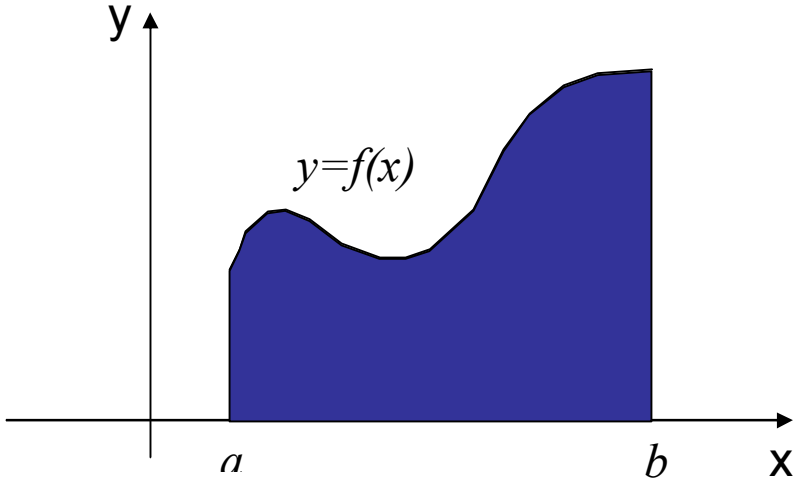
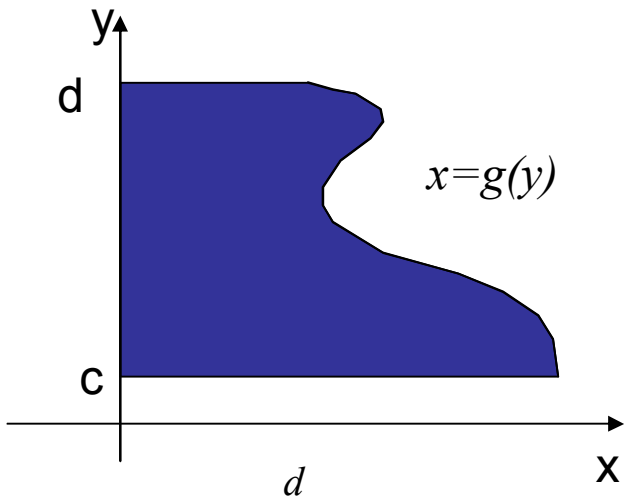
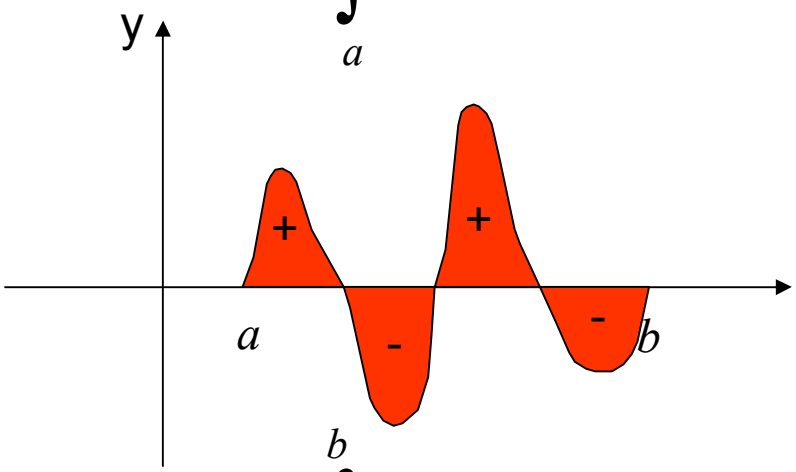
Вычисление площадей в декартовой системе координат



