Nuclear Instruments and Methods in Physics Research B 309 (2013) 88–91

Contents lists available at SciVerse ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Coherent bremsstrahlung from neutrons

Yu.P. Kunashenko *

Dept. of Theoretical and Experimental Physics, National Research Tomsk Polytechnic University, 634050 Tomsk, Russia
Dept. of Theoretical Physics, Tomsk State Pedagogical University, 634041 Tomsk, Russia

A R T I C L E I N F O

Article history:
Received 2 December 2012
Received in revised form 17 February 2013
Accepted 18 February 2013
Available online 22 March 2013

Keywords:
Neutron interaction with electromagnetic field
Scattering of neutrons in the crystals
Coherent bremsstrahlung

A B S T R A C T

Theory of coherent bremsstrahlung from neutrons in crystals is developed in the framework of method of virtual photons. For this purpose we reduced the cross-section of scattering of a photon by a neutron. On the basis of derived equations we calculated the differential over an emitted photon energy cross-section for coherent bremsstrahlung from neutrons in the tungsten crystal.

1. Introduction

The neutron has a spin and anomalous magnetic moment and, therefore, it can interact with electromagnetic field. Schwinger [1] was the first who predicted that fast neutrons can be scattered by atom electric field due to their magnetic moment. The physics of this scattering is explained in the following way: in a neutron rest frame due to the Lorentz transformation of atom electric field, the magnetic field appears and the neutron magnetic moment interacts with it. The Schwinger scattering of fast neutrons by the atoms was experimentally proved in 1956 [2]. The paper [3] dedicated to semi – centenary anniversary of the discovery of Schwinger scattering of fast neutrons contains a review of theoretical and experimental works on this subject, and possible new experiments was given.

Another mechanism of neutron interaction with electromagnetic field is photon emission from neutrons. For the first time, an emission of the photons from neutrons in an external magnetic field has been theoretically studied in [4–6]. This new type of radiation produced at interaction of the anomalous magnetic moment and magnetic field was named "spin" light [6].

It is well known that when the fast charged particle interacts with an aligned crystal, there may appear coherent scattering (a thin crystal, the Born approximation, or two-wave diffraction) or channeling effect (thicker crystal, when many-wave diffraction is replaced by channeling).

A coherent effect in the charged particle scattering by a crystal arises due to periodical arrangement of the crystal atoms, when the scattering amplitudes of the particle on individual atoms are summing up in a phase. As a result of this summation an interferential multiplier appears in the scattering cross – section which leads to appearance of coherent peaks at the fixed parameters (beam energy, angles of incidence into a crystal with respect to crystallographic axes or planes, scattering angles).

The channeling phenomenon appears when the fast charged particle passes through crystal at small angle with respect to a crystal axis or plane and the crystal thickness is rather large, so that the Born approximation or many-wave diffraction approach do not work. In this case a fast charged particle motion in a crystal is governed only by the averaged (continuous) potential of crystal axis or plane while the periodic part of the crystal potential is neglected [7].

The coherent scattering or channeling of relativistic electrons in a crystal lead to emission of coherent bremsstrahlung [8–9] or channeling radiation [11–13].

In analogy with scattering of the charged particles by a crystal, we may suppose that when the fast neutron interact with the crystal, one can expect both the coherent scattering of neutrons and the channeling of neutrons. The coherent Schwinger scattering and channeling of fast neutrons in the crystals has been considered in [14–19].

In present paper we have developed the theory of coherent bremsstrahlung from neutrons passing through a crystal at small angle with respect to the crystal axis.

The paper is organized as follows: in Section 1 we consider scattering of photons by neutron; in Section 2 we study the coherent

* Address: Dept. of Theoretical and Experimental Physics, National Research Tomsk Polytechnic University, 634050 Tomsk, Russia.
E-mail address: kunashenko@tpu.ru

0168-583X/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.nimb.2013.02.031

bremsstrahlung from neutrons; and in Conclusion we discuss the obtained results.

