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From "narrow pairs" to relativistic positronium atom: evolution of coherent peaks for type-B photoproduction of electron–positron pairs in a crystal

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Abstract

It is shown, that the brilliance of coherent peak for type-B photoproduction in a crystal increases significantly for the pairs emitted with close momenta and is maximal for the case of coherent photoproduction of bound e^+e^- pair – the relativistic positronium atom.

1. Introduction

During the last years, interest in the coherent pair photoproduction in the crystals has been focused on the coherent process of type-B, when the photon momentum is parallel to the crystal axis. In the present report it is shown that under detection of pairs with restricted emission angles ("narrow pairs") the coherent peaks become more brilliant, with larger height and narrower width [1]. Most brilliant coherent peaks [2] take place with the production of the relativistic positronium atom (a bound state of the e^+e^- pair), when created e^+e^- have equal and parallel momenta. The relativistic positronium atom means a positronium atom traveling at a relativistic speed. The relative height of coherent peaks in comparison with "ordinary" coherent pair production (integrated over emission angles) reaches to 10 in the case of "narrow pairs" and up to 10^2 in the case of the relativistic positronium atom.

2. "Narrow pairs"

The cross section of e^+e^- pair creation in a crystal can be expressed as usual [3] by the sum of coherent $d\sigma_{coh}$ and incoherent $d\sigma_{incoh}$ parts. We considered the type-B process (the photon momentum $\boldsymbol{k}$ is parallel to the crystal axis OZ) and for simplicity took into account the interaction of the photon only with single crystal string (the separate string approximation). In this approximation one has

$$\frac{d\sigma_{coh}}{d\Omega_+ d\Omega_- d\varepsilon_+} = \frac{d\sigma_1}{d\Omega_+ d\Omega_- d\varepsilon_+} \times \exp\left(-q^2\overline{u^2}\right) |S(q_\parallel)|^2 \left| \sum_{n=0}^{N-1} \exp(i q_\parallel nd) \right|^2,$$

$$\frac{d\sigma_{incoh}}{d\Omega_+ d\Omega_- d\varepsilon_+} = \frac{d\sigma_1}{d\Omega_+ d\Omega_- d\varepsilon_+} N\left[1 - \exp\left(-q^2\overline{u^2}\right)\right]. \tag{1}$$

Here, the system of units in which $\hbar = c = 1$ is used, $d\sigma_1/d\Omega_+ d\Omega_- d\varepsilon_+$ is the pair photoproduction cross section on a separate atom of a crystal, $\varepsilon_+(\varepsilon_-)$, and $p^+(p)$ are positron (electron) energy and momenta, $d\Omega_+(d\Omega_-)$ the elementary solid angle, $\exp(-q^2\overline{u^2})$ is the Debye–Waller factor taking into account the thermal vibrations of the crystal atoms, $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{p}^+ - \boldsymbol{p}^- = \boldsymbol{q}_\parallel + \boldsymbol{q}_\perp$, is the momentum transferred with $\boldsymbol{k}$ being photon momenta, $\boldsymbol{q}_\perp$ being perpendicular to $\boldsymbol{k}$, and $\boldsymbol{q}_\parallel$ being parallel to $\boldsymbol{k}$, $S(\boldsymbol{q}_\parallel)$ is the structure factor of the crystal axis and N is the number of atoms in an axis, with d being the crystal lattice constant. The sum of exponents in Eq. (1) is a well known interference multiplier [3]

$$I(q_\parallel) = \left| \sum_{n=0}^{N-1} \exp(i q_\parallel nd) \right|^2 = \frac{\sin^2(N q_\parallel d/2)}{\sin^2(q_\parallel d/2)}, \tag{2}$$

which takes into account the coherent effect, connected with periodic arrangement of the atoms in a string. The

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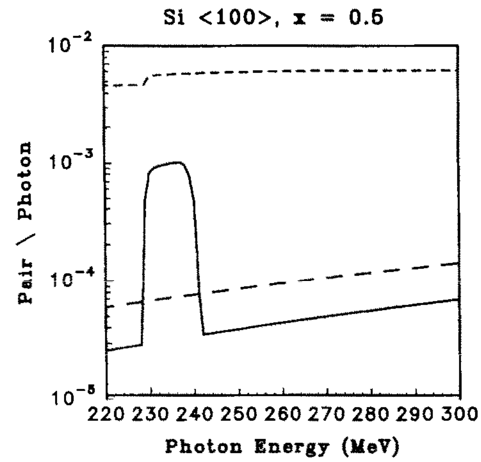

