact, the current flow from one metal into another and the energy barrier is built. Assume that the displacement of electrons is about an inter-atomic distance, i.e. ~3 Å. How many electrons are displaced from a surface of 1 cm^2?

4. Find the equilibrium electron current through a semiconductor-metal contact at room temperature. The surface electron density of the semiconductor is $n_S = 10^{14}$ cm^{-3}.

Find the contact voltage across a p - n transition in germanium at room temperature. The donor concentration in n-region is $N_d = 10^{14}$ cm^{-3}. The acceptor concentration in p-region is $N_a = 10^{14}$ cm^{-3}. Assume that the diffuse rectification theory is true.

Section 8

1. Prove that the torque **M**, acting upon a dipole **p** in the field **E,** is as follows: **M = [pE]**. If the dipole density is N, the entire torque is $\mathbf{M}_N = [N\mathbf{pE}] = [\mathbf{PE}]$. Thus the quantity **P** is the dipole moment of the unit volume.

2. The distance between an argon atom and electron is 15 Å.
a) Find the induced dipole moment.
б) Explain what kind of a force acts between the atom and electron.

4. In table P8.3, the dielectric loss in ThO_2 (with Ca impurity) is listed.

Table P8.3

f = 695 Hz				f = 6950 Hz			
T K	tgδ×10^{-2}	T K	tgδ×10^{-2}	T K	tgδ×10^{-2}	T K	tgδ×10^{-2}
555	2.3	509	8.6	631	2.6	581	8.6
543	4.2	503	8.1	621	3.6	568	8.6
532	7.0	494	6.3	612	4.3	543	5.5
524	8.6	485	4.2	604	5.5	518	2.5
516	9.2	475	2.9	590	7.3	498	1.0

a) Draw the temperature dependence and find the maximums for every frequency.
б) The experiment shows that the dielectric constant at room temperature is $\varepsilon_\infty\left(T = 25^\circ C\right) \approx 16.2\varepsilon_v$. Prove that it coincides with the quantity $\varepsilon_s - \varepsilon_\infty \approx 3\varepsilon_v$.
в) Assume that the temperature dependence of the relaxation time is an exponent $\tau = \tau_0 \exp(H/Rt)$, find parameters H and τ_0.

Section 9

1. In table P9.1, the experimental data on diffusion of radioactive silver into alloy silver-indium (16.7% In) are listed. The heat treatment was made at 728.5° C during $5.9 \cdot 10^4$ seconds.

Таблица P9.1

The penetration depth, 10^{-4} m	Specific radioactivity. Arbitrary units	The penetration depth, 10^{-4} m	Specific radioactivity, Arbitrary units
0	600	3.29	88
0.84	540	3.76	50
1.32	450	4.25	25
1.83	360	4.70	12
2.30	250	5.20	5
2.79	160	5.68	2

Find the diffusion coefficient.

2. The diffusion coefficients of zinc atoms into copper crystals are listed in the following table.

T,K	1322	1253	1176	1007	878
D, м^2/с	$1.0 \cdot 10^{-12}$	$4.0 \cdot 10^{-13}$	$1.1 \cdot 10^{-13}$	$4.0 \cdot 10^{-15}$	$1.6 \cdot 10^{-16}$

Find the activation energy and frequency factor D_0.

3. To make the endurance of the steel construction surface greater, it is undergo the action of high temperature in carbon medium. In the process of treatment, carbon atoms diffuse into the surface layer of steel. The layer enriched by carbon atoms is undergone the following heat treatment at lower temperature. As a result, the hard layer of great surface endurance (cemented layer) is produced. The coefficient of carbon-steel diffusion is given as follows:

$$D = 0.12 \cdot 10^{-4} \exp\left(\frac{-32000\,\mathrm{cal/mol}}{RT}\right) \mathrm{m}^2/\mathrm{c}.$$

What time is needed to build the cemented layer of the width 0.5mm on a steel rod with radius 2.cm at the temperature of diffusion heat treatment 925° C?

Solution of control problems

Module 3, problem 7

Let A_1 and A_2 be the periods of lattice. Obviously:

$$D_1 = \frac{\sqrt{2}}{2} A_1 \quad \textit{and} \quad D_2 = \frac{\sqrt{3}}{2} A_2.$$

There are four molecules per a side-centered crystal cell and only two molecules per a volume crystal lattice cell. We assume that the volume per a molecule does not change while transforming: $A_1^3/4 = A_2^3/2$. Thus:

$$D_1/D_2 = \sqrt{2/3}(2)^{1/2} = 1{,}029$$

Module 7, problem 3

The harmonic oscillator energy is $E_n = n\hbar\omega$. For simplicity we do not take into consideration the zero oscillation energy, which does not depend on temperature. The average population number can be written as follows:

$$\langle n\rangle = \frac{\sum_n n e^{-E_n/k_BT}}{\sum_n e^{-E_n/k_BT}} = \frac{\sum_n n e^{-n\hbar\omega/k_BT}}{\sum_n e^{-n\hbar\omega/k_BT}} = \frac{\frac{d}{d(\hbar\omega/k_BT)}\sum_n^{\infty} e^{-n\hbar\omega/k_BT}}{\sum_n^{\infty} e^{-n\hbar\omega/k_BT}} =$$

$$= \frac{e^{-\hbar\omega/k_BT}\left(1-e^{-\hbar\omega/k_BT}\right)^{-2}}{\left(1-e^{-\hbar\omega/k_BT}\right)^{-1}} = \frac{1}{\left(e^{\hbar\omega/k_BT}-1\right)}.$$