2. Cross-section of a photon scattering by a neutron

In the reference of the fast neutron passing through the crystal the electric field of an atom is perpendicular to its magnetic field. According to the Lorentz transformations, the fields are compressed in the longitudinal direction. Therefore, in a moving reference frame the electromagnetic field of the crystal is similar to the fields of a plane wave (if neutron relativistic factor $\gamma >> 1$). Therefore the electromagnetic field of the crystal can be replaced by a flux of virtual photons. Further, a process of bremsstrahlung can be described in terms of the virtual photons scattered by a neutron [8].

The scattering of photons by neutron has been studied earlier (see, e.g. [20]), but the change of photon energy during scattering has been neglected. We don't use this approximation.

It is convenient to consider scattering of photons by neutrons in a coordinate system moving with the neutron. In the moving reference frame one can use non relativistic approach to the scattering process. The scattering process is described by the second-order perturbation theory [20,21]. The matrix element of the photon scattering by the neutron is:

$$M_{fi} = \sum_{\alpha} \int \left\{ \frac{\langle f|\hat{V}|\alpha_0\rangle\langle\alpha_1|\hat{V}|i\rangle}{(E_i + \hbar\omega) - E_\alpha} + \frac{\langle f|\hat{V}|\alpha_0\rangle\langle\alpha_1|\hat{V}|i\rangle}{(E_i + \hbar\omega) - (E_\alpha + \hbar\omega + \hbar\omega')} \right\} d\vec{k}_\alpha \tag{1}$$

Here, the following notations are introduced:

$$\begin{aligned} |i\rangle &= \mathrm{X}_i \exp[i\mathbf{k}_i\mathbf{r}]|1\rangle, \quad |f\rangle = \mathrm{X}_f \exp[i\mathbf{k}_f\mathbf{r}]|1\rangle \\ |\alpha_0\rangle &= \mathrm{X}_\alpha \exp[i\mathbf{k}_\alpha\mathbf{r}]|0\rangle, \quad |\alpha_1\rangle = \mathrm{X}_\alpha \exp[i\mathbf{k}_\alpha\mathbf{r}]|1\rangle, \end{aligned} \tag{2}$$

with $|i\rangle$, $|f\rangle$ and $|\alpha\rangle$ being an initial, final and intermediate states of the considered system (neutron and photon), $\mathrm{X}_{i(f,\alpha)}$ are the neutron spin wave functions, $\boldsymbol{k}_{i(f,\alpha)}$ are the neutron wave vectors, $|0\rangle$ and $|1\rangle$ are the photon wave functions in the second-quantization representation, $\hbar\omega$ and $\hbar\omega'$ are the energies of incident and scattered photons.

The operator of the neutron interaction with an electromagnetic radiation field has the form [14]:

$$\begin{aligned} \hat{V} &= \boldsymbol{\mu}\mathbf{H}_{rad} = \boldsymbol{\mu}\, rot\mathbf{A}_{rad} \\ &= \boldsymbol{\mu}\sum\sqrt{\frac{2\pi\hbar c^2}{\omega_k}}([i\mathbf{k},\mathbf{e}_k]\hat{a}_k \exp[i\mathbf{kr}] - [i\mathbf{k},\mathbf{e}_k]\hat{a}_k^+ \exp[-i\mathbf{kr}]). \end{aligned} \tag{3}$$

Here, $\boldsymbol{\mu} = \mu_n\boldsymbol{\sigma}$ is the operator of a neutron magnetic momentum with the Pauli matrix $\boldsymbol{\sigma}$, $\mu = -1.91\mu_n$ is the anomalous magnetic moment of a neutron, $\mu_n = e\hbar/2mc$ nuclear magneton, m is the neutron rest mass, ω_k is the photon frequency, $\mathbf{e}_k$ is the photon polarization vector and $\mathbf{k}$ is the wave vector, $\hat{a}_k^+$ is the photon creation operator and $\hat{a}_k$ is the photon annihilation operator, $\mathbf{H}_{rad}$ is the magnetic field and $\mathbf{A}_{rad}$ is the vector potential of the radiation field, $\hbar$ is the Plank constant, c is the speed of light in a vacuum, e is the elementary charge.

