

## DYNAMIC MODEL OF WIND SPEED LONGITUDINAL COMPONENT

Yermek Zh. Sarsikeev<sup>a</sup>, Boris V. Lukutin<sup>b</sup>, Danil Yu. Lyapunov<sup>c</sup>,  
Mikhail A. Surkov<sup>d</sup> and Sergey G. Obuhov<sup>e</sup>

National Research Tomsk Polytechnic University 30, Lenin Ave., Tomsk, Russia, 634050 (zip code)

<sup>a</sup>sarsikeev@tpu.ru, <sup>b</sup>lukutin48@mail.ru, <sup>c</sup>lyapunov@tpu.ru, <sup>d</sup>masur@yandex.ru, <sup>e</sup>serob99@mail.ru

**Keywords:** renewable energy sources, wind power plant, wind speed, longitudinal component, air flow, dynamic model.

**Abstract.** The paper deals with the development of the dynamic model of wind by modeling the longitudinal component of wind speed. Statistical description of turbulent wind component using Kaimal's function is represented. All necessary parameters for model performing are calculated. The results of modeling in the form of dynamic characteristics are presented.

### Introduction

There are a lot of technical problems to solve in the process of wind power plants design. The problems are connected with the choice of optimal relation between the power of energy sources and energy storages power, tuning of system's automatics, optimization of control system, etc. Most of the problems can be solved only on the basis of modeling the wind power plants operating modes. For that reason the dynamic wind model is needed.

Complicated multi-dimension wind models usage is redundant for the problems identified above. They are based on the differential Navier-Stocks equations which describe non-stationary movement of air flow in space and time.

From the position of wind power engineering the longitudinal component of wind speed is of great interest. Present paper is dedicated to modeling of the component under consideration. The aim of investigations conducted is universal model of wind speed longitudinal component development, which can be implemented in popular mathematical software packages.

### Theoretical justification

Oscillations of basic meteorological factors such as wind speed, pressure, humidity, etc. contain components with periods in the range from fractions of a second to tens of thousands of years. From the point of view of wind power plants operating modes modeling time intervals from seconds up to several hours are of interest.

Micrometeorological oscillations of wind speed with periods from fractions of a second to several minutes are defined by small-scale turbulence. Its energy spectrum in surface atmosphere layer has maximum at about 1 minute.

In mezometeorological time interval with periods from minutes to hours intensive oscillations of meteorological components are relatively seldom. It allows to get stable average values of wind speed, temperature, etc., using the period averaged for 10...20 minutes.

Spectral distribution of horizontal wind speed plotted by Van-der-Hoven according to measurement readings at 125-meter meteorological tower in Bruckhaven, given in [1, 2] shows that the maximums of the energy spectrum of synoptic and daily oscillations of wind speed are significantly different from the maximum of high-frequency oscillations of turbulence. That allows the use of independent time sampling followed by the superposition for mathematical description of the oscillations. At given assumptions dynamic wind model can be represented by the expression:

$$V(t) = \bar{V} + v(t), \quad (1)$$

where  $\bar{V}$  is average wind speed in 10 minutes interval,  $v(t)$  is dynamic or turbulent wind speed component.

For statistical description of turbulent wind component in wind power engineering empirical models of the spectral density  $S(f)$  are mainly used. The most famous are functions of Davenport, Karman and Kaimal. For wind velocity dynamic component modeling in the present work recommended by international standard spectral Kaimal's model is used [3].

In accordance with the model of normal turbulence, it is assumed that the turbulent fluctuations of the wind speed field is stationary field of random vectors. Their components have Gaussian random distribution with zero expectation.

Spectral densities of components' power in normalized form for the Kaimal's model are described by expression:

$$\frac{f \cdot S(f)}{\sigma^2} = \frac{4 \cdot f \cdot L / \bar{V}}{(1 + 6 \cdot f \cdot L / \bar{V})^{5/3}}, \quad (2)$$

where  $f$  is frequency in Hz;  $S(f)$  – one-sided spectrum of the longitudinal wind speed vector component,  $\sigma$  – standard RMS deviation of longitudinal wind speed vector component;  $L$  – integral turbulence scale parameter.

Spectral decomposition shows stationary random function decomposed into harmonic oscillations of different frequencies  $f_1, f_2, \dots, f_k, \dots$ , while the harmonic amplitudes are random values.

According to the Fourier theorem, any function with period  $\pi$  can be represented as a series:

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \cdot \cos(\omega_k t + \varphi_k), \quad (3)$$

where  $A_k$  is amplitude of  $k$ -th harmonic oscillation,  $\omega_k$  is circular frequency of harmonic oscillation;  $\varphi_k$  is initial phase of the  $k$ -th oscillation;  $A_0$  is free component representing the expectation of the function in the interval  $0 \dots \pi$ .

