

6.13 The Kinetic Phenomena in Semiconductors

The elementary theory of semiconductor electric conduction and scattering of charge carriers was discussed previously. We are going to discuss more correct conceptions, which take into account that the directed motion of the carriers is not equilibrium and hence a certain non-equilibrium function has to be used. As a fundamental equation of the transport phenomena, the Boltzmann kinetic equation will be used.

6.13.1 The Boltzmann Kinetic Equation in a Weak Electric Field

When temperature and crystalline structure is uniform and the electric field ε in x-direction is applied to the crystal, the distribution function follows the equation:

$$\left(\frac{df}{dt}\right)_{\varepsilon} = \frac{df}{dt} + \frac{df}{dp_x} \cdot \frac{dp_x}{dt}. \quad (6.124)$$

Obviously:

$$\frac{dp_x}{dt} = F_x = -e\varepsilon. \quad (6.125)$$

Hence:

$$\left(\frac{df}{dt}\right)_{\varepsilon} = \frac{df}{dt} - e\varepsilon \frac{df}{dp_x}. \quad (6.126)$$

The external field ε is usually much less than the intrinsic field, and the change of the equilibrium distribution function is small. We can write:

$$f = f_0 + f_1. \quad (6.127)$$

Here $f_0 = f_{\Phi}$ or f_{M-B} (for degenerated or non-degenerated gas), f_1 is a small addend, which describes the transport phenomenon.

If the external field is switched off the equilibrium distribution is reinstated (because of collisions between electrons and scattering centers). We assume that the disturbance is not great and the quantity $(df/dt)c$ due to collisions is proportional to deviation:

$$\left(\frac{df}{dt}\right)_{cm} = -\frac{f_1}{\tau} = -\frac{f - f_0}{\tau}, \quad (6.128)$$

τ is the relaxation time.

Having in mind that f_0 does not depend on time:

$$\frac{d(f - f_0)}{f - f_0} = -\frac{dt}{\tau}. \quad (6.129)$$

And we get:

$$f_1 = (f_1)_0 e^{-t/\tau}. \quad (6.130)$$

Here $(f_1)_0$ is the function f_1 at the zero moment (the time when the field is switched off).

In general case $\tau = \tau(\mathbf{k})$, the function form depends on scattering. Hence, in a semiconductor under action of an electric field, there are two processes. The first process is changing the distribution function with the speed $(df/dt)_\varepsilon$. The second process is a relaxation one, which tends to return the system to the equilibrium state with the speed $(df/dt)_c$.

The condition of the stationary state is:

$$(df/dt)_\varepsilon = (df/dt)_{cm} \quad (6.131)$$

In accordance with (6.125) and (6.128):

$$-\frac{df}{dt} + e\varepsilon \frac{df}{dp_x} = \frac{f - f_0}{\tau(\mathbf{k})}. \quad (6.132)$$

For the stationary state:

$$\frac{df}{dt} = 0. \quad (6.133)$$

Hence according to (6.132) and (6.133):

$$f = f_0 + e\varepsilon \tau \frac{df}{dp_x} \quad (6.134)$$

Having in mind that the difference between f and f_0 is small, we write (6.134) as follows:

$$f = f_0 + \frac{e\varepsilon\tau}{m_n} \frac{df_0}{dp_x}. \quad (6.135)$$

Equation (6.135) can be considered as the possible form of the Boltzmann kinetic equation for an electron gas.

Differentiating function f_0 relative energy we get:

$$f = f_0 + e\varepsilon\tau v_x \frac{df_0}{dE}. \quad (6.136)$$

Let us investigate the transport phenomenon (electric conduction) using the Boltzmann equation.

6.13.2 The Electric Conductivity of Electron Semiconductors

The electron concentration in the energy interval dE of the spherical surface in k-space:

$$.dn = N(E) 2 f(E) dE = \frac{4\pi(2m_n)^{3/2}}{h^3} (E - E_c)^{1/2} f(E) dE, \quad (6.137)$$

Here m_n — the electron effective scalar mass, $m_n = m^* = m_{on}$. Assume that $E_c = 0$.

The charge carried by drifting electrons in x-direction per unit square and time is:

$$-e v_x dn = -\frac{4\pi e(2m_n)^{3/2}}{h^3} E^{1/2} f(E) dE. \quad (6.138)$$

The current density:

$$j = -\frac{4\pi e(2m_n)^{3/2}}{h^3} \int_0^\infty v_x E^{1/2} f(E) dE. \quad (6.139)$$

Having substituted (6.136) in (6.139) we get the sum of two integrals. The first one is zero because it expressed the electric current under equilibrium condition. The second integral:

$$j = -\frac{4\pi e(2m_n)^{3/2}}{h^3} \varepsilon \int_0^\infty v_x^2 \tau E^{1/2} \frac{df_0}{dE} dE. \quad (6.140)$$

