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THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING

A guide to laboratory exercises on the course

«Theoretical fundamentals of electrical engineering»

for the second year students of the Institute of Power Engineering

following the majors

13.03.02 «Electrical engineering and electrotechniques»

The guide is examined and approved for publication on a methodic seminar of the Department of Electrical Grids and Electrical Engineering of the Institute of Power Engineering

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CONTENTS

INTRODUCTION

This manual is your guide to the first electronics laboratory in the electrical engineering program. It is assumed that by completing the first electronics laboratory course you are familiar with basic electronic measurements and instrumentation, as well as with elements of data analysis, presentation of results, and reporting. Professional engineering practice requires using proper experimental methods and procedures. They include not only good measurement techniques, but also proper recording of all relevant information, preparing tables and graphs, etc. Almost as important as obtaining good data is their proper presentation which often determines success in this laboratory course as it does in engineering practice. Upon completion of the first laboratory course you should be very familiar with effective laboratory practices and professional style data presentation. They will be a great asset in your future.

The experiments in this lab manual are designed to give the student practical experience in working with DC and AC circuits operating in steady-state and transient modes, with three-phase circuits, and with nonlinear circuits. The laboratory will complement and support the theory taught in the lectures, and should help the student to apply his knowledge of electrical engineering.

Every week before lab, each student should read over the laboratory experiment and work out the various calculations, etc. that are outlined in the guide.

During experimental work in laboratory a student should observe the following rules:

1. Return parts and jumper wires to correct bins when you are finished with them.
2. Do not put suspected defective parts back in the bins. Give them to the Lab Technician for testing or disposal.
3. Report all equipment problems to Lab Instructor or Lab Technician.
4. Most experiments have several parts; students must alternate in doing these parts as they are expected to work in group.
5. Each student must have a laboratory notebook. The notebook should be a permanent document that is maintained and witnessed properly, and that contains accurate records of all lab sessions.
6. Laboratory and equipment maintenance is the responsibility of not only the Lab Technician, but also the students. A concerted effort to keep the equipment in excellent condition and the working environment well-organized will result in a productive and safe laboratory.

Safety in the Laboratory

To minimize electric shock hazard, the experiments are designed for low-voltage; however one should never assume that electric circuits are safe. Few milliamps of current through the body can be lethal. For your safety you must follow safety rules particularly:

- Turn off power before working on circuits.
- Know the location of emergency power-off switch.
- Make sure that the transformers and equipments are plugged into utility lines, have no exposed wiring. Check with the instructor if you are not certain about the procedure.

Laboratory Notebook

The laboratory notebook is a record of all work pertaining to the experiment. This record should be sufficiently complete so that you or anyone else of similar technical background can duplicate the experiment and data by simply following your laboratory notebook. Record everything directly into the notebook during the experiment. Do not use scratch paper for recording data. Do not trust your memory to fill in the details at a later time.

Guidelines for Laboratory Notebook

- State the objective of the experiment.
- Draw the ciruit digram and mention the values of resistances etc. which are used.
- Make a note of all the measuring instruments you have used.
- Mention the formulas used.
- Create a table and write down the readings, including the units.
- Show all your calculation neatly and SYSTEMATICALLY. Do this is an organized manner.
- Attach graph if any.
- Be concise. Complete sentences are not necessary as long as the context is clear.
- If mistakes are made, they should not be erased. Just bracket them and make a short note explaining the problem.
- Make entries as the lab progresses; don't assume you can fill it in later. The instructor will ask to see it during the lab.

- Draw the figure using pencil before you come to the lab so that you can make corrections to it in case you need to do so by erasing and redrawing. This will ensure tidy and neat work.

- Prepare the READING TABLE using pencil and ruler and not just by sketching lines. Sketching gives rise to crooked lines and gives the lab notebook a haphazard look.

General Lab Report Format

Following the completion of each laboratory exercise in Electrical Engineering courses, a report must be written and submitted for grading. The purpose of the report is to completely document the activities of the design and demonstration in the laboratory.

Reports should be complete in the sense that all information required to reproduce the experiment is contained within. Writing useful reports is a very essential part of becoming an engineer. In both academic and industrial environments, reports are the primary means of communication between engineers. There is no one best format for all technical reports but there are a few simple rules concerning technical presentations which should be followed. Adapted to this laboratory they may be summarized in the following recommended report format:

• Title page

• Introduction

• Experimental Procedure

• Experimental Data

• Conclusions

Detailed descriptions of these items are given below.

Title Page:

The title page should contain the following information

• Your name

• Course number (including section)

• Experiment number and title

• Date submitted

• Instructors Name

Introduction:
It should contain a brief statement in which you state the objectives, or goals of the experiment. It should also help guide the reader through the report by stating, for example, that experiments were done with three different circuits or consisted of two parts etc. or that additional calculations or data sheets can be found in the appendix,
or at the end of the report.

The Procedure:
It describes the experimental setup and how the measurements were made. Include here circuit schematics with the values of components. Mention instruments used and describe any special measurement procedure that was used.

Results/Questions:
This section of the report should be used to answer any questions presented in the lab handout. Any tables and/or circuit diagrams representing results of the experiment should be referred to and discussed/explained with detail. All questions should be answered very clearly in paragraph form. Any unanswered questions from the lab handout will result in loss of points on the report. The best form of presentation of some of the data is graphical. In engineering presentations a figure is often worth more than a thousand words. There are some simple rules concerning graphs and figures which should always be followed. If there is more than one figure in the report, the figures should be numbered. Each figure must have a caption following the number. For example, *"Figure 1.1: TTL Inverter"*

Conclusion:
A brief conclusion summarizing the work done, theory applied, and the results of the completed work should be included here. Data and analyses are not appropriate for the conclusion.

Notes

- Any drawings in the report done by hand must be done with neatness, using a straight edge and drawing guides wherever possible. Free hand drawings will not be accepted.

- It is your responsibility to obtain the instructor's signature and to include the signed sheet with your final experiment report.

- Each student must submit an individual report based on an individual effort.

Laboratory exercise No. 1
INVESTIGATION OF A TWO-MESH DC CIRCUIT

The objective of the exercise: Experimental verification of Kirchhoff's laws, superposition principle, and the linear relationship theorem.

Theoretical Background

Kirchhoff's current law. The algebraic sum of currents in a node of electrical circuit is equal to zero. The currents flowing into the node are usually taken with negative sign, and the currents flowing out of the node are usually taken with positive sign:

$$\sum I = 0.$$

Kirchhoff's voltage law. The algebraic sum of voltage in a contour is equal to the algebraic sum of electromotive forces (e.m.f) in this contour. In the left-hand part of the equation voltage drops across the elements should be taken with positive sign if the direction of the contour tracing coincides with the current direction in the element. In the right-hand side of the equation the e.m.f are positive if arrows of e.m.f sources coincide with the direction of the contour tracing:

$$\sum IR = \sum E.$$

Equations for currents are independent for all nodes in a circuit except for one node. Equations for voltages are independent for all contours of plane (planar) circuit except for the external contour.

Superposition principle. The current in any branch of a linear electrical circuit including several sources can be represented as the sum of currents generated by each source.

Linear relationship theorem. When one parameter of a linear electric circuit (e.m.f, resistance, or current) is changed, any other two parameters of this circuit (currents or voltages) is bound by a linear relation of the form $y = ax + b$.

Electrical Circuit

To perform the experiments of exercise the electric circuit represented in Fig. 1.1 is used. Sources of electric power are accumulator batteries internal resistance of which is small in comparison with the resistances R_1, R_2, and R_3 so the batteries can be considered as sources of e.m.f. E_1 and E_2.
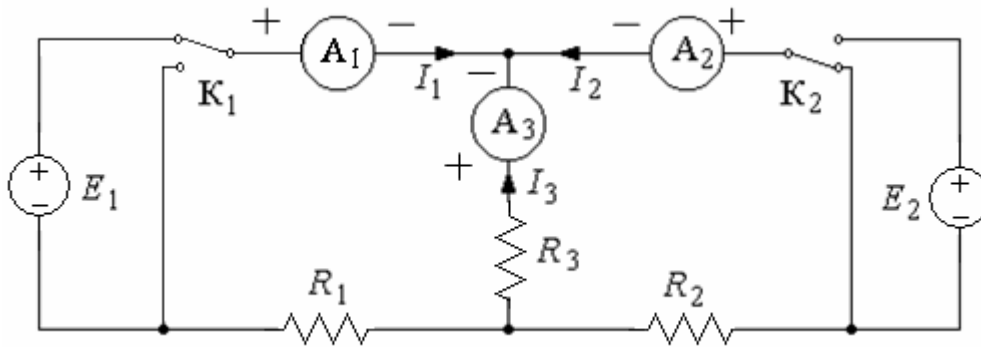
Fig. 1.1

As e.m.f. sources E_1 the battery should be taken with controlled output voltage. E.m.f. of source E_2 does not change in all experiments.

With the use of keys K_1 and K_2 these sources can be switched on or off the circuit or substituted by a conductor with zero resistance.

Before starting experiments voltages generated by the sources of e,m.f. should measure by voltmeters. The voltage of E_1 must be set according the variant of the exercise given in Table 1.1.

Table 1.1

Variant		1	2	3	4	5	6	7	8
E_1	V	5	6	7	8	9	10	11	12
R_1	Ω	100	150	220	100	150	220	100	220
R_2	Ω	150	100	100	220	220	150	220	100
R_3	Ω	220	220	150	150	100	100	150	330

While assembling the circuit a special attention should be paid to the polarity of e.m.f sources and measuring instruments.

Review Questions

Study theoretical material and answer the following questions:
1. In which way should the contours be chosen to make independent the Kirchhoff's equations of the circuit?
2. In Fig. 1.2 a schematic circuit diagram is presented (branches are shown by lines, nodes by points). How many independent equations should be written for this circuit according to the Kirchhoff's current and voltage laws?
3. In Fig. 1.3 the circuit under investigation is shown without meters. For this circuit write the equations according to Kirchhoff's current and voltage laws.

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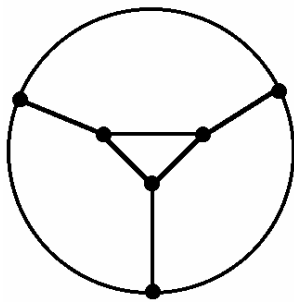
Fig. 1.2
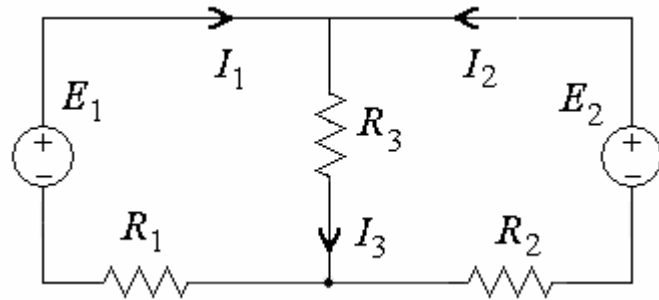
Fig. 1.3

4. What are the rules of sign definition in Kirchhoff's current and voltage laws?
5. What is the essence of superposition principle? How can one verify this principle using the circuit drawn in Fig. 1.1? Can this principle be applied to calculation of powers?
6. Explain the superposition principle using the circuit diagram shown in Fig. 1.3. Write the equations for analytical verification of the superposition principle (prove the equivalence of the equations for two currents).

