

Channeling radiation from polarized electron



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ABSTRACT

In the present paper we calculate the cross-section of radiation from axially channeled polarized electrons. We consider two types of transitions: with and without electron spin flip. It is shown that channeling radiation from polarized electron without spin flip is 2–3 times greater than one with electron spin flip.

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1. Introduction

When a relativistic charged particle enters into a crystal at small angle with respect to a crystal plane or axis its interaction with crystal can be described by continuous (averaged) potential of the crystal plane or axis [1]. The relativistic charged particle can be captured into channeling state [1–3]. The channeled particle has the discrete transverse energy levels. During transition from one transverse energy level to another level the channeled particle emits photons and channeling radiation (CR) appears. CR has been investigated theoretically and experimentally [3–6].

If the electron has high energy its motion should be described by the Dirac equation. Studying the channeling the Dirac equation is usually reduced to the Schrödinger like equation. But in this case the information about particle spin direction is lost. In order to take into account electron spin we have solved the Dirac equation [7] for axially channeled electron. In this paper we have demonstrated that the longitudinal polarization manifests itself as a splitting of every energy level into two sublevels that correspond to two possible spin projections onto the electron momentum. As a result the spectral lines of CR should be split. This splitting can be experimentally recorded, if the experimental equipment could resolve these lines.

In the present paper we calculate the intensity of radiation from axially channeled polarized electrons. We consider two types of CR from electron: with and without electron spin flip.

The radiation from polarized electrons in crystal has been studied in the frame work of semiclassical approach [8]. In this paper the energies of channeled electrons were greater than electron energies considered in our paper. The goal of the paper [8] was investigation of self-polarization of electron beam during radiation in a crystal. The semiclassical method provides good numerical results of the calculations, but sometimes difficult physical interpretation of the results.

2. Theory

In our calculation we follow the Ref. [6], but the main difference is that we use the wave function of a channeled electron with fixed spin orientation.

The probability of spontaneous radiation due to transition between the energy levels of channeled electron $i \rightarrow f$ is described by well known expression (see, e.g. [9])

$$W_{if} = \frac{ce^2}{2\pi\hbar} \int F_\lambda \frac{\delta(\omega_{if} - \omega)}{k} d^3k, \quad (1)$$

where $k = |\vec{k}| = \omega/c$ is the photon wave vector. In Eq. (1) c is the velocity of light, e is the charge of electron, \hbar is the Plank constant, and

$$\omega_{if} = \frac{E_i - E_f}{\hbar}, \quad (2)$$

where E_i and E_f are channeled total electron energies in initial (i) and final (f) states, and

$$F_\lambda = (\vec{\alpha}_{if}^* \vec{\epsilon}_\lambda^*)(\vec{\alpha}_{if} \vec{\epsilon}_\lambda) \quad (3)$$

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$\vec{\epsilon}_\lambda$ is the photon polarization vector and $\lambda = \pm 1$ (the plus sign indicates the right circular polarization and the minus sign indicates the left one). For both left and right circular photon polarizations one has [6,9]:

$$F_\pm = [\vec{n}, \vec{\alpha}_{if}][\vec{n}, \vec{\alpha}_{if}]/2 \mp i\vec{n}[\vec{\alpha}_{if}, \vec{\alpha}_{if}]/2. \quad (4)$$

Here \vec{n} is the unit vector directed as the emitted photon momentum

$$\vec{n} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \quad (5)$$

with θ and φ being the photon emission angles. In Eqs. (3) and (4) α_{if} are the transition matrix elements:

$$\vec{\alpha}_{if} = \int \left(\int U_f^* \gamma_0 \vec{U}_i e^{-i\vec{k}_r \cdot \vec{r}_\perp} d\vec{r}_\perp \right)^{i(p_{iz}-p_{fz}-k_z)z} dz. \quad (6)$$

here $\vec{k}_r = (k_x, k_y)$ is the photon wave vector perpendicular to the crystal axis, k_z is the projection of photon wave vector onto the crystal axis, p_{iz}, p_{fz} are the longitudinal electron momenta in the initial and final states, $\gamma_0, \vec{\gamma}$ are the Dirac matrices. We assume that the electron moves in a crystal along OZ axis of a Cartesian coordinates system (OZ axis is chosen parallel to the crystal axis).