After performing a standard algebra we obtain the matrix element of the photon absorption by the neutron:

$$\begin{aligned} \langle 0,\alpha|\hat{V}|\beta,1\rangle &= \sum\sqrt{\frac{2\pi\hbar c^2}{\omega_k}}\langle \mathrm{X}_\alpha, 0|\ \boldsymbol{\mu}\ [i\mathbf{k},\mathbf{e}_k] \\ &\quad \int \exp[-i\mathbf{k}_\alpha\mathbf{r}]\exp[i\mathbf{kr}]\exp[i\mathbf{k}_\beta\mathbf{r}]d\mathbf{r}\ \hat{a}_k|\mathrm{X}_\beta, 1\rangle \\ &= \langle \mathrm{X}_\alpha, |\ \mu\ [\mathrm{ik,e}_k].|\mathrm{X}_\beta\rangle\delta(-k_\alpha + \mathrm{k} + \mathrm{k}_\beta), \end{aligned} \tag{4}$$

and the matrix element of photon emission from the neutron:

$$\begin{aligned} \langle 1,\alpha|\hat{V}|\beta,0\rangle &= \sum\sqrt{\frac{2\pi\hbar c^2}{\omega_k}}\Big\langle \mathrm{X}_\alpha, 1|\boldsymbol{\mu}[i\mathbf{k},\mathbf{e}_k] \\ &\quad \int \exp[-i\mathbf{k}_\alpha\mathbf{r}]\exp[-i\mathbf{kr}]\exp[i\mathbf{k}_\beta\mathbf{r}]d\mathbf{r}\ \hat{a}_k^+|\mathrm{X}_\beta, 0\Big\rangle \\ &= \Big\langle \mathrm{X}_\alpha|\boldsymbol{\mu}[i\mathbf{k},\mathbf{e}_k].|\mathrm{X}_\beta\Big\rangle\delta(-\mathbf{k}_\alpha + \mathbf{k} + \mathbf{k}_\beta), \end{aligned} \tag{5}$$

In Eqs. (4) and (5) it is took into account that the matrix elements $\langle 0|\hat{a}|1\rangle = 1, \langle 1|\hat{a}+|0\rangle = 1$, and other matrix elements of the photon creation operator ($\hat{a}_k^+$) and photon annihilation operator ($\hat{a}_k$) are equal to zero.

In Eq. (1) the summation over intermediate spin states of the neutron is carried out by using the completeness of the spin wave functions: $\sum_n|\mathrm{X}_n\rangle\langle\mathrm{X}_n| = 1$.

Assuming that a neutron in the initial state is at rest, and neglecting the photon momenta in comparison with the neutron momentum, we find:

$$\begin{aligned} M_{fi} &= \frac{2\pi\hbar c^2}{\sqrt{\omega\omega'}} \\ &\left(\frac{\langle \mathrm{X}_f|(\boldsymbol{\mu}[i\mathbf{k},\mathbf{e}])(\boldsymbol{\mu}[i\mathbf{k}',\mathbf{e}'])|\mathrm{X}_i\rangle}{\hbar\omega} - \frac{\langle \mathrm{X}_f|(\boldsymbol{\mu}[i\mathbf{k}',\mathbf{e}'])(\boldsymbol{\mu}[i\mathbf{k},\mathbf{e}])|\mathrm{X}_i\rangle}{\hbar\omega'}\right) \\ &\times \delta(\mathbf{k} - (\mathbf{k}_f - \mathbf{k}')) \end{aligned} \tag{6}$$

Here, $\mathbf{k}$ is the wave vector, $\mathbf{e}$ is the polarization vector and ω is the frequency of the incident photon while $\mathbf{k}', \mathbf{e}', \omega'$ are the wave vector, polarization vector and frequency of the scattered photon, respectively.