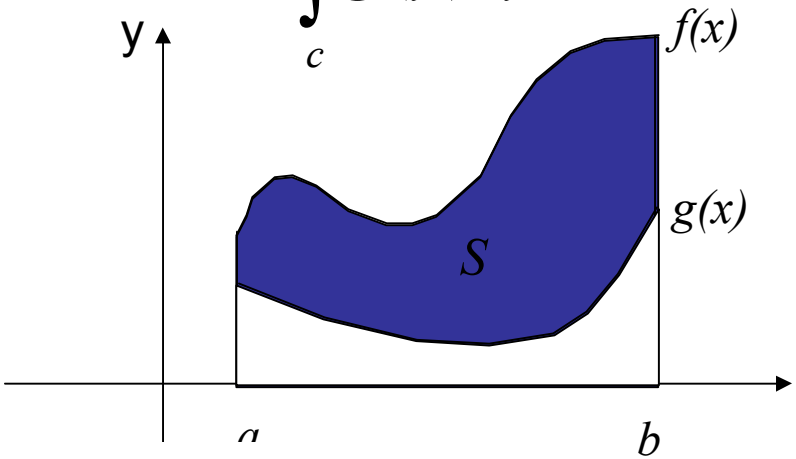
$$S = \int_a^b f(x)dx$$



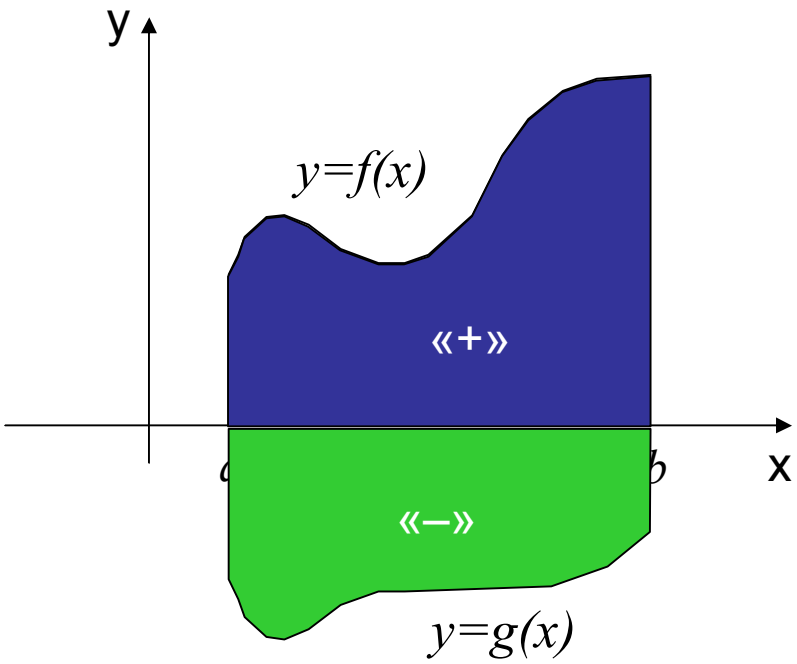
$$S = \int_c^d g(y)dy$$



$$S = \int_a^b |f(x)|dx$$



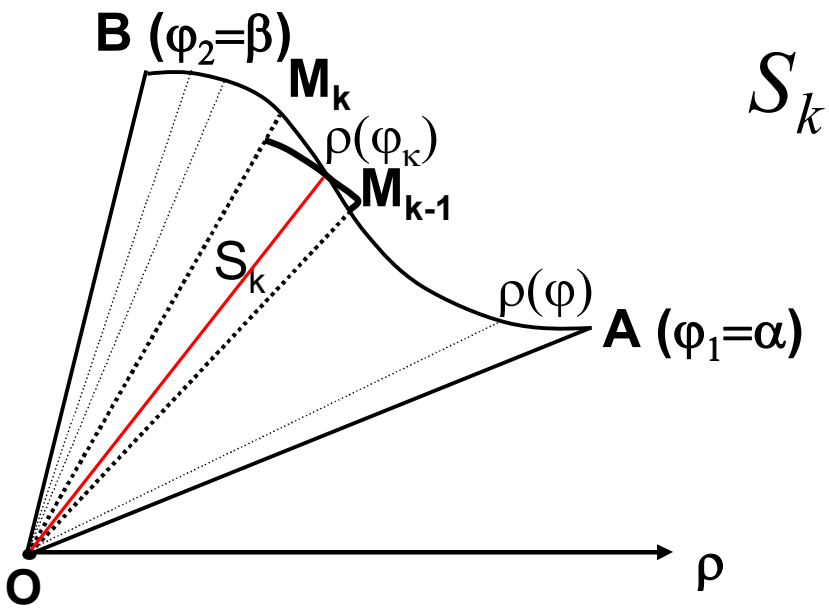
$$S = \int_a^b [f(x) - g(x)]dx$$



$$S = \int_a^b f(x)dx + \int_a^b |g(x)|\,dx = \int_a^b [f(x) - g(x)]dx$$

$$S = \int_a^b |f(x) - g(x)|\,dx \qquad \textbf{(5)}$$

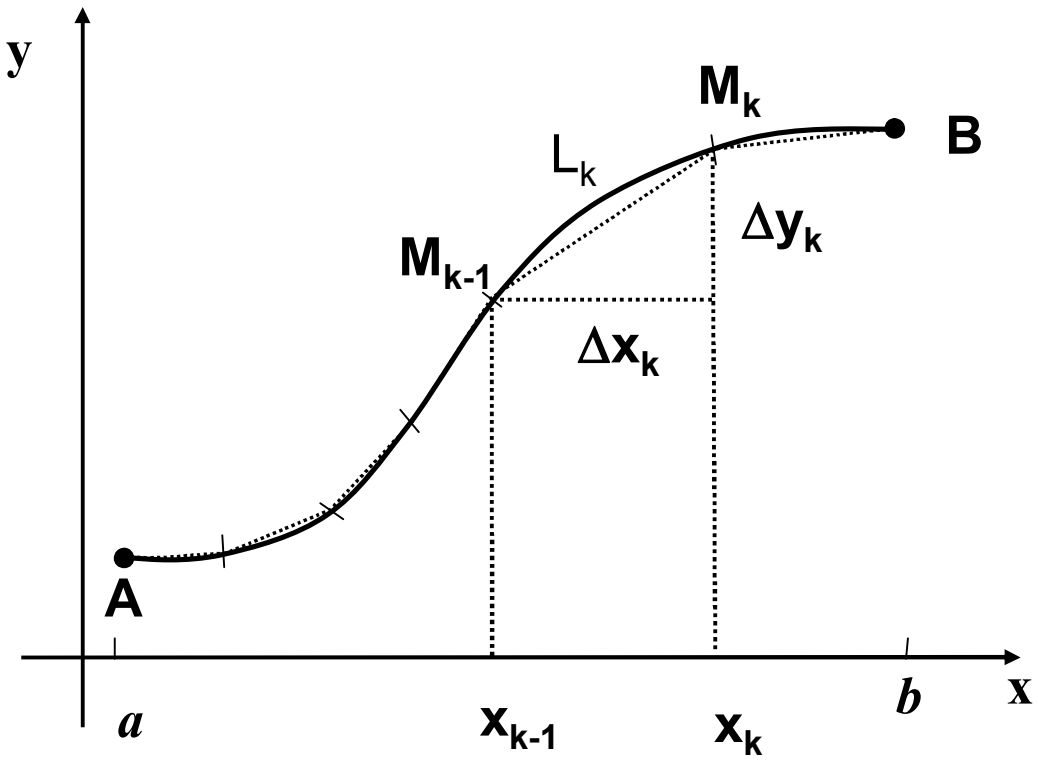
Вычисление площадей в полярной системе координат