Experimental Investigations

1. Measuring the circuit parameters.
Using the multimeter as a voltmeter measure the voltage of the source E_2 and set e.m.f. of E_1 according the variant of exercise (Table 1.1). In all experiments the value of this voltage should be fixed. Using the multimeter as an ohmmeter measure resistances R_1, R_2, and R_3. The obtained results write down to Table 1.2.

Table 1.2

E_1	E_2	R_1	R_2	R_3
V	V	Ω	Ω	Ω

2. Experimental verification of the first and second Kirchhoff's law and the superposition principle. The corresponding electrical circuit is presented in Fig. 1.1.

Experiment 1. E.m.f. E_1 is switched on, E_2 is switched off. Key K_1 is churned up, K_2 is churned down.

Experiment 2. EMF E_2 is switched on. K_2 is churned up.

Experiment 3. The both sources of EMF are switched on.

Write down the readings of the devices to Table 1.3. In the fourth row of the table write the sum of readings obtained in experiments 1 and 2. Compare this result with the readings of devices taken in experiment 3.

Sum up values of currents measured in experiments 1, 2 and 3 according to the Kirchhoff's current law. Write down the result to the right column of the table. Requirements of the Kirchhoff's current law should be met in all experiments.

Calculate the sum of voltages across the passive elements in the left and right contours of the circuits for all three experiments. The results should be equal to the voltages of e.m.f. sources.

<div align="right"><i>Table 1.3</i></div>

Experi-ments	Voltages of e.m.f. sources		Reading of devices			Results of calculations		
	U_1	U_2	I_1	I_2	I_3	$\sum IR(1)$	$\sum IR(2)$	$\sum I$
	V	V	mA	mA	mA	V	V	mA
1	E_1	0						
2	0	E_2						
3	E_1	E_2						
Calcula-tions	E_1	E_2				Validation of superposition principle		

3. Verification of the linear relationship theorem for currents I_2 and I_3. EMF E_1 is varied, E_2 remains constant.

Experiment 4. Change the polarity of e.m.f. E_1 (the source E_2 is connected to the circuit) and write down the measured currents to Table 1.4.

To the same table write the results of experiments 2 and 3 (Table 1.3). Draw the dependence of I_3 on I_2 by the data in Table 1.4. Note that this dependence is linear ($I_3 = aI_2 + b$) and calculate the coefficients *a* and *b*.

<div align="right"><i>Table 1.4.</i></div>

No.	U_1	U_1	I_3	I_2	Notes
	V	V	mA	mA	
4	-E_1	E_2			Coefficient of the equation
3	E_1	E_2			$I_3 = aI_2 + b$
2	0	E_2			$a = ..., b = ...$

4. Draw general conclusions to the laboratory exercise. In the conclusions

you should point out the fulfillment of the objectives. Explain the discrepancy between theoretical and experimental data if such discrepancy has been registered.

Laboratory exercise No. 2

INVESTIGATION OF AN ACTIVE TWO-TERMINAL CIRCUIT AND EXPERIMENTAL VERIFICATION OF THEVENIN'S THEOREM

The objectives of the exercise: Experimental verification of Thevenin's theorem and calculation of equivalent circuit parameters.

Theoretical Background

Thevenin's Theorem states that any linear circuit containing several voltages and resistances can be replaced by just a single voltage in series with a single resistor (Fig. 2.1, *b*). In other words, it is possible to simplify any liner circuit, no matter how complex, to an equivalent circuit with just a single voltage source in series with a resistance connected to a load as shown below.

Norton's Thereom (Fig. 2.1, *c*) is identical to Thevenin's Theorem except that the equivalent circuit is an independent current source in parallel with an impedance (resistor). Therefore, the Norton equivalent circuit is a source transformation of the Thevenin equivalent circuit.

$$a \qquad\qquad b \qquad\qquad c$$
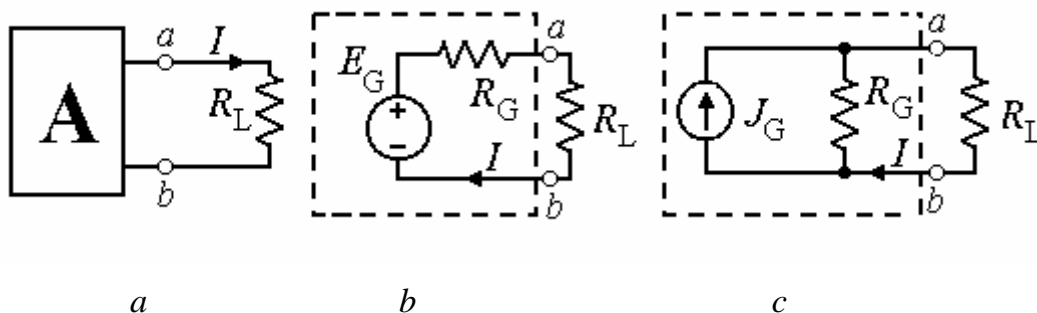
Fig. 2.1

In Thevenin representation e.m.f. of the equivalent circuit (equivalent generator) is equal to the voltage between points *ab* measured in the open-circuit mode (resistance R_G equals infinity), while resistance of the circuit is equal to the relative resistance between points *ab* (it should be noted that resistance of e.m.f. sources equals zero and resistance of current sources equals infinity). Then the equivalent circuit is represented according the scheme in

Fig. 2.1, c, the current of the source J_G can be found as a short-circuit current ($R_L = \infty$): $J_G = I_{SC} = E_G / R_G$.

Parameters of equivalent generator can be found experimentally by measuring the voltage across the load and current in the load for two different resistances of R_L. It is so called the method of two loads. The next possibility to find the parameters of the generator is measuring the short-circuit current ($R_L = 0$) and open-circuit voltage ($R_L = \infty$).

Calculation of generator parameters in the method of two loadings is performed according the following equations:

$$E_G = \frac{U_2 I_1 - U_1 I_2}{I_1 - I_2}; \quad R_G = \frac{U_2 - U_1}{I_1 - I_2}, \tag{2.1}$$

where U_1 and U_2 are readings of the voltmeter corresponding to two loads, and I_1 and I_2 are reading of amperemeter corresponding to the same loads.

Parameters of the generator also can be calculated out of short-circuit and open-circuit regimes as

$$E_G = U_{OC}, \quad R_G = U_{OS} / I_{SC} \tag{2.2}$$

where U_{OC} are readings of the voltmeter in the open-circuit mode ($I = 0$) and I_{SC} are reading of the voltmeter in the short-circuit mode ($U_{ab} = 0$). Calculation of the load current can be performed according the equation (for circuit in Fig. 2.1 b):

$$I = \frac{E_G}{R_G + R_L} \tag{2.3}$$

or by

$$I = \frac{J_G}{1 + R_L / R_G} \tag{2.4}$$

(circuit in Fig. 2.1, c). Here $R_L = U_{ab} / I$ is the resistance of load, R_G is the resistance of equivalent generator, and $I_G = I_{SC}$.

Power dissipated in the load and the coefficient of efficiency are the following: $P_L = I^2 R_L$ and $\eta = P_L / (I_1 E)$.
Their values depend on ratio of R_G and R_L.

The external characteristic of the generator $U_{ab} = f(I)$ is the dependence of U_{ab} and I. This characteristic allows one to find graphically the current and voltage of load. To do so we need to draw the volt-ampere characteristic of the load R_L ($U_L = R_L I$) in the same axes as the characteristic of the generator. The point of intersection gives us U_L and I_L.

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Electrical Circuit

The circuit diagram is drawn in Fig. 2.2.
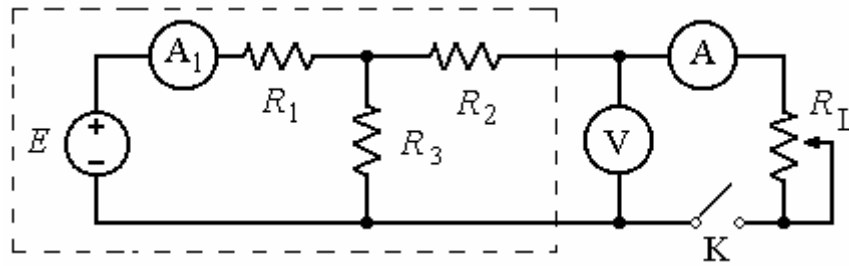
Fig. 2.2

The part of the circuit marked by a dashed line represents an active two-terminal element including source of e.m.f. $E = 15$ V and resistors R_1, R_2, R_3. Parameters of these resistors should be taken from Table 1.1 in accordance to the variant of exercise.

As a load the variable resistor $R_L = 0 \div 1000$ Ohms should be taken. DC current and voltage on the output of the two-polar elements are measured by voltmeter with permissible limit of 20 V and by ampermeter with limit of 200 mA.

When the switch K is opened, the open-circuit regime is realized in the circuit ($I = 0$). Closed switch and zero resistance of load give us the short-circuit regime ($U_{ab} = 0$).

Review Questions

1. Write down the equations which allow one to calculate the parameters of the equivalent generator out of open and short-circuit regimes.
2. Assuming that the parameters of the circuit in Fig. 2.2 (E, R_1, R_2, and R_3) are known obtain the formulae for calculation of E_G and R_G.
3. Show analytically that the maximum power dissipated in the load is observed when $R_L = R_G$.

Note. For maximum power $\dfrac{\partial P_L}{\partial R_L} = 0$.

Experimental Investigations

1. Measure actual values of resistors R_1, R_2, and R_3 and the voltage of e.m.f. source E. Write down the obtained values of parameters to Table 2.1.
2. Build the electrical circuit according to Fig. 2.2. Measure voltage U_{ab}, current I and I_1 in short-circuit and open-circuit modes.

3. Measure U_{ab}, I, and I_1 for six different resistances of load. Measurements should be done with equal intervals of load changes.
4. Calculate e.m.f. of the equivalent generator and resistance of the generator. Calculations should be done
   (a) using the measurements in short-circuit and open-circuit modes (Eq. 2.2):
   (b) according to the method of two loads (Eq. 2.1).
      Results of calculation (E_G и R_G) write down to Table 2.1

Table 2.1

$E =\dots$ V	$R_1 =\dots$ Ω		$R_2 =\dots$ Ω				$R_3 = \dots$ Ω		
Experiment	I_1	I	U_{ab}	R_L	E_G	R_G	$\ln(R_L/R_G)$	P_L	η
	mA	mA	V	Ω	V	Ω	–	W	%
Open-circuit		0		∞			∞	0	100
1									
2									
. . .									
6									
Short-circuit			0	0			-∞	0	0

5. In all experiments calculate values of the load according Ohm's law, power P_L dissipated in the load, efficiency coefficient η, and function $\ln(R_L / R_G)$. The results write down to the Table.
6. Draw external characteristic of the generator $U_{ab} = f(I)$ and CV characteristic of the load. Define graphically the value of current in the load I and voltage U_{ab} across the load. For the same load calculate current I according Eq. 2.3. Compare the results.
7. Using the data from Table 2.1 draw the functions $P_L = f\left[\ln(R_L / R_G)\right]$ and $\eta(I)$. Find out the resistance of load corresponding to the maximum power.
8. Using the parameters of the circuit (E, R_1, R_2, and R_3) calculate analytically e.m.f. E_G and the internal resistance of the generator R_G. Compare the data with results of experiments.
9. Draw conclusions to the laboratory exercise.