On the other hand the dispersion of a stationary random function is equal to the sum of dispersions of all harmonics of its spectral decomposition:

$$D = \sigma^2 = \int_0^{\infty} S(f) \cdot df = \sum_{k=0}^{\infty} D_k = \sum_{k=0}^{\infty} S(f_k) \cdot \Delta f, \quad (4)$$

If one and the same set of frequencies for the spectral decomposition of function and for Fourier series is used, it follows from Ex. (3)–(4) that the amplitude of  $k$ -th harmonic oscillation of the Fourier series equals to the standard RMS deviation of corresponding spectrum harmonic:

$$A_k = \sqrt{D_k} = \sqrt{S(f_k) \cdot \Delta f}, \quad (5)$$

where  $\Delta f$  is the spacing between adjacent frequencies.

Performing transformations and passing to a finite number of frequencies  $N$ , we obtain the equation for the longitudinal wind speed component defined within time interval  $T$ :

$$V(t) = \bar{V} + \sum_{k=1}^N A_k \cdot \cos(\omega_k t + \varphi_k), \quad (6)$$

where  $\bar{V}$  is wind speed, averaged in 10 minutes interval.

In Ex. (6) simulation time  $T$  corresponds to a half period of the fundamental harmonic:  $T = \pi$ . Thus the number  $N$  determines the sampling frequency of the timing signal:

$$\Delta t = \frac{T}{N}; \quad \Delta \omega = \frac{\pi}{T}; \quad \Delta f = \frac{1}{2 \cdot T}; \quad t = k \cdot \Delta t \quad k=1 \dots N \quad f_k = \frac{k}{2 \cdot T} \quad (7)$$

Ex. (2)–(6) allow to construct simulation model of longitudinal wind speed component if spectral parameters of turbulence are known.

The amplitude of the spectral density for the respective frequency  $f_k$  is determined by Ex. (2). Phase angle  $\varphi_k$  is a random number ranging from 0 to  $2\pi$ .

Spectral parameters of the turbulence for Kaimal's model are determined in accordance with the requirements specified in [3]. According to the requirements all wind power plants in conformity with the turbulence intensity are divided into three sub-classes A, B, C. Each sub-class is characterized by its expected value of the intensity of air flow turbulence at the height of the wind wheel axis  $I_{ref}$ .

Sub-class A corresponds to wind power plants with increased turbulence ( $I_{ref}=0,16$ ) and is adopted for urban areas, where the roughness of the earth's surface is  $z_0>0,3$  [4]. Sub-class B corresponds to a more open area ( $0,1<z_0<0,3$ ) and its turbulence intensity is assumed to be equal to  $I_{ref}=0,14$ . Sub-class C is characterized by open countryside ( $z_0<0,1$ ) with turbulence intensity  $I_{ref}=0,12$ .

Standard RMS deviation of the wind speed longitudinal component at the height of wind wheel axis for 90% quantile of the standard wind power plant is given by the expression:

$$\sigma = I_{ref} \cdot (0,75 \cdot \overline{V}_{hub} + b), \quad b=5,6 \text{ m/s}, \quad (8)$$

where  $\overline{V}_{hub}$  is average wind speed at the height of wind wheel axis.

Longitudinal scale factor of turbulence airflow  $\Lambda$  at the height of wind wheel axis  $Z$  is expressed by the relation:

$$\Lambda = \begin{cases} 0,7 \cdot Z & Z \leq 60 \text{ m} \\ 42 \text{ m} & Z > 60 \text{ m} \end{cases}, \quad (9)$$

For integral scale factor of longitudinal wind speed vector component  $L$  calculation the next expression is used:

$$L = 8,1 \cdot \Lambda, \quad (10)$$

Initial data for the turbulence parameters calculation are the sub-class of wind power plant, which is determined by the place of its location, the height of the wind wheel axis  $Z$  and the average wind speed  $\overline{V}_{hub}$  for a given simulation time interval.

### Practical implementation

Software implementation of the model developed above is carried out in two stages. At the first stage the values of amplitudes  $A_k$  and phase angles  $\phi_k$  corresponding to harmonic components of simulated signal are calculated. At the second stage the synthesis of the signal is performed.

To perform the calculations it is necessary to create two numeric data arrays:  $M1[m, N]$ ,  $M2[N, Nt]$ .