$$S_k = \frac{1}{2}\rho^2(\varphi_k)\Delta\varphi_k$$

$$S = \lim_{\lambda \to 0} \sum_{k=1}^{n} \frac{1}{2}\rho_k^2 \Delta\varphi_k = \frac{1}{2}\int_{\varphi_1}^{\varphi_2} \rho^2(\varphi)d\varphi \qquad (6)$$

Вычисление длин дуг кривых в декартовых координатах.



$$L = \int_a^b \sqrt{1 + (y')^2}\, dx \qquad (7)$$

длина дуги кривой, заданной параметрически.

$$L = \int_{t_1}^{t_2} \sqrt{(\varphi')^2 + (\psi')^2 + (\zeta')^2}\, dt \qquad (8)$$

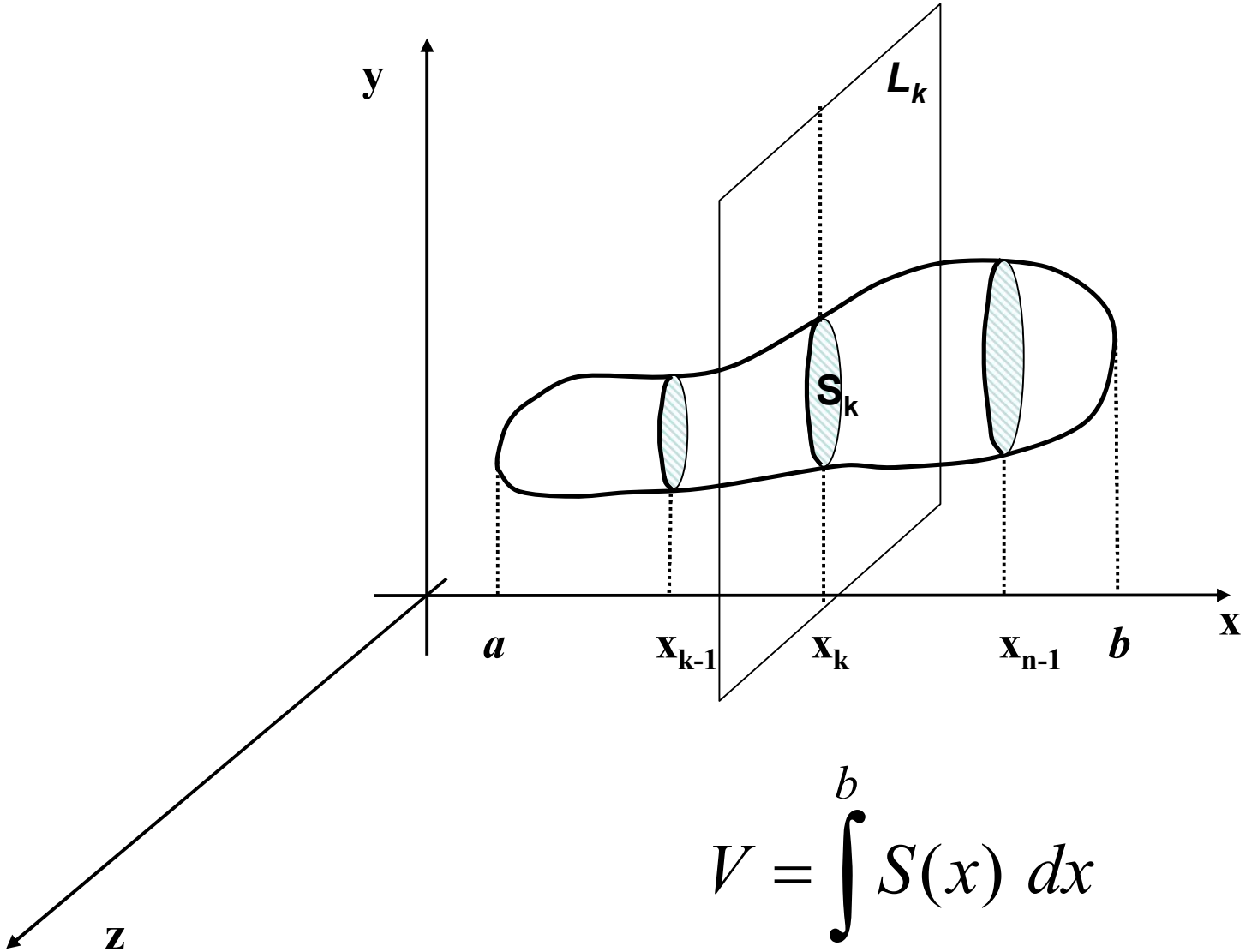
В частности
для плоской кривой:

$$L = \int_{t_1}^{t_2} \sqrt{(\varphi')^2 + (\psi')^2}\, dt \qquad (8^*)$$

длина дуги в полярной системе координат.