Hence the electric current is due to the addend f_1 .
Assume that:

$$v_x^2 \approx v_y^2 \approx v_z^2 \approx \frac{2}{3} \frac{E}{m_n}. \quad (6.141)$$

In accordance with (6.139):

$$j = -\frac{8\pi e^2(2m_n)^{3/2}}{3h^3 m_n} \varepsilon \int_0^\infty \tau E^{3/2} \frac{df_0}{dE} dE. \quad (6.142)$$

For non-degenerated gas $f_0 = f_{M-B}$. Hence:

$$j = \frac{8\pi e^2(2m_n)^{3/2}}{3h^3 m_n kT} e^{\frac{E_\phi}{kT}} \varepsilon \int_0^\infty \tau E^{3/2} e^{-\frac{E_\phi}{kT}} dE. \quad (6.143)$$

Taking into account that $n_0 = N_c e^{-\frac{E_c - E_F}{kT}}$ we get ($E_c = 0$):

$$e^{\frac{E_\phi}{kT}} = \frac{n_0}{N_c} = \frac{n_0 h^3}{2(2\pi m_n kT)^{3/2}}. \quad (6.144)$$

Multiplying and dividing (6.143) by $\int_0^\infty E e^{-\frac{E}{kT}} E^{1/2} dE$, and having in mind (6.144) we get:

$$j = \frac{n_0 e^2 \langle \tau \rangle}{m_n} \varepsilon. \quad (6.145)$$

The quantity $\langle \tau \rangle$ is the average relaxation time:

$$\langle \tau \rangle = \frac{\int_0^{\infty} \tau E e^{-\frac{E}{kT}} E^{1/2} dE}{\int_0^{\infty} E e^{-\frac{E}{kT}} E^{1/2} dE} \quad (6.146)$$

$$\int_0^{\infty} E e^{-\frac{E}{kT}} E^{1/2} dE = \frac{3}{4} \sqrt{\pi} (kT)^{5/2}, \quad (6.147)$$

Equation (6.145) is the Ohm law for non-degenerated electron gas. Hence the specific electric conductivity:

$$\sigma = \frac{n_0 e^2}{m_n} \langle \tau \rangle. \quad (6.148)$$

The electron mobility:

$$\mu_n = \frac{e}{m_n} \langle \tau \rangle. \quad (6.148a)$$

The electric conductivity of degenerated electron gas depends only on the electrons, which are located near the Fermi level.

The relaxation time of those electrons may be considered a constant quality τ_F , which corresponds to electrons occupying the Fermi level. The situation becomes simplified, and you are just to introduce τ_F in formula (6.148) and (6.148a).

6.13.3 The Electron Scattering from Heat Oscillations

When an electron (or hole) interacts with the lattice oscillation (phonon) the transmission of energy and the birth of a new phonon or the inverse process can be observed. The energy and quasi-momentum conservation law is true.

In the process of photon generation:

$$E(\mathbf{k}) = E_1(\mathbf{k}_1) + \hbar\omega_q; \quad (6.149)$$

$$\mathbf{k} = \mathbf{k}_1 + \mathbf{q}, \quad (6.150)$$

$E(\mathbf{k})$ is the electron energy before collision; \mathbf{k} is the wave number; $E_1(\mathbf{k}_1)$ is the electron energy after the birth of a new photon with the wave vector \mathbf{q} and energy $\hbar\omega_q$.

When an electron absorbs a phonon:

$$E_1(\mathbf{k}_1) = E(\mathbf{k}) + \hbar\omega_q; \quad (6.151)$$

$$\mathbf{k}_1 = \mathbf{k} + \mathbf{q}, \quad (6.152)$$

$E_1(\mathbf{k}_1)$ –the electron energy after phonon absorption; \mathbf{k}_1 – the electron vector.

The quantity of phonons depends on temperature. Thus the scattering of electrons from the heat oscillations of a lattice depends on temperature too. The multi phonon scattering is less probable.

The relaxation time of the electron scattering from longitudinal acoustic phonons in atomic semiconductors:

$$\tau(\mathbf{k}) = \frac{\tau_0}{(m^*)^{3/2} E^{1/2} T}, \quad (6.153)$$

The quantity τ_0 characterizes the crystal; m^* is the effective electron mass; E is the electron energy.

The electron mobility:

$$\mu_{TK} = \frac{4e\tau_0}{3\sqrt{\pi}k^{1/2}(m^*)^{5/2}T^{3/2}} = bT^{-3/2}. \quad (6.154)$$

The temperature dependence of the (6.154) type is often observed in a rather wide temperature interval.

6.13.4 Thermoelectric Phenomena

The effects of Zeebek, Peltier, and Thompson are known as the thermoelectric effects.

The generation of thermal E.M.F. in a circuit of two solids of different properties when the temperature of the soldered joints is different is called the Zeebek phenomenon.

