

Будем вычислять производные

$$\begin{aligned}
 m \frac{d}{dt} \left(\frac{d\vec{r}}{d\tau} \right) &= m \frac{d}{dt} \left(\frac{\vec{v}}{\sqrt{1 - (v^2/c^2)}} \right) = \\
 &= \frac{m}{\sqrt{1 - (v^2/c^2)}} \frac{d\vec{v}}{dt} + m\vec{v} \left[-\frac{1}{2} \left(1 - \frac{v^2}{c^2} \right)^{-3/2} \left(-\frac{2|\vec{v}|}{c^2} \frac{d|\vec{v}|}{dt} \right) \right] = \\
 &= \tilde{m}\vec{a} + m \frac{\vec{v}}{c^2} \frac{|\vec{v}|}{\sqrt{1 - (v^2/c^2)}^3} \frac{d|\vec{v}|}{dt} = \tilde{m}\vec{a} + m|\vec{v}| \vec{e}_\tau \frac{|\vec{v}|}{c^2 \sqrt{1 - (v^2/c^2)}^3} |\vec{a}_\tau| = \\
 &= \tilde{m}\vec{a} + \tilde{m}v^2 \frac{\vec{a}_\tau}{c^2 (1 - (v^2/c^2))} = \tilde{m}\vec{a} + \tilde{m}\vec{a}_\tau \frac{v^2}{c^2 - v^2}
 \end{aligned}$$