ВАРИАНТ 11

Проинтегрировать уравнения

1. $y' = (2y+1)ctg\,x$;

2. $(2x+1)dy + y^2\,dx = 0$;

3. $(y-x)y\,dx + x^2 dy = 0$;

4. $x^3 dy + (4y^3 + 3x^2 y)dx = 0$;

5. $y' - 4y = e^{4x}$;

6. $(x^2+1)y' + xy = x(x^2+1)$;

7. $xy' + 2y = (x+3)xy^3$;

8. $\dfrac{xdy - ydx}{x^2 + y^2} = 0$;

9. $(4y^2 + 2xy + 3x^2)dy + (y^2 + 6xy + 2x^2)dx = 0$

10. $xdy + (x+y)dx = 0, \quad y(1) = 1$;

11. $y' = e^{y'}(y'-1)$;

12. $y = x(y')^2 + (y')^2$;

13. $xy'' = y'\ln\dfrac{y'}{x}$;

14. $y''(y+1) = (y')^2 + y'$;

15. $y'' + 2y' - 15y = 0,$
    $y(0) = 1, \quad y'(0) = 0$;

16. $y''' + 6y'' + 11y' + 6y = x^2 + 3$;

17. $y''' - 3y' - 2y = -4xe^x$;

18. $y'' + y = 2\cos 7x - 3\sin 7x$;

19. $y'' + y = 4ctg\,x,$
    $y\left(\dfrac{\pi}{2}\right) = 4, \quad y'\left(\dfrac{\pi}{2}\right) = 4$;

20. Указать структуру общего решения уравнения
$$9y'' + 24y' + 16y = xe^x + \cos x;$$

ВАРИАНТ 12

Проинтегрировать уравнения

1. $\sqrt{y^2+1}\,dx - xy\,dy = 0$;

2. $(x^2+1)\,y' + 4xy = 0$;

3. $xy\,y' = x^2 + y^2$;

4. $xy' = 2y(\ln y - \ln x)$;

5. $(x+1)y' - 2y = (x+1)^4$;

6. $y = xy' - x^2\cos x$;

7. $x^2y^2 + 1 = x^3yy'$;

8. $\left[3x^2(1+\ln y)\right]dx + \left(\dfrac{x^3}{y} - 2y\right)dy = 0$;

9. $(x^2 + y^2 + x)e^{2x}dx + ye^{2x}dy = 0$;

10. $2xy' - y = 3x^2$, $\quad y(1) = 1$;

11. $(y')^2 - 3xy' + 3x^2 - p = 0$;

12. $xy' - y = \ln y'$;

13. $y''(2y+3) - 2(y')^2 = 0$;

14. $x^3y'' + x^2y' = 1$;

15. $y'' - 2y' - 8y = 0$,
    $y(0) = 2, \quad y'(0) = 5$;

16. $y^{IV} + 4y''' + 8y'' + 8y' + 3y = 3x^2 - 1$

17. $y'' + y' + y = 3xe^{2x}$;

18. $y'' + 4y' + 4y = e^x(\sin x + \cos x)$;

19. $y'' + 6y' + 8y = \dfrac{4e^{-2x}}{2 + e^{2x}}$,
    $y(0) = 0, \quad y'(0) = 0$;

20. Указать структуру общего решения уравнения
$$y'' - 2y' = 2ch2x;$$

ВАРИАНТ 13

Проинтегрировать уравнения

1. $y' - xy^2 = 2xy$;

2. $\dfrac{(10 - x^2)xy}{5 - y^2} = y'$;

3. $y'x = y(\ln y - \ln x + 1)$;

4. $ydx + (2\sqrt{xy} - x)dy = 0$;

5. $xy' - y = x^2 \cos x$;

6. $(x + 1)y' - y = e^x(x + 1)^2$;

7. $xy' - 2x^2\sqrt{y} = 4y$;

8. $[3x^2 + \cos(x + y)]dx + [\cos(x + y) - 3y^2]dy = 0$

9. $(2x + y)dx + (x + 2y)dy = 0$;

10. $xy' = y\ln\dfrac{y}{x}, \quad y(1) = 1$;

11. $(y')^2 - \dfrac{xy}{a^2} = 0$;

12. $y = 4xy' - (y')^3$;

13. $y''\sin^4 x = \sin 2x$;

