ВАРИАНТ 1

Проинтегрировать уравнения

1. $(xy^2 + x)dx + (x^2 y - y)dy = 0$;
2. $y'\sin^2 x = y\ln y$;
3. $(x + 2y)dx = xdy$;
4. $x^2 dy - (2xy - y^2)dx = 0$;
5. $(1 + x^2)y' - 2xy = (1 + x^2)^2$;
6. $(xy' - 1)\ln x = 2y$;
7. $xy' - 3y = -x^4 y^2$;

8. $(4x^3 e^y + y^4 e^x)dx + (x^4 e^y + 4y^3 e^x)dy = 0$
9. $(\cos y \cdot \sin x + 1)dx + (\sin y \cdot \cos x - 1)dy = 0$;

10. $(x^2 - y^2)y' = 2xy, \quad y(0) = 1$;
11. $(y')^2 - y'(e^{x+y} + x^2 y) + e^{x+y} x^2 y = 0$;   13. $x^3 y'' + x^2 y' = 1$;
12. $(y')^3 - y + x = 0$;   14. $y'(1 + (y')^2) = y''$;

15. $y'' + 3y' + 2y = 0,$ 18. $y'' + 2y' = 10e^x(\sin x + \cos x)$;
     $y(0) = 2, \quad y'(0) = -3$;
16. $y''' - y'' + y' - y = x + 5$;   19. $y'' + 3y' + 2y = \dfrac{e^{-x}}{e^x + 2}$,
17. $y'' - y' + 2y = e^x(x^2 - 1)$; $y(0) = 0, \quad y'(0) = 0$;

20. Указать структуру общего решения уравнения
$$y'' - 8y' + 16y = 12x^2 - 28x + e^{4x};$$

ВАРИАНТ 2

Проинтегрировать уравнения

1. $y'\cos x = \dfrac{y}{\ln y}$;

2. $3e^x tg\, y\, dx + (1 - e^x)\cos^{-2} y\, dy = 0$

3. $xy' = y - x\, e^{\frac{y}{x}}$;

4. $y^2 + x^2 y' = xyy'$;

5. $x^2 y' = 2xy + 3$;

6. $x^2 y' + 2xy = \ln x$;

7. $xy' + y = 2y^2 \ln x$;

8. $\left(\dfrac{x}{\sqrt{x^2 - y^2}} - 1 \right)dx - \dfrac{y\, dy}{\sqrt{x^2 - y^2}} = 0$;

9. $x(y^2 - 3)dx + (x^2 y - 2\sqrt{y})dy = 0$;

10. $y'\cos x = (y + 1)\sin x, \quad y(0) = -1$;

11. $(y')^4 - 3(y')^3 + 7y' - 6 = 0$;

12. $y = xy' + y' - (y')^2$;

13. $(y + 1)\cdot(y + 2)y'' = (y')^2$;

14. $xy''' - y'' - x + 1 = 0$;

15. $y'' - 8y' + 17y = 0,$
    $y(0) = 0, \quad y'(0) = 1$;

16. $y^{IV} + 18y'' + 81y = 2x + 1$;

17. $y'' - 5y' + 4y = 4x^2 e^{2x}$;

18. $y'' + 6y' + 13y = e^{-3x}\cos 4x$;

19. $y'' + 9y = \dfrac{9}{\sin 3x}$,

    $y\left(\dfrac{\pi}{6}\right) = 4, \quad y'\left(\dfrac{\pi}{6}\right) = \dfrac{3}{2}\pi$;

20. Указать структуру общего решения уравнения
$$y''' - 16y' = 48e^{4x} + 64\cos 4x - 64\sin 4x;$$

ВАРИАНТ 3

Проинтегрировать уравнения

1. $y'\,ctg\,x + y = 2$;

2. $\sec^2 x \cdot ctg\,y\,dx + \sec^2 y \cdot tg\,x\,dy = 0$;

3. $xy' = y(\ln y - \ln x)$;

4. $(x^2 - 2y^2)\,dx + 2xy\,dy = 0$;

5. $xy' + y - x - 1 = 0$;

6. $y' - y\,ctg\,x = \dfrac{1}{\sin x}$;

7. $x^2 y' - 2xy + y^2 = 0$;

8. $2x\cos^2 y\,dx + (2y - x^2 \sin 2y)dy = 0$;

9. $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \dfrac{x}{y})dy = 0$;

10. $xy' + y = y^2, \quad y(1) = 0{,}5$;

11. $y = (y')^2$;

13. $2x(y')^2 + y'' = 0$;

12. $y = x(y')^2 - (y')^2$;