Laboratory exercise No. 3

A CAPACITOR AND INDUCTOR IN AC CIRCUIT

Objectives of the exercise: To measure experimentally parameters of a capacitor and inductor and to verify Kirchhoff's laws in AC circuit.

Theoretical Background

In a real capacitor some losses (energy dissipation in the form of heat) are always present due to isolation imperfection. In analysis of electrical circuits such a capacitor is represented as a equivalent circuit with elements connected in parallel. Parameters g and C of the equivalent circuit can be defined.

In case of parallel connection of R, L and C elements according to Ohm's and Kirchhoff's laws one can write the following equation for the input current in complex representation:

$$\dot{I} = \dot{I}_R + \dot{I}_L + \dot{I}_C = \dot{U}\dot{Y},$$

where $\dot{Y} = g - jb = y\exp(-j\varphi)$ is the complex conductivity of the circuit, g is the active, $b = b_L - b_C$ is the reactive, and y is the whole conductivity; $\varphi = \arctan(g/\omega C)$ is the angle of phase shift between voltage and current, $b_L = 1/(\omega L)$ is inductive conductivity and $b_C = \omega C$ is the capacitive one. Voltage across the capacitor lags in phase behind the current (the angle of phase shift is negative, ($-90° < \varphi < 0$, $b = -b_C$).

A real inductance coil also has thermal losses in contrast to the ideal one. The equivalent circuit for such an inductor is usually represented as R and L elements connected in series. These parameters can be found experimentally using the readings of devices and a phasor diagram corresponding to the circuit.

In case of series connection of R, L and C elements according to Ohm's and Kirchhoff's laws one can write in complex representation the following equation for the input voltage:

$$\dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C = \dot{I}\dot{Z},$$

where $\dot{Z} = R + jR = z\exp(+j\varphi)$ is the impedance, R is the active resistance, and $X = X_L - X_C$ is the reactance of the circuit; $\varphi = \arctan(X/R)$ is the angle of phase shift between voltage and current, $X_L = \omega L$ is the inductive resistance and $X_C = 1/(\omega C)$ is the capacitive one.

Current in the inductor lags in phase behind the voltage (the angle of phase shift is positive, $0 < \varphi < 90°$, $X = X_L$).

Review Questions

1. Which physical phenomena are reflected in the equivalent circuit of the capacitor by g and C elements and in the equivalent circuit of the inductor by R and L elements?

2. Define the limits in which can be varied the phase shift between voltage and current at the input of the passive two-polar element.

3. Write down Kirchhoff's current law for the circuits represented in Fig. 3.1 and Kirchhoff's voltage law for the circuits represented in Fig. 3.2. The equations should be written both for instantaneous and complex values of voltages and currents.

Electrical Circuit

Electrical circuits including an active-capacitive and active-inductive elements which should be investigated in the laboratory exercise are drawn in Figs 3.1 and 3.2 correspondingly.
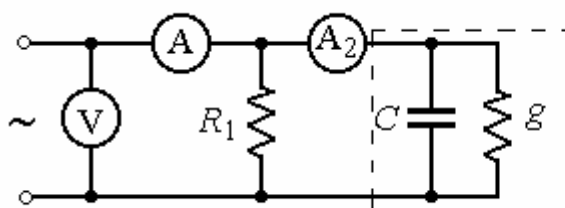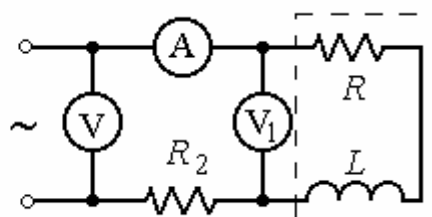
Fig. 3.1   Fig. 3.2

By dashed lines in these pictures a capacitor with parameters g and C and inductor with parameters R and L are marked. Both circuits are connected to a generator of sinusoidal voltage with frequency f and rms voltage U which can be changed.

Table 3.1

Variant		1	2	3	4	5	6	7	8	9	10
U	V	4	4.5	5	4	4.5	5	4.5	4	4.5	5
f	Hz	500	1000	2000	1000	500	500	1000	2000	1000	500
R_1	Ω	330	150	100	220	470	680	470	220	330	470
C	µF	1	1	1	1	1	0,47	0,47	0,47	0,47	0,47
R_2	Ω	680	680	1000	1000	470	220	220	470	470	150
L	mH	100	100	100	100	100	40	40	40	40	40

Values of U and f as well as other parameters of circuits can be taken from Table 3.1 according to the variant of the exercise. Measurements of currents are done with an amperemeter with limit of 200 mA and measurements of voltage by a voltmeter with limit of 20 V.

Experimental Investigations

A) Investigation of an active-capacitive circuit.

1. Build the circuit shown in Fig. 3.2 with parameters chosen according the variant of exercises (Table 3.1).

2. Set the frequency and the voltage of the generator and write down the parameters of the circuit to Table 3.2.

Table 3.2

$f =$	Hz,	$\omega = 2\pi f =$		rad/s,	$R_1 =$	Ohms,		$C =$		μF
Experiment			**Results of calculations**							
U	I	I_2	I_1	φ_2	g	C	$\dot{I}$	$\dot{I}_2$	$\dot{I}_1$	$\dot{I}_1 + \dot{I}_2$
V	mA	mA	mA	grad	Sm	μF	mA	mA	mA	mA

3. Calculate the current through resistor R_1 assuming that the initial phase angle of input voltage equals zero. Draw phasors of these current and voltage along the axis of real numbers in a phasor diagram. Draw other two currents of the circuit in the diagram. Vectors $\dot{I}_1$ and $\dot{I}_2$ form a parallelogram with diagonal represented by current $\dot{I}$. Vertex of this parallelogram can be found using a pair of compasses. After drawing the diagram measure the phase angle of current $\dot{I}_2$ and write it down to table 3.2.

4. Write complex values of all current in Table 3.2 and calculate parameters g and C of the capacitor.

B) Investigation of an active-inductive circuit.

1. Assemble the circuit shown in Fig. 3.1 with parameters chosen according the variant of exercises (Table 3.2).

2. Set the frequency and the voltage of the generator and write down the parameters of the circuit to Table 3.3.

Table 3.3

$f =$	Hz,	$\omega = 2\pi f =$		rad/s,	$R_2 =$	Ohms,		$L =$		mH
Experiment			**Results of calculations**							
U	I	U_1	U_2	φ_1	R	L	$\dot{U}$	$\dot{U}_1$	$\dot{U}_2$	$\dot{U}_1 + \dot{U}_2$
V	mA	V	V	grad	Ω	mH	V	V	V	V

3. Calculate the voltage across resistor R_2 assuming that the initial phase angle of the input current equals zero $\dot{U}_2 = U_2 = R_2 I$. Draw vectors of these current and voltage along the axis of real numbers in a phasor diagram. Draw vectors $\dot{U}_1$ and $\dot{U}$ in the diagram. These vectors form a triangle one vertex of which can be found using a pair of compasses.

4. Measure the angle φ_1 between the current and voltage of the inductor in the diagram and calculate parameters of the inductor (R and L). The obtained results write down tot Table 3.3.

5. Using the diagram calculate the complex rms values of voltages $\dot{U}$, $\dot{U}_1$, and $\dot{U}_2$ and write the results to Table 3.3. Compare the value of voltage $\dot{U} = \dot{U}_1 + \dot{U}_2$ with the corresponding value measured experimentally.

6. Draw the conclusions to the exercise.

Laboratory exercise No. 4

WYE CONNECTION OF THE LOAD IN A THREE-PHASE CIRCUIT

The objective of the exercise: Investigation of a three-phase circuit with balanced and unbalanced loads. Revealing the influence of neutral on distribution of voltages in the circuit. Drawing phasor diagrams corresponding to experimental results.

Theoretical Background

In a three-phase circuit with Y connection of the load line currents are equal to phase currents and line voltages to the differences of phase voltages: $\dot{U}_{AB} = \dot{U}_A - \dot{U}_B$. In a balanced circuit $U_L = \sqrt{3} U_{Ph}$. Some voltage drop between neutral points of generator and load appears in an unbalanced three-phase circuit without a neutral wire. In this circuit phase voltages are different.

In a three-phase circuit with the neutral the current of the neutral is not equal zero if the load is unbalanced, but all phase voltages are identical.

Note that we can use different expressions to characterize the type of circuit configuration:

The Wye = Y = "Star" connection.

Electrical Circuit

The electrical circuits investigated in this exercise are shown in Figs 4.1a (the circuit without neutral) and 4.1b (the circuit with neutral).

The circuits are supplied by a three-phase generator with balanced system of e.m.f. The rms value of phase voltage is 7 volts and frequency is 50 Hz. Parameters of the load correspond to the variant of the exercise, they are shown in Table 4.1. In balanced load resistors of 1 kΩ should be included in all branches of the circuit. The load is changed in a phase known as a "special phase". In this phase we include resistors corresponding to the variant (Table 4.1), a capacitor and inductor. Also we investigate the open-circuit mode (the load in the special phase equals infinity), and the short-circuit mode (**only in the circuit without a neutral**).

a)   b)
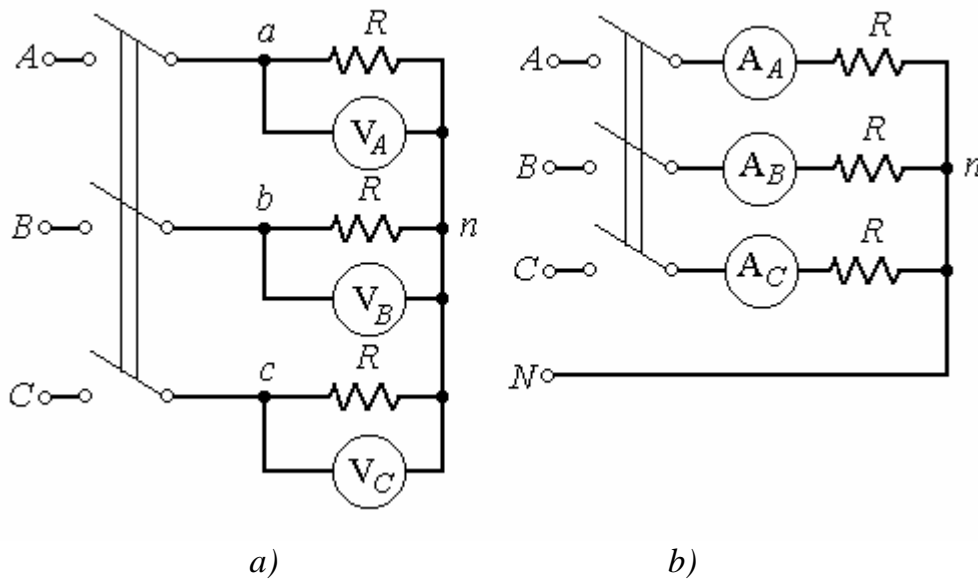
Fig. 4.1

In the circuit without a neutral we measure phase voltages with a voltmeter with limits of measurements of 20 V. Phase currents in this circuit are calculated according Ohm's law. The voltage between neutral points should be defined with the use of phasor diagram. Also with the diagram we define the current of the special phase when a capacitor or inductor is included in the circuit. The phase angle of this current should be used in drawing of the diagram corresponding to circuits with the same load and zero wire.

In the circuit including the neutral we measure phase currents with a amperemeter with limits of measurements of 200 mA. The current of the zero wire should be defined out of the phasor diagram.

Review Questions

Study theoretical material and answer the following questions:
1. A load of what kind is called a balanced load?