Fig. 1. The yield of symmetrical e^+e^- pairs under photons interaction with ⟨100⟩ Si axis ($T = 273$ K, $L = 0.35$ μm), into emission angles cone $\Theta_{m1} \le 10^{-2}$ (dashed line) and $\Theta_{m2} \le 10^{-3}$ (solid line). The dashed lines represent the cross section in an amorphous target for the same thickness and into emission angle cone $\Theta_{m2} \le 10^{-3}$.

interference multiplier has narrow sharp peaks equal to N^2 at

$$q_{\parallel} = k - p_{\parallel}^{+} - p_{\parallel}^{-} = g_n, \quad n = 1, 2, 3 \cdots, \tag{3}$$

where $\omega = k$ is the photon energy, $p_{\parallel}^{\pm}$ is the projection of electron (positron) momentum on the crystal axis, $g_n = g_0 n$ ($g_0 = 2\pi/d$) is the reciprocal lattice vector. In the relativistic limit ($\omega \gg m$) the emission angles of electron and positron are small: $\Theta_{\pm} \sim m/\omega \ll 1$ and the relation (3) takes the form:

$$\omega - p^{+}\left(1 - \Theta_{+}^{2}/2\right) - p^{-}\left(1 - \Theta_{-}^{2}/2\right) = g_n, \quad n = 1, 2, 3 \cdots. \tag{4}$$

As it follows from Eq. (4), if $\mathrm{d}\sigma_1/\mathrm{d}\Omega_+\mathrm{d}\Omega_-\mathrm{d}\varepsilon_+ \neq 0$, then for every fixed momenta of the electron p^- and the positron p^+ one can find those values of ω, Θ_+, Θ_- (or for fixed $p^+\Theta_+$ one can find the necessary ω and $p^-\Theta_-$, etc. when the interference multiplier (2) equals to N^2 and the cross section has a sharp peak.

It is obvious that the measurement of any differential cross section under the condition (4) can lead to an observation of the sharp coherent peaks. In a real experiment, due to angular and energy resolutions of a detecting system, one can only approach to the exact condition (4) and the sharp maxima will be "washed away". One of the simplest ways to observe the increase of brilliance of coherent peak could be the detection of pairs with the fixed value of $x = \omega/\varepsilon_+$, emitted at angles less than a chosen Θ_m (some kind of the collimation, or momenta correlation).

Fig. 1 shows the results of the calculations of the yield of symmetrical pairs ($\varepsilon_- = \varepsilon_+ = \omega/2$; $x = 0.5$) dependent on the photon energy in vicinity of the first coherent peak ($k = 1$, ⟨100⟩ Si at the photon energy $\omega = 240$ MeV and crystal thickness $L = 0.35$ μm), for two values of maximum emission angle $\Theta_{m1} = 10^{-2}$ and $\Theta_{m2} = 10^{-3}$. As it follows from Fig. 1, the coherent peak for Θ_{m2} becomes more brilliant, the maximum value of the peak is of the same order as for Θ_{m1} and the width becomes sufficiently narrow. The total pair yield for Θ_{m2} decreases in comparison with the case of Θ_{m1} approximately by an order of magnitude (mainly due to the suppression of the incoherent part of the cross section). The results of our calculations are in qualitative agreement with the recent experiment at Tomsk synchrotron "Sirius" [4].

3. Coherent photoproduction of a positronium traveling at a relativistic speed

The specific case of symmetric "narrow" pairs (Eq. (4)) production in a crystal is the coherent photoproduction of a positronium atom traveling at a relativistic speed (A_{2e}) [2]. The photon can create in the Coulomb field of an atom only the para-positronium (singlet state), and the cross section of this process was found in Ref. [5,6].

For the coherent process of type-B in a crystal, the cross section (in the separate string approximation) has the sharp peaks at the photon energies defined by

$$\omega_k = \left(4m^2 + g_n^2\right)/2g_n, \quad n = 1, 2, 3 \cdots, \tag{5}$$

which follows from Eqs. (3) and (4) assuming $p^+ = p^-$. We neglected in Eq. (5) the small binding energy ε_n of electron and positron in a positronium atom in comparison with energy E of the relativistic motion because $\varepsilon_n \ll E \simeq$

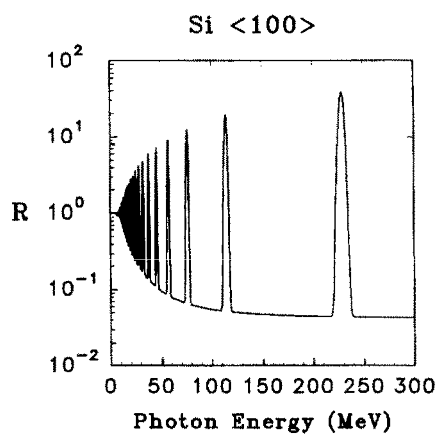

Fig. 2. The relation of the differential cross sections of photoproduction of positronium atom (bound e^+e^- pair) on the crystal axis consisting of N atoms (Si⟨100⟩, $N = 10^3$, $T = 273$ K) to the same cross section on N atoms in an amorphous target. The energy spread of incident photon beam $\Delta\omega/\omega = 10^{-2}$ is taken into account.