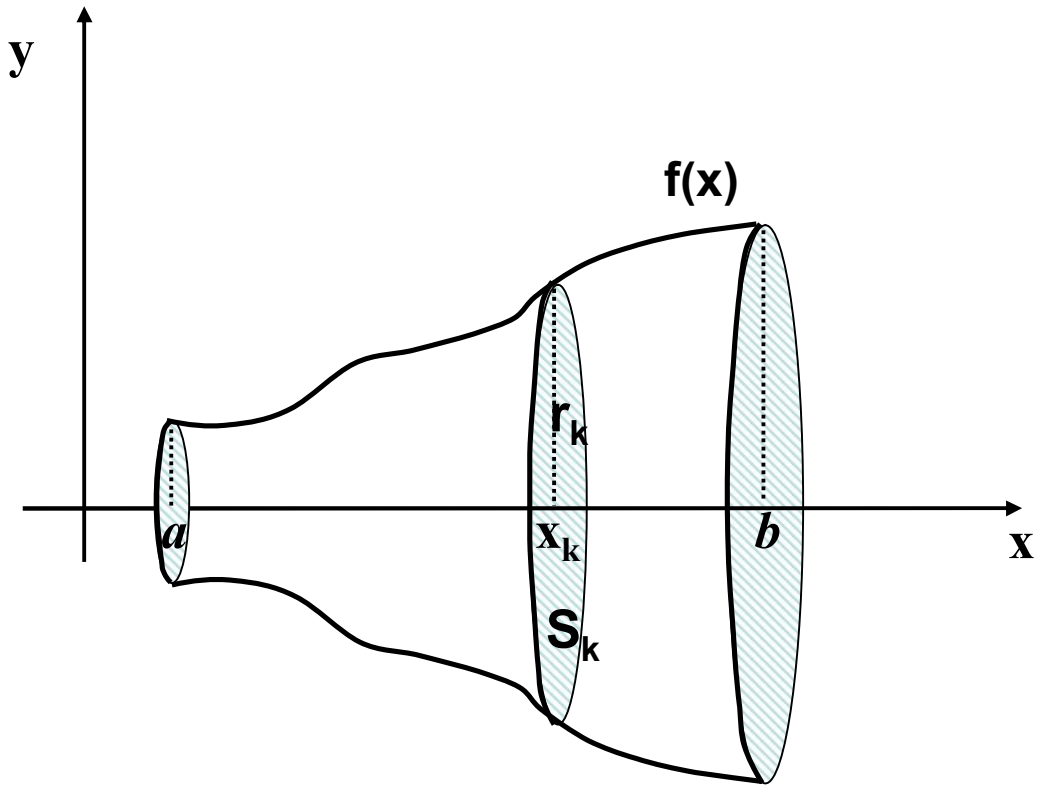
$$L = \int_{\varphi_1}^{\varphi_2} \sqrt{(\rho')^2 + \rho^2}\; d\varphi \qquad (9)$$

