(shape and relationship to the temperature and density of ions) is qualitatively consistent with experimental observations.

LITERATURE CITED

1. K. O. Bizli, D. G. Lominadze, and A. B. Mikhailovskii, Fiz. Plaamy, 2, 170 (1976).
2. B. Lewis and G. von Elbe, Combustion, Flames, and Explosions of Gases, Academic Press, New York (1961).
3. V. N. Gladyshev, in: Proc. Tenth All-Union Acoustics Conf. [in Russian], Sec. Zh., Moscow (1983), pp. 25-28.
4. T. H. Stix, The Theory of Plasma Waves, McGraw-Hill, New York (1962).
5. D. A. Frank-Kamenetskii, Lectures on Plasma Physics [in Russian], 3nd ed., Atomizdat, Moscow (1968), p. 166.
6. L. A. Artsimovich, Controlled Thermonuclear Reactions [in Russian], 2nd ed., rev., Fizmatgiz, Moscow (1963), p. 14.
7. D. J. Rose and M. Clark, Plasmas and Controlled Fusion, MIT Press, Cambridge, MA (1961).
8. V. N. Gladyshev, Akust. Zh., 30, No. 3, 391 (1984).

COHERENT BREMSSTRAHLUNG OF RELATIVISTIC ELECTRONS IN ANTIFERROMAGNETS

Yu. P. Kunashenko

UDC 537.533.539.12:540.145

The coherent bremssstrahlung (CB) of relativistic electrons in antiferromagnetic crystals is studied theoretically. It is shown that the internal magnetic field of an antiferromagnet has a negligible effect on CB. The complete CB spectrum is calculated for hematite, α—Fe_2O_3.

The present work is devoted to studying the influence of internal magnetic fields in antiferromagnetic crystals on the spectrum of coherent bremsstrahlung (CB).

The analogous problem of radiation channeling in magnetic crystals has been considered in [1]. A qualitative estimate of CB in periodic electric and magnetic crystal fields has been given in [2]. These studies were only qualitative and did not calculate the radiation spectrum. Radiation either in a magnetic or in an electric field was considered in [2]. Yet in actual crystals charged particles interact with both the electric and the magnetic fields of the antiferromagnet at the same time, giving rise to interference terms [3] which may considerably exceed the purely magnetic contribution.

THEORY

In order to describe the interaction of relativistic particles with crystals it is convenient to use the simple pseudo-photon approximation [4], in which one essentially replaces the particle fields by equivalent electromagnetic fields.

We consider the interaction of a relativistic electron with an antiferromagnetic atom possessing a nonzero magnetic moment (a "magnetic" atom). Let the x-axis be parallel to the magnetic field vector of the atom $\mathbf{H} = (H, 0, 0)$. We may take the spatial distribution of H to be Gaussian:

$$H = H_0 \exp\left[-\frac{(z-z_i)^2}{2z_0^2} - \frac{\rho_i^2}{2\rho_0^2} - \frac{(\varphi-\varphi_i)^2}{2\varphi_0^2}\right]. \quad (1)$$

where H_0 is the maximum value $|\mathbf{H}|$; $\mathbf{r}_i = (x_i, y_i, z_i)$ is the radius vector of the atom, $\rho_i^2 = x_i^2 + y_i^2$; $\varphi_i = \begin{cases} 0, & H > 0 \\ \pi, & H < 0 \end{cases}$. ρ_0 and z_0 are assumed to be of the order of the screening radius. The Coulomb potential of the nucleus, including the screening of the atomic electrons, is taken to be [5]:

Scientific-Research Institute of Nuclear Physics, S. M. Kirov Polytechnic Institute, Tomsk.

Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 2, pp. 57-61, February, 1988. Original article submitted June 9, 1986.