The temperature change at the soldered joint of two metals when the electric current is flowing is called the Peltier effect. The phenomenon does not depend on the Joule-Lenz heat.

The liberation or absorption of the heat additional to the Joule-Lenz heat when the current runs through a uniform conductor with the temperature gradient is called the Thompson effect.

The Peltier and Zeebek phenomena are observed usually in metals. In semiconductors those phenomena are stronger. The effects are greater than in metals by the factor of orders and are of very much practical importance. In particular, the semiconductor pairs of great E.M.F. are used as the electric power sources; the Peltier phenomenon is used in refrigerators

The Thompson effect is mainly of theoretical interest. The temperature difference through a semiconductor produces the concentration gradient and hence the diffuse current. The volume electric charges are generated. If the volume charge field is against an external field, the last one make a work against the intrinsic field and the additional is liberated. When the direction of both fields coincides the intrinsic field make a work to initiate the charge carriers drift. The heat energy of the semiconductor becomes less and it cools.

The Zeebek Phenomenon in Non-degenerated Semiconductors

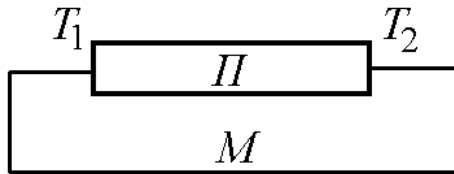


Fig.6.17. The metal-semiconductor contact

A differential thermoelectric E.M.F is defined as follows: $\alpha = \frac{d\varepsilon}{dT}$, (6.155)

dT - the temperature difference of soldered joints; $d\varepsilon$ - E.M.F. It follows that:

$$\varepsilon = \int_{T_1}^{T_2} \alpha(T) dT. \quad (6.156)$$

The semiconductor quantity α is often measured relative a metal. In Fig.6.17, a semiconductor is closed from both sides by a metal plate. The temperature of contacts is T_1 and T_2 . An electric current runs through the circuit. The main addend of the thermal electromotive force is the voltage produced inside the semiconductor; the contact potential difference is small.

The equilibrium charge carriers concentrations of a non-degenerated impurity semiconductor are the exponential functions of temperature. At the ends of the semiconductor, the concentrations are:

$$n_{01} = N_c e^{-\frac{E_c - E_F}{kT_1}}; \quad (6.157)$$

$$p_{01} = N_v e^{-\frac{E_F - E_v}{kT_1}}; \quad (6.158)$$

$$n_{02} = N_c e^{-\frac{E_c - E_F}{kT_2}}; \quad (6.159)$$

$$p_{02} = N_v e^{-\frac{E_F - E_v}{kT_2}}. \quad (6.160)$$

The temperature difference between the semiconductor ends is small and we can assume that the quantity E_ϕ is constant. .

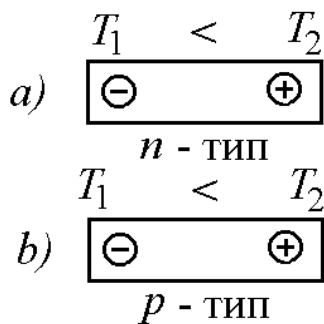


Fig.6.18. The volume charge in semiconductors: n-type (a), and p-type (b)

If $T_2 > T_1$, $n_{02} \gg n_{01}$ and $p_{02} \gg p_{01}$. Hence, when the temperature difference ($T_2 - T_1$) is established the charge carriers diffusion from the hot end to the cool one begins. It changes the equilibrium concentration of carriers and generates volume charges. Those charges produce the opposite electric field and the dynamical equilibrium is established.

The drift current produced by the intrinsic field (the field of the volume charges) will equilibrate the diffuse current of the charge carriers. As a result there will be no current at all.

In the n-type semiconductor (the equilibrium electron concentration is much greater than that one of the holes), the diffusion of electrons is only of importance. In p-semiconductors, the situation is opposite.

In an electron semiconductor, the diffusion produces

a negative electric volume charge at the cold joint and a positive volume charge at the hot joint. The quantity α is negative (Fig.18.a).

In a hole semiconductor, the diffusion produces a positive electric volume charge at the cold joint and a negative volume charge at the hot joint. The quantity α is positive (Fig.6.18b).

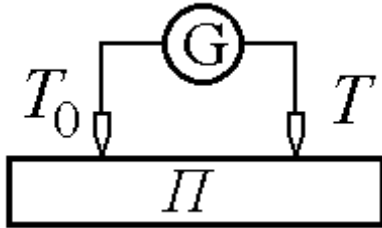


Fig.6.19. Thermal probe

Thus in hole semiconductors, the cold joint is positive; in electron semiconductors, the cold joint is negative.

The method of measuring with a thermal probe is shown in Fig.6.19. One of the contact-probe is heated, the temperature of the other contact-probe is the same as that one of the semiconductor. The direction of current is controlled. If the heated contact-probe is positive the semiconductor is of n -type, if the heated contact-probe is negative the semiconductor is if p -type.

Dependence of α on the charge carriers concentration was established by N.L.Pisarenco in forties years of the last century. To get that dependence let us use the thermodynamic ideas. The Peltier heat Q_{Π} is defined as follows:

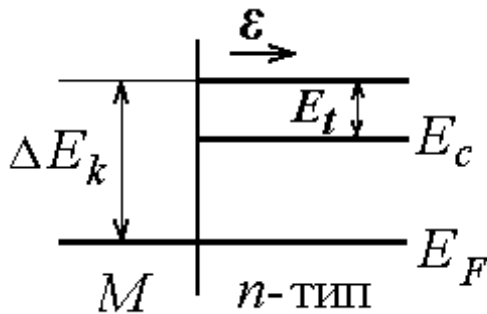


Fig.6.20. The zone diagram of the metal-semiconductor interface

$$Q_{\Pi} = \Pi I t, \quad (6.161)$$

$$\Pi = \alpha T. \quad (6.162)$$

The quantity I is a direct (constant) current, which runs through the semiconductor in a time interval t ; Π – the Peltier coefficient. The heat liberated in joint is due to the transition of the charge carrier energy to the lattice when the carriers transit through the interface. When the current is of opposite direction, the lattice energy is transmitted to the charge carriers and the semiconductor is cooled.

The zone diagram of the metal-semiconductor interface is shown in Fig.6.20. Ha When equilibrium, the Fermi levels are identical. When the electric field ϵ is applied, the electrons of the semiconductor conduction zone transit into metal. Assuming that the field is weak and the zones are horizontal, and neglecting the rectification effect at the interface, the reducing of the electron energy is ΔE_{κ} .

Thus:

$$\Delta E_{\kappa} = Q_{\Pi} = \Pi e = \alpha T e. \quad (6.163)$$

If the current is of opposite direction, the electrons are moving from the Fermi level to levels of the conduction zone. The electrons will be heated by the quantity ΔE_{κ} . That heating can be done by the energy transition from the lattice and leads to cooling of the joint.

We can write:

$$\Delta E_{\kappa} = (E_c - E_F) + E_T, \quad (6.164)$$

E_T is the mean heat energy of a non-degenerated electron gas. It is convenient to measure E_T from E_c

$$E_T = (r+2)kT. \quad (6.165)$$

The quantity r is defined by:

$$l = CE^r, \quad (6.166)$$

The quantity l is the mean free path; E – the total energy of charge carrier. The parameter r depends on the scattering mechanism:

$$0 \leq r \leq 2. \quad (6.167)$$

When scattering from acoustic oscillations, $r = 0$; from optic oscillations of an ion lattice, $r = 1$; from impurity ions, $r = 2$.

In accordance with (6.163)-(6.165):

$$\alpha Te = E_c - E_F + (r+2)kT. \quad (6.168)$$

The quantity $(E_c - E_{\phi})$ is defined by

$$n_0 = N_c e^{-\frac{E_c - E_F}{kT_1}} \quad (6.169)$$

$$E_c - E_F = kT \ln \frac{N_c}{n_0}. \quad (6.170)$$

Taking into account (6.168) we get:

$$\alpha = \frac{k}{e} \left(r + 2 + \ln \frac{N_c}{n_0} \right). \quad (6.171)$$

For an electron semiconductor, $\alpha < 0$, and the sign of (6.171) must be changed. Thus for an electron semiconductor, the Pisarenko formula can be written as follows:

$$\alpha_n = -\frac{k}{e} \left(r + 2 + \ln \frac{N_c}{n_0} \right); \quad (6.172)$$

For a hole semiconductor:

$$\alpha_p = \frac{k}{e} \left(r + 2 + \ln \frac{N_v}{p_0} \right). \quad (6.173)$$

Equation (6.172) and (6.173) coincides with the experimental data. The quantity α is of order of mV/degree; for metallic pairs is of order of mkV/degree. In accordance with those formulas, the parameter α of a semiconductor does not depend on the type of metal in contact.

While deducing formula (6.162) we use only one semiconductor-metal contact. It is possible because of (6.173). While discussing the Peltier effect, there is no need to take into consideration the second contact. While establishing relation between the Peltier coefficient and that one of E.M.F., we assumed that $\left(T = \frac{T_1 + T_2}{2}\right)$ and $dT = T_2 - T_1$ is small (the joint temperature can be considered equal T).

Thus α_n и α_p are functions of charge carrier concentration and temperature. The quantity α_p and α_n decreases with temperature. For intrinsic and mixed electric conduction:

$$\alpha = \frac{\alpha_p \mu_p P_0 + \alpha_n \mu_n n_0}{\mu_p P_0 + \mu_n n_0} \quad (6.174)$$

$$\alpha_i = \frac{\alpha_p \mu_p + \alpha_n \mu_n}{\mu_p + \mu_n} \quad (6.175)$$

According to (6.172) and (6.173), $\alpha_n < 0$ and $\alpha_p > 0$; and because the numerator of (6.174) is a difference of corresponding quantities it can be very small. The sign of α depends on absolute magnitudes of addends in the numerator.

While approaching the intrinsic conductivity region of an acceptor semiconductor the sign of α_p changes (if $\mu_n > \mu_p$). In a donor semiconductor, the absolute magnitude of α_n decreases with temperature.

At high temperature the quantity α_i does not depend on donors or acceptors and its absolute magnitude is small (when $\mu_n > \mu_p$ the sign is negative).

Deducing of formula for t.e.m.f. coefficient with the aid of the Boltzmann equation

The Boltzmann equation can be written as follows:

$$(\mathbf{v}_n \nabla_r f) + \frac{1}{\hbar} (\mathbf{F} \nabla_k f) = -\frac{f_1}{\tau}, \quad (6.176)$$

\mathbf{v}_n - the electron velocity; \mathbf{F} - the force; ∇_r and ∇_k - the Hamilton operators:

$$\text{grad}_r = \nabla_r = \frac{\partial}{\partial x} \mathbf{i} + \frac{\partial}{\partial y} \mathbf{j} + \frac{\partial}{\partial z} \mathbf{k}; \quad (6.177)$$

$$\text{grad}_k = \nabla_k = \frac{\partial}{\partial k_x} \mathbf{i} + \frac{\partial}{\partial k_y} \mathbf{j} + \frac{\partial}{\partial k_z} \mathbf{k}, \quad (6.178)$$

$\mathbf{i}, \mathbf{j}, \mathbf{k}$ - The unit orsts.

We can write:

$$(\mathbf{v}_n \nabla_r f) = v_{nx} \frac{\partial f}{\partial x} + v_{ny} \frac{\partial f}{\partial y} + v_{nz} \frac{\partial f}{\partial z};$$

$$\frac{1}{\hbar} (\mathbf{F} \nabla_k f) = \left(\frac{dk}{dt} \nabla_k f \right) = \frac{\partial k_x}{\partial t} * \frac{\partial f}{\partial k_x} + \frac{\partial k_y}{\partial t} * \frac{\partial f}{\partial k_y} + \frac{\partial k_z}{\partial t} * \frac{\partial f}{\partial k_z}.$$

When the decrement $f_1 = f - f_0$ is small in the left side of (6.176), f can be substituted by f_0 :

$$(\mathbf{v}_n \nabla_r f_0) + \frac{1}{\hbar} (\mathbf{F} \nabla_k f_0) = -\frac{f_1}{\tau}. \quad (6.179)$$

$$\mathbf{F} = -e\boldsymbol{\varepsilon} = e\nabla_r \varphi, \quad (6.180)$$

The quantity $\boldsymbol{\varepsilon}$ is the electric field strength of the volume charges; φ - the electrostatic potential. The distribution function of the non-degenerated electron gas is: a

$$f_0 = e^{-\frac{E-E_F}{kT}} = f_0(E, E_\phi, T), \quad (6.181)$$

The electron energy:

$$E = \frac{\hbar^2 k^2}{2m^*}. \quad (6.182)$$

Assume that m^* is a scalar (the spherical energy surface). The electron velocity:

$$\mathbf{v}_n = \frac{1}{\hbar} \nabla_k E(\mathbf{k}) = \frac{\hbar}{m^*} \mathbf{k}. \quad (6.183)$$

We are to have in mind that $T = T(x, y, z)$ and $E_F = E_F(x, y, z)$.

Thus

$$\nabla_r f_0 = \frac{\partial f_0}{\partial T} \nabla_r T + \frac{\partial f_0}{\partial E_F} \nabla_r E_F = \frac{\partial f_0}{\partial E} \left(\frac{E_\phi - E_0}{T} \nabla_r T - \nabla_r E_\phi \right); \quad (6.184)$$

$$\nabla_k f_0 = \frac{\partial f_0}{\partial E} \nabla_k E = \hbar \frac{\partial f_0}{\partial E} \mathbf{v}_n; \quad (6.185)$$

Introducing those quantities in (6.179) we get:

$$f_1 = -\tau(k) \frac{\partial f_0}{\partial E} \left(\frac{E_F - E_0}{T} \nabla_r T - \nabla_r (E_F - e\varphi) \right) \mathbf{v}_n. \quad (6.186)$$

When the charge carriers are scattered by the acoustic oscillations and the free path lengs does not depend on energy ($r = 0$):

$$\tau = \frac{l}{v_n} = \frac{m^* l}{\hbar k}. \quad (6.187)$$

The electron component of current density:

$$j_n = \frac{e\hbar l}{3\pi^2 m^*} \left(\frac{E_F}{T} \nabla T - \nabla_r (E_F - e\phi) \right) \int_0^\infty \frac{\partial f_0}{\partial E} k^3 dk - \frac{e\hbar l}{3\pi^2 m^* T} \nabla T \int_0^\infty \frac{\partial f_0}{\partial E} E k^3 dk.$$

The differential t.e.m.f.:

$$\alpha = \frac{\left| \nabla \left(\phi - \frac{E_F}{y} \right) \right|}{|\nabla T|}. \quad (6.188)$$

Using equation (6.154):

$$\mu_{TK} = \frac{4el}{3(2\pi k m^* T)^{1/2}}. \quad (6.189)$$

Using the known formulas we get at last:

$$\alpha = \frac{k}{e} * \frac{\mu_p p_0 \left(2 + \ln \frac{N_c}{n_0} \right) - \mu_n n_0 \left(2 + \ln \frac{N_v}{p_0} \right)}{\mu_p p_0 + \mu_n n_0} \quad (6.190)$$

A glance at (6.190) shows that it is the formula (6.174) at $r = 0$.
For the intrinsic semiconductor having in mind that $N_c = N_v$:

$$\alpha = -\frac{k}{e} * \frac{\mu_n / \mu_p - 1}{\mu_n / \mu_p + 1} \left(2 + \frac{\Delta E_0}{2kT} \right). \quad (6.191)$$

Thus the quantity α of an intrinsic semiconductor depends on the ratio of electron and hole mobility, the width of the forbidden zone ΔE_0 , and temperature. It should be noted that while deducing (6.190) and (6.191) we did not take into consideration the capture of the carriers by phonons.

When there is the temperature gradient in a semiconductor, phonons are moving from the hot regions to cool ones. While colliding the phonons transmit their momentum to electrons.

Those collisions produce the additional drift of electrons and hence the additional t.e.m.f. The effect is observed at low temperature. At high temperature, the phonon-phonon scattering is reconstructed and the rapture effect diminishes.

6.13.5 The Galvanic Effects. The Hall Effect

When an electric \mathbf{E} and magnetic field \mathbf{B} acts simultaneously, the electric and heat conduction can be changed in \mathbf{E} -direction, and the transversal temperature difference can be produced. Those phenomena are called the **galvanic phenomena**. The Hall effect (the transversal voltage generation), the magnetic resistance effect, the Ettingshausen effect (the transversal temperature gradient generation), the Nernst effect (the longitudinal temperature gradient generation).

The expression ‘**transversal**’ and ‘**longitudinal**’ are used relative to the temperature gradient direction.

An effect is called ‘**uneven**’ if the effect sign is not changed when the magnetic field is reversed. When the magnetic field is reversed but the sign of an effect is the same, the effect is called ‘**even**’. The Hall and Ettinghausen effects are uneven. The Nernst effect and the magnetic resistance effect are even.

The Lorentz force is given by:

$$\mathbf{F} = e\mathbf{E} + e[\mathbf{v}, \mathbf{B}]. \quad (6.192)$$

When ($\mathbf{E} \perp \mathbf{B}$) the trajectory is a cycloid. In general case, two motions form the trajectory. The first one is rotation in the plane perpendicular to the magnetic field:

$$r_0 = m^* \mathbf{E} / e\mathbf{B}^2 \quad (6.193)$$

The second one is a drift along the electric field with velocity:

$$v_d = \mu\mathbf{E}. \quad (6.194)$$

The criterion of a magnetic field magnitude is the ratio between the radius of curvature r_0 (6.193) and the free path length λ . If

$$r_0 \gg \lambda, \quad (6.195)$$

The electron motion is slightly disturbed and the field is called the **weak field**.

If

$$r_0 \ll \lambda, \quad (6.196)$$

The field is called the **strong field**.

The condition (6.195) - (6.196) can be substituted by others if we compare the relaxation time τ with a cyclic period (the time of one turn by the circle in the magnetic field) T_c :

The field is weak if

$$\tau \ll T_c \quad (6.197)$$

The field is strong if

$$\tau \gg T_c. \quad (6.198)$$

The quantity T_c is defined by:

$$\omega_c = 2\pi / T_c = e\mathbf{B} / m^*. \quad (6.199)$$

The cyclic frequency ω_c is called the **cyclotron frequency**.

Thus according to (6.197) and (6.198) in the weak field

$$\mu\mathbf{B} \ll 1; \quad (6.200)$$

In the strong field:

$$\mu\mathbf{B} \gg 1. \quad (6.201)$$

It is clear that the magnetic field criterion does not depend on the external magnetic field strength but on the charge carrier mobility. The same magnetic field can be considered in one crystal weak and in the other strong.

For example, in germanium at high temperature, $\mu_n \approx 3000 \text{ cm}^2/\text{V}\cdot\text{s}$. The magnetic field strength of 10000 Gauss satisfies to criterion (6.200) of the weak field. The same field at low temperature ($\sim 10 \text{ K}$) and electron mobility of $10^4 - 10^5 \text{ cm}^2/\text{V}\cdot\text{s}$ is strong (satisfies the condition (6.201)).

Let us discuss the magneto-galvanic phenomena in weak fields. It should be noted that generation of the Hall electric field (perpendicular to \mathbf{E} and \mathbf{B}) is caused by the component of the Lorentz force $e[\mathbf{v}_d, \mathbf{B}]$. The Hall electric field \mathbf{E}_X will increase till the Lorentz force compensates it. The transversal force becomes zero and electrons again move along the straight line.

The current density \mathbf{j} is oriented along the x-axis, but the summarized electric field (taking into account the Hall electric field) will be turned relative to the current density vector at certain angle φ , which is called the **Hall angle** [see Fig.6.21].

$$\text{tg}\varphi = \mathbf{E}_X/\mathbf{E} = R[\mathbf{j}\mathbf{B}]/\mathbf{E} = RB\sigma\mathbf{E}/\mathbf{E} = \mu\mathbf{B}. \quad (6.202)$$

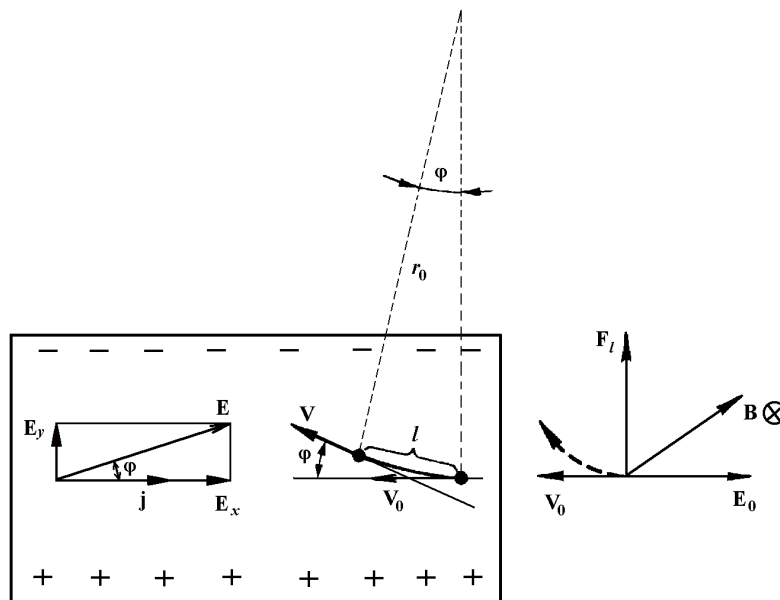


Fig.6.21. Curving of the charge carrier trajectory and formation of an angle between \mathbf{j} and \mathbf{E} in Hall effect

While describing the Hall effect we did not take into account the scattering processes. An analysis shows that the Hall coefficient depends on it:

$$R = \pm A/en, \quad (6.203)$$

A is constant, which depends on scattering mechanism; n is the charge carrier concentration.

Introducing $\sigma = en\mu$, we can write:

$$\mu_d = |R|\sigma/A, \quad (6.204)$$

The quantity $A\mu_d$ is called the **Hall mobility**.

$$\mu_X = A\mu_d. \quad (6.205)$$

In covalent crystals when scattering from acoustic phonons, the electron free path does not depend on energy ($r = 0$):

$$A = 3\pi/8; R = 3\pi/8 \cdot 1/en \approx 1.17 \frac{1}{en}. \quad (6.206)$$

For ionic semiconductors at the temperature below the Debye temperature while scattering from optic phonons, $r = 1/2$:

$$A \approx 0.99; R \approx 0.99 \frac{1}{en}. \quad (6.207)$$

When the temperature is higher than that of Debye, $r = 1$:

$$A \approx 1.11; R \approx 1.11 \frac{1}{en}. \quad (6.208)$$

While scattering from impurity ions, $r = 2$:

$$A \approx 315/512 \approx 1.93; R \approx 1.93 \frac{1}{en}. \quad (6.209)$$

When the electrons scatter from holes:

$$A \approx 315/512; R \approx 1.93 \frac{1}{en}. \quad (6.210)$$

When the charge carriers scatter from neutral impurity atoms:

$$A = 1; R = \frac{1}{en}; \quad (6.211)$$

The quantity A is evaluated using the Boltzmann kinetic equation.

In metals and degenerated semiconductors, only electrons at the highest levels create the electric conduction. Thus the energy distribution may not be taken into consideration, and we assume that the relaxation time is constant ($A = 1$):

$$\mu_X = \mu_d. \quad (6.212)$$

The situation is rather complicated in a semiconductor with mixed conduction. The electrons and holes deflect in the same direction. Because their mobility is different, there is no complete charge compensation. The transversal electric field is produced. It accelerates the charges of one sign and breaks the others. The Hall field would increase till the flows of electrons and holes become equal. The theoretical consideration leads to:

$$R = \frac{A}{e} \frac{\mu_p^2 p - \mu_n^2 n}{(\mu_p p + \mu_n n)^2}. \quad (6.213)$$

It follows from that equation that in intrinsic ($n = p$), the electron and hole mobility is identical the Hall field is zero. The measurement of electric conductivity and the Hall constant is not sufficient to find the mobility and concentration of charge carriers.

A magnetic field affects the electric resistance. The effect is characterized by the ratio between the specific resistance increment $\Delta\rho = \rho(\mathbf{B}) - \rho(0)$ and the resistance when there is no magnetic field:

$$H = \frac{\rho(\mathbf{B}) - \rho(0)}{\rho(0)} = \frac{\Delta\rho}{\rho(0)}, \quad (6.214)$$

The quantity H is called the **kinetic magnetic resistance coefficient**.

The phenomenon is due to the fact that there are the velocity distribution, relaxation time and effective mass anisotropy, when the energy surfaces are not spherical. Indeed if all charge carriers move with the same speed, were of the same isotropy mass, and their free pass were the same, they would move linearly as a result of compensation of the Lorenz force $e[\mathbf{v}_d, \mathbf{B}]$ by the Hall force.

In reality in a magnetic field, the electron trajectory is not linear because the compensation takes place only for charges, which move with the average speed. The carriers with the speed less than the mean one are affected strongly by the Hall electric field. The carriers with the speed greater than the average one are affected strongly by the magnetic component of Lorenz force $e[\mathbf{v}_d, \mathbf{B}]$, which deflects the carriers in the opposite direction.

In both cases, there is an increment of the charge carrier speed from the average one. It leads to decreasing of the free path. Indeed if when there was not a magnetic field, was moving along an *l-segment* (see Fig.6.21). When the field is switched in the trajectory changes:

$$l_x = l \cos \varphi \approx l \left(1 - \varphi^2 / 2\right) = l \left(1 - \mu^2 B^2 / 2\right). \quad (6.215)$$

The mobility is proportional to the free path in \mathbf{E} -direction and the electric conductivity is proportional to mobility. The decreasing of the path leads to the less electric conductivity. The resistance in the magnetic field becomes greater

The speed dispersion of carriers handicaps the influence of slow and fast carriers on conductivity. According the statistic theory, the kinetic magnetic resistance coefficient of impurity semiconductors:

$$H = C \mu_{n,p} B^2, \quad (6.216)$$

The quantity C depends on the scattering mechanism. When scattering from acoustic phonons ($p = -1/2$), $C = \pi/10$. When scattering from ionic impurity ($p = 3/2$), $C = 1$.

In Ettingshausen effect, the temperature gradient is produced in direction perpendicular to the magnetic field and the current density.

$$\nabla T = A_E [\mathbf{jB}]. \quad (6.217)$$

$$\nabla_z T = A_E \mathbf{B}_x \mathbf{j}_x. \quad (6.218)$$

The Ettingshausen coefficient of a non-degenerated electron semiconductor:

$$A_E = \frac{k_0 T}{e} \frac{2r-1}{2} \frac{R_\sigma}{\chi_q + \chi_e}. \quad (6.219)$$

The effect is due to the different action of The Hall and magnetic force upon slow and fast charges.

The carries with the speed greater the average one would be deflected by the magnetic field stronger then by transversal electric Hall field. They would be collected on one of the sides of the sample (z-direction). The charge carriers with a speed less the average one would be deflected in opposite direction (the Hall field acts stronger then magnetic field). The more is the number of carriers, the higher is the temperature of the side. The opposite side would be cooled, because the electrons take the lattice energy while transmitting to a thermodynamic equilibrium. Thus besides the transversal Hall effect, the transversal temperature gradient is produced.

The Nernst effect is also due to the different action of the Hall and Lorenz force on the charge carriers with different speed. The slower is the carrier, the stronger its trajectory is screwed. The fast carriers would accumulate at the sample side and heat it.. The slow carriers would cool the opposite side. As a result, the longitudinal temperature gradient (along **E**-direction) is produced:

$$\nabla_x T = A_N \mathbf{B}_y \mathbf{j}_x, \quad (6.220)$$

The quantity A_N is called the **Nernst kinetic coefficient**.

It should be noted that the gradient sign does not depend on the magnetic field direction but on current direction.

The more serious consideration of the galvanic-magnetic phenomena is based on the Boltzmann kinetic equation.