Вычисление объема по известному параллельному сечению



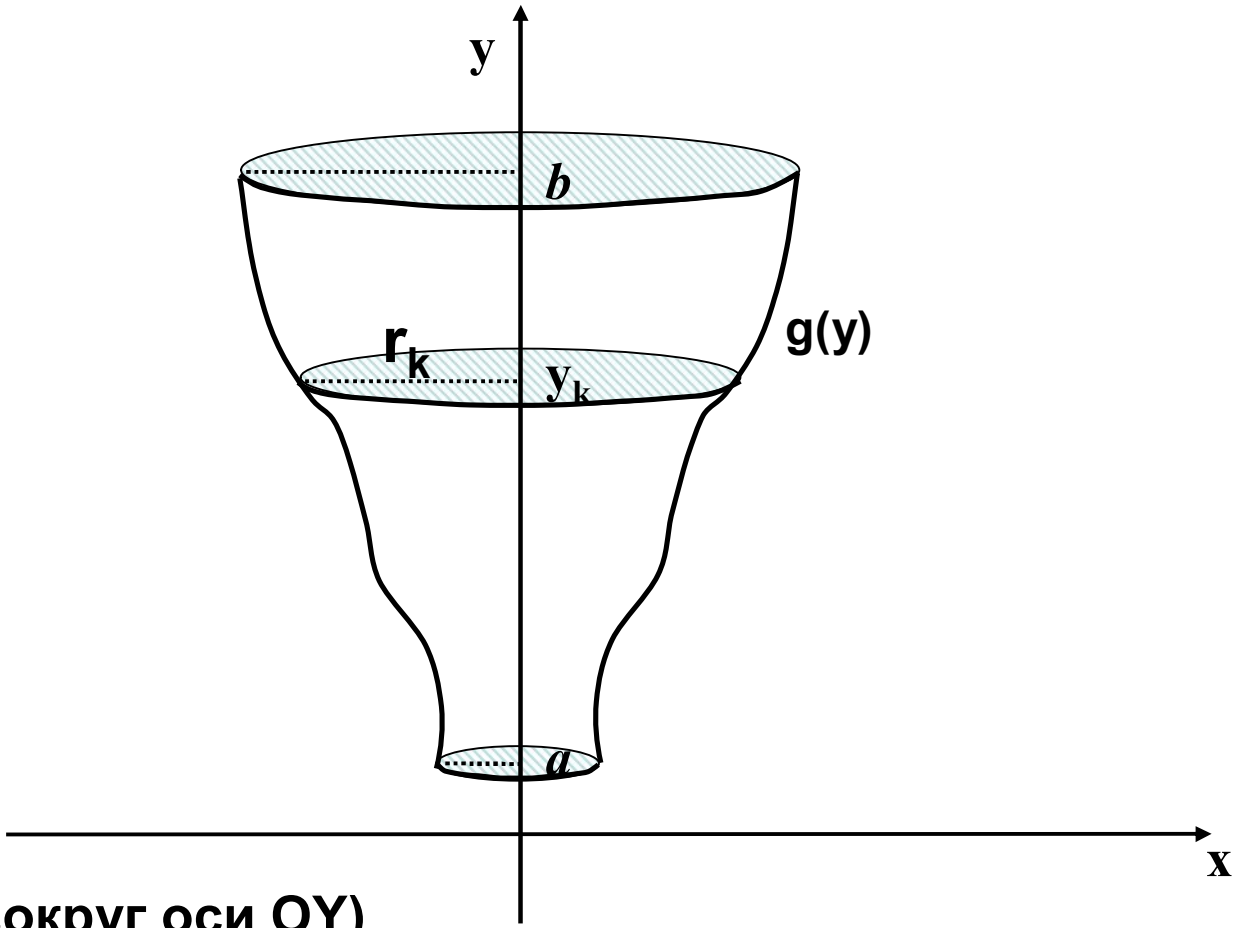
$$V = \int_a^b S(x)\, dx \qquad (10)$$

Объем тела вращения



$$V = \pi \int_a^b f^2(x)\, dx \qquad (11)$$

(вращение вокруг оси OX)



(вращение вокруг оси OY)

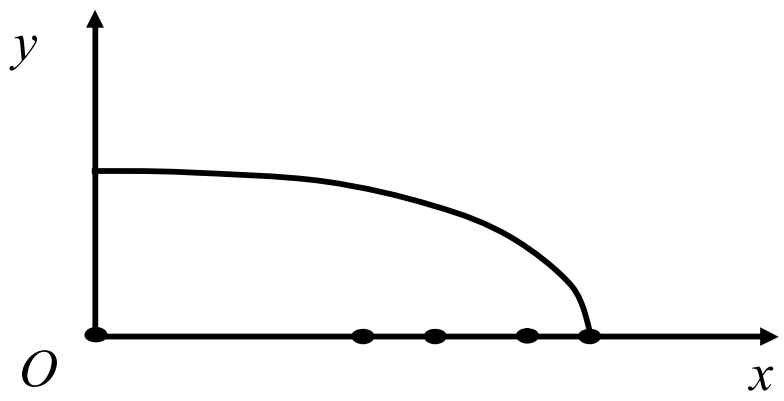
$$V = \pi \int_a^b g^2(y)\, dy \qquad (11^*)$$

Построение графиков функций, заданных параметрически

Построить кривую, заданную уравнением $\begin{cases} x=a\cdot\cos(t), \\ y=b\cdot\sin(t) \end{cases}$ **в I четверти**

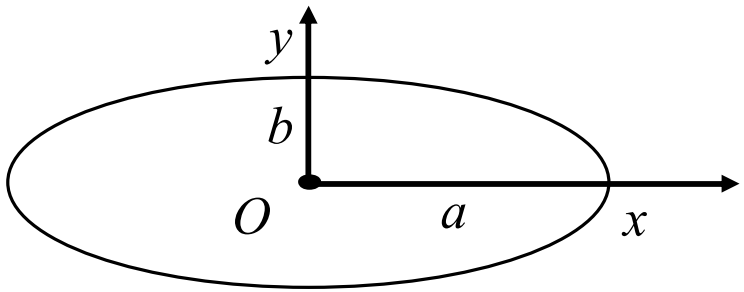
Подготовим таблицу значений x и y

t	x	y
0	a	0
$\pi/6$	$\sqrt{3}/2$ a	½ b
$\pi/4$	$\sqrt{2}/2$ a	$\sqrt{2}/2$ b
$\pi/3$	½ a	$\sqrt{3}/2$ b
$\pi/2$	0	b

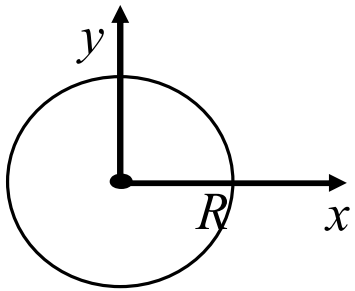


В параметрической форме

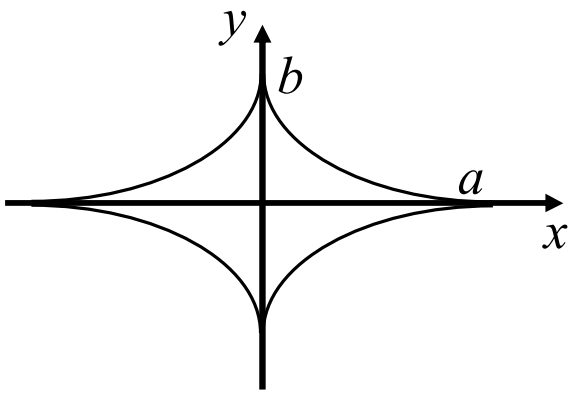
Эллипс $\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$