0038-5697/88/3102-0132$12.50

$$V(\boldsymbol{r}) = V_0 \exp\left[-(\boldsymbol{r}-\boldsymbol{r}_i)^2/R^2\right],\ V_0 = Ze/B. \tag{2}$$

Here Ze is the nuclear charge, R and B are constants having units of length. We thus obtain in the laboratory frame

$$\boldsymbol{H} = \boldsymbol{n}_x H_0 \exp\left[-\frac{(z-z_i)^2}{2z_0^2} - \frac{\rho_i^2}{2\rho_0^2} - \frac{(\varphi-\varphi_i)^2}{2\varphi_0^2}\right], \tag{3}$$

$$\boldsymbol{E} = V_0 \exp\left[-(\boldsymbol{r}-\boldsymbol{r}_i)^2/R^2\right](\boldsymbol{r}-\boldsymbol{r}_i)/R^2. \tag{4}$$

Let the electron move along the z-axis with velocity $-\mathbf{v}$. This corresponds to the physically most interesting case, namely, when the motion of the electron is perpendicular to the magnetic field.

We calculate the number of equivalent photons lying between ω and $\omega + d\omega$ in the usual way [4]:

$$n(\omega) = n_E(\omega) + n_H(\omega) + n_{EH}(\omega), \tag{5}$$

where

$$n_H(\omega) = \frac{H_0^2 z_0^2 \rho_0^2}{\pi\beta\hbar c}\cdot\exp\left[-2\Omega^2 z_0^2\right] f(\varphi_i), \tag{6a}$$

$$\Omega = \hbar\omega/\gamma\beta c\hbar,$$

$$f(\varphi) = \varphi_0(\pi/4)^{1/2}\left[\operatorname{erf}(2\pi-\varphi) - \operatorname{erf}(\varphi)\right], \tag{6b}$$

$$n_E(\omega) = \frac{V_0^2 R^2}{4\beta\hbar c}\exp\left[-\Omega^2 R^2\right], \tag{7}$$

$$n_{EH}(\omega) = \frac{V_0 H_0 (R^2+4z_0^2)}{\pi\beta^2\hbar c R^2}\left[\frac{R\cdot\rho_0}{(0.5R^2+\rho_0^2)^{1/2}}\right]^3 \exp\left[-\Omega^2(R^2+4z_0^2)\right]\Phi, \tag{8}$$

$$\Phi = F(2\pi-\varphi_i) - F(-\varphi_i),$$

$$F(\varphi) = \varphi_0\pi e^{-\varphi^2}\left\{\cos\varphi_i \operatorname{Im}\operatorname{erf}\left[\frac{1}{\sqrt{2}}\left(\frac{\varphi}{\varphi_0}+i\varphi_i\right)\right] - \sin\varphi_i \times \right.$$

$$\left. \times \operatorname{Re}\cdot\operatorname{erf}\left[\frac{1}{\sqrt{2}}\left(\frac{\varphi}{\varphi_0}-i\varphi_i\right)\right]\right\}.$$

The total number of pseudo-photons consists of three terms: $n_E(\omega)$ due to the electric field, $n_H(\omega)$ pertaining to the magnetic field H, and $n_{EH}(\omega)$ determined by the product EH.

Since in an antiferromagnet one usually has $|E| \gg |H|$, the effects of the interaction of the electron with the magnetic field will be small compared to those due to the electric field. The interference term $n_{EH}(\omega)$ is proportional to the product EH, so that $n_{EH} \gg n_H$, where $n_H \sim H^2$. The interference term is therefore essential in detecting the interaction of an electron with the weak magnetic field of an antiferromagnetic crystal. The bremsstrahlung cross section is determined in terms of the cross section $d\Phi(\omega, \omega')$ of the Compton scattering of the equivalent photon by the electron:

$$d\sigma = \int_{\omega_1}^{\omega_2} n(\omega)\, d\Phi(\omega, \omega')\, d\omega = 2\pi r_0^2 \int_{\omega_1}^{\omega_2} n(\omega)\left[2 + \frac{\hbar\omega'}{m^2c^2}\delta - \frac{2\delta}{\omega}\gamma\beta c + \left(\frac{\delta}{\omega}\gamma\beta c\right)^2\cdot 2\right] d\omega d\omega'/\omega\lambda. \tag{9}$$