2. Write equations relating line and phase voltages for a circuit with balanced and unbalanced loads connected as a star.

3. Let's assume that we have a symmetric three-phase circuit loaded by three resistors. Draw the phasor diagram corresponding to this circuit. Show the shift of point n (the neutral of the load) in the diagram if the load in phase A is increased (decreased).

4. Draw the phasor diagram corresponding to unbalanced circuit with a neutral loaded by three unequal resistors.

Experimental Investigations

1. Measure the line and phase voltages of the given three-phase generator. Write the values of these voltages to column "Notes" in tables 4.2 and 4.3.

Table 4.1

Variant		1	2	3	4	5	6	7	8	9	10
R_1	kΩ	4,7	2,2	4,7	2,2	4,7	2,2	4,7	2,2	4,7	2,2
R_2	Ω	680	680	680	680	680	470	470	470	470	470
L	mH	200	140	100	100	200	140	100	100	200	140
C	μF	1	1,22	1,47	1	1,22	1,47	1	1,22	1,47	1
Special phase		A	A	B	C	B	C	A	B	C	A

2. Assemble the circuit shown in Fig. 9.1, *a* (**the circuit without a neutral**) and measure the voltages in a balanced mode. Write the reading of voltmeters to the upper row of Table 4.1. Calculate the ratio U_L/U_{Ph}.

Table 4.2

Experimental data			Results of calculation				The load of the special phase	Notes $U_{Ph} = \quad$ V
U_A	U_B	U_C	I_A	I_B	I_C	U_{nN}		
V	V	V	mA	mA	mA	V		$U_L = \quad$ V
							1 kΩ	Balanced load
							R_1	
							R_2	
							L	Unbalanced load
							C	
							∞	Open circuit
							0	**Short circuit**

3. Measure the voltages in the three-phase circuit without neutral with the following loads in the special phase:

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a) decrease of the active load; resistor R in the special phase should be changed for resistor $R_1 > R$;

b) increase of the active load; resistor R in the special phase should be changed for resistor $R_2 < R$;

c) inductive load L in the special phase;

d) capacitive load C in the special phase;

e) infinite load (open circuit) in the special phase;

f) zero load (short circuit) in the special phase; remove the resistor out of the special phase and include in it a wire.

The results obtained in experiments 3 and 4 write down to Table 4.2.

4. Calculate the currents in all phases. Write the results to Table 4.2.

5. Draw phasor diagrams for all experiments (here 7 vector diagrams should be drawn). Begin the drawings with a triangle of line voltages. In this case the position of neutral point n can be found using a pair of compasses.

6. Define the voltage U_{nN} between neutral points of the generator and load out of vector diagrams. The results write down to Table 4.2.

7. Assemble the circuit with a neutral wire and balanced load (Fig. 4.1, *b*) and measure the currents. All currents should be identical and the current of the neutral is equal to zero.

8. Make all experiments as previously, **except the short circuit (!)**. The obtained currents should be written to Table 4.3.

Table 4.3

Experiments and calculations				The load of the special phase	Notes
I_A	I_B	I_C	I_N		$U_{Ph} =$ V
mA	mA	mA	mA		$U_L =$ V
				R_1	Balanced load
				R_2	Unbalanced load
				L	
				C	
				∞	

9. Draw phasor diagrams corresponding to all experiments (5 diagrams should be drawn).

10. Analyze the diagrams and formulate conclusions to this exercise.

Laboratory exercise No. 5

DELTA CONNECTION OF THE LOAD IN A THREE-PHASE CIRCUIT

The objective of the exercise: Investigation of a three-phase circuit with balanced and unbalanced loads in delta connection. Drawing phasor diagrams corresponding to experimental results.

Theoretical Background

In a three-phase circuit with delta configuration of a load a line voltage is equal to phase voltage, and line currents are equal to the difference of phase currents, for example, $\dot{I}_A = \dot{I}_{AB} - \dot{I}_{CA}$. If a load is a balanced one, $I_L = \sqrt{3} I_{Ph}$.

Electrical Circuit

The circuit diagram is presented in Fig. 5.1. The circuit is supplied by blanced three-phase generator with phase voltage of 7 V and frequency of 50 Hz. Line currents are measured by amperemeters with limits of 200 mA, phase currents should be calculated according Ohm's law or obtained with the use of phasor diagrams.
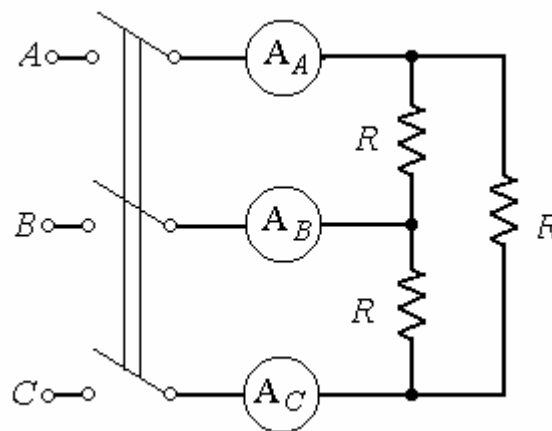
Fig. 5.1

In balanced load 1 kΩ resistors (R) are included into all phases. In unbalanced load we include resistance or reactance into the special phase. The active load of a special phase is formed by resistance lesser than R ($R_1 < R$) or greater than R ($R_2 > R$). Reactance is formed by an inductor or by capacitor. Parameters of the devices are given in Table 5.1. Above that, we should break a wire in the special phase (open circuit) and break a line.

Table 5.1

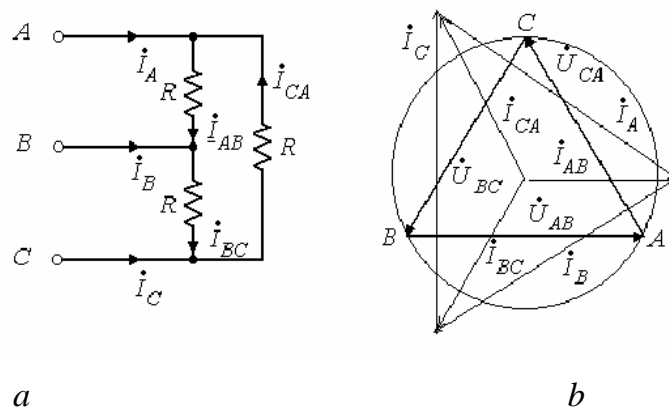| Variant		1	2	3	4	5	6	7	8	9	10
R_1	kΩ	4,7	2,2	4,7	2,2	4,7	2,2	4,7	2,2	4,7	2,2
R_2	Ω	680	680	680	680	680	470	470	470	470	470
L	mH	200	140	100	100	200	140	100	100	200	140
C	μF	1	1,22	1,47	1	1,22	1,47	1	1,22	1,47	1
Special phase		AB	AB	BC	CA	BC	CA	AB	BC	CA	AB

a   *b*

Fig.5.2

Review Questions

Study theoretical material and answer the following questions:
1. Show line and phase currents in electrical circuit presented in Fig. 5.1.
2. Write equations relating line and phase currents in balanced and unbalanced circuits.
3. A phasor diagram corresponding to a three-phase circuit with balanced delta connected load is shown in Fig. 5.2. Explain how we should change the diagram if phase *AB* is broken.
4. Explain how we should change the diagram if a linear wire *A* is broken.

Experimental Investigations

1. Measure all line voltages. The mean value of these voltages write down to Table 5.2.
2. Build electrical circuit according to Fig. 5.1 and measure all currents of a balanced load. Reading of the amperemeters write down to Table 5.2.
   a) reduce active load in the special phase ;
   b) increase active load in the special phase ;
   c) remove the load out of the special phase (break the special phase);

d) include an inductor into the special phase ;
e) include a capacitor into the special phase ;
f) break the line that is not connected with the special phase.

Results of all experiments write down to Table 5.2. Also write to this table the values of currents calculated according Ohm's law.

Table 5.2

I_A	I_B	I_C	I_{AB}	I_{BC}	I_{CA}	$U_L=$ (V)
mA	mA	mA	mA	mA	mA	The load of the special phase
						Balanced active
						Unbalanced active, $R_1 < R$
						Unbalanced active, $R_2 > R$
						The load is excluded ($R = \infty$)
						An inductor
						A capacitor
						The line is broken

3. Draw phasor diagrams corresponding to all experiments.
4. Analyze the obtained results and formulate conclusions to the laboratory exercise.

Laboratory exercise No. 6

INVESTIGATION OF SERIES RESONANCE CIRCUIT

The objective of the exercise: Investigation of resonance mode in a circuit including inductor and capacitor connected in series. Registration of circuit characteristics with different generator frequencies.

Theoretical Background

Resonance is such a mode of electrical circuit including energy storing elements (inductors and capacitors) in which phase shift between the input current and input voltage equals zero.

If a circuit includes inductors and capacitors connected in series, at some frequency reactive components of circuit impedance compensate each other. In this case we obtain series resonance in the circuit. Let's consider the simplest circuit with only one inductor characterized by parameters R and L and a single capacitor C (Fig. 6.1). If the circuit is supplied by a harmonic source of voltage $u(t) = U\sqrt{2}\sin(\omega t)$ and has the parameters

$$z = \sqrt{R^2 + X^2}, \quad X = X_L - X_C,$$

$$X_L = \omega L, \quad X_C = \frac{1}{\omega C}, \quad \varphi = \arctan \frac{X}{R}.$$

a harmonic current $i(t) = I\sqrt{2} \sin(\omega t - \varphi)$ with rms value $I = U/z$ flows in this circuit.

It is clear that the phase difference is equal to zero only if $X = 0$ or $X_L = X_C$, $\omega^2 LC = 1$. So to obtain resonance in the circuit we need to change frequency ω, inductance L, or capacitance C. For given inductance and capacitance the resonance develops at frequency $\omega_{RES} = \dfrac{1}{\sqrt{LC}}$.

Rms values of voltage across the inductor ($U_L = X_L I$) and across the capacitor ($U_C = X_C I$) are equal in resonance, and for large quality factors $Q = \dfrac{\rho}{R} > 1$ their magnitudes are much larger than the magnitude of the input voltage U.

Electrical Circuit

The electrical circuit of the exercise is shown in Fig. 6.1. The circuit is supplied by a generator with alternating frequency $f = \dfrac{\omega}{2\pi}$. Changes of frequency may result in changes of the generator voltage, in this case the rms value of voltage should be tuned according the requirements of the experiment.
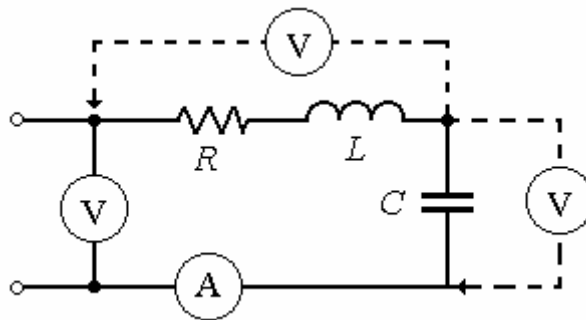

Fig. 6.1

The inductor is represented in the circuit as an active resistor and an ideal inductor connected in series. This active resistance R can be measured by the ohmmeter while parameters of circuit elements should be chosen according the variant of the exercise (Table 6.1).