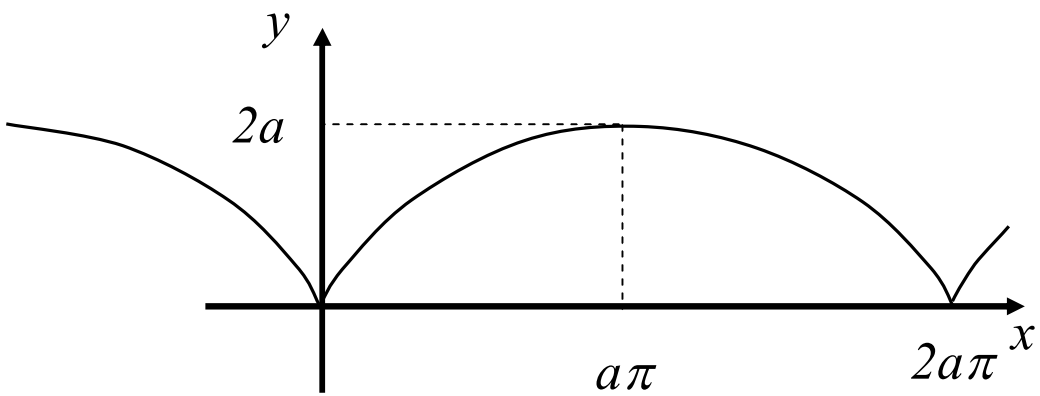
Окружность $\begin{cases} x = R\cos t \\ y = R\sin t \end{cases}$



Астроида $\begin{cases} x = a\cos^3 t \\ y = b\sin^3 t \end{cases}$



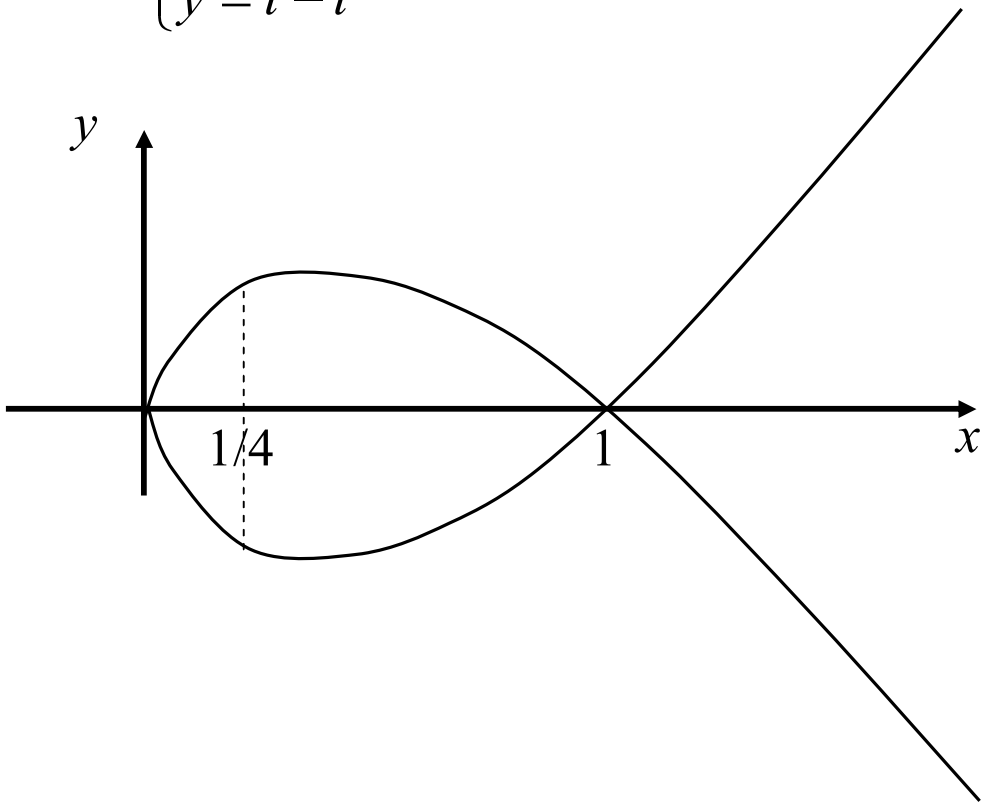
Циклоида $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$