Here $\delta = mc^2\,\hbar\omega[2\lambda\varepsilon_1(\varepsilon_1 - \hbar\omega)]^{-1}$, $r_0 = e^2/mc^2$ is the classical radius of the electron, $\varepsilon_1 = \gamma mc^2$, $\lambda = \hbar/mc$ is the Compton wavelength, ω is the pseudo-photon frequency, ω' is the frequency of the radiated photon

$$\omega_1 = \delta\gamma\beta c, \tag{10a}$$

$$\omega_2 = 2\hbar\omega'\gamma^2\beta c/\varepsilon_1\lambda. \tag{10b}$$

The transition from interaction with individual atoms to interaction with the crystals is achieved by replacing atomic field by crystal fields. Summing (3) over all atoms of the lattice results [4] in an extra factor of

$$\left|\sum_{i=1}^{M}\exp\left[-i\hbar\kappa\cdot r_i\gamma\beta c\hbar\right]\right|^2,$$

multiplying $n(\omega)$, where r_i is the position vector of the i-th atom, κ is the wave vector of the pseudo-photon, and M is the total number of atoms in the crystal.

As a concrete example let us take the typical antiferromagnet hematite α-Fe_2O_3 as the target.

Hematite has a corumdum (α-Al_2O_3) type structure [6, 7]. The unit cell of this structural type belongs to the rhombohedral syngony, the cell containing two Fe_2O_3 units, with a space group D_{3d}^6.

Consider now the motion of a relativistic electrom along the <111> axes of which the "magnetic" iron atoms lie. To explain the CB spectrum in such a crystal one need discuss only the interaction of the electron with a single axis. As shown in Fig. 1, along any axis one may select a unit cell containing four Fe atoms and six O atoms. The direction of the magnetic moment are shown by arrows.

The summation over atoms along a crystal axis is conveniently carried out in two stages: we first sum over atoms of the unit cell, and then over all the unit cells.

Because of the strong screening of the Coulomb potential and the magnetic field of the atom, Eqs. (1) and (2), one may neglect the interference of fields from different atoms. The total number of equivalent photons in such a unit cell will th refore be given by

$$n_S(\omega) = n_{ES}^{Fe}(\omega) + n_{ES}^{0}(\omega) + n_{EHS}(\omega) + n_H(\omega). \tag{11}$$

Comparing (5) and (11) we see that there is a new term, coming from the Coulomb potential of the cell's oxygen atoms, which will be naturally combined with n_{ES}^{Fe} [see below, Eq. (17)].

It can be shown that

$$n_{\alpha S}(\omega) = n_\alpha(\omega)\cdot|S_\alpha(\omega)|^2, \tag{12}$$

where α = {E, H, EH}, n_α is given by (5)-(8), N_E^O, n_E^{Fe} are characterized by the values of Z, B, R (see Table 1), and $S_\alpha(\omega)$ is the structure factor. Having due regard for the unit cell type, a calculation leads to the following expressions for the structure factor:

$$\begin{aligned}
&|S_E^O(\omega)|^2 = 6,\\
&|S_E^{Fe}(\omega)|^2 = 4\cos^2(\omega a/\gamma\beta c)\cos^2[\omega(a+b)2\gamma\beta c],\\
&|S_H(\omega)|^2 = 4\cos^2[\omega(a+b)2\gamma\beta c]\,|f(0) - \exp[i\omega a\gamma\beta c]f(\pi)|^2,\\
&|S_{EH}(\omega)|^2 = 4\cos^2(\omega(a+b)/2\gamma\beta c]\,|F(0) - \exp[i\omega a/\gamma\beta c]F(\pi)|^2.
\end{aligned} \tag{13}$$

Summing $n_{\alpha S}(\omega)$ over all the cells of the axis

$$n_\alpha(\omega) = n_{\alpha S}(\omega)\left|\sum_{n=1}^{N}\exp[-i\Omega nd]\right|^2, \tag{14}$$

we obtain for the CB cross section:

$$d\sigma = \int_{\omega_1}^{\omega_2}\sum_{\alpha}n_\alpha|S_\alpha|^2\,|\exp[-i\Omega nd]|^2\,d\Phi(\omega,\omega')\,d\omega. \tag{15}$$