ω. This means that our consideration, in principal, is correct for any values of the principal quantum numbers n.

Fig. 2 shows sharp and large peaks for the ratio

$$R = \frac{\mathrm{d}\sigma_{\mathrm{coh}}(A_{2e}) + \mathrm{d}\sigma_{\mathrm{incoh}}(A_{2e})}{N\, \mathrm{d}\sigma_{\mathrm{l}}(A_{2e})},$$

of the differential cross section of coherent photoproduction of A_{2e} on a separate crystal string consisting of N atoms, to the differential cross section on N atoms in an amorphous target, calculated according to Ref. [2], taking into account the energy spread of the initial photon beam $\Delta\omega/\omega = 10^{-2}$. The emission angle of A_{2e} is taken $\theta_{A_{2e}} = \lambda/2\gamma_{A_{2e}}R$, (where $\gamma_{A_{2e}}$ is the relativistic factor of A_{2e}, $\theta_{A_{2e}}$ is an angle for which the cross section $\mathrm{d}\sigma_{\mathrm{coh}}(A_{2e})$ is maximum for a given photon energy). Further, calculations show that the quantity

$$R(\Delta_{A_{2e}}) = \frac{\int_0^{2\pi}\int_0^{\Delta_{A_{2e}}}(\mathrm{d}\sigma_{\mathrm{coh}}(A_{2e}) + \mathrm{d}\sigma_{\mathrm{incoh}}(A_{2e}))\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi}{N\int_0^{2\pi}\int_0^{\Delta_{A_{2e}}}\mathrm{d}\sigma_{\mathrm{l}}(A_{2e})\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi}, \quad (6)$$

which characterizes the increase of the yield in comparison with an amorphous target as a function of the angular size $\Delta_{A_{2e}}$ of the detector, drops from $R(\Delta_{A_{2e}}) = 34$ at $\Delta_{A_{2e}} = 10^{-4}$ to $R(\Delta_{A_{2e}}) = 17$ at $\Delta_{A_{2e}} = 1 \times 10^{-3}$ and approaches $R(\Delta_{A_{2e}}) = 13.7$ at $\Delta_{A_{2e}} \to \pi$ (for the first resonance peak $k = 1$). The total cross section of coherent positronium photoproduction (in the ground state) is $\sigma_{\mathrm{coh}}(A_{2e}) + \sigma_{\mathrm{incoh}}(A_{2e}) = 8.7$ mb (Si⟨100⟩, $N = 10^3$, $T = 273$ K), for the photon energy $\omega = 229$ MeV near the first coherent peak.

Comparing the enhancement due to the coherence effect under measurements of the total pair yield, one can see the impressive increase of the coherent peak brilliance (ratio of peak height to background) from $\sim 10^{-1}$ in the case of "usual" pairs up to ~ 10 in the case of "narrow" pairs (Fig. 2) and up to ~ 10–10^2 in the case of photoproduction of relativistic A_{2e}.

The physical reason for the relative increase of the coherent peak for the "narrow pairs" is as follows. If the emission angles region $\Delta\theta = 0 \div \theta_m$ contains the coherent peak (within definite range of ω values, according to Eq. (4)), that means that part of the amplitudes satisfies Eq. (4) and its contribution to the cross section is proportional to N^2; the other part of amplitudes does not satisfy Eq. (4) and its contribution to the cross section is proportional to N. If one decreases $\Delta\theta$, still keeping the coherent peak inside $\Delta\theta$ (the case of "narrow" pairs) the relative yield of incoherent ($\sim N$) process decreases and integrated over $\Delta\theta$ cross section in a crystal becomes more and more proportional to N^2, compare dotted and solid lines in Fig. 1. At the same time in an amorphous target the cross section integrated over $\Delta\theta$ is always proportional to N. The width of the peak $\Delta\omega$ can be easily estimated for every given $\Delta\theta_m$, since the width of coherent peak in $I(q_\parallel)$ is of the order of $q_\parallel d/N$, where $q_\parallel$ defined by Eq. (4) is the function of ω, θ_+, θ_-, and p_+ and p_- are determined by the given $x = \varepsilon_+/\omega$ value.

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