The electron interaction with a crystal axis is described by the model potential of a crystal axis

$$V(r) = AZe/r, \quad (7)$$

where Z is the atomic number of crystal atom, r is the distance from axis, A is the fitting constant. At calculation we use the same number of the energy levels as in a real axis potential and fit potential (7) to real one that the main energy levels coincide. The potential (7) was used for example in [10].

The channeled electron wave functions (Dirac spinors) U_i for the fixed spin orientation have been obtained in [7] and can be written in the following form (v_n is the electron angular momentum quantum number, $n = \{i, f\}$ denotes initial or final electron states)

$$U_f^* = (e^{-iv_f\phi} u_{0f}^*, e^{-i(1+v_f)\phi} u_{1f}^*, e^{-iv_f\phi} Y_f u_{0f}^*, e^{-i(1+v_f)\phi} u_{1f}^*/Y_f), \quad (8)$$

$$U_i = \begin{pmatrix} e^{-iv_i\phi} u_{0i} \\ e^{-i(1+v_i)\phi} u_{1i} \\ e^{-iv_i\phi} Y_i u_{0i} \\ e^{-i(1+v_i)\phi} u_{1i}/Y_i \end{pmatrix}. \quad (9)$$

here the functions u_{0n} and u_{1n} are

$$u_{0n} = C_{0n} e^{-r/b_n} \frac{r^{s-1}}{Y_n \mu_n} \left(-\frac{n}{q_n + v_n} + 1 \right) U \left(-n, 2s_n, \frac{2r}{b_n} \right) + \frac{2r}{b_n} \times \frac{n}{q_n + v_n} U \left(1 - n, 1 + 2s_n, \frac{2r}{b_n} \right), \quad (10)$$

$$u_{1n} = C_{0n} e^{-r/b_n} r^{s_n-1} \left(\frac{n}{q_n + v_n} + 1 \right) U \left(-n, 2s_n, \frac{2r}{b_n} \right) - \frac{2r}{b_n} \times \frac{n}{q_n + v_n} U \left(1 - n, 1 + 2s_n, \frac{2r}{b_n} \right), \quad (11)$$

with $U(a, b, c)$ being the confluent hypergeometric function and

$$C_{0n} = (2/b_n)^{s_n} Y_n (q_n + v_n) \mu_n / ((n-1)! \Gamma(n+2s_n)(n-1 + 2s_n)n^2 + n(q_n + v_n)^2(1 - \mu_n^2(1 + Y_n^2))) \quad (12)$$

normalizing coefficient and $\Gamma(n)$ is the gamma function. In Eqs. (8)–(12) we use the following notations:

$$\begin{aligned} s_n &= \frac{1}{2} + \sqrt{\left(\frac{1}{2} + v_n\right)^2 - \alpha_z^2}, & Y_n &= \sqrt{\frac{\xi_n \gamma - 1}{\xi_n \gamma + 1}}, \\ \mu_n &= \sqrt{(\chi_n - 1)/(\chi_n + 1)}, \\ b_n &= \hbar/mce \left(1 + \gamma Y_n \sqrt{\gamma^2 - 1} \right) \sqrt{\chi_n^2 - 1}, \\ q_n &= \left(1 + \sqrt{(2s_n + 2n - 1)^2 + 4\alpha_z^2} \right) / 2, \end{aligned} \quad (13)$$

where

$$\chi_n = \sqrt{1 + \alpha_z^2/\gamma} \sqrt{n + (1/2 + v_n)^2 - \alpha_z^2}, \quad (14)$$

and γ is the electron relativistic factor, $\xi_n = 1$ if the electron spin is directed parallel to longitudinal momentum (parallel OZ axis), and $\xi_n = -1$ for opposite spin direction, $\alpha_z = Ze^2/\hbar c$, m is the electron rest mass. It is very important for the following calculation that the wave functions for different electron spin orientation have the same form and differ only by the sign of the parameter ξ_n .