Substituting (5)-(8), (12), and (13) into (15), and using the equation [4]:

TABLE 1

	Z	R	B
Fe	26	0.1 Å	0.19 Å
O	8	0.08 Å	0.17 Å

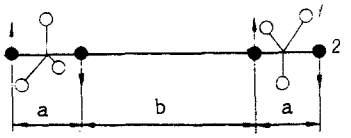

Fig. 1

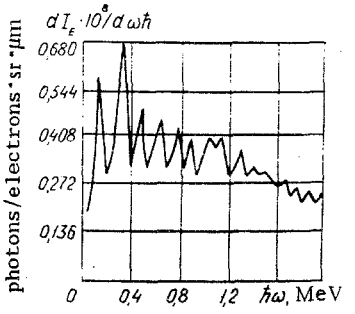

Fig. 2

Fig. 1. Unit cell of hematite: 1) O atoms; 2) Fe atoms; arrows show the direction of the magnetic field. $H_0 \sim 10^6$ Oe, a = 2.6 Å, b = 4.2 Å.

Fig. 2. Usual spectrum of CB, due to the Coulomb crystal fields.

$$\left|\sum_{n=1}^{N}\exp\left[-i\Omega nd\right]\right|^2 = \omega_0 N\sum_{m=-\infty}^{\infty}\delta\left(\hbar\omega - m\hbar\omega_0\right),\quad \omega_0 = 2\pi\gamma\beta c\hbar/2\left(a+b\right),$$

we find

$$d\sigma = \omega_0 N\sum_{\alpha}\sum_{m=m_1}^{m_2} n_\alpha\left(\hbar\omega_0 m\right)\cdot\left|S_\alpha\left(m\omega_0\right)\right|^2\Phi_m d\hbar\omega'. \tag{16}$$

Here

$$m_1 = [\omega_1/\omega_0],\ m_2 = [\omega_2/\omega_0],\ \Phi_m = [2 + (\hbar\omega')^2/(\varepsilon_1(\varepsilon_1 - \hbar\omega')) + 2\beta\varepsilon_1\hbar\omega'/(\gamma\hbar\omega_0 m(\varepsilon_1 - \hbar\omega)) + \beta\varepsilon_1\hbar\omega'/(\gamma\hbar\omega_0 m(\varepsilon_1 - \hbar\omega'))]\cdot(\hbar\omega_0 m)^{-2}.$$

The CB intensity I will then be given by

$$\frac{dI_\alpha}{\hbar d\omega} = \frac{d\sigma_\alpha}{\hbar d\omega}\cdot jl^{-1} = \frac{\omega_0 Nj}{l}\sum_{m_1}^{m_2} n_\alpha\left(\hbar\omega_0 m\right)\left|S_\alpha\left(m\omega_0\right)\right|^2\Phi_m,$$

where j is the electron current with unit density and l is the crystal thickness.

The effect of thermal vibrations may be incorporated into the problem by averaging the cross section over the initial states of the crystal lattice [4]. As is well known, such averaging gives rise to an incoherent part, which is of no interest in our discussion.

RESULTS

Combining the contributions of the electric field from the oxygen and iron atoms $d\sigma_E^O + d\sigma_E^{Fe}$, we have

$$d\sigma = d\sigma_E + d\sigma_H - d\sigma_{EH}. \tag{17}$$

Consequently, the total intensity of the CB,

$$\frac{dI}{\hbar d\omega} = \frac{dI_E}{\hbar d\omega} + \frac{dI_H}{\hbar d\omega} + \frac{dI_{EH}}{\hbar d\omega},$$

consists of the electric $dI_E/\hbar d\omega$, magnetic $dI_H/\hbar d\omega$, and mixed $dI_{EH}/\hbar d\omega$ contributions.