If the incident photon moves along the OZ axis then its wave vector is defined as follows:

$$\boldsymbol{k} = \{0, 0, k\}, \quad k = \omega/c,$$

while the wave vector of a scattered photon can be written as:

$$\mathbf{k}' = \{k'\sin\Theta\cos\Phi, k'\sin\Theta\sin\Phi, k'\cos\Theta\}, \quad k' = \omega'/c$$

Here, Θ and Φ are the photon scattering angles. The polarization vectors of the incident photon are:

$$\mathbf{e}_1 = \mathbf{e}_x, \quad \mathbf{e}_2 = \mathbf{e}_y$$

For the scattered photon we choose the polarization vector in a form [22]:

$$\mathbf{e}_1' = \frac{\mathbf{n}\times\mathbf{e}_z}{\sqrt{1-(\mathbf{n}\cdot\mathbf{e}_z)^2}}, \quad \mathbf{e}_2' = \frac{\mathbf{n}(\mathbf{n}\cdot\mathbf{e}_z) - \mathbf{e}_z}{\sqrt{1-(\mathbf{n}\cdot\mathbf{e}_z)^2}}, \quad \mathbf{n} = \frac{\mathbf{k}}{k}.$$

Here $\boldsymbol{e_x}$, $\boldsymbol{e_y}$, $\boldsymbol{e_z}$ are the unit vectors directed along the axes of the chosen coordinate system.

According to the Fermi golden rule the cross-section of photon scattering by the neutron can be defined by:

$$d\sigma_{fi} = \frac{2\pi}{\hbar c}|M_{fi}|^2\delta(\hbar\omega - T_f - \hbar\omega')\frac{(\hbar\omega')^2 d\hbar\omega'}{(2\pi\hbar c)^3}d\Omega. \tag{7}$$

There are two possible ways of the photon scattering by a neutron: in the first case the orientation of the neutron spin does not change, in the second one the scattering of the photon is accompanied by spin-flip of the neutron.

Assuming that the neutron spin is parallel to the initial momentum of the neutron (in the laboratory frame) we may write the neutron spin wave functions in the form:

$$\mathrm{X}_1 = \begin{pmatrix}1\\0\end{pmatrix}, \quad \mathrm{X}_2 = \begin{pmatrix}0\\1\end{pmatrix}.$$

After averaging over the polarizations of the incident photons, summation over the polarizations of the scattered photons and performing some algebra, we find the cross-section of photons scattering by neutrons with conservation of the neutron spin orientation:

$$\frac{d\sigma_{\uparrow\uparrow}}{d\Omega} = \left(\frac{\mu_n}{\hbar c}\right)^4 \frac{\hbar\omega'\left((\hbar\omega)^2 + (\hbar\omega')^2\right)}{\hbar\omega}(3 + \cos 2\Theta) \tag{8}$$

The scattering cross-section of a photon by a neutron with a neutron spin-flip is:

$$\frac{d\sigma_{\uparrow\downarrow}}{d\Omega} = \left(\frac{\mu_n}{\hbar c}\right)^4 \frac{\hbar\omega'(\hbar\omega + \hbar\omega')^2}{\hbar\omega} \sin^2\Theta \tag{9}$$

The frequency of the incident and scattered photons are related by the well known Compton formula:

$$\frac{\hbar}{mc^2}\omega' = \frac{\omega}{mc^2/\hbar + \omega(1 - \cos\Theta)}, \tag{10}$$

where m is now the neutron rest mass.

The Fig. 1 shows two angular distributions of photons scattered by the neutrons: without a neutron spin flip (Fig. 1a), and with a neutron spin flip (Fig. 1b). The energy of the incident photon is $\hbar\omega = 10$ MeV.