В параметрической форме

Петля

$$\begin{cases} x = t^2 \\ y = t - t^3 \end{cases}$$

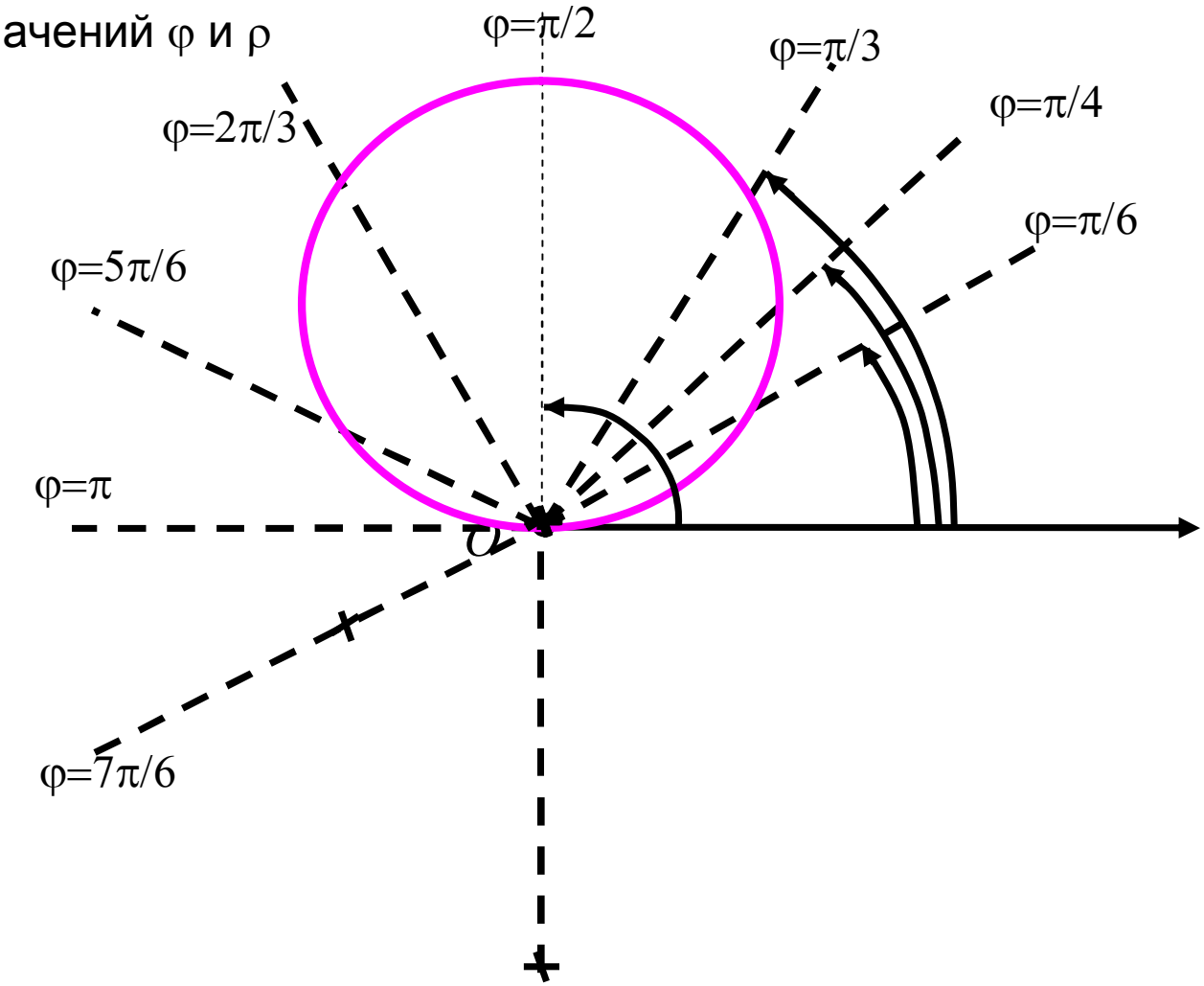


Построение графиков функций в полярной системе координат

Построить кривую, заданную уравнением $\rho = \sin\varphi$

Подготовим таблицу значений φ и ρ

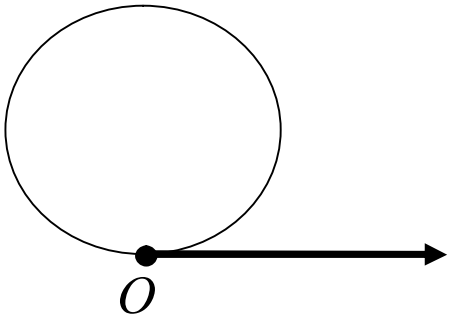
φ	ρ
0	0
$\pi/6$	1/2
$\pi/4$	$\sqrt{3}/2$
$\pi/3$	$\sqrt{2}/2$
$\pi/2$	1
$2\pi/3$	$\sqrt{3}/2$
$5\pi/6$	1/2
π	0
$7\pi/6$	$-\sqrt{3}/2$