Figures 2-4 illustrate the CB intensity $dI_\alpha/\hbar d\omega$ whenthe electron moves along the <110> axes of a hematite crystal of thickness l = 0.65 m; the electron energy was ε_1 = 5.7 MeV, and the electronic current density was normalized to 1 electrons/cm^2.

We observe that the interference term $dI_{EH}/\hbar d\omega$ is much larger than $dI_H/\hbar d\omega$, e.g., for the maximum at $\hbar\omega$ = 0.32 MeV their ratio is

$$\frac{dI_{EH}\,\hbar d\omega}{dI_H/\hbar d\omega} \simeq 0.23\cdot 10^5. \tag{18}$$

The following additional ratios are significant:

$$\frac{dI_E/d\omega\hbar}{dI_H/d\omega\hbar} \simeq 0.4\cdot 10^5,\quad \frac{dI_E/d\omega\hbar}{dI_{EH}/d\omega\hbar} \simeq 1.7\cdot 10^5. \tag{19}$$

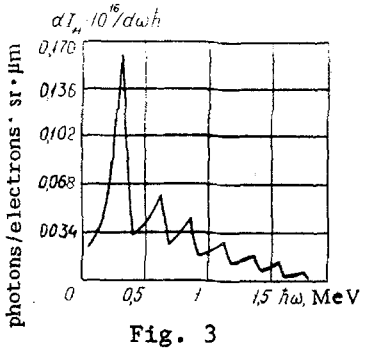

Fig. 3

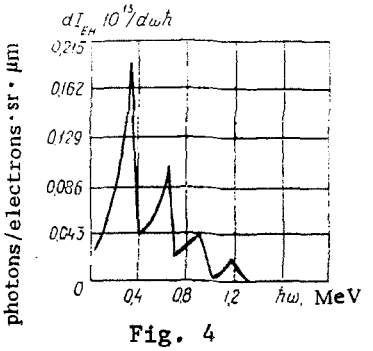

Fig. 4

Fig. 3. Spectrum of radiation intensity due to the magnetic lattice term of the radiation spectrum.

Fig. 4. The interference term $dI_{EH}/\hbar d\omega$ of the ratiation spectrum.

The various corresponding maxima of $dI_E/\hbar d\omega$, $dI_H/\hbar d\omega$, $dI_{EH}/\hbar d\omega$ occur at the same frequency, e.g., the first maxima are at $\hbar\omega = 0.32$ MeV. This is due to the fact that the magnetic and crystalline unit cells of hematite coincide [7]. We see from (19) that $dI_{EH}/\hbar d\omega$ $dI_E/\hbar d\omega$ and is practicaly undetectable compared to $dI_E/d\omega\hbar$. In crystals where the magnetic and crystalline unit cells do not coincide $dI_E/\hbar d\omega$ and $dI_{EH}/\hbar d\omega$ will assume maxima at different frequencies. For this case $dI_{EH}/d\omega\hbar$ is detectable against the $dI_E/d\omega\hbar$ background.

Simple estimates show that the magnetic and interference radiations both lie in the xy plane.

It would be interesting to investigate the polarization characteristics of our problem.

LITERATURE CITED

1. B. Hafizi, Phys. Rev. A, 30, 1514 (1984).
2. G. Mayer, Nucl. Instrum. Methods, 153, 557 (1978).
3. Yu. P. Kunashenko, Thesis Proc. of the 15th All-Union Conf. on the Physics of Charged Particle Interactions with Crystals [in Russian], Izd. Mosk. Gos. Univ. (1985), p. 63.
4. M. A. Ter-Mikaélyan, The Influence of Matter on Electromagnetic Processes [in Russian], Izv. Akad. Nauk Arm. SSR (1969).
5. Yu. V. Gott, Interaction of Particles with Plasma [in Russian], Atomizdat, Moscow (1969).
6. Yu. A. Izyumov and R. P. Ozerov, Magnetic Meutrograpy [in Russian], Nauka, Moscow (1966).
7. S. Shur, V. Shtrauzer, and G. V. Vallan, in: Antiferromagnetism [Russian translation], Inostr. Lit., Moscow (1956), pp. 420-425.