For further consideration it is convenient introduce the channeled electron transverse energies by subtracting the energy of the longitudinal motion $mc^2\gamma$ from the total energy:

$$\begin{aligned} \varepsilon_{Nn} &= E_N \begin{cases} \gamma, \xi_n = 1 \\ (\gamma + 2), \xi_n = -1 \end{cases}, \\ E_N &= Amc^2 \alpha_z^2 / 2(N - 1/2)^2 \end{aligned} \quad (15)$$

where $N = 1 + v + n$ is principal quantum number. As it follows from Eq. (15), for the channeled electron with different spin orientation the energy levels differ.

After some algebra, we find matrix elements (4) in the dipole approximation

$$\begin{aligned} \alpha_{xif} &= \frac{i}{2} \frac{\omega_{if}}{c} \alpha_{01} (Y_f + Y_i) \left(\delta_{v_f, v_i-1} + \delta_{v_f, v_i+1} \right) \delta_{p_{iz}, p_{fz} + k_z}, \\ \alpha_{yif} &= \frac{i}{2} \frac{\omega_{if}}{c} \alpha_{01} (Y_f + Y_i) \left(\delta_{v_f, v_i-1} - \delta_{v_f, v_i+1} \right) \delta_{p_{iz}, p_{fz} + k_z}, \\ \alpha_{zif} &= -ik_r \alpha_{01} (1 - Y_f Y_i) \left(\delta_{v_f, v_i-1} + \delta_{v_f, v_i+1} \right) \delta_{p_{iz}, p_{fz} + k_z}, \end{aligned} \quad (16)$$

$$\vec{\alpha}_{if} = (\alpha_{xif}, \alpha_{yif}, \alpha_{zif}) \quad (17)$$

In Eq. (16) the Kroneker symbol $\delta_{p_{iz}, p_{fz} + k_z}$ takes into account the longitudinal momentum conservation, and additionally Kroneker symbol $\delta_{v_f, v_i \pm 1}$ gives $v_f = v_i \pm 1$. In Eq. (16) we introduce the following notations:

$$\alpha_{01} = ((Y_f Y_i)(I_{1if} + Y_f Y_i I_{0if}))^{-1}, \quad (18)$$

$$\omega_{if} \cong \Omega_{if} + k_z \beta, \quad (19)$$

here ω_{if} is the frequency of emitted photon. In the dipole approximation we obtain:

$$\begin{aligned} I_{1if} &= \int u_{1i} u_{1f}^* \rho^2 d\rho, \\ I_{0if} &= \int u_{0i} u_{0f}^* \rho^2 d\rho. \end{aligned} \quad (20)$$

Eq. (19) follows from (2) with accuracy up to terms of the second order $(\hbar k/p_z)^2$, with p_z being the channeled electron longitudinal momentum (for more detail see [6]), $\beta = (1 - 1/\gamma^2)^{1/2}$ and

$$\Omega_{if} = \varepsilon_{Nf} - \varepsilon_{Ni} \quad (21)$$

is the change of the electron energy due to the transition from one energy level to another one.

After substitution of the matrix element $\vec{\alpha}_{if}$ into (4) and then into (1), summing over photon polarization and performing a standard algebra (similar to calculation in Ref. [6]) we find the formula for the probability of CR, $dW_{if}/d\Omega d\omega$. We do not give this formula because it is a very large. It is more convenient to consider the

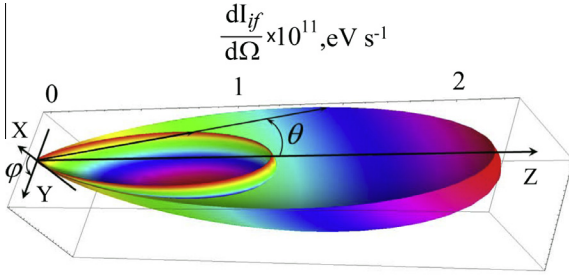


Fig. 1. Angular distribution of CR photon intensity (indicatrix) from polarized electron with energy $E \approx 10$ MeV ($\gamma = 20$) channeled along a single crystallographic axis $\langle 100 \rangle$ in Si. It is shown two types of transition: with electron spin flip (outside surface), and without electron spin flip (inside surface). For convenience we increase the scale of the OX and OY axes 5 times. The axis OZ is directed as a crystal axis.