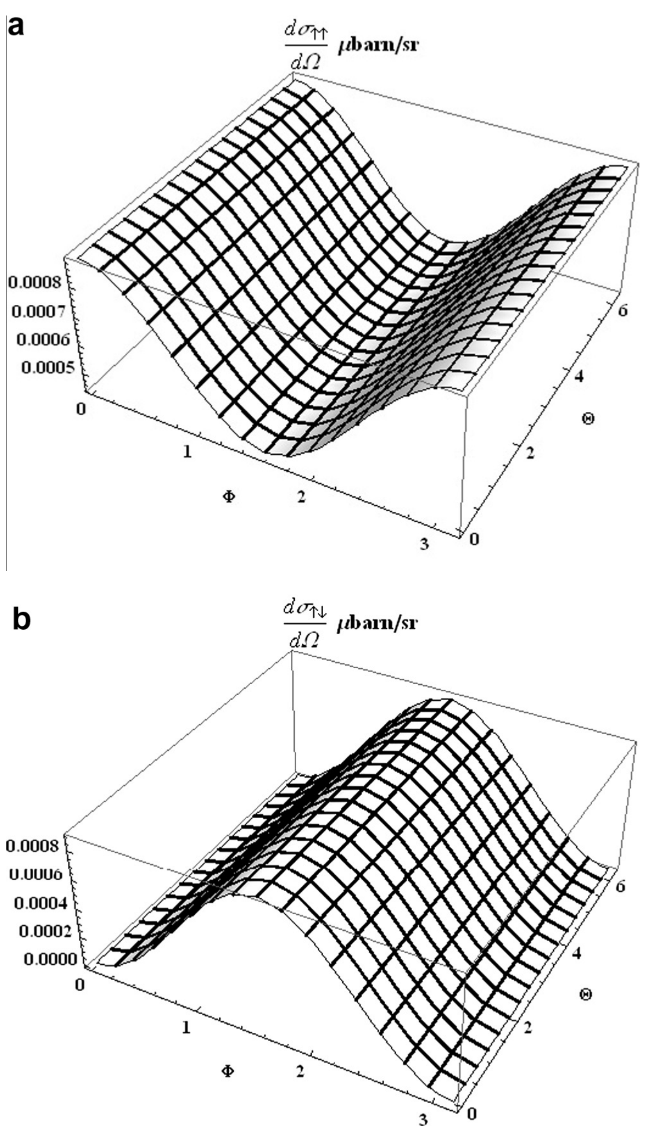

Fig. 1. The cross-section of the photons scattering by the neutrons: (a) the photons scattering without a neutron spin flip, (b) the photons scattering with a neutron spin flip (b).

As it seen from the figures, the cross-sections of the photons scattering by neutrons without and with neutron spin flip differ in angular distributions of the scattered photons.

After integration of Eqs. (9) and (10) over emission angles of scattered photon we arrive at expression:

$$\sigma_{\uparrow\uparrow} = \frac{16\pi}{3}\left(\frac{\mu_n}{\hbar c}\right)^4 \frac{\hbar\omega'\left((\hbar\omega)^2 + (\hbar\omega')^2\right)}{\hbar\omega}, \tag{11}$$

for the total cross-section of photons scattering, without a neutron spin flip, and at

$$\sigma_{\uparrow\downarrow} = \frac{8\pi}{3}\left(\frac{\mu_n}{\hbar c}\right)^4 \frac{\hbar\omega'(\hbar\omega + \hbar\omega')^2}{\hbar\omega}, \tag{12}$$

for the total cross-section of photons scattering, with a neutron spin flip.

It follows from Eqs. (11) and (12) that total cross-sections of the photons scattering by a neutron without and with a neutron spin flip differ. If one neglects the change of a photon energy at scattering than the both cross-sections are become equal:

$$\sigma_{\uparrow\uparrow(\uparrow\downarrow)} = \frac{32\pi}{3}\left(\frac{\mu_n}{\hbar c}\right)^4 (\hbar\omega)^2.$$

The last formula coincides with the result of Ref. [20].

3. The cross section of the coherent bremsstrahlung from neutrons

When the particle is moving parallel to the axis of the crystal, it is enough to take into account its interaction with single crystallographic axis [23]. The electrostatic potential of the crystal axis is a sum of the Coulomb potentials of the individual atoms forming the axis and is equal to:

$$V(\vec{r}) = \sum_{i=1}^{N} V_1(|\vec{r} - \vec{r}_i|),\ V_1(r) = \frac{Ze}{r}\exp\left(-\frac{r}{R}\right) \tag{13}$$

Here V_1 is the potential of a single atom, R is the screening radius, N is the number of atoms in the axis, Z is atomic number.