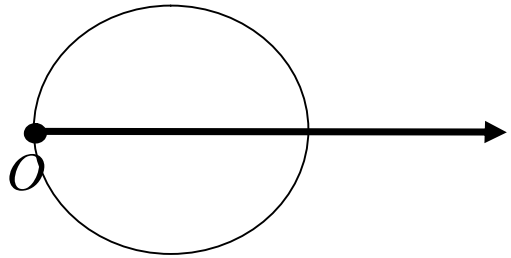
В полярной системе координат

Окружности

$$\rho = a\sin\varphi$$

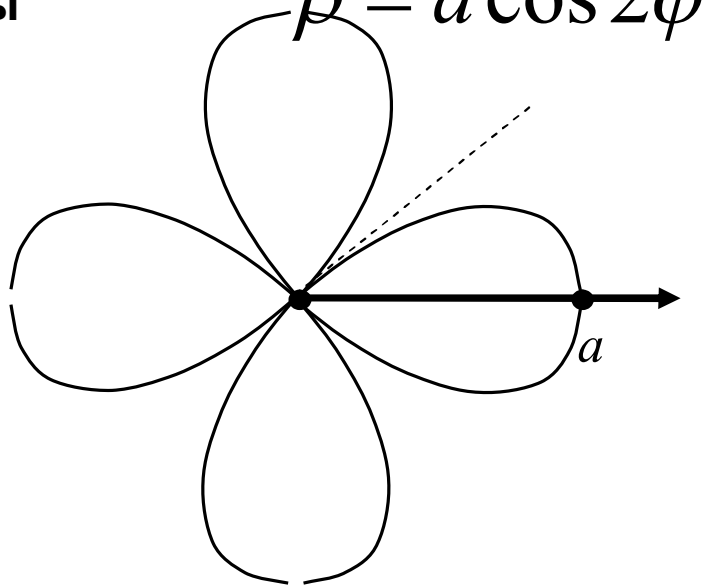


$$\rho = a\cos\varphi$$

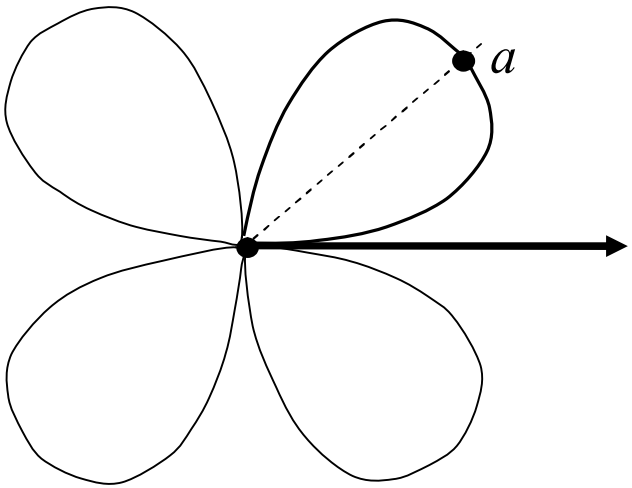


Розы

$$\rho = a\cos 2\varphi$$



$$\rho = a\sin 2\varphi$$

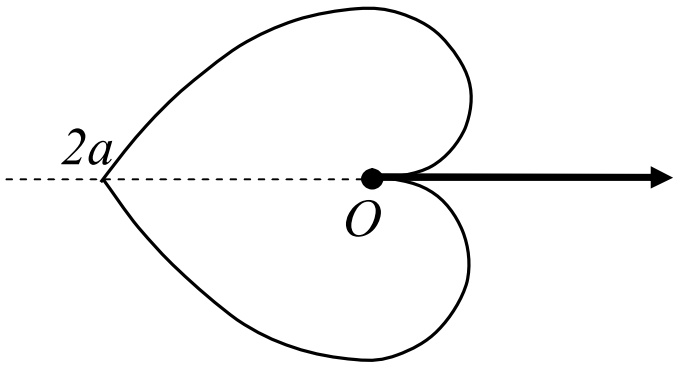


$$\rho = a\cos 3\varphi$$

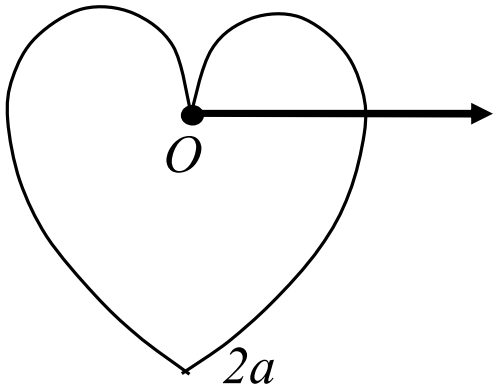
$$\rho = a\sin 3\varphi$$

В полярной системе координат

Кардиоиды $\rho = a(1-\cos\varphi)$



$\rho = a(1-\sin\varphi)$



$\rho = a(1+\cos\varphi)$

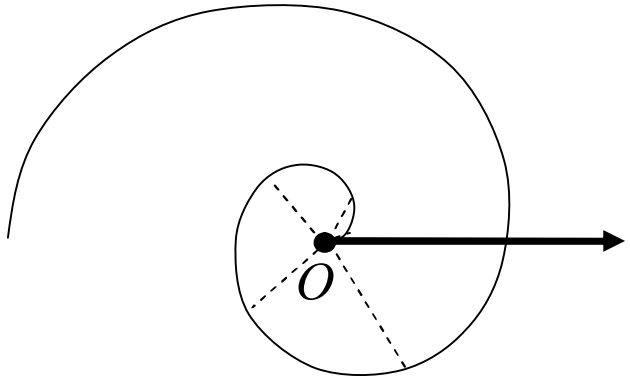
$\rho = a(1+\sin\varphi)$

Розы $\rho = a\cos 2\varphi$

$\rho = a\sin 2\varphi$

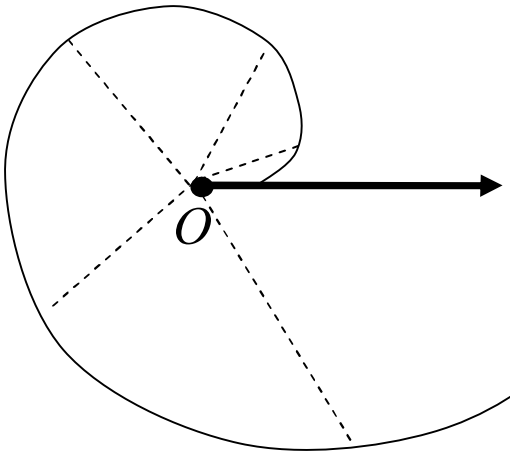
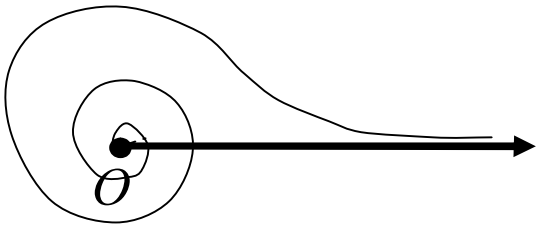
В полярной системе координат

Спираль Архимеда $\rho = a\varphi$



Логарифмическая и гиперболическая спирали

$$\rho = e^{\varphi}$$



$$\rho = \frac{a}{\varphi}$$