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Table 6.1

Variant		1	2	3	4	5	6	7	8	9
U	V	6	6	6	6	5	5	5	5	1,5
L	mH	100	100	100	100	40	40	40	40	10
C	µF	1	0,47	0,22	0,1	1	0,47	0,22	0,1	1

Measurements of all voltages and currents are performed by voltemeter with 20 volt limit of measurements and by amperemeter with 200 mA limit, correspondingly.

Review Questions

Answer the following questions:
1. What mode of electrical circuit is called resonance mode?
2. Which parameters we can change in the circuit shown in Fig. 6.1 to obtain resonance?
3. How we can calculate resonance frequency for the circuit shown in Fig. 7.1? Calculate the resonance frequency for your variant.
4. How changing frequency and measuring the input current in the circuit we can tell that resonance is achieved?
5. Draw qualitatively phasor diagrams for the circuit in Fig. 7.1 corresponding to frequencies $f = f_{RES}, f < f_{RES}$, and $f > f_{RES}$.

Experimental Investigations

1. Measure the active resistance of the inductors with the ohmmeter. Calculate resonance frequency f_{RES} and quality factor Q corresponding to the variant of exercise. Write parameters of the circuit and results of calculations to Table 6.2
2. Assemble the circuit shown in Fig. 6.1 and register the readings of devices with frequency $f = f_{RES}$, with two values of frequency greater than f_{RES}, and with two values lesser than f_{RES}. With the second voltmeter measure the voltages across the inductor and capacitor. Write down the obtained results to Table 6.3.
3. For all frequencies calculate voltages $U_R = RI$ and $U_L = 6.28fLI$ and phase angle $\varphi = \pm \arccos(U_R / U)$. Positive sign in this equation corresponds to frequency $f > f_{RES}$, negative to $f < f_{RES}$. The results of calculations write down to Table 6.3.
4. Using the data of Table 6.3 draw dependences $I(f)$, $\varphi(f)$, $U_L(f)$, and $U_C(f)$. The last two graphs draw in the same coordinate system.

5. Calculate currents and voltages in the circuit corresponding to three frequencies taken from Table 6.3 ($f < f_{RES}$, $f = f_{RES}$, and $f > f_{RES}$). The results of calculations write down to Table 6.4.
6. Draw three phasor diagrams corresponding to data of Table 6.4.
7. Compare theoretical and experimental results and draw general conclusion to the exercise.

Table 6.2

R	L	C	f_{RES}	Q
Ohm	mH	μF	Hz	–

Table 6.3

U	f	I	U_{IND}	U_R	U_L	U_C	φ
V	Hz	mA	V	V	V	V	grad

Table 6.4

U	f	X_C	X	z	I	U_C	U_L	U_R	U_{KIN}	φ
V	Hz	Ω	Ω	Ω	mA	V	V	V	V	grad

Laboratory exercise No. 7

INVESTIGATION OF PARALLEL RESONANCE CIRCUIT

The objective of the exercise: Investigation of resonance in a circuit with inductor and capacitor connected in parallel. Registration of circuit characteristics with different generator frequencies.

Theoretical Background

In resonance mode of a circuit the phase difference between the input current and input voltage is equal to zero. Resonance in a circuit with reactive

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elements connected in parallel is called *parallel resonance*. The simplest circuit where resonance is possible to observe is a circuit including an inductor (parameters R and L) and capacitor (C) connected in parallel (Fig 7.1).

If harmonic voltage $u = U\sqrt{2}\sin(\omega t)$ is applied to this circuit, the current $i = I\sqrt{2}\sin(\omega t - \varphi)$ in it is also a harmonic function. Here

$$I = yU, \quad y = \sqrt{g^2 + b^2}, \quad g = R / z_{IND}^2, \quad b = b_K - b_C,$$

$$b_C = \omega C, \quad b_{IND} = \omega L / z_{IND}^2, \quad z_{IND} = \sqrt{R^2 + (\omega L)^2}, \quad \varphi = \arctan(b / g).$$

These equations illustrate the main property of the resonance mode: the input current of the circuit has the same phase angle as the input voltage ($\varphi = 0$) and the imagine part of the input conductivity equals zero ($b = 0$). Resonance in such a circuit is possible to obtain by varying frequency of the input voltage or by varying capacitance or inductions of elements. For a circuit with given parameters R, L and C the resonance frequency is defined by the equation

$$\omega_{RES} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}, \quad f_{RES} = \frac{\omega_{RES}}{2\pi}.$$

In the resonance mode rms value of the current running in the inductor ($I_{IND} = U / z_{IND}$) and in the capacitor ($I_C = \omega C U$) can be much greater than the value of the input current ($I = Uy$).

Electrical Circuit

The circuit investigated in the current laboratory exercise is shown in Fig. 7.1. It's supplied by the source of sinusoidal voltage the frequency of which is possible to change by a switch labeled *Frequency*. Rms value of the voltage can be changed with a switch *Amplitude*. In all experiments the magnitude of voltage should be kept at 6 V.
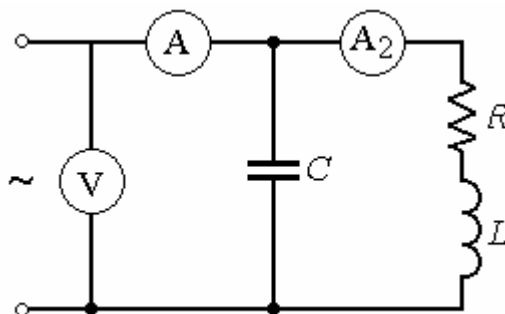

Fig. 7.1

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The inductor in the circuit is represented by active resistor and ideal inductance coil characterized by parameters R and L connected in series. Parameters C and L are given in Table 7.1, while active resistance R of the inductor should be measured by the ohmmeter.

Table 7.1

Variant		1	2	3	4	5	6	7	8	9
L	mH	40	40	40	40	40	100	100	100	100
C	μF	1	0,47	0,57	0,22	1,22	0,47	1	1,1	0,57

The rms value of input voltage is measured by voltmeter with 20 V limit of measurements, currents in inductor and capacitor are measured by amperemeters with limit of 200 mA. In this circuit the current in the capacitor can be calculated according Ohm's law.

Review Questions

Study theoretical material and answer the questions:
1. What is the difference between parallel and series resonance?
2. Write the equations describing the resonance mode in circuits drawn in Figs 7.1 and 7.2. Which parameters we can change to obtain resonance in these circuits?
3. How we can tell that the resonance mode is achieved in the circuit if we can measure only the input current?
4. Draw three phasor diagrams corresponding to the circuit in Fig. 7.1 if $f = f_{RES}, f < f_{RES}$, and $f > f_{RES}$.
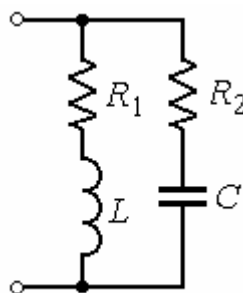

Fig. 7.2

Experimental Investigations

1. Measure the active resistance R of the inductor by the ohmmeter. Calculate the resonance frequency and write this frequency and other parameters of the circuit to Table 7.2.

Table 7.2

R	L	C	f_{RES}
Ω	mH	μF	kHz

2. Assemble the circuit shown in Fig. 7.1 and measure currents and voltages corresponding to five frequencies (for $f = f_{RES}$, for two values of frequency $f < f_{RES}$, and two values $f > f_{RES}$). Current in the capacitor calculate according Ohm's law. Phase shift between the input current and voltage should be calculate as

$$\varphi = \arcsin \frac{I^2 + I_C^2 - I_{IND}^2}{2 I_C I}.$$

The results of calculation write to Table 7.3

3. Calculate currents in the circuit for three values of frequency ($f = f_{RES}$, $f < f_{RES}$, and $f > f_{RES}$) using equations given in paragraph *Theoretical background*. Phase shift between the input current and input voltage calculate as $\varphi = \arctan(b/g)$. In calculations assume that exact values of U, R, L, and C are given. The obtained results write to table 7.4 and compare with experimental data.

Table 7.3

U	f	I	I_{IND}	I_C	φ
V	kHz	A	A	A	grad

Table 7.4

U	f	b_C	z_K	y	I	I_{IND}	I_C	φ
V	kHz	Ohm^{-1}	Ohm	Ohm^{-1}	A	A	A	grad

4. Draw phasor diagrams according to Table 7.4.

5. Draw the functions $\varphi(f)$, $I(f)$, $I_{IND}(f)$, and $I_C(f)$ using the data from Table 7.4. All functions of current draw in the same coordinate system.
6. Compare theoretical and experimental results and draw conclusions to the laboratory exercise.

Laboratory exercise No. 8

EXPERIMENTAL VALIDATION OF SWITCHING RULES

The objective of the exercise: Experimental verification of switching rules, calculation of energy dissipated at switching.

Theoretical Background

In circuits with inductors and capacitors very quick initial changes of variables can be observed in some situations, and afterward variables approaches slowly the steady-state magnitudes. To simplify analysis of transients these short-duration changes can be viewed as an instantaneous process with zero duration. But in this assumption we need to agree with violation of switching rules stating continuous changes of induction currents and capacitor voltages. At the same time Kirchhoff's rules should be satisfied along with the general rules of switching which can be formulated as follows:

The magnetic flux in an inductor just after switching ψ(0₋) is equal to the flux just before switching ψ(0₊).

The electric charge in a capacitor just after switching q(0₋) is equal to the electric charge just before switching q(0₊).

If a circuit includes only capacitors, we can characterize switching by the following equations:

$$\Sigma q(+0) = \Sigma q(-0) \text{ or } \Sigma Cu_C(+0) = \Sigma Cu_C(-0),$$

and for a circuit including only inductors :

$$\Sigma\psi(+0) = \Sigma\psi(-0) \text{ or } \Sigma Li_L(+0) = \Sigma Li_L(-0).$$

At the beginning of transient process very large (theoretically, infinitely large) and very short (again, infinitely short) pulses of current develop in capacitors. Correspondingly large and short pulses of voltages develop in inductors if such elements are included into a circuit. At the same time, we observe finite changes of energy stored in a capacitor ($W_E = Cu_C^2 / 2$) or in an inductor ($W_M = Li_L^2 / 2$). In assumption of instantaneous switching the energy changes in a jump (this process is possible to call "the incorrect switching"),

and the part of energy "lost" at switching ($\Delta W = \sum W(-0) - \sum W(+0)$) dissipates in a form of spark between contacts of the switch or in wires of the circuit.

Electrical Circuit

The circuit investigated in the current exercise is shown in Fig. 8.1. The capacitor C_1 (100 µF) is connected to the direct e.m.f. source of 15 volts through key 1 (the key is in position 1), at the same time key 3 should be closed. Opening key 3 we disconnect the capacitor from the source, but its charge conserves.
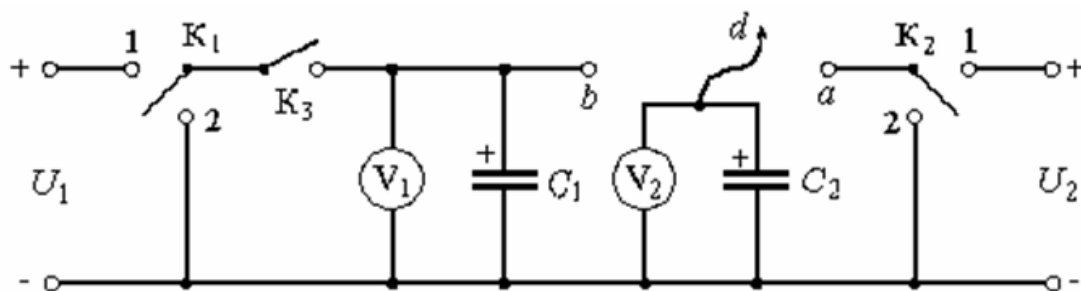

Fig. 8. 1

In the right-hand part of the circuit capacitor C_2 (10 or 470 µF) is connected to the second source of e.m.f. if key "*d*" is in position "*a*" and key 2 in position 1. The voltage U_2 of the second source should be set according the variant of the exercises, magnitude of this voltage is given in Table 8.1.