The spectrum of virtual photons of crystal axis as it seen in the rest frame of neutron moving parallel to the axis is given by [15]:

$$\begin{aligned} n(\omega)d\omega &= \frac{Z^2e^2}{\pi^2}N \times \left\{\left[L - B\exp\left[-\left(\frac{\hbar\omega}{\gamma\hbar c}\bar{u}\right)^2\right]\right]\right. \\ &\left. + \frac{2\pi}{d}\sum_{g_n} B\exp\left[-\left(\frac{\hbar\omega}{\gamma\hbar c}\bar{u}\right)^2\right]\delta(k_1 - g_n)|S|^2\right\}\frac{d\omega}{\omega}, \\ L &= \pi\ln\left[\frac{a\lambda^{-2}}{(\hbar\omega/\gamma\hbar c)^2 + R^{-2}}\right], \quad a \approx 1, \\ B(x) &= \pi\{-(1 + x)e^x Ei(-x) - 1\}, \\ x &= \left[\left(\frac{\hbar\omega}{\gamma\hbar c}\right)^2 + R^{-2}\right]\bar{u}^2. \end{aligned} \tag{14}$$

Here d is the lattice constant, ω is the frequency of virtual photon, $\lambda = mc/\hbar$ the is Compton wave length of a neutron, $\bar{u}^2$ is the mean-square displacement of a crystal atom from equilibrium position, $Ei(-x)$ is the exponential integral function, $g_n = 2\pi n/d$ is the 1D reciprocal lattice vector, $|S|$ is the structure factor of a crystal axis, γ is the neutron relativistic factor. The spectrum of virtual photons of a separate atom is:

$$n_{am}(\omega)d\omega = \frac{Z^2e^2}{\pi^2}L\frac{d\omega}{\omega} \tag{15}$$

If we neglect the terms $(\hbar\omega/\gamma\hbar c)$ in the functions L and B in Eq.(14) than we obtain the result of Ter- Mikaelian for virtual photons spectrum [8].

Let us transform the photons scattering cross-section by the neutrons into the laboratory coordinate system. For this purpose we use the Compton formula [10] and the Lorentz transformation for the photon energy (the Doppler shift):

$$\hbar\omega_r = \gamma\,\hbar\omega(1-\beta\cos\Theta). \tag{16}$$

Here we denote $\beta = v/c$, and v is the neutron velocity in the laboratory coordinate system, $\hbar\omega_r$ is the energy of emitted photon. The cross-section of photons scattering by neutrons without neutron spin flip is:

$$\begin{aligned}\frac{d^2\sigma_{\uparrow\uparrow}(\omega_r,\omega)}{d\omega_r d\omega} &= \frac{2\pi\mu_n^4}{\hbar^2c^4}\frac{\omega_r}{\gamma^3\omega^2} \\ &\times\frac{\left(2\omega\omega_r\gamma\left(1+(1+\beta)\omega\frac{\hbar}{mc^2}\right)-\omega^2(1+\beta^2)\gamma^2-\omega_r^2\left(1+2\omega\frac{\hbar}{mc^2}\left(1+\omega\frac{\hbar}{mc^2}\right)\right)\right)}{\left(\beta\gamma-\frac{\hbar}{mc^2}\omega_r\right)\left(\beta-\frac{\hbar}{mc^2}\omega(1-\beta)\right)} \\ &\times\left(\left(\frac{mc^2}{\hbar}\right)^2(\omega_r^2+\gamma^2\omega^2)-2\beta\gamma\frac{\hbar}{mc^2}\right. \\ &\times\left.\left(\omega_r+\gamma\omega_1\left(1+\omega\frac{\hbar}{mc^2}\right)\right)+\gamma^2\beta^2\left(2+\omega\frac{mc^2}{\hbar}\left(2+\omega\frac{\hbar}{mc^2}\right)\right)\right)\end{aligned}$$