14. $(y')^2 + yy'' = 0$;

15. $y'' - 5y' + 4y = 0$,
    $y(0) = 0, \quad y'(0) = 1$;

18. $y'' - 4y' + 8y = e^x(5\sin x - 3\cos x)$;

19. $y'' + y = 2\,ctg\,x$,

16. $y^{IV} + y''' + 4y'' + 4y' = x^2 + 3$;

$\quad y\left(\dfrac{\pi}{2}\right) = 1, \quad y'\left(\dfrac{\pi}{2}\right) = 2$;

17. $y'' - 4y' + 20y = 5e^{3x}$;

20. Указать структуру общего решения
$$y'' - 4y = 8\sin 2x + 3e^{2x};$$

ВАРИАНТ 4

Проинтегрировать уравнения

1. $xy' + y - 3 = 0$;

2. $\sqrt{6y - y^2}\, dx - (4 + x^2)\, dy = 0$;

3. $xy' + xe^{\frac{y}{x}} - y = 0$;

4. $2x^3 y' = y(2x^2 - y^2), \quad y(1) = 1$;

5. $y' - 2y\, tg\, x = \sin x$;

6. $y'\cos^2 x + y = 1$;

7. $x^2(x - 1)y' - y^2 - xy(x - 2) = 0$;

8. $\dfrac{2x(1 - e^y)}{(1 + x^2)^2}\, dx + \dfrac{e^y}{1 + x^2}\, dy = 0$;

9. $(2 - 9xy^2)\, x\, dx + (4y^2 - 6x^3)\, y\, dy = 0$;

10. $y'\, x\, y\, (1 + x^2) = 1 + y^2, \quad y(1) = 0$;

11. $y'(x - \ln y') = 1$;

12. $y = xy' + a\sqrt{1 + (y')^2}$;

13. $y'' = xe^{2x}$;

14. $1 + (y')^2 = yy''$;

15. $y'' - 2\sqrt{2}\, y' + 2y = 0,$
    $y(0) = 1, \quad y'(0) = 0$;

16. $y^{IV} + 4y'' + 3y = 2x + 5$;

17. $9y'' + 42y' + 49y = 5e^{-\frac{7}{3}x}$;

18. $y'' + 2y' = -2e^x(\sin x + \cos x)$;

19. $y'' - 9y' + 18y = \dfrac{9e^{3x}}{1 + e^{-3x}},$
    $y(0) = 0, \qquad y'(0) = 0$;

20. Указать структуру общего решения уравнения
$$y'' + 2y' + y = 5e^{-x} + x\sin 2x;$$

ВАРИАНТ 5

Проинтегрировать уравнения

1. $y'\cos x = (y+1) \cdot \sin x$;

2. $\dfrac{dx}{x(y-1)} + \dfrac{dy}{x+2} = 0$;

3. $xy' - y = \sqrt{x^2 + y^2}$;

4. $xy'(\ln y - \ln x + 4) = y(\ln \dfrac{y}{x} + 5)$;

5. $y'x + 2y = x^3$;

6. $(1 + e^x)(y' + y) = 1$;

7. $y'x - 3y = -x^4 y^2$;

8. $(x + \ln y)dx + (1 + \dfrac{x}{y} + \sin y)dy = 0$;

9. $(2xy - 5)dx + (3e^y + x^2)dy = 0$;

10. $x^2 y' + y^2 + xy + x^2 = 0$, $y(1) = 0$;

11. $y^2((y')^2 + 1) = 1$;

12. $y = \dfrac{2}{3}xy' + \dfrac{1}{3}(y')^2$;

13. $x^2 y'' + xy' = 1$;

14. $y'' \, tg \, y = 2(y')^2$;

15. $y'' - 8y' + 16y = 0$,
    $y(0) = 1$, $y'(0) = 2$;

16. $y''' - y'' - y' + y = x - 1$;

17. $y''' - 2y'' - 3y' = (8x - 14)e^{-x}$;

18. $y'' - 4y' + 4y = e^{2x} \sin 6x$;

19. $y'' + 16y = \dfrac{16}{\cos 4x}$,
    $y(0) = 3$, $y'(0) = 0$;

20. Указать структуру общего решения уравнения
$$y'' - 2y' + y = xe^{3x} + x^2 - 1;$$

ВАРИАНТ 6

Проинтегрировать уравнения

1. $(1-x^2)y' = xy$;

2. $y' = 2\sqrt{y}\cdot\ln x$;

3. $y' + \dfrac{x+y}{x+2y} = 0$;

4. $(y^2 - 2xy)dx + x^2 dy = 0$;