Table 8.1

Variant	1	2	3	4	5	6	7	8	9	10
	2	3	4	3	2	3	4	5	2	4
U_2, V	6	5	6	7	5	6	7	8	4	8
	10	7	8	11	8	9	10	11	6	12
	14	10	10	14	11	12	13	14	10	14

Switching the key "*d*" (actually, a conductor) from position "*a*" to position "*b*" we connect two capacitors with different charges and voltages. Such an experiment allows us to validate the laws of switching formulated in the general form, stating conservation of the total charge of the system. The voltages across the capacitors are controlled by DC voltmeters V_1 and V_2 with limits of 20 V. In this laboratory exercise electrolytic capacitors are used, and

in the circuit they should be connected exactly as it is shown in the circuit diagram (Fig. 8.1).

Review Questions

1. Formulate the laws of switching in the general for and write corresponding equation for the circuit in Figs 8.2 *a* and *b*.
2. Write the equation defining the total charge of capacitors C_1 and C_2 before switching (Fig. 8.1) assuming that voltages across the capacitors are U_1 and U_2 correspondingly. Write the equation characterizing the total charge after switching when the capacitors have the same voltage $U_C(0_+)$.
3. Write the equations for energy of capacitors in the circuit presented in Fig. 8.1 before ($W(0_-)$) and after switching ($W(0_+)$).
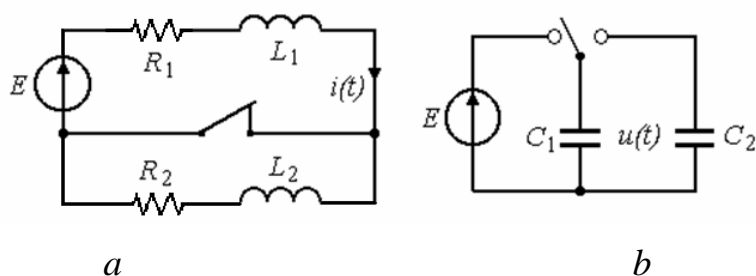

a   *b*

Fig 8.2

Experimental Investigations

1. Build the circuit shown in Fig. 8. 1 including capacitor C_2 of 10 μF. Switch key 1 (K_1) to position 1, key 2 (K_2) to position 2, and close key 3 (K_3). The reading of voltmeter V_1 (voltage U_1) write to table 8.2. In the first experiment $U_2 = 0$.
2. Open key 3 (K_3) simultaneously changing position of key "d" from "*a*" to "*b*". Write readings of voltmeter V_2 registered right after the switching (voltage U_1) to Table 8.2.
3. Reconstruct the initial setting of the circuit, i.e., set key K_2 to position 1, conductor "*d*" to position "*a*", and redo the experiment with different voltages U_2 taken from Table 8.1 according the variant of the exercise. Voltages registered before and right after switching write to table 8.2.
4. Redo the experiments with $C_2 = 470$ μF.
5. In all experiments calculate the total charge of capacitors and energy of electric field stored in them before and after switching. Calculate the difference of energy (ΔW) registered before and after switching. The results of calculations write to Table 8.2.
6. Analyze the obtained results and draw conclusions to the laboratory exercise.

Table 8.2

$U_1 = \ldots$ V	**Before switching**			**After switching**			
$C_1 = 100\ \mu F$	U_2	$\Sigma q(0_-)$	$\Sigma W(0_-)$	$U_C(0_+)$	$\Sigma q(0_+)$	$\Sigma W(0_+)$	ΔW
	V	μC	mJ	V	μC	mJ	mJ
	0						
$C_2 = 10\ \mu F$							
	0						
$C_2 = 470\ \mu F$							

Laboratory exercise No. 9

INVESTIGATION OF TRANSIENTS IN A FIRST-ORDER CIRCUIT

The objective of the exercise: Investigation of transients in a circuit including a capacitor and resistor (a first-order circuit). Comparison of experimental and analytical results.

Theoretical Background

Transients are changes of currents or voltages in a circuit from one steady-state to other. Usually transients develop at switching. Power of a real source is always finite, so variables associated with changes of energy (current of the inductor or voltage across terminals of a capacitor) cannot be changed instantaneously. Conservation of these variables is defined by rules of switching.

Solution to the system of differential equations describing the transient mode of a circuit includes two components. The first is called *the forced response* of the circuit, and the second is *the natural response* of the circuit. For example, a current can be written as a sum $i(t) = i_{\mathrm{Fr}}(t) + i_{\mathrm{Nr}}(t)$.

The forced response of the circuit is calculated as a steady-state current or voltage in the circuit after switching ($t = \infty$). The natural response is a sum of exponential functions:

$$i_{\mathrm{Nr}}(t) = \sum A_k e^{p_k t} \ ,$$

obtained with different roots p_k of characteristic equation. Here coefficients A_k are integration constants calculated with the use of independent and dependent initial conditions.

The equation describing the natural response in a circuit with only one inductor or capacitor includes only one exponential function and characteristic equation has one root. For example, the natural response for the circuit drawn in Fig. 9.1, *a* is the following:

$$i_{\mathrm{Nr}}(t) = Ae^{pt}.$$

The forced response for the circuit in Fig. 9.1 supplied by DC source is equal to zero $(i_{\mathrm{Fr}}(t) = 0)$ because DC current does not flow through a capacitor. Before switching the capacitor has zero charge, and according to the second rule of switching

$$U_C(0_+) = U_C(0_-) = 0.$$

So

In this case changes of current and voltage of the capacitor are described by equations

$$i(t) = \frac{U}{R}\exp(-t/RC); \quad U_C(t) = U - i(t)R = U\left(1 - \exp(-t/RC)\right). \quad (9.1)$$

Here $p = -1/RC$ is the root of characteristic equation and $\tau = 1/|p| = RC$ is the time constant. This constant can be found with the use of a tangent line to the graph obtained experimentally (Fig. 9.1*b*).
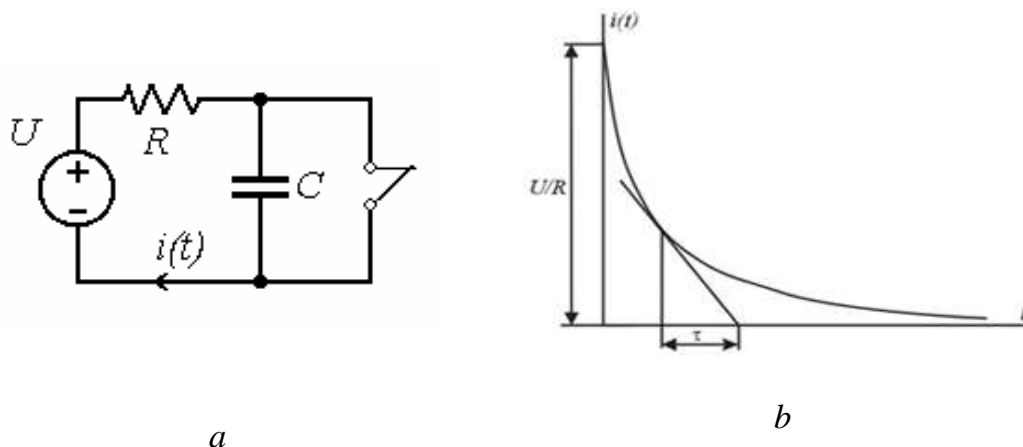

a

b

Fig. 9.1

The equations describing current and voltage of the discharge can be obtained in the same way. They are the following:

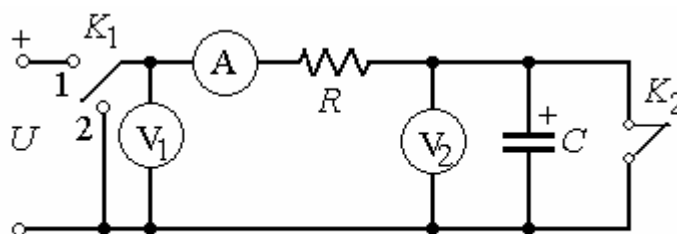$$i(t) = -\frac{U}{R}\exp(-t/RC), \quad U_C(t) = U\exp(-t/RC).$$ (9.2)

Fig. 9.2

Charge and discharge of the capacitor are investigated with the use of the circuit shown in Fig. 9.2. The circuit is supplied from a DC source the voltage of which is given in Table 9.1. Also in this table values of R and C are written. Special attention should be paid to the polarity of capacitor.

Table 9.1

Variant		1	2	3	4	5	6	7	8
U	V	10	11	12	13	14	3	4	4
R	kΩ	100	122	133	147	200	22	22	33
C	μF	100	100	100	100	100	470	470	470

Two voltmeters with 20 V limits of measurement are included into the circuit along with an amperemeter (200 μA). The first voltmeter allows one to control the voltage of the source, the second voltmeter shows changes of voltage across terminals of capacitor. Current of charge and discharge is measured by the amperemeter.

Before development of transients key K_1 is connected with point 1 (the source is included into the circuit), key K_2 is closed. In this geometry of the circuit voltage across the capacitor equals zero ($U_C(0) = 0$) and the current is $i_C(t) = U/R$. At the moment of switching key K_2 opens, voltage preserves its magnitude, and the current is still equal to the ratio of voltage and resistance ($i_C(t) = U/R$). So this formula gives us the current at the first moment after switching ($t = 0_+$). After that the current and voltage should be registered simultaneously with equal intervals of time (usually 5 seconds).

Then the capacitor is charged, key K_1 is switched to position 2. Now we register discharge of the capacitor in the same manner as it was done earlier.

Review Questions

Study theoretical material and answer the questions:
1. How forced response of the circuit can be calculated along with independent and dependent initial conditions?
2. How the time constant can be found theoretically and experimentally?
3. Calculate the root of characteristic equation, time constant, and the total time of transients for the circuit used in your experiments.

Experimental Investigations

1. Build the circuit shown in Fig 9.2 and measure the value of current and voltage corresponding to $t = 0_+$. In this experiment key K_2 is closed, key K_1 is in position 1. Write the current and voltage to Table 9.2.
2. Open K_2 and measure currents and voltages in the circuit with equal temporal intervals. Write the obtained results to Table 9.2.
3. Switch key K_2 to position 2 and measure currents and voltages of discharge. Write the results to Table 9.2.
4. Draw graphs of current and voltage (four graphs $i_C(t)$ and $U_C(t)$), define graphically the time constant for each graph, and calculate the mean value of the constant.
5. Calculate analytically the time constant as $\tau = RC$. Compare analytical and experimental results.
6. Calculate values of functions $i_C(t)$ and $U_C(t)$ for moments of time $t = \tau, 2\tau, 3\tau, 4\tau, 5\tau$. Mark obtained points in the same coordinate system as the experimental graphs.
7. Analyze the results and write conclusions to the exercise.