14. $2(y')^2 = (y - 1)y''$;

15. $9y'' - 12y' + 4y = 0$,
    $y(0) = 0, \quad y'(0) = 1$;

16. $y''' + y'' = x^2 + 2$;

17. $y''' - 3y' + 2y = (4x + 9)e^{2x}$;

18. $y'' + 2y' = 3e^x(\sin x + \cos x)$;

19. $y'' + 4y = \dfrac{4}{\cos 2x}$,
    $y(0) = 2, \quad y'(0) = 0$

20. Указать структуру общего решения уравнения
$$y'' - 2y' + y = 2e^x - 8\cos 2x;$$

ВАРИАНТ 14

Проинтегрировать уравнения

1. $\dfrac{x}{x^2+1}dx - \dfrac{y^2}{y^3+1}dy = 0$;

2. $\sqrt{1+\ln x}\,dx - xy\,dy = 0$;

3. $2y'\cdot x^2 = y^2 + 6xy + 4x^2$;

4. $(x^2+y^2)dx - 2xydy = 0$;

5. $y' - y = e^x \sin x$;

6. $xy' + y = x^3 \ln x$;

7. $y' + 2xy = 2x^3 y^3$;

8. $\left(\ln^3 y - \dfrac{3y\ln^2 x}{x}\right)dx + \left(\dfrac{3x}{y}\ln^2 y - \ln^3 x\right)dy = 0$

9. $(5x+3y)dx + (3x-7y)dy = 0$;

10. $y' + y\cos x = \dfrac{1}{2}\sin 2x, \quad y(0) = 0$;

11. $(y')^3 - 5(y')^2 + y' + 4 = 0$;

12. $y = xy' - (y'+2)$;

13. $y'' + y'\,tg\,x = \sin 2x$;

14. $yy'' - (y')^2 = 0$;

15. $y'' - 6y' + 10y = 0$,
    $y(0) = -1, \quad y'(0) = 0$;

16. $y''' + 2y'' - y' - 2y = 2x - 3$;

17. $y'' - 2y' - y = 4xe^x$;

18. $y'' - 4y' + 8y = e^x(2\sin x - \cos x)$;

19. $y'' - 6y' + 8y = \dfrac{4}{2+e^{-2x}}$,

    $y(0) = 1 + 3\ln 3, \quad y'(0) = 10\ln 3$

20. Указать структуру общего решения уравнения
$$y''' + y'' = 10\sin x + 6\cos x + 4e^x;$$

ВАРИАНТ 15

Проинтегрировать уравнения

1. $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$;

2. $\dfrac{dx}{\cos^2 x \cos y} = -ctg\,x \cdot \sin y\,dy$;

3. $y'\cos\dfrac{y}{x} = \dfrac{y}{x}\cos\dfrac{y}{x} - 1$;

4. $(x^2 - 3y^2)dx + 2xy\,dy = 0$;

5. $y' - ctg\,x \cdot y = \sin^2 x$;

6. $y' + y\,tg\,x = \sec x$;

7. $y' + 2y = y^2 e^x$;

8. $\left(\ln y - \dfrac{y}{x}\right)dx + \left(\dfrac{x}{y} - \ln x\right)dy = 0$;

9. $(tg\,x + y + \sin y)dx + (ctg\,y + x + x\cos y)dy = 0$

10. $xy' + y = 2y^2 \ln x, \quad y(1) = 0{,}5$;

11. $y = a\sqrt{1+(y')^2}$;

12. $y(y')^2 = 2x(y')^3 + 1$;

13. $y'' = \dfrac{\cos 2x}{\sin^2 2x}$;

14. $y\,y'' = y'' + 2(y')^2$;

15. $y'' + 4y' + 5y = 0$,
    $y(0) = -2, \quad y'(0) = 1$;

16. $y^{IV} - 6y''' + 9y'' = 3x - 1$;

17. $y''' + 4y'' + 5y' + 2y = (2x+16)e^x$;

18. $y'' + 2y' + 5y = -\cos x$;

19. $y'' + \dfrac{1}{4}y = \dfrac{1}{4}ctg\dfrac{x}{2}$,
    $y(\pi) = 2, \quad y'(\pi) = 0{,}5$

20. Указать структуру общего решения уравнения
$$y'' - 2y' + 10y = \sin 3x + e^x;$$