5. $y' + y\,tg\,x = \sin 2x$;

6. $xy' + y = x(2\ln x + 1)$;

7. $y' - xy + y^3 e^{-x^2} = 0$;

8. $2x\cos^2 y\,dx + 8(\sqrt[3]{y} - \dfrac{x^2}{8}\sin 2y)dy = 0$;

9. $2xy^{-3}dx + (y^2 - 3x^2)y^{-4}dy = 0$;

10. $x\,y' + y - x - 1 = 0, \quad y(1) = 0{,}5$;

11. $y = y' + \dfrac{e^x}{y'}$;

12. $y' = \ln(xy' - y)$;

13. $2y\,y'' - (y')^2 = 0$;

14. $x\ln x\cdot y'' = y'$;

15. $y'' + 14y' + 53y = 0,$
    $y(0) = 1, \quad y'(0) = -5$;

16. $y''' - 2y'' - 3y' = 2x^2 + 1$;

17. $12y'' - 7y' + y = (2x^2 - x + 3)e^x$;

18. $y'' + 2y' + 5y = -17\sin 2x$;

19. $y'' - 3y' + 2y = \dfrac{1}{2 + e^{-x}}$,
    $y(0) = 1 + 3\ln 3, \qquad y'(0) = 5\ln 3$

20. Указать структуру общего решения уравнения
$$y'' + 3y' = 2sh3x;$$

ВАРИАНТ 7

Проинтегрировать уравнения

1. $\sqrt{y^2 + 2} \cdot x\,dx + y(1 + x^2)\,dy = 0$;

2. $\left(\sqrt{xy} - \sqrt{x}\right)dx + \left(\sqrt{xy} + \sqrt{y}\right)dy = 0$

3. $xy\,dy + (x^2 - 2y^2)\,dx = 0$;

4. $\left(x + y\sin\dfrac{y}{x}\right)dx - x\sin\dfrac{y}{x}\,dy = 0$;

5. $y'x + 2y = x^4$;

6. $x(x+1)y' - y = x^2$;

7. $xy^2 y' = x^2 + y^3$;

8. $\dfrac{y}{x}\,dx + (\ln x - y^3)\,dy = 0$;

9. $(3x^2 + 6x^2 y + 3xy^2)\,dx + (2x^3 + 3x^2 y)\,dy = 0$

10. $x\,y' + x\,e^{\frac{y}{x}} - y = x, \quad y(1) = 0$;

11. $(y')^6 + 2(y')^4 + 3(y')^2 + y' - 5 = 0$;

12. $y = 2x\,y' + (y')^3$;

13. $x\,y'' + \dfrac{1}{x}\,y' = \dfrac{1}{x}$;

14. $y'' + y' + 2 = 0$;

15. $y'' + 2y' + 2y = 0,$
    $y(0) = 0, \quad y'(0) = 1$;

16. $y^{IV} - 10y'' + 9y = x + 3$;

17. $y''' + 2y'' - 3y' = (8x + 6)e^x$;

18. $y'' - 4y' + 4y = e^{-2x}\sin 6x$;

19. $y'' + 4y = 8\,ctg\,2x,$
    $y\left(\dfrac{\pi}{4}\right) = 5, \quad y'\left(\dfrac{\pi}{4}\right) = 4$;

20. Указать структуру общего решения уравнения

$$49y'' + 2y' + y = e^{-\frac{x}{2}}(x + 1) + \sin x;$$

ВАРИАНТ 8

Проинтегрировать уравнения

1. $\dfrac{e^{-y^3}}{x^2-9}dy + \dfrac{x}{y^2}dx = 0$;

2. $(1+e^{2x})\,y^2\,dy - e^x\,dx = 0$;

3. $x^2\,dy + y^2\,dx = xy\,dy$;

4. $(y+\sqrt{x\,y})dx = x\,dy$;

5. $e^x(y+y') = 1$;

6. $y'x + 2y = x^3$;

7. $y' - \dfrac{2xy}{1+x^2} = 4\dfrac{\sqrt{y}}{\sqrt{1+x^2}}\,arctg\,x$;

8. $2x\left(\sqrt{x^2-y}+1\right)dx - \sqrt{x^2-y}\,dy = 0$;

9. $\left(\dfrac{1}{y}+tg\,x\right)dx - \dfrac{x}{y^2}dy = 0$;

10. $(1+x^2)\,y' - 2xy = (1+x^2)^2, \quad y(0) = 0$;

11. $3(y')^5 - 6(y')^4 + (y')^2 - 8 = 0$; 　　13. $y(y+1)y'' = (y')^2$;