Parameter  $m$  in the array  $M1$  is determined by the amount of calculated variables:  $f_k, S(f_k), A_k$ , etc. The value  $N$  determines the number of harmonic oscillations, which are taken into account for the decomposition of function. At small value of  $N$  we obtain the low accuracy of the calculation, while the large value of  $N$  requires additional resources of PC. For the class of problems the number of harmonics taken into account  $N = 100$  is quite acceptable.

Parameter  $Nt$  corresponds to a given number of calculation points of the process used for output. Importantly, the synchronization of frequencies adopted during the development of the model requires the fulfillment of certain relations between  $N$  and  $Nt$ .

For example, the purpose of modeling is to simulate the wind speed longitudinal component in the time interval  $T_{mod}=100$  s sampled at  $\Delta t_{ref}=1$  s. If directly  $T = 100$  s is accepted, then at  $N = 100$  according to Ex. (7), we obtain  $\Delta t = 1$  s, which greatly exceeds the desired sampling interval.

For considered example, it is necessary to determine the period of expansion as  $T=\Delta t_{ref} \cdot N$ , then calculate the parameter  $c$  which determines the size of the output array  $Nt=c \cdot N$  and the number of cycles that must be done for the complete filling of the array:

$$c = \frac{T_{mod}}{T}. \quad (11)$$

Normalized spectral density according to Kaimal determined by Ex. (2) is represented graphically in Fig. 1. Fig. 2 shows an example of modeling the wind speed longitudinal component using the developed technique.

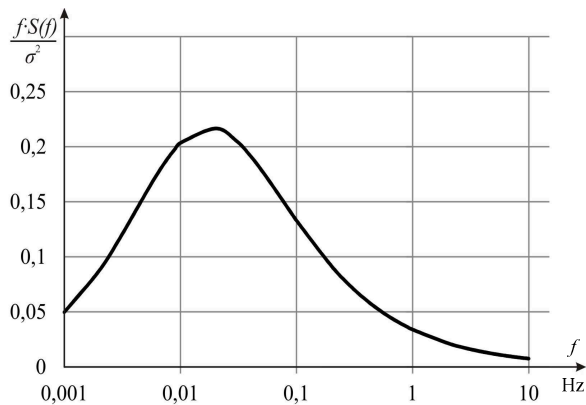


Fig. 1. Spectral density according to Kaimal

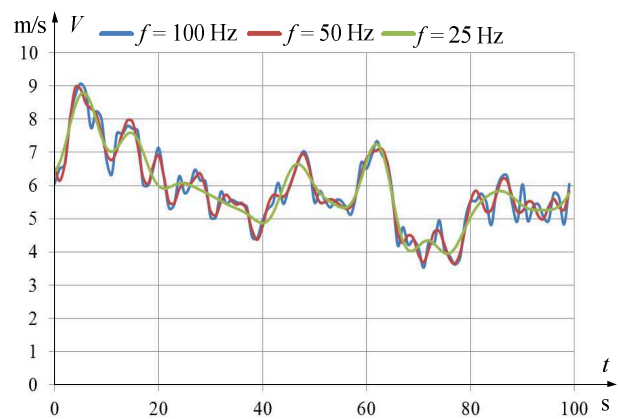


Fig. 2. Dynamic characteristic of longitudinal wind speed component

## Summary

For wind speed longitudinal component modeling in the present work spectral Kaimal's model is used. The model developed allow to tune the modeling degree of accuracy by presetting the sampling step and frequency spectrum. The modeling results obtained are in good agreement with the results of other authors' investigations involved in the development of wind mathematical models for wind power engineering. The model proposed has a simple structure, easy to implement by means of software applications and may find practical implementation in scientific researches devoted to small wind power engineering.

## Acknowledgements

This work was financially supported by Ministry of Education and Science of Russian Federation according to priorities for the development of science and technology (R&D registration number 7.5245.2011, state registration number 01201254010, March 15th, 2012).

## References

- [1] T. Burton, D. Sharpe, N. Jenkins, E. Bossanyi, Wind energy handbook. John Wiley & Sons, New-York, 2001.
- [2] I. Van Der Hoven, Power spectrum of horizontal wind speed in the frequency range from 0.0007 to 900 cycles per hour, J. Meteor. 14 (1957) 160-164.
- [3] Standard IEC 61400-1: 2005, Wind Turbines – Part 1: Design requirements. Information on [http://www.neti.com.tr/Energy/Wind\\_turbines\\_design\\_IEC\\_61400\\_1.pdf](http://www.neti.com.tr/Energy/Wind_turbines_design_IEC_61400_1.pdf)
- [4] Erich Hau, Wind Turbines: Fundamentals, Technologies, Application, Economics, second ed. Springer-Verlag, Berlin, Heidelberg, 2006.