ВАРИАНТ 16

Проинтегрировать уравнения

1. $y' = \dfrac{y^2 - 2y}{2x}$;

2. $(1 - x^2)dx + 3x\sqrt[3]{y}\,dy = 0$;

3. $2x^3 dy - y(2x^2 - y^2)dx = 0$;

4. $(y^2 - 3x^2)dy + 2xy\,dx = 0$;

5. $y' - 2xy = (x+1)e^{x^2}$;

6. $xy' - y = -2x\ln x$;

7. $(1 + x^2)y' = xy + x^2 y^2$;

8. $(3x^2 + 6xy^2)dx + (6x^2 y + 4y^3)dy = 0$;

9. $(2xy + 3y^2)dx + (x^2 + 6xy - 2y)dy = 0$;

10. $x^2 y' = 2xy + 3, \quad y(1) = 0$;

11. $x = 2y' + 3(y')^2$;

12. $y = y'(x + 1 - y')$;

13. $2yy'' = 1 + (y')^2$;

14. $xy'' = y' + x^2$;

15. $y'' + 2y' - 3y = 0,$
    $y(0) = 6, \quad y'(0) = 2$;

16. $y''' + 3y'' + 2y' = 3x^2 + 2x$;

17. $y^{IV} + 8y'' + 16y = xe^x$;

18. $y'' - 4y' + 8y = e^x(-3\sin x + 4\cos x)$;

19. $y'' + 3y' = \dfrac{9e^{3x}}{1 + e^{3x}}$;
    $y(0) = \ln 4, \quad y'(0) = 3(1 - \ln 2)$

20. Указать структуру общего решения уравнения
$$y''' - 1000y = 20e^{10x} - 100\cos 10x\,;$$

ВАРИАНТ 17

Проинтегрировать уравнения

1. $(1 + x^2)y' = x \sin^2 y$;

2. $y\,dx - (4 + x^2) \ln y\,dy = 0$;

3. $y'x = x e^{\frac{y}{x}} + y$;

4. $y - xy' = x \sec \dfrac{y}{x}$;

5. $x^2 y' + 2xy - \ln x = 0$;

6. $y' + y\,tg\,x = \cos^3 x$;

7. $y'x = y - x^4 y^4$;

8. $\left(arctg\,y + \dfrac{y}{x^2 + 1} \right)dx + \left(arctg\,x + \dfrac{x}{y^2 + 1} \right)dy = 0$;

9. $2xy\,dx + (x^2 - y^2)dy = 0$;

10. $x^2 y' + y^2 - 2xy = 0, \quad y(1) = 0$;

11. $y = 0{,}25(y')^2$;

12. $y(y')^3 + x = 1$;

13. $y'''(x - 1) - y'' = 0$;

14. $2yy'' = (y')^2$;

15. $y'' - 6y' + 8y = 0,$
    $y(0) = 1, \quad y'(0) = 0$;

16. $y''' - y' = 3x^2 - 2x + 1$;

17. $y''' - 7y'' + 15y' - 9y = (8x - 12)e^x$;

18. $y'' + 2y' + 5y = -2\sin x$;

19. $y'' + y = \dfrac{1}{\cos x}$;

    $y(0) = 1, \quad y'(0) = 0$

20. Указать структуру общего решения уравнения
$$y'' - y' + 4y = xe^x + \cos 2x ;$$

ВАРИАНТ 18

Проинтегрировать уравнения

1. $(5x^3 - 1)y' + 3x^2 y^4 = 0$;

2. $\sin^2 x \cos^2 y\, dx - \cos^2 x\, dy = 0$;

3. $(y^2 - xy)dx + (x^2 - 2xy)dy = 0, \quad y(1) = 1$;

4. $xdy - ydx = ydy$;

5. $y'\cos x = (y+1)\sin x$;

6. $(1 + x^2)y' + 2xy = 2x$;

7. $y' + xy = (1+x)e^{-x} y$;

8. $[(y+1)e^x + e^y]dx + [(x+1)e^y + e^x]dy = 0$;

9. $(y - \sin x)dx + (x+1)dy = 0$;

10. $\cos y \cdot \sin x\, dx - \sin y \cdot \cos x dy = 0, \quad y(0) = \dfrac{\pi}{3}$;

11. $y = x - \dfrac{4}{9}(y')^2 + \dfrac{8}{27}(y')^3$;