A molecule has three oscillation modes:

$$\langle E\rangle = 3N\hbar\omega\Big/\left(e^{\hbar\omega/k_BT}-1\right)$$

At high temperature: $E = 3Nk_BT = 3nRT$, where *n* – the number of moles, *R* – the gas constant. At low temperature:

$$E = 3N\hbar\omega e^{\hbar\omega/k_BT}$$

Thus at high temperature:

$$C = \frac{1}{n}\cdot\frac{\partial E}{\partial T} = 3R \approx 6\,кал/(моль\cdot К).$$

At low temperature:

$$C = 3R\left(\frac{\hbar\omega}{k_BT}\right)^2 e^{-\hbar\omega/k_BT}.$$

The specific heat at high temperature coincides with the Dulong and Petit law. At low temperature $C \to 0$ in accordance with classic model but the detailed dependence is different than the classic one. The Debye approximation leads to better results

Module 7, problem 4

The oscillation energy of a lattice is:

$$U = \int \frac{\hbar\omega g(\omega)d\omega}{e^{\hbar\omega/k_BT}-1}$$

The quantity $g(\omega)$ is the phonon density of states. At low temperature, the small frequencies are of the importance. For a three-dimensional lattice:

$$g(\omega)d\omega = \frac{d^3k}{(2\pi)^3} = \frac{4\pi k^2dk}{(2\pi)^3} = \frac{4\pi\omega^2d\omega}{(2\pi)^3c^3},$$

The quantity *c* is a sonic speed. Thus $g(\omega)\sim\omega^2$, and it follows that the energy is proportional to T^4, and the specific heat is proportional T^3.

When a solid is composed of two-dimensional crystals (graphite), the density is proportional to frequency: $g(\omega)\sim\omega$. It leads to a square dependence of the specific heat on temperature.

Module 9, problem 7

Let a metal sample undergo acceleration **-a**. In the sample coordinate system, electrons are accelerated in reverse direction and the equivalent electric field produced is $\mathbf{E} = \frac{m\mathbf{a}}{e}$. The field generates an electric current with density $\mathbf{j} = \frac{m\sigma\mathbf{a}}{e}$. Thus we can measure the ratio *e/m*.

Module 13, problem 5

The electric conductivity can be written as follows: $\sigma = e(n_e\mu_e + n_p\mu_p)$, where n_e and μ_e (n_p and μ_p) correspondingly the density and mobility of electrons and holes. In intrinsic semiconductor: $n_e = n_p$. Thus:

$$n_e = \frac{1}{h^3}\int \frac{p^2 dp}{\exp\{(\Delta + p^2/2m_e - E_F)/k_B T\} + 1}.$$

The probability $\rho_p(E)$ that the hole populates the level with energy E is equal the probability that an electron does not populate that level.

$\rho_p(E) = 1 - \rho_e(E)$. $E = -p^2/2m_p$. Hence:

$$n_p = \frac{1}{h^3}\int \frac{p^2 dp}{\exp\{(\Delta + p^2/2m_p + E_F)/k_B T\} + 1}.$$

The quantity Δ = 0.1eV (the energy slit), E_F is the Fermi level. If $(\Delta - E_F)/k_B T >> 1$ and $E_F >> k_B T$, we have:

$$n_e = \frac{1}{h^3}\int p^2 e^{-\frac{\Delta + (p^2/2m_e) - E_F}{k_B T}} dP,$$

$$n_p = \frac{1}{h^3}\int p^2 e^{-\frac{(p^2/2m_p) + E_F}{k_B T}} dP$$

Condition $n_e = n_p$ leads to:

$$E_F = \frac{\Delta}{2} + k_B T\left(\frac{m_p}{m_e}\right)^{3/4}$$

Thus the conduction electron density:

$$n_e = AT^{3/2}e^{-\Delta/k_B T}$$

A is a constant. The temperature dependence of conductivity can be written as follows: $\sigma = \sigma_0 T^{3/2} e^{-\Delta/2k_B T}$. Thus:

$$\frac{\sigma(T_1)}{\sigma(T_2)} = \left(\frac{T_2}{T_1}\right)^{3/2} e^{-\Delta(T_2 - T_1)/2k_B T_2 T_1} = \frac{\sigma(T_1)}{\sigma(T_2)} = 6.5.$$

Module 14, problem 5

Let the axis *y* coincides with the current and magnetic field is directed along the axis *z*. There is no a Hall effect in germanium. It means that the electric current in *x*-direction is zero. The velocity of electrons and holes in *y*-direction is: $v_e = -\mu_e E, v_p = \mu_p E$. The magnetic field force along the *x*-axis:

$$F_e = -ev_e H \quad and \quad F_p = ev_p H.$$

The transversal component of velocity:

$$v'_e = -\mu_e v_e H = +\mu_e^2 HE \quad u \quad v'_p = \mu_p^2 HE,$$

The current in x-direction:

$$I' = en_p v'_p - en_e v'_e = eHE\left(n_p \mu_p^2 - n_e \mu_e^2\right)$$

There is no current when: $n_p \mu_p^2 = n_e \mu_e^2$ (1)

The current in *y*-direction: $I = e\left(n_p \mu_p + n_e \mu_e\right)$. The electron component of the current:

$$f = \frac{\mu_e n_e}{\left(\mu_e n_e + \mu_p n_p\right)} = \left(1 + \frac{\mu_p n_p}{\mu_e n_e}\right)^{-1}$$

Taking into account condition (1):

$$f = \left(1 + \frac{\mu_e}{\mu_p}\right)^{-1} = \frac{2}{7}$$