The cross-section of photons scattering by neutrons with neutron spin flip is:

$$\begin{aligned}\frac{d^2\sigma_{\uparrow\downarrow}(\omega_r,\omega)}{d\omega_r d\omega} &= \frac{2\pi\mu_n^4}{\hbar^2c^4}\frac{\omega_r}{\gamma^3\omega^2} \\ &\times\frac{(\omega_r-\gamma\omega_1(1-\beta))\left(\omega_r-\gamma\omega(1+\beta)+2\omega\omega_r\frac{\hbar}{mc^2}\right)\left(\frac{\hbar}{mc^2}(\omega_r+\gamma\omega)-\beta\gamma\left(2+\frac{\hbar}{mc^2}\omega\right)\right)}{\left(\beta\gamma-\frac{\hbar}{mc^2}\omega_r\right)\left(\beta-\frac{\hbar}{mc^2}\omega(1-\beta)\right)}\end{aligned}$$

In accordance with the virtual photons method, the cross-section of coherent bremsstrahlung by neutrons is:

$$\frac{d\sigma^{CR}_{\uparrow\uparrow(\uparrow\downarrow)}(\omega_r)}{d\omega_r} = \int_{\omega_{MIN}}^{\omega_{MAX}}\frac{d^2\sigma_{\uparrow\uparrow(\uparrow\downarrow)}(\omega_r,\omega)}{d\omega_r d\omega}n(\omega)d\omega,$$

and the cross-section of bremsstrahlung by neutrons in amorphous target is

$$\frac{d\sigma^{AM}_{\uparrow\uparrow(\uparrow\downarrow)}(\omega_r)}{d\omega_r} = N\int_{\omega_{MIN}}^{\omega_{MAX}}\frac{d^2\sigma_{\uparrow\uparrow(\uparrow\downarrow)}(\omega_r,\omega)}{d\omega_r d\omega}n_{am}(\omega)d\omega,$$

The limits of integration are determined by the condition $-1 \leqslant \cos\Theta \leqslant 1$ and equal:

$$\omega_{MIN} = \frac{\omega}{\gamma+\beta\gamma+\frac{mc^2}{\hbar}\omega}, \quad \omega_{MAX} = \frac{\omega}{\gamma(1-\beta)}$$

4. Conclusions

In order to demonstrate the coherent effect in Fig. 2 we plot the ratio R of the cross-section of coherent bremsstrahlung from neutron to that in an amorphous target containing the same number of atoms N as the axis of the crystal

$$R = \frac{d\sigma^{CR}_{\uparrow\uparrow(\uparrow\downarrow)}(\omega_r)}{d\omega_r}\Big/\frac{d\sigma^{AM}_{\uparrow\uparrow(\uparrow\downarrow)}(\omega_r)}{d\omega_r}.$$

In our calculation, the $\langle 100\rangle$ tungsten crystal axis was chosen, and relativistic factor of neutron was set equal to $\langle 100\rangle$.

It is interesting to not that, despite the fact that the Eqs. (14) and (15) for bremsstrahlung from neutrons with and without spin flip are different, we have the same ratio R.

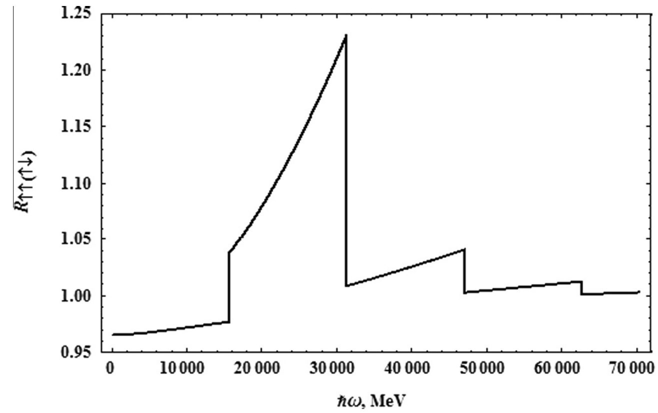

Fig. 2. The ratio R of the cross-section of the coherent bremsstrahlung from the neutron to the cross-section of the neutron bremsstrahlung in an amorphous target.