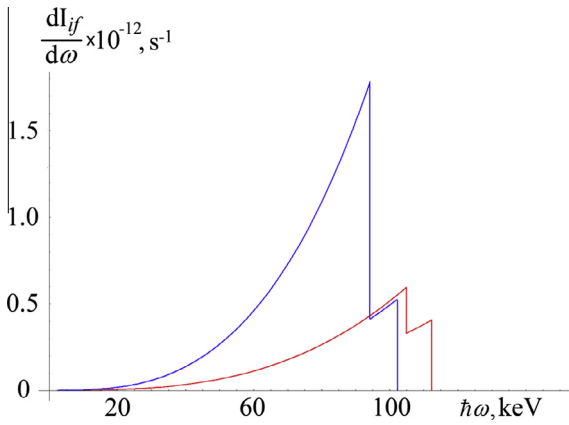


Fig. 2. Energy spectrum of CR from polarized electrons. The upper curve shows CR without electron spin flip. The lower curve shows CR with electron spin flip. The other parameters are as in Fig. 1.

probability integrated over photon emission angles or over photon frequencies.

After integration over photon frequencies we obtain the angular distribution of CR intensity:

$$\frac{dI_{if}}{d\Omega} = \frac{dW_{if}}{d\Omega} \frac{\Omega_{if}}{1 - \beta \cos \theta} = \alpha \Omega_{if}^2 \frac{3 + \cos 2\theta}{8(1 - \beta \cos \theta)^5} \left(\frac{Y_f + Y_i}{Y_f^2 Y_i^2} \right)^2 \frac{\Omega_{if}^2}{c^2} (I_{1if} + Y_f Y_i I_{0if})^2. \quad (22)$$

After integration of probability over emitted photon angles θ and φ we reduce the formula for the spectral distribution of CR intensity:

$$\begin{aligned} \frac{dI_{if}}{d\omega} &= \frac{dW_{if}}{d\omega} \omega \\ &= \frac{\alpha}{4\beta^3} \omega \left(1 - 2 \frac{\omega}{\Omega_{if}} + (1 + \beta^2) \frac{\omega^2}{\Omega_{if}^2} \right) \left(\frac{Y_f + Y_i}{Y_f^2 Y_i^2} \right)^2 \frac{\Omega_{if}^2}{c^2} (I_{1if} + Y_f Y_i I_{0if})^2. \end{aligned} \quad (23)$$

Eqs. (22) and (23) describe CR with and without electron spin flip.

3. Results of calculations and discussion

During calculation of angular distribution and spectral distribution of the emitted CR photons we summed up Eqs. (22) and (23) over all possible transitions $i \rightarrow f$ of channeled electron from one energy level to another one contributed to radiation process.

Fig. 1 shows the angular distribution of CR photon intensity (indicatrix) from circular polarized electron with energy $E \approx 10$ MeV ($\gamma = 20$) channeled along a single crystallographic axis $\langle 100 \rangle$ in Si. It is shown two types of transitions: with electron spin flip (outside surface), and without electron spin flip (inside surface). As follows from Fig. 1, the value of angular distribution $dI/d\Omega$ of CR without electron spin flip is approximately 2 times greater than CR with electron spin.

In Fig. 2 we plot energy spectrum of CR from circular polarized electrons. The upper curve corresponds to CR radiation without electron spin flip. The lower curve corresponds to CR radiation with electron spin flip. The other parameters are as in Fig. 1.

Fig. 2 demonstrates that CR without electron spin flip is approximately three times greater than CR with electron spin flip. From Figs. 1 and 2 one can see that CR without electron spin flip is 2–3 times greater than one with electron spin flip.

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