12. $y\,y' - x(y')^2 = 1$; 　　　　　　　　14. $2y'' = \dfrac{y'}{x} + \dfrac{x^2}{y'}$;

15. $10y'' - 7y' + y = 0,$ 　　　　　　　18. $y'' + y = 2\cos 5x + 3\sin 5x$;
    $y(0) = 7, \quad y'(0) = 2$;

16. $y''' - 4y'' + 5y' - 2y = 4x$; 　　　19. $y'' - 2y' = \dfrac{4e^{-2x}}{1+e^{-2x}},$

17. $y'' + 6y' + 34y = (3-4x)e^x$; 　　　$y(0) = \ln 4, \quad y'(0) = \ln 4 - 2$;

20. Указать структуру общего решения уравнения
$$y'' + y = 2\sin x - 6\cos x + 2e^x;$$

ВАРИАНТ 9

Проинтегрировать уравнения

1. $x^3(y^4+4)dx + y^3(x^4+4)dy = 0$

2. $\dfrac{x\,dx}{y+1} - \dfrac{y\,dy}{x+1} = 0$

3. $x\,dy + \left(x\cdot\sqrt{\dfrac{y}{x}-1} - y\right)dx = 0$

4. $x\,y' - y = (x+y)\ln\dfrac{x+y}{x}$;

5. $\cos x \cdot y' - y\sin x = x\,e^{-x^2}$;

6. $(x^4-1)y' + 2xy(x^2+1) = 1$;

7. $(1-x^2)y' + xy = x(1-x^2)\sqrt{y}$; $\left(\dfrac{x}{\sin y}+2\right)dx + \dfrac{(x^2+1)\cos y}{\cos 2y - 1}\,dy = 0$;

8. $(10xy - 8y - 3)dx + (5x^2 - 8x + 6)dy = 0$;

9. $(2x^3 - xy^2)dx + (2y^3 - x^2 y)dy = 0$, $\quad y(0)=1$;

10. $x = e^{2y'} - (y')^2$;

11. $xy'(y'+2) = y$;

12. $y'' = \dfrac{y'}{x}(\ln\dfrac{y'}{x}+1)$;

13. $y\,y'' = (y')^2 - (y')^3$;

15. $2y'' + 5y' + 2y = 0$, $\quad y(0)=0, \quad y'(0)=1$;

16. $y^{IV} + 4y''' + 10y'' + 12y' + 5y = x+8$;

17. $y''' - y'' - 9y' + 9y = (12-16x)e^x$;

18. $y'' + 6y' + 13y = e^{-3x}\cos 5x$;

19. $y'' + \pi^2 y = \dfrac{\pi^2}{\cos\pi x}$

$\quad y(0)=3, \quad y'(0)=0$;

20. Указать структуру общего решения уравнения

$$15y'' - 11y' + 2y = x + e^x(\sin x + \cos x);$$

ВАРИАНТ 10

Проинтегрировать уравнения

1. $3tg\,y\,dx - 2x\ln x\,dy = 0$;

2. $y' - y = 2(1 + x^2 y')$;

3. $(x^2 + y^2)dx + xy\,dy = 0$;

4. $y' = \dfrac{x - y}{x + y}$;

5. $(x^2 + 1)y' - 2xy = x(x^2 + 1)^{3/2}$;

6. $xy' - y = x(\ln x - 2)$;

7. $y'x + y = 2y^2 \ln x$;

8. $3x^2 \sin^2 y\,dx + (y + x^3 \sin 2y)dy = 0$;

9. $(e^x + y - 1)dx + (x - e^y + 3)dy = 0$;

10. $(\sin^2 y + x\,ctg\,y)y' = 1, \quad y(1) = \dfrac{\pi}{2}$;

11. $\arcsin\dfrac{x}{y'} = y'$;

12. $(y')^2 - x\,y' + y = 0$;

13. $y'' = y'(1 + y')$;

14. $y'' - \dfrac{y'}{x - 1} = x(x - 1)$;

15. $y'' - 4y' + 3y = 0$,
    $y(0) = 6, \quad y'(0) = 10$;

16. $y^{IV} + 2y''' + 4y'' - 2y' - 5y = 2x$;

17. $y'' - y' + y = (x^2 - 3x + 1)e^{-x}$;

18. $y'' + 2y' + 5y = 10\cos x$;

19. $y'' - 3y' + 2y = \dfrac{1}{1 + e^{-x}}$,
    $y(0) = 1 + 2\ln 2, \quad y'(0) = 3\ln 2$;

20. Указать структуру общего решения уравнения
$$y'' + y = 2\cos 7x + 3\sin x;$$