12. $2y(y')^2 = 2x(y')^3 + 1$;

13. $y'' = 2xy'(x^2 + 2)^{-1}$;

14. $y''x\ln x = y'$;

15. $y'' + 3y' + 2y = 0,$
    $y(0) = 1, \quad y'(0) = -1$;

16. $y^{IV} + 2y''' + y'' = 2 - 3x^2$;

17. $y'' + 2y' + 2y = (x+2)e^x$;

18. $y'' + y = 2\cos 3x - 3\sin 3x$;

19. $y'' - 6y' + 8y = \dfrac{4}{1 + e^{-2x}}$;
    $y(0) = 1 + 2\ln 2, \quad y'(0) = 6\ln 2$

20. Указать структуру общего решения уравнения
$$y'' + 4y' = 16sh4x ;$$

ВАРИАНТ 19

Проинтегрировать уравнения

1. $y' = 10^{x+y}$;

2. $y'e^{-x} = x - 1$;

3. $x^2 y' + y^2 = xyy'$;

4. $(x^2 + y^2) y' = 2xy$;

5. $y' - y\,tg\,x = e^x \sec x$;

6. $(x + 2) y' - y = x(x + 2)^2$;

7. $y'x^3 \sin y = xy' - 2y$;

8. $(3x^2 y^2 + 7)dx + 2x^3 ydy = 0$;

9. $(x^2 + 3y^2)xdx + (y^2 + 3x^2)ydy = 0$;

10. $(xy' - 1)\ln x = 2y, \quad y(e) = 1$;

11. $(y')^2 - \dfrac{1}{x} = 0$;

12. $y = 6xy' + 2(y')^3$;

13. $(y')^2 + yy'' = yy'$;

14. $y''(e^x + 1) + y' = 0$;

15. $2y'' - 7y' + 3y = 0,$
    $y(0) = 0, \quad y'(0) = 0{,}5$;

16. $y''' - y'' = 6x^2 + 3x$;

17. $y''' - y'' - 5y' - 3y = -(18x + 4)e^x$;

18. $y'' + 6y' + 13y = e^{-3x} \cos 8x$;

19. $y'' + 4y = 4\,ctg\,2x$;
    $y\left(\dfrac{\pi}{4}\right) = 3, \quad y'\left(\dfrac{\pi}{4}\right) = 2$

20. Указать структуру общего решения уравнения
$$y'' - y' + 0{,}25y = e^{4x} \sin 2x + 1;$$

ВАРИАНТ 20

Проинтегрировать уравнения

1. $x^2 y' - \cos 2y = 1$;

2. $y'(x + \sqrt{x}) = \sqrt{1 - y}$;

3. $(4x^2 - y^2)dx + 2xydy = 0$;

4. $y' = \dfrac{y}{x} + \sin\dfrac{y}{x}$;

5. $y' - y\,ctg\,x = \sin^3 x \cdot \cos x$;

6. $y' - y\,ctg\,x = 2x\sin x$;

7. $3xy' + 2y + xy^2 = 0$;

8. $\dfrac{(x + 2y)dx + ydy}{(x + y)^2} = 0$;

9. $(x^2 + y^2 + 2x)e^x dx + 2ye^x dy = 0$

10. $(x^3 - 3xy^2)dx + (y^3 - 3x^2 y)dy = 0, \ y(0) = 1$

11. $(y')^9 + 6(y')^5 + (y')^2 - y' + 4 = 0$;   13. $y''(x^2 + 1) = 2xy'$;

12. $y = xy' - y' - (y')^2$;   14. $yy'' - (y')^2 = y^3$;

15. $y'' - 10y' + 26y = 0$,
$y(0) = 5, \quad y'(0) = 1$;

18. $y'' - 2y' + 10y = e^{\frac{3}{2}x}(2\cos x - \sin x)$

16. $y^{IV} - 3y''' + 3y'' - y' = x - 3$;   19. $y'' - 3y' = \dfrac{9e^{-3x}}{3 + e^{-3x}}$;

17. $y'' - 5y' = 4xe^{5x}$;

$y(0) = 4\ln 4, \quad y'(0) = 3(3\ln 4 - 1)$

20. Указать структуру общего решения уравнения
$$y''' - 36y' = 36e^{6x} - 72(\cos 6x + \sin 6x);$$