Our calculations show that during the passage of the neutrons through oriented crystals coherent bremsstrahlung from neutrons may appear. The value of total cross-section of the coherent bremsstrahlung from neutron in a crystal can exceed the bremsstrahlung cross-section in amorphous target by 20%. It is typical value for ether coherent effects for the axial orientation.

Brighter coherent effects could be expected in the study of differential cross-section of coherent bremsstrahlung from neutron in crystals, as in the case for the coherent production of electron – positron pairs by high energy photons in a crystal [24].

Acknowledgments

The author is grateful to Prof. Yu.L. Piviovarov for fruitful discussions.

The work is partially supported by the Russian Fund for Basic Research, Grant 10-02-01386-a; by the Russian Education Federal Agency program «Support of Scientific Schools», contract 224.2012.2.

References

[1] J. Schwinger, Phys. Rev. 73 (1948) 407–409.
[2] Yu.A. Alexandrov, I.I. Bondarenko, ZhETP 31 (1956) 726 (in Russian).
[3] Yu.A. Alexandrov, Preprint E3–2006-142 JINR.-2006.- 7 c.
[4] I.M. Ternov, V.G. Bagrov, A.M. Khapaev, Soviet Physics JETP 21 (1965) 613.
[5] V.L. Lyuboshitz, Yad. Fiz. 4 (1966) 269.
[6] V.A. Bordovitsyn, I.M. Ternov, V.G. Bagrov, Phys. Usp. 38 (1995) 1037.
[7] J. Lindhard, Kgl. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 34 (1965) 14.
[8] M.L. Ter-Mikelian, High Energy Electromagnetic processes in Condensed Media, Wiley Interscience, New-York, 1972.
[9] H. Uberall, Phys. Rev. 103 (1956) 1055.
[10] A.W. Saenz, H. Uberall, Theory of coherent bremsstrahlung, in: A.W. Saenz, H. Uberall (Eds.), Coherent Radiation Sourses, Springer-Verlag, Berlin, 1985, pp. 5–32.
[11] V.A. Bazylev, N.K. Zhevago, Radiation of Fast Particles in Matter and External Fields, Nauka, Moscow, 1987 (in Russian).
[12] V.N. Baier, V.M. Katkov, V.M. Strakhovenko, Electromagnetic Processes at High Energies in Oriented Single Crystals, World Scientific Publishing Comp., 1994.
[13] A.I. Akhiezer, N.F. Shul'ga, Electrodynamics oh high energy in medium, Nauka, Moscow, 1993 (in Russian).
[14] A.N. Dyumin, I.Ya. Korenblit, V.A. Ruban, B.B. Tokarev, JETP Lett. 31 (1980) 413.
[15] V.G. Baryshevskii, A.M. Zaitzeva, Izv. VUZov 3 (1985) 103 (in Russian).
[16] Yu.P. Kunashenko, Il Nuovo Cimento. 34C (2011) 453.
[17] A.N. Dyumin, V.A. Ruban, B.B. Tokarev, M.F. Vlasov, JETP Lett. 1985 (42) (1985) 61.
[18] V.I. Vysotskii, R.N. Kusmin, ZhETP 82 (1982) 177 (in Russian).
[19] K.B. Korotchenko, Yu.P. Kunashenko, Il Nuovo Cimento. 34C (2012) 537–545.
[20] V.N. Galitskii, V.I. Karnakov V.N. Kogan, Problems in Quantum Mechanics, M.: Наука, 1981 (648c).
[21] L.D. Landau, E.M. Lifshiys, Quantum Mechanics, Pergamon, Oxford, 1975.
[22] A.A. Sokolov, I.M. Ternov, Relativistic electron, Nauka, Moscow, 1974 (in Russian).
[23] Yu.P. Kunashenko, Yu.L. Pivovarov, Nucl. Ins. Meth. B 119 (1996) 137.
[24] I. Endo, Y. Okazaki, Yu.P. Kunashenko, Yu.L. Pivovarov, Nucl. Ins. Meth. 179 (2001) 343.