Table 9.2

	Experimental data					Results of calculations					
	Charge			Discharge			Charge			Discharge	
t	$i(t)$	$u_C(t)$	t	$i(t)$	$u_C(t)$	t	$i(t)$	$u_C(t)$	t	$i(t)$	$u_C(t)$
s	μA	V	s	μA	V	s	μA	V	s	μA	V
0											
5											
10											
15											
20											
25											
…											
60											

Laboratory exercise No. 10

INVESTIGATION OF TRANSIENTS IN A SECOND-ORDER CIRCUIT

The objective of the exercise: Investigation of transients in a circuit including capacitor, inductor and resistor connected in series (a second-order circuit). Comparison of experimental and analytical results.

Theoretical Background

An example of a first-order circuit is given in Fig. 10.1. After switching this circuit is described by the following equation written according the Kirchhoff's voltage law:

$$U_R + U_L + U_C = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = E \ . \tag{10.1}$$
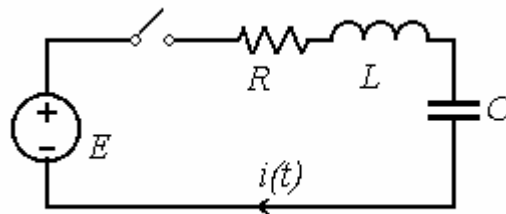

Fig. 10.1

After calculation of derivatives in the left and right-hand parts of this formula we obtain differential equation of the second order:

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0. \tag{10.2}$$

In solution to this equation is possible to obtain with classical approach, the force response of the circuit is absent in the solution and form of the natural response depends on the roots p_1 and p_2 of characteristic equation:

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \ . \tag{10.3}$$

Here three variants are possible. The roots are:
1. Real, negative, and different ($p_1 \neq p_2 < 0$). In this case we observe over-dumped response of the circuit.
2. Real, negative, and identical ($p_1 = p_2 = -\delta = -R/2L$) and critically dumped transient process.

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3. Complex conjugative roots $p_{1,2} = -\delta \pm j\omega_{Free}$ give us underdumped response. Here $\delta = \dfrac{R}{2L}$ is the attenuation factor and $\omega_{Free} = \sqrt{(LC)^{-1} - \delta^2}$ is the cyclic frequency of free oscillations. In this case changes of current are described by equation:

$$i(t) = A\exp(-\delta t)\cos(\omega_{Free}t + \alpha) = \dfrac{E}{\omega_{Free}L}\exp(-\delta t)\sin(\omega_{Free}t). \quad (10.4)$$

In Eq. 10.4 the amplitude of oscillations is the following:

$$A = \dfrac{E}{\omega_{Free}L}.$$

The current corresponding to underdumped response is presented in Fig. 10.2.
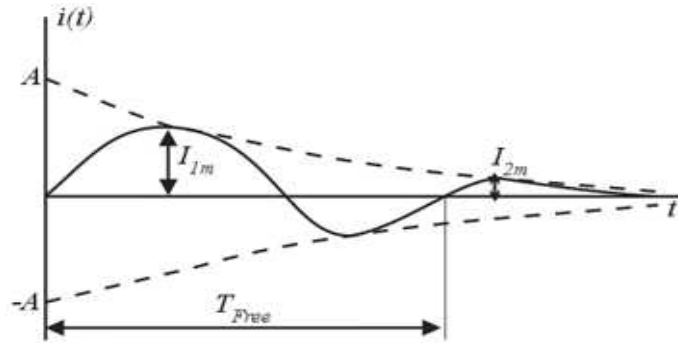

Fig. 10.2

To evaluate the rate of the current changes the decrement of oscillations Δ is usually used. This parameter is equal to the ratio of current magnitudes at two moments of time taken with the temporal interval between them of one period (T_{Free}):

$$\Delta = \dfrac{i(t)}{i(t + T_{Free})}.$$

The logarithmic decrement is also possible to use for this purpose:

$$\Theta = \ln\Delta = \delta T_{Free} = T_{Free} / \tau_{Free}.$$

Here τ_{Free} is the time constant of the oscillation envelope (Fig. 10.2). To find the decrement experimentally we can use amplitudes of the first and second periods:

$$\Delta = \dfrac{I_{1m}}{I_{2m}}.$$

Review Questions

Study theoretical material and answer the questions:

1. How is possible to write characteristic equation corresponding to differential equation of the second order?
2. What types of transients are possible in a second-order circuit including an inductor, resistor, and capacitor?
3. How is possible to calculate independent and dependent initial conditions in a second-order circuit?
4. Give the definition of the oscillation decrement and explain how this parameter is possible to find experimentally.

Electrical Circuit

Transients in circuits including capacitors and inductors are so fast that they usually cannot be registered with amperemeters and voltmeters. But repetition of the process allows one to obtain representation of voltages in a screen of oscilloscope.

Table 10.1

Variant		1	2	3	4	5	6	7	8
L	mH	10	10	40	40	40	40	100	100
C	µF	0,47	1	0,1	0,22	0,47	1	0,1	0,22
R_{Shunt}	Ω	22	22	100	100	100	100	220	220

Electrical circuit investigated in this exercise is shown in Fig. 10.3. The circuit is supplied from a source generating rectangular pulses of voltage with frequency of 200 Hz and amplitude of 8 V (Fig. 10.3, *b*). This voltage is measured by a DC voltmeter with 20 V limits of measurements. Note that amplitude of 8 V corresponds to the rms value of 4 V registered by the voltmeter.

Parameters of the circuit should be taken from Table 10.1 according the variant of the exercise. R_{Ind} is the internal resistance of the inductor. Voltage proportional to the current of transient process is registered by the oscilloscope connected in parallel with resistor R_{Shunt}.

Experimental Investigations

1. Build the circuit shown in Fig 10.3, *a*. Values of R_{shunt}, L, and C write down to Table 10.2. Set the frequency of pulses equal to 200 Hz and amplitude equal to 8 V.
2. Write the setting of oscilloscope (the time scale m_t and scale of voltage m_u) to Table 10.2. Calculate the scale of current as $m_i = 10^3 m_u / R_{Shunt}$ and write it

to the same table. Copy the graph of current from the screen of oscilloscope.

3. Using the graph of the current find the period of oscillation T_{Free} and the time constant τ_{Free} (the time constant can be found with the use of a tangent line to oscillation envelope).

4. Measure amplitudes of oscillations corresponding to the first I_{1m} and second I_{2m} periods. Calculate decrement of oscillations Δ and logarithmic decrement of oscillations Θ.

5. Calculate the attenuation factor $\delta = \tau_{Free}^{-1}$, cyclic frequency of oscillations $\omega_{Free} = 2\pi/T_{Free}$ and the integration constant $A = E/(\omega_{Free}L)$. Results of calculations write to Table 10.2.

6. According to Eq. 10.4 calculate values of the current at moments of time: $t = 0$, $T_{Free}/4$, $T_{Free}/2$, $3T_{Free}/2$, and T_{Free}, and draw corresponding points in the graph of current.

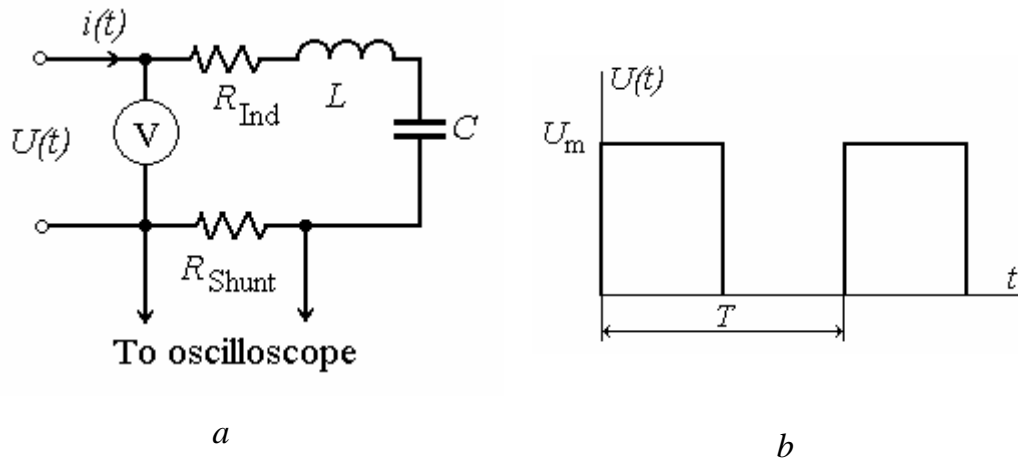7. Analyze the results and write conclusions to the exercise.

a   b

Fig. 10.3

Table 10.2

Parameters	$E = 8$ V	$L =$		$C =$		$R_{shunt} =$	
Scales	$m_t =$ ms/div		$m_u =$ ms/div		$m_i =$ mA/div		
Experimental results (data from the graph)	$T_{Free} =$		$\tau_{Free} =$		$I_{1m} =$		$I_{2m} =$
Results of calculations	$\Delta =$	$\Theta =$		$\delta =$		$\omega =$	$A =$

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Laboratory exercise No. 11

INVESTIGATION OF A NONLINEAR DC CIRCUIT

The objective of the exercise: Experimental investigation of nonlinear elements. Application of Zener diode to stabilize voltage. Calculation of equivalent characteristics of series-parallel connection of nonlinear elements.

Theoretical Background

To analyze properties of nonlinear DC circuit we need to know current-voltage characteristics (CVC) of all elements included into this circuit. Such characteristics obtained with DC currents are called static characteristics. If a circuit consists of several nonlinear elements with different types of connection, we can simplify it by representing groups of elements by equivalent CV characteristic.

For example, instead of several elements connected in series we can introduce an equivalent element with characteristic obtained on the base of Kirchhoff's voltage law:

$$U(I) = \sum_{k=1}^{n} U_k(I).$$

Here summation of voltages (points of the characteristic) is done for several chosen values of currents.

If elements are connected in parallel, according to Kirchhoff's current law we need to sum up currents with fixed values of voltages:

$$I(U) = \sum_{k=1}^{n} I_k(U).$$

Some nonlinear elements is possible to connect in such a way that the registered changes of voltage in the circuit would be much smaller comparing with changes of the input variables. Such a circuit is called a voltage stabilizer. On the other hand, we can build a nonlinear circuit in which current in some branch does not depend on resistance of this branch. This devise is called a current stabilize.

Electrical Circuit

Electrical circuit parts of which are shown in Fig. 11.1, *a*, *b*, *c*, and *d* is used in the exercise. The voltage of the supply is varied in the interval $U_1 = 0 \div 15$ V. The part *a* in Fig. 11.1 is the common for all experiments.
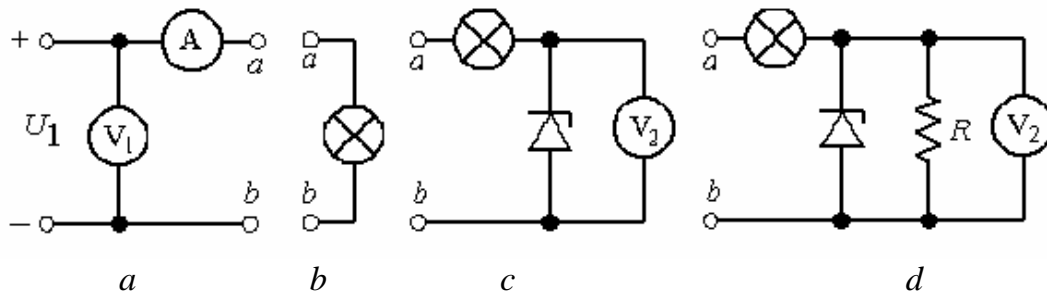
Fig. 11.1

To terminals *ab* of this part we connect a lamp (Fig. 11.1, *b*), Zener diode and a lamp (*c*), or the combination of these elements and a linear resistor (*d*). In corresponding experiments we can obtain CV characteristic of the lamp $U_L(I_L)$, of Zener diode $U_{ZD}(I_{ZD})$, and equivalent characteristic of the circuit $U_{Eq}(I_{Eq})$. In the last two experiments we can also register the dependence of voltage across the diode on the input voltage $U_2(U_1)$ in the open-circuit mode (Fig. 11.1, *c*) and with some load (Fig. 11.1, *d*). The resistance of the load and the voltage of operation point are given in Table 10.1. Currents and voltages should be measured with DC amperemeters and voltmeters with limits of 20 mA and 20V.

Table 11.1

Variant		1	2	3	4	5	6	7
R	kΩ	1	2,2	1,68	2,53	3,2	3,67	1,22
U_{1P}	V	5	6	7	8	9	10	11

Review Questions

Read the theory to the laboratory exercise and answer the following questions:

1. What element can be called *a nonlinear resistor*? Describe the properties of this element.
2. What laws are valid for nonlinear circuits?
3. How we can obtain equivalent CV characteristic of several nonlinear elements connected in series? In parallel?
4. Which element is possible to use for stabilization of voltage?
5. Which element is possible to use for stabilization of current?

Experimental Investigations

1. Experimental registration of a lamp characteristic. The circuit is shown in Fig. 11.1, *a* and *b*. The points of characteristic should be registered with equal intervals of current variations. The largest possible current is 35 mA. The results must be written to Table 11.2.
2. Experimental registration of Zener diode characteristic. The circuit is shown in Fig. 11.1, *a* and *c*. The points of characteristic should be registered with equal intervals of voltage variations. The largest possible voltage is 15 V. The results must be written to Table 11.2.
3. Experimental registration of equivalent characteristic for series-parallel connection of elements. The circuit is shown in Fig. 11.1, *a* and *d*. The points of characteristic should be registered with equal intervals of voltage variations. The largest possible voltage is 15 V. The results must be written to Table 11.2.
4. In the same system of coordinates draw the graphs $U_L(I_L)$, $U_{ZD}(I_{ZD})$, and $U_{Eq}(I_{Eq})$ according to data written to table 11.2. In the same system draw the characteristic of the linear resistor $U_R=RI_R$.
5. In the same system of coordinates draw the equivalent characteristic of a circuit obtained as a result of summation. Compare it with experimental characteristic.
6. Find in the graphs the currents $I_L = I_{Eq}$, I_{ZD}, I_R and voltages $U_Л$, U_{ZD}, U_R corresponding to the point of operation(voltage U_{1P}).
7. Calculate static resistance of the circuit at the operation point.
8. Draw two curves $U_2(U_1)$ obtained in the open-circuit mode and with the load *R*. For each curve find the intervals of stabilization and calculate the coefficient of stabilization

$$k_{CT} = \frac{\Delta U_1 / U_{1M}}{\Delta U_2 / U_{2M}},$$

here ΔU is variation of voltage in interval of stabilization, U_{CM} is the mean voltage corresponding to this interval.
9. Write conclusions to the exercise.

Table 11.2

A lamp		*A Zener diode*			*Series-parallel connection*		
$U_1= U_L$	$I= I_L$	U_1	$I= I_{ZD}$	$U_2= U_{ZD}$	$U_1= U_{Eq}$	$I= I_{Eq}$	U_2
V	mA	V	mA	*V*	V	mA	*V*

Laboratory exercise No. 12

AN INDUCTOR WITH FERROMAGNETIC CORE IN A CIRCUIT OF AC CURRENT

The objective of the exercise: Experimental investigation of nonlinear inductor.

Theoretical Background

Inductors with ferromagnetic cores are often used in modern devices to amplify magnetic flux. Magnetic reluctance of ferromagnetic core depends on the magnetic field strength **H**. So an inductor with a core is a nonlinear element characterized by Weber-Ampere characteristic $\psi(i)$.

If we assume that $\psi=BSw$ and $wi=Hl$ (**B** is the inductance of magnetic field, **S** is the square of core cross-section, **l** is the length of core), we obtain the curve $\psi(i)$ similar to the function **B(H)**, called the magnetic curve of material.

For ferromagnetic material these curves are possible to obtain experimentally. An example of the curve is shown in Fig. 12.1. The inductor is inertialess element, and if we apply sinusoidal voltage to it, we obtain aharmonic current in its wires. We assume that the voltage is described by the equation

$$u(t) = U_m \cos(\omega t).$$

In this case the magnetic flux can be found as

$$\psi(\omega t) = \int u dt = \Psi_m \sin(\omega t),$$

where $\Psi_m = U_m/\omega = U\sqrt{2} / \omega$ is the amplitude of magnetic flux.
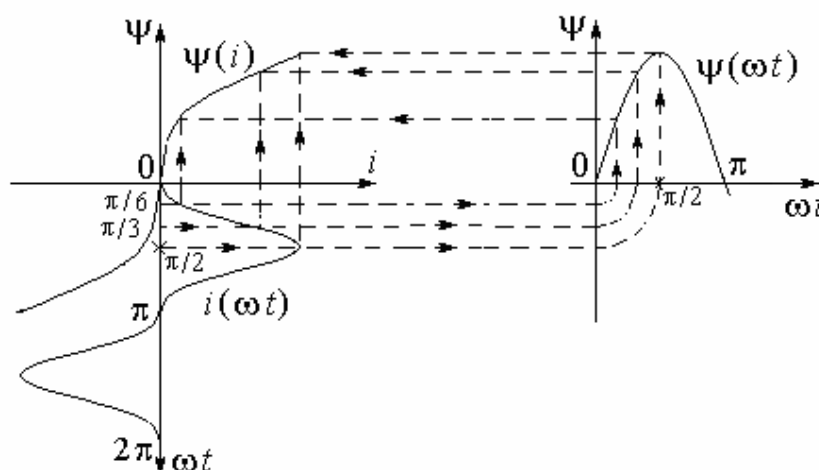

Fig.12.1

Electrical Circuit

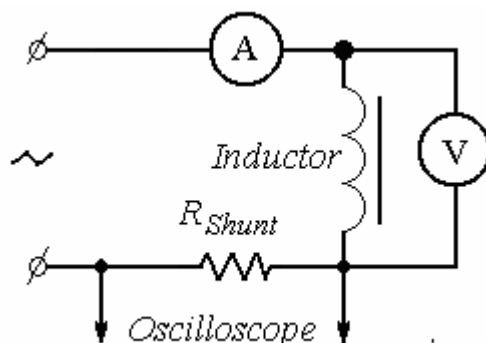To obtain the dependence of current on time $i(\omega t)$ the circuit is used shown in Fig. 12.2.

Fig. 12.2

The circuit is supplied by a source of sinusoidal voltage with frequency of 50 Hz and rms value of 24 V. As a nonlinear inductor the primary coil of a transformer is used (number of turns $w = 300$). The shunt is a linear resistor so the dependence of current through it on time $i(\omega t)$ is the same as dependence of voltage $u(\omega t)$.

Rms values of current and voltage is measured by AC amperemeter and voltmeter with limits of measurements of 200 mA and 20 V correspondingly.

The oscilloscope scales for voltage and time should be chosen in such a way that the curve fills the whole screen. The scale of current m_i in mA/cm can be calculated as $m_i = 10^3 m_u /(10 R_{Shunt})$, where m_u is the scale of voltage.

Review Questions

Read the theory to the laboratory exercise and answer the following questions:
1. Why the dependence of magnetic flux on current in inductors with ferromagnetic core is a nonlinear function?
2. How losses can be minimized in the core of inductor?

Experimental Investigations

1. Build the circuit presented in Fig. 12.2. Resistor R_{Shunt} should be taken from Table 12.1 according the variant of the exercise.

Table 12.1

Variant		1	2	3	4	5	6	7	8	9	10
R_{Shunt}	Ω	47	57	69	80	55	65	90	100	110	122

2. Switch on the oscilloscope. Draw half-wave of the voltage on paper. Calculate the scale of current. This scale and reading of voltmeter and amperemeter write to Table 12.2.

Table 12.2

U	I	m_i
V	mA	mA/mm

3. Calculate the amplitude of magnetic flux Ψ_m and draw the sinusoidal function $\psi(\omega t) = \Psi_m \sin(\omega t)$ with the same temporal scale as the scale of curve obtained with oscilloscope. In the same picture draw the curve of current $i(\omega t)$ in a way shown in Fig. 12.1.

4. Draw Weber-Ampere characteristic of the inductor $\psi(i)$ (the example is given in Fig. 13.1).

5. Write conclusions to the exercise.

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Glossary

Attenuation factor – коэффициент затухания
Balanced load – симметричная нагрузка (обычно в трехфазной цепи)
Bipolar junction transistor – биполярный транзистор
Branch – ветвь (электрической цепи)
Characteristic equation –характеристическое уравнение
Circuit diagram – схема (электрической) цепи
Connected in parallel – соединены параллельно
Connected in series – соединены последовательно
Core of the inductor – сердечник катушки индуктивности
Critically dumped transient process – критический (граничный) переходный процесс
Current-voltage characteristic (CVC) – вольт-амперная характеристика (ВАХ)
Delta connection of the load – соединение нагрузки треугольником (обычно в трехфазной цепи)
Forced response of the circuit – принужденная составляющая (выражение используется при математическом описании переходных процессов)
Fourier series – ряд Фурье
Full-wave rectifier – двухполупериодный выпрямитель
Half-wave rectifier – однополупериодный выпрямитель
Impedance – импеданс, комплексное сопротивление, полное сопротивление
Natural response of the circuit – свободная составляющая (выражение используется при математическом описании переходных процессов)
Neutral, neutral wire – нейтраль, нулевой провод (в трехфазных цепях)
Node – узел
Oscilloscope – осциллограф
Overdumped transient process – апериодический (затухающий) переходный процесс
Parallel resonance – резонанс напряжений
Phase, initial phase – фаза, начальная фаза
Phasor diagram – векторная диаграмма
Rectification – выпрямление (токов)
Rectifier – выпрямитель
RMS, current (voltage) RMS – действующее значение, действующее значение тока (напряжения)
Root of characteristic equation – корень характеристического уравнения
Semiconductor – полупроводник
Series resonance – резонанс токов

Shunt – шунт
Source of e.m.f. – источник ЭДС
Superposition principle – принцип наложения (суперпозиции)
Switching rules – законы коммутации
The first Kirchhoff's rule, Kirchhoff's current law (KCL) – первый закон Кирхгофа
The second Kirchhoff's rule, Kirchhoff's voltage law (KVL) – второй закон Кирхгофа
Time constant – постоянная времени (переходного процесса)
Transients, transient processes – переходные процессы
Underdumped transient process – колебательный переходный процесс
 Wye connection of the load соединение нагрузки звездой (в трехфазной цепи)
Zener diode – стабилитрон

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THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING

A guide to laboratory exercises on the course

«Theoretical fundamentals of electrical engineering»

for the second year students of the Institute of Power Engineering

following the majors

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