Electrical Conduction in Metals

- in a perfect crystal lattice, the conduction electrons would not experience any resistance in their motion

- a crystal lattice is never perfect

- Violations of the periodicity of a lattice are due to impurities or vacancies of crystalline lattice, and also to thermal oscillations of the lattice. Scattering of the electrons on the impurity atoms and on phonons leads to the appearance of electrical resistance of metals

the rule of Matthiessen

- The resistivity of metals obey the rule of Matthiessen

$$\rho = \rho_{osc} + \rho_{imp}$$

- where ρ_{osc} is the resistivity due to thermal oscillations of the lattice, and ρ_{imp} is the resistivity due to the scattering of electrons on the impurity atoms.

- at a low concentration of the impurities does not depend on the temperature and forms the so-called *residual resistivity* of a metal (i.e. resistivity which a metal has at 0 K).

The average velocity

- The average velocity of electrons called the *drift velocity* is

$$\mathbf{v}_{dr} = \frac{1}{n}\sum_{i=1}^{n}\mathbf{v}_i$$

- the force of resistance of the medium proportional to the velocity of the body

$$\mathbf{F}_f = -r\mathbf{v}_{dr}$$

equation of motion of an electron

$$m^* \frac{d\mathbf{v}_{dr}}{dt} = -eE - r\mathbf{v}_{dr}$$

- If the external field **E** = 0

$$m^* \frac{d\mathbf{v}_{dr}}{dt} + r\mathbf{v}_{dr} = 0$$

- The solution of this equation is

$$\mathbf{v}_{dr} = \mathbf{v}_{0dr} \exp\left(-\frac{r}{m^*}t\right)$$

the relaxation time

$$\tau = \frac{m^*}{r}$$

- the relaxation time characterizes the process of the establishment of equilibrium between the electrons and the lattice violated by the action of the external field **E**

- friction force **F**$_f$ is

$$\mathbf{F}_f = -\frac{m^*}{\tau}\mathbf{v}_{dr}$$

drift velocity

- Steady value of the drift velocity takes place when the net force on an electron is zero

$$-eE - \frac{m^*}{\tau}\mathbf{v}_{dr} = 0$$

$$\mathbf{v}_{dr} = -\frac{e\mathbf{E}\tau}{m^*}$$

current density

$$\mathbf{j} = \mathbf{v}_{dr}(-e)n = \frac{ne^2\tau}{m^*}\mathbf{E}$$

- the conductivity σ is

$$\sigma = \frac{ne^2\tau}{m^*}$$

validity of the equation

- The equation is valid both in the classical interpretation of the motion of conduction electrons in a metal and in the quantum mechanical interpretation.

- distinctions between these two interpretations are in difference of the concentration of electrons disturbed